

pair  $(K, f)$  where  $K$  is a simplicial complex and  $f$  is an identification map of  $|K|$  onto a space  $X$  which maps each open simplex of  $K$  homeomorphically and is such that if  $f(|s_p^i|)$  and  $f(|s_q^j|)$  meet, then  $p = q$  and  $f$  identifies  $|\bar{s}_p^i|$  and  $|\bar{s}_q^j|$  under linear homeomorphism; the chain groups of  $(K, f)$  are obvious quotients of those of  $K$ . One can adapt the simplicial approximation theorem to pseudo-simplicial complexes; then the latter will have the advantage over block dissections (p. 128) that they can be used for cup products and the fundamental group, and blocks might be omitted on a first reading.

Sections 2.1-2.3 are disappointing, but the beginner must learn the notation of the book for chains, etc. Cohomology is introduced in an admirable way (2.6, 2.7); it is called *contrahomology* in the book because it transforms contravariantly (with respect to maps of spaces). The treatment of relative homology and exact sequences (2.8, 2.9) is excellent. Having read to p. 116, one can justify pseudo-dissection by acyclic carriers (as suggested on p. 133); block-dissections and their application to real projective space pp. 128-135) could be omitted. Cup products are introduced on pp. 140-152 with interesting applications to maps of spheres.

I believe the beginner should next turn to p. 313 and study singular homology. On first reading, normalised and cubical theories might be omitted. After 8.1 (for which a little of 5.6 is needed) one can read pp. 330-40 and 361-4.

Pages 228-246 deal with the fundamental group and lead naturally to pp. 345-49 (for which pp. 318-329 are needed) where the Hurewicz isomorphism theorem is proved. Having got so far the beginner will be able to find his own way about the book.

There are several departures from customary notation. As already mentioned cohomology is called *contrahomology* in this book; it seems hardly worth making the change, for who will forget that cohomology is contravariant? The cohomology ring is denoted by  $R^*(X)$  instead of  $H^*(X)$ ; the same letter  $J$  is used for the additive group of integers and for the ring of integers, so why not similarly in cohomology? It is also confusing to find  $V^n$  used for the closed unit  $n$ -cell. Maps of spaces are written on the right, so  $Xfg$  is the image of  $X$  by the map  $f$  followed by  $g$ .

The printing is very good.

The book achieves the purpose of providing an introduction which reaches the developing parts of the subject, and for those who already know a little algebraic topology is by far the best textbook for further study.

D. G. PALMER

KREYSZIG, E., *Differential Geometry* (Mathematical Expositions No. 11, Toronto, and Oxford University Press, 1959), pp. xiv + 352, 48s.

Differential Geometry is today one of the most popular subjects of research. It experienced a decline after the impetus from General Relativity in the 1920's and 30's had died down, but it has now received new life, mainly through the union with Topology from which there emerged the two closely related subjects of Differential Topology (the study of differentiable manifolds) and Differential Geometry in the Large. One change is that the various classical techniques, based largely on tensor calculus, which were at one time an indispensable part of differential geometry, are now out of fashion and have been replaced by new "suffix-free" methods. Nevertheless the older tools still have their uses, and anyone who wishes to research in geometry or its applications is well advised to have some knowledge of tensor calculus and of the classical differential geometry with which it is so closely associated. It often happens that a new result is first discovered through old-style calculations which are afterwards replaced by an argument along more modern lines.

Because of the popularity of modern differential geometry many universities are

again offering courses in tensor calculus and the study of curves, surfaces and Riemannian spaces—courses which are also proving useful to research students in such other fields as Applied Mathematics, Physics and Engineering. This has shown up the need for a textbook written in a modern style, and several books have been published in the past year or two in an attempt to meet this need; the book under review is one of them. It is a free translation of the author's *Differentialgeometrie* published in Leipzig, and is admirably written in a style that will appeal to both British and American readers. Although the book is restricted to curves and surfaces embedded in euclidean 3-space, tensor calculus is used throughout and it is indicated how and where results may be extended to Riemannian  $n$ -space. There is no systematic treatment of tensor calculus as such, e.g. through tensor algebra which is the way many teachers now prefer to do it, but the various ideas and formulae of the calculus are introduced as they are needed for the development of the geometry.

After chapters on geometrical preliminaries and the theory of curves, a chapter is given to the concept of a surface and properties of the first fundamental form, followed by a chapter on the second fundamental form. It is perhaps unfortunate that the important idea of Gaussian curvature is introduced with the second rather than the first fundamental form where it properly belongs as an intrinsic property of the surface; this is one of the places where the historical development of the subject is not in line with what is now generally regarded as desirable. This comment applies also to the treatment of geodesics and geodesic curvature in the next chapter, intrinsic ideas treated in what is admittedly the more concise non-intrinsic way. This chapter also includes an interesting account of the Gauss-Bonnet theorem and its applications, which is one of the important links between classical and modern Differential Geometry. The next chapters are on mappings of various kinds and on absolute differentiation and Levi-Civita parallelism, and finally there is a long chapter describing a number of special surfaces, ruled surfaces and developables, and surfaces of constant Gaussian curvature.

The author has been careful throughout to focus attention on geometry without any sacrifice to analytic rigour, a sacrifice that marred so many of the earlier texts on curves and surfaces. It is perhaps a pity that the book contains no systematic treatment of Riemannian or other generalised spaces, but in a book of this size the author must decide whether to do this or to give space to special problems and special surfaces. The latter is perhaps of less significance in relation to modern interests but it will certainly be useful to have such an account handy for reference, particularly for applications outside geometry.

Many useful problems are given and there is a carefully prepared section on their solutions. The book is excellently printed and produced in this country by the Oxford University Press.

A. G. WALKER

SEMPLE, J. G., AND KNEEBONE, G. T., *Algebraic Curves* (Clarendon Press: Oxford University Press, 1959), 361 pp., 45s.

The stated aim of this book is to introduce the reader to some of the fundamental concepts and methods of modern algebraic geometry by a discussion of the theory of algebraic curves. In the reviewer's opinion this aim has been very successfully achieved.

The book is well written with many useful and interesting historical notes (including Appendix C) and with summaries at several places of earlier parts