The Singular Nature of Spacetime

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We consider to what extent the fundamental question of spacetime singularities is relevant for the philosophical debate about the nature of spacetime. After reviewing some basic aspects of the spacetime singularities within general relativity, we argue that the well known difficulty to localize them in a meaningful way may challenge the received metaphysical view of spacetime as a set of points possessing some intrinsic properties together with some spatiotemporal relations. Considering the algebraic formulation of general relativity, we argue that the spacetime singularities highlight the philosophically misleading dependence on the standard geometric representation of spacetime.

1. Introduction. Despite Earman's (1995) invitation to consider more carefully the question of spacetime singularities, only a little literature in spacetime philosophy has been devoted to this foundational issue. (Some notable exceptions are Earman 1996, Curiel 1999, and Mattingly 2001.) This paper aims to take up this invitation and to carry out philosophical investigations about spacetime singularities in the framework of the contemporary debate about the status and the nature of spacetime. Indeed, there are two main positions with respect to spacetime singularities and their generic character due to the famous singularity theorems: first, they can be thought of as physically meaningless, only revealing that in these cases the theory of general relativity (GR) breaks down and must be superseded by another theory (like a future theory of quantum gravity, for instance).[1] Therefore spacetime singularities as such do not tell us anything physically relevant. Second, spacetime singularities can be taken more 'seriously': they can well be considered as physically problematic but nevertheless as involving some fundamental features of spacetime. In this sense, their careful study at the physical, mathematical and conceptual

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1. They may merely not occur in our universe if one of the (necessary) hypotheses of the singularity theorems were violated (Mattingly 2001). For a detailed physical discussion of these hypotheses, see Senovilla 1997.

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levels may be helpful in order to understand the nature of spacetime as described by GR. This paper aims to investigate this line of thought. In this framework, the question of spacetime singularities is actually a fascinating one, which may be related at the same time to the question of the 'initial' state of our universe and to the question of the fundamental structure of spacetime.

Roughly, the main question of this paper is the following one: in a scientific realist perspective, and assuming that the spacetime singularities tell us something about the nature of spacetime (again, this assumption is not evident), what do they tell us? The (tricky) problem of the very definition of spacetime singularities is an essential part of the question.

In Section 2, we will review the main concepts necessary to give an account of spacetime singularities within GR. In particular we will see that the various attempts to define the spacetime singularities in terms of local entities (like some kind of 'holes' or 'missing points' for instance) fail. We will then argue in Section 3.1 that this may constitute a strong argument for considering spacetime singularities rather as a nonlocal property of spacetime. The central part of the paper consists in evaluating the possible consequences of spacetime singularities for the metaphysical conception of spacetime.

Even if taken 'seriously', spacetime singularities are however not a satisfactory part of GR. In this perspective, we will briefly consider in Section 3.2 some recent theoretical developments within the algebraic formulation of GR regarding the spacetime singularities. These developments draw some possible physical (and indeed mathematical) consequences of the above mentioned aspects. This algebraic approach to spacetime takes the nonlocal aspects of the spacetime singularities as revealing that spacetime is nonlocal and pointless at the fundamental level. The considerations about the algebraic formulation of GR underline the fact that the metaphysical conception of spacetime should not be dependent on a particular formulation (like the inherently pointlike standard geometric one for instance), as it seems to be often the case.

2. Some Aspects of the Singular Feature of Spacetime.

2.1. Extension and Incompleteness. At the present state of our knowledge, it seems to be quite commonly accepted in the relevant physics literature that there is no satisfying general definition of a spacetime singularity (for instance, Wald 1984, 212). In other terms, the notion of a spacetime singularity covers various distinct aspects that cannot be all captured in one single definition. We certainly do not pretend to review all these aspects here. We rather want to focus on the first two fundamental notions that are at the heart of most of the attempts to define spacetime singularities.

The first is the notion of *extension* of a spacetime Lorentz manifold (together with the interrelated notion of continuity and differentiability conditions).[2] The idea is to ensure that what we count as singularities are not merely (regular) 'holes' or 'missing points' in our spacetime Lorentz manifold that could be covered ('filled') by a 'bigger' but regular spacetime Lorentz manifold with respect to some continuity and differentiability conditions (or C^k-conditions). These latter conditions (together with the notion of extension) are therefore essential for any characterization of spacetime singularities. But, at this level, there are two major ambiguities that are part of the difficulties to define spacetime singularities. First, extensions are not unique and all possible extensions must be carefully considered in order to discard (regular) singularities that can be removed by a mere regular extension. Given some C^k-condition, we will always consider maximal spacetime Lorentz manifolds.[3] A spacetime singularity will therefore be defined with respect to certain C^k-conditions (and indeed should be called a C^k-singularity; these conditions are often implicit and not always mentioned). This fact leads to the second difficulty: it is not clear what exactly the necessary and sufficient continuity and differentiability conditions are for a spacetime Lorentz manifold to be physically meaningful.[4]

Strongly related to the idea of extension, the second essential notion in order to give an account of spacetime singularities is the notion of *curve incompleteness*, which is the feature that is widely recognized as the most consensual characterization so far of spacetime singlarities (see, for instance, Wald 1984, Section 9.1). Moreover, it is actually curve incompleteness that is predicted by the singularity theorems as the generic singular behavior for a wide class of solutions.[5] The broad idea is that we should look at the behavior of physically relevant curves (namely, geodesics and curves with a bounded acceleration) in the spacetime Lorentz

2. An extension of a spacetime Lorentz manifold (M, g) is any spacetime Lorentz manifold (M', g') of the same dimension where (M', φ) is an envelopment of M and such that $\varphi^*(g) = g'|_{\varphi(M)}$ holds.

3. A spacetime Lorentz manifold (M, g) is maximal with respect to some C^k-condition if there is no extension (M', g') where the metric g' is C^k at the boundary $\partial' M$ of $\varphi(M)$ in M'.

4. A possible guideline would be to require that these conditions secure that the fundamental laws of GR—that is, the Einstein field equations and the Bianchi identity, are well defined; see Earman 1995, Section 2.7.

5. However, the notion of curve incompleteness does not encompass all aspects of spacetime singularities (like for instance certain aspects linked with the violation of the cosmic censorship).

manifold for ‘detecting’ spacetime singularities (which actually do not belong to the spacetime Lorentz manifold). In particular, the idea is that an (inextendible) half-curve of finite length (with respect to a certain generalized affine parameter) may indicate the existence of a spacetime singularity. The obvious intuition behind this idea is that, roughly, the (inextendible) curve has finite length because it ‘meets’ the singularity (it must be clear that this way of speaking is actually misleading in the sense that the ‘meeting’ does not happen in the spacetime Lorentz manifold). Pictorially, anything moving along such an incomplete (nonspacelike) curve (like an incomplete geodesic or an incomplete curve with a bounded acceleration) would literally ‘disappear’ after a finite amount of proper time or after a finite amount of a generalized affine parameter (again, we must be very careful when using such pictures; for instance, the event of the ‘disappearance’ itself is not part of the spacetime Lorentz manifold). In more formal terms, a (maximal) spacetime Lorentz manifold is said to be *b*-complete if all inextendible C^1-half curves have infinite length as measured by the generalized affine parameter (and it is *b*-incomplete otherwise).[6] The link with the initial intuition comes from the fact that it can be shown that *b*-completeness entails the completeness of geodesics and of curves with a bounded acceleration (but not vice versa).

2.2. Boundary. The most widely accepted standard definition of a singular spacetime is the following one: a (maximal) spacetime (Lorentz manifold) is said to be singular if and only if it is *b*-incomplete. However, *b*-incompleteness refers only indirectly (if at all) to spacetime singularities in the sense of localized singular parts of spacetime (like spacetime points where something ‘goes wrong’). Spacetime singularities are actually not part of the spacetime Lorentz manifold (M, g) representing spacetime (within GR) in the sense that they cannot be merely represented by points $p \in (M, g)$ (or regions) where some physical quantity related to the spacetime structure (like the scalar curvature, for instance) goes to infinity.[7]

Boundary constructions can be understood as attempts to describe spacetime singularities directly in terms of local properties that can be ascribed to certain boundary points ‘attached’ to the spacetime Lorentz manifold. The physical motivation is to do local physics, that is, to study

6. The generalized affine parameter, u, for a C^1-half-curve, $\gamma(t)$, is defined by $u := \int_0^t (\sum_{\alpha=0}^{3} (V^\alpha(t))^2)^{1/2} dt$, where $V(t') = V^\alpha(t') e_\alpha(p)$, and $p = \gamma(t')$ is the tangent vector expressed in the parallel propagated orthonormal basis e_α.

7. Spacetime is represented within GR by a pair (M, g), where M is in general assumed to be a ‘nice’ (paracompact, connected, Hausdorff, oriented) 4-dimensional differentiable manifold and g is a C^k ($k \geq 2$ in general) Lorentz metric, a solution of the Einstein field equations and defined everywhere on M.

the spacetime structure 'near' (or even 'at') the singularities. It will suffice for our purpose here to consider only briefly some aspects of the so called *b*-boundary and *a*-boundary constructions.

The main idea of the *b*-boundary construction is to consider the *b*-incomplete curves to define (singular) boundary points (as their endpoints) that can be 'attached' to the spacetime Lorentz manifold. Schmidt's procedure provides a way to construct such a (singular) boundary ∂M (called a *b*-boundary) using the equivalence between the *b*-completeness of the spacetime Lorentz manifold (M, g) and the Cauchy completeness of the total space OM of the orthonormal frame bundle $\pi : OM \rightarrow M$ (Schmidt 1971). In order to make physical sense of the idea of localizing the spacetime singularities with the help of these boundary points, it is necessary to endow the singular boundary with some differential or at least some topological structure. But it has been shown that the *b*-boundary of the closed Friedman-Lemaître-Robertson-Walker (FLRW) solution, which is part of the 'standard model' of contemporary cosmology, consists of a single point that is not Hausdorff separated from points of the spacetime Lorentz manifold (Bosshard 1976; Johnson 1977). Being not Hausdorff separated from points of M, this unique boundary point, which should represent the two singularities of the closed FLRW model, is actually 'arbitrarily close' to the points of M.[8] It is then very difficult to give physical meaning to such a behavior in terms of local entities or properties since any (regular) point $p \in (M, g)$ has the singular boundary point in its (arbitrarily small) neighborhood:[9] at least any (usual) sense of localization of the singularities, which is indeed one of the main motivations for boundary points, seems then to be lost (Earman 1995, 36–37). Moreover, such bad topological behavior has been shown to be a feature of all boundary constructions that share with the *b*-boundary construction certain natural (and rather weak) conditions (Geroch, Can-bin, and Wald 1982).

With the help of the central notion of extension or envelopment, the *a*-boundary construction aims to truly capture the idea of 'missing points', according to which spacetime singularities have to be considered as points in a 'bigger' manifold. More precisely, the motivation of the *a*-boundary construction is that singularities in a spacetime Lorentz manifold have to be considered as points (or subsets) of the topological boundary of the

8. A topological space $\overline{M}$ is Hausdorff if $\forall\, p, q \in \overline{M}, p \neq q, \exists$ open sets $U, V \subset \overline{M}$ such that $p \in U$ and $q \in V$ and $U \cap V = \emptyset$; p and q are said to be Hausdorff separated. So, if two points of a topological space (like $\overline{M}$) are not Hausdorff separated, it is not possible for them to have two ('arbitrarily small') disjoint neighborhoods (open sets): it is in this topological sense that they can be considered as 'arbitrarily close'.

9. The same problem arises in the case of the Schwarzschild solution.

(image of the) manifold with respect to an envelopment (such subsets are called boundary sets). In order to overcome the already mentioned difficulty of the nonuniqueness of the possible envelopments of a given manifold (Section 2.1), the *a*-boundary is defined as a set of equivalence classes of boundary sets (with respect to different envelopments) under a relevant equivalence relation (called the 'mutual covering relation'; see Scott and Szekeres 1994). The *a*-boundary points representing (essential) spacetime singularities are further defined with respect to incomplete curves. Avoiding the technical details, it is sufficient for our purpose here to emphasize that a spacetime singularity is then represented by an equivalence class of boundary sets, most of which are in general not singletons (and not even necessarily connected). In this framework, any interpretation of a spacetime singualrity as a pointlike or local spacetime entity to which local properties could be ascribed seems problematic also (Curiel 1999, 133–136).

3. Singular Feature and Spacetime Metaphysics.

3.1. A Nonlocal Feature of Spacetime. We have seen that spacetime singularities cannot be described merely by spacetime Lorentz manifold points (or regions) where something 'goes wrong' (where the scalar curvature 'blows up' for instance). This is actually intimately related to the dynamical nature of the spacetime structure as described by GR: spacetime singularities are indeed singularities of the spacetime structure itself and there is no a priori fixed (spacetime) structure or entity with respect to which the spacetime singularities could be defined. The point is that the very characterizations of spacetime singularities within GR, in terms of curve (*b*-)incompleteness or with the help of boundary constructions, do not enable us to conceive spacetime singularities as meaningful *local* entities or properties, which are defined here to be those that can be associated with (and determined at) a spacetime point and its (arbitrarily small) neighborhood. In order to be physically meaningful, such definition of local entities and propeties requires that a (topological) separation assumption holds among the spacetime points. For instance, it seems meaningful to require that distinct local properties can be associated with (determined at) distinct spacetime points together with their disjoint (arbitrarily small) neighborhood, no matter how close to one another they are (Hausdorff condition). Such a (topological) separation assumption then lies at the heart of the definition of local entities and properties and is in general part of the standard differential geometric representation of spacetime, in which, therefore, "a significant amount of locality is being presupposed" (Earman 1987, 453). As we have seen above, this aspect of locality can be violated by spacetime singularities in some physically important cases. In general, there is no clear and necessary link between the singular behavior of spacetime and the existence of any particular local entities like spacetime (boundary) points or local properties instantiated at particular spacetime (boundary) points. (This is clearly underlined in Curiel 1999; see also Dorato 1998, 340.) In this sense, this singular behavior seems to constitute an irreducible *nonlocal* feature of spacetime.[10] More precisely, and as a consequence of this nonlocal aspect, the singular behavior of spacetime cannot be reduced to (is not supervenient on) some intrinsic properties instantiated at some particular spacetime (boundary) points.[11] In this perspective, it bears some analogy with some other nonlocal aspects of the spacetime structure, like the gravitational energy and, in a certain sense, some (irreducible) global topological properties. But it does not merely amount to the widely recognized (Cleland 1984) nonsupervenience of spacetime relations on intrinsic properties of the spacetime points (or pointlike bits of matter; the main claim here does not side with any position in the debate between substantivalism and relationalism, but for simplicity we mainly use the spacetime points talk). Whereas a particular spacetime relation needs to be instantiated between particular spacetime points, what we want to stress here is that spacetime may possess some fundamental features that are actually independent of the existence of any particular spacetime points (and of any intrinsic properties instantiated at particular spacetime points). Such nonlocal features of spacetime may therefore challenge the received atomistic (local) view of spacetime (and of the world) as a set of points possessing some intrinsic properties together with some spacetime relations (as within Lewis's thesis of Humean supervenience). (To include merely the nonlocal features of spacetime in the supervenience basis would be a rather ad hoc solution.) So, it seems that not only quantum physics, but also classical general relativistic physics may threaten this traditional metaphysical conception of the world.[12]

This should prevent us from limiting our ontological considerations about spacetime only to local properties and entities. In particular, we should not put too much ontological weight on local and intrinsic properties of spacetime points (as well as on spacetime points themselves). Indeed, this sceptical attitude towards spacetime points and their possible

10. From the semantical point of view, this then favors the "adjective conceptions of spacetime singularities" (Earman 1995, 28).

11. In the standard view, intrinsic properties are those whose instantiation is independent of accompaniment or loneliness; see Langton and Lewis 1998.

12. Butterfield (2006) has recently argued that classical mechanics also excludes this atomistic conception about spacetime and the world, which, following Lewis, he calls 'pointillisme'.

intrinsic properties may well receive support from the GR principle of active general covariance (or of invariance under active diffeomorphisms) and the related hole argument. Due to this fundamental physical principle, a wide range of philosophers of physics and physicists agree on the fact that, within GR, spacetime points cannot be physically individuated (and therefore 'localized'), possessing intrinsic properties, for instance, independently of the spacetime relations (structure) as represented by the metric (for instance, Dorato 2000; Rovelli 2004, Chapter 2).

As regards the ontological status of spacetime, taking into consideration ('seriously') the singular feature of spacetime (and more generally nonlocal—or global—aspects of spacetime) seems to favor a nonatomistic spacetime metaphysics, be it substantivalist or relationalist. Such a conception is understood in the broad sense of an ontology that does not give priority to local entities, like spacetime points or pointlike bits of matter, with or without intrinsic properties, over the global structure in which they are embedded, like the spacetime or world structure with its nonlocal aspects. Of course, this broad metaphysical framework can be refined, according to the 'ontological space' one leaves to local entities.[13] We argue that both substantivalism and relationalism need to accommodate these nonatomistic (or structural) and nonlocal (or global) aspects.[14] Indeed, within a scientific realist perspective, this focus on structures has strong flavors of a structural realist metaphysics.

According to a rather radical approach to the question of the singular behavior of spacetime, it may be the case that the moral of the 'spacetime singularities problem' is that the very concept of a spacetime point (or pointlike bits of matter) is challenged at the fundamental level. (This amounts to reducing the 'ontological space' of local entities to zero, i.e., rejecting them from our ontology.) The singular feature would then reveal the fundamental nonlocal and nonpointlike (or pointless) nature of spacetime, which would need to be described in other mathematical (nonpointlike or pointless) terms. These could be algebraic.

3.2. Algebraic Approaches. If the philosophical analysis of the singular feature of spacetime is able to shed some new light on the possible nature of spacetime (as we have tried to show), one should not lose sight of the

13. For instance, we have just seen that there is a strong argument against intrinsic properties of spacetime points, therefore reducing their ontological weight: at best, they are on the same ontological footing as spacetime relations, their identity being entirely determined by relational properties; see Esfeld and Lam 2008.

14. Indeed, it seems that it is exactly what it is done in the recent 'sophisticated' substantivalist and relationalist positions that seek to account for the hole argument; see Rickles and French 2007 and references therein.

fact that, although connected to fundamental issues in cosmology, like the 'initial' state of our universe, spacetime singularities involve unphysical behavior (like, for instance, the very geodesic incompleteness implied by the singularity theorems or some possible infinite value for physical quantities) and therefore constitute a physical problem that should be overcome. (However this does not entail that GR is either false or incomplete; see Earman 1996.) We now want to consider some recent theoretical developments that directly address this problem by drawing some possible physical (and mathematical) consequences of the above considerations.

Indeed, according to the algebraic approaches to spacetime, the singular feature of spacetime is an indicator of the fundamental nonlocal character of spacetime: it is conceived actually as a very important part of GR that reveals the fundamental pointless structure of spacetime. This latter cannot be described by the usual mathematical tools like standard differential geometry, since, as we have seen above, it presupposes some 'amount of locality' and is inherently pointlike. The mathematical roots of such considerations are to be found in the full equivalence of, on the one hand, the usual (geometric) definition of a differentiable manifold, M, in terms of a set of points with a topology and a differential structure (compatible atlases) with, on the other hand, the definition using only the algebraic structure of the (commutative) ring $C^\infty(M)$ of the smooth real functions on M (under pointwise addition and multiplication; indeed $C^\infty(M)$ is a (concrete) algebra). For instance, the existence of points of M is equivalent to the existence of maximal ideals of $C^\infty(M)$.[15] Indeed, all the differential geometric properties of the spacetime Lorentz manifold (M, g) are encoded in the (concrete) algebra $C^\infty(M)$. Moreover, the Einstein field equations and their solutions (which represent the various spacetimes) can be constructed only in terms of the algebra $C^\infty(M)$. (The original idea is due to Geroch [1972].) Now, the algebraic structure of $C^\infty(M)$ can be considered as primary (in exactly the same way in which spacetime points or regions, represented by manifold points or sets of manifold points, may be considered as primary) and the manifold M as derived from this algebraic structure. Indeed, one can define the Einstein field equations from the very beginning in abstract algebraic terms without any reference to the manifold M, as well as the abstract algebras, called the 'Einstein algebras', satisfying these equations. The standard geometric description of spacetime in terms of a Lorentz manifold (M, g) can then be considered as inducing a mathematical (Gelfand) representation of an Einstein algebra.

15. A maximal ideal of a commutative algebra $\mathcal{A}$ is the largest proper subset of—indeed a subgroup of the additive group of—$\mathcal{A}$ closed under multiplication by any element of $\mathcal{A}$. The corresponding maximal ideal of $C^\infty(M)$ to a point $p \in M$ is the set of all vanishing functions at p.

Without entering into too many technical details, the important point for our discussion is that Einstein algebras and sheaf-theoretic generalizations thereof reveal the above discussed nonlocal feature of (essential) spacetime singularities from a different point of view.[16] In the framework of the *b*-boundary construction $\overline{M} = M \cup \partial M$ (see Section 2.2), the (generalized) algebraic structure C corresponding to M can be prolonged to the (generalized) algebraic structure $\overline{C}$ corresponding to the *b*-completed $\overline{M}$ such that $\overline{C}_M = C$, where $\overline{C}_M$ is the restriction of $\overline{C}$ to M; then in the singular cases (like the closed FLRW solution), only constant functions (and therefore only zero vector fields) can be prolonged. (In the algebraic formalism, vector fields are abstract 'derivations'.) This underlines the nonlocal feature of the singular behavior of spacetime, since constant functions are nonlocal in the sense that they do not distinguish points. This fundamental nonlocal feature suggests noncommutative generalizations of the Einstein algebraic formulation of GR, since noncommutative spaces are highly nonlocal (see, for instance, Demaret, Heller, and Lambert 1997 and references therein). We will not discuss this matter here. It is sufficient for us to stress that, in general, noncommutative algebras have no maximal ideals, so that the very concept of a point has no counterpart within this noncommutative framework. Therefore, according to this line of thought, spacetime, at the fundamental level, is completely nonlocal (pointless indeed). Then it seems that the very distinction between singular and nonsingular is not meaningful anymore at the fundamental level; within this framework, spacetime singularities are 'produced' at a less fundamental level, together with standard physics and its standard differential (commutative) geometric representation of spacetime (see Heller 2001 and references therein).

Although these theoretical developments are rather speculative, it must be emphasized that the algebraic representation of spacetime itself is "by no means esoteric" (Butterfield and Isham 2001, Section 2.2.2). Starting from an algebraic formulation of the theory, which is completely equivalent to the standard geometric one, it provides another point of view on spacetime and its singular behavior that should not be dismissed too quickly. At least it underlines the fact that our interpretative framework for spacetime should not be dependent on the traditional atomistic and local (pointlike) conception of spacetime (induced by the standard dif-

16. There are indeed several algebraic approaches to GR. For instance, according to the Abstract Differential Geometry program of Mallios and Raptis (2003), spacetime singularities are merely direct artifacts of our mathematical (C^∞-)representation of spacetime: indeed, they simply disappear once GR is written in purely algebraic (sheaf-theoretic) terms. In the following, we rather brielfy consider the (less radical) approach of Heller (2001), which emphasizes some interesting points for our discussion.

ferential geometric formulation). Indeed, this misleading dependence on the standard differential geometric formulation seems to be at work in some standard arguments in contemporary philosophy of spacetime, as in the hole argument (recently discussed by Bain [2003] within the framework of the algebraic formulation of GR), or in the field argument (Field 1980, 35). According to the latter argument, field properties occur at spacetime points or regions, which must therefore be presupposed. Such an argument seems to fall prey to the standard differential geometric representation of spacetime and fields, since within the algebraic formalism of GR, (scalar) fields—elements of the algebra C^∞—can be interpreted as primary and the manifold (points) as a secondary, derived notion (and this does not even take into account the fact that, within sheaf-theoretic or noncommutative generalizations, the very concept of a point may be challenged at the fundamental level).

4. Conclusion. Taking up Earman's invitation to consider spacetime singularities 'seriously' has led us to deal with fundamental issues about the nature of spacetime. Indeed, we have seen that spacetime may possess some fundamental nonlocal features, like the singular feature, that challenge the traditional atomistic view about spacetime (as in Lewis's Humean supervenience thesis). According to this received view, spacetime is conceived as a set of points, at which intrinsic properties are instantiated, together with the spacetime relations. Indeed, the very concept of a spacetime point seems to lie at the heart of the challenge. It cannot be merely postulated anymore (as in the field argument), since it is indeed a secondary, derived, notion within the algebraic formulation of GR. This latter formualtion may with reason be considered as deserving to play a role in the interpretative issues about spacetime—at least to the same extent as the standard differential geometric formulation does. Actually, the alleged interpretational problems with respect to spacetime singularities may find part of their roots in the misleading dependence on the atomistic and local conception of spacetime, which is actually induced by this standard differential geometric representation of spacetime. And this gives a structuralist flavor to spacetime as described by GR and independently of the formulation. But this is a story for another time.

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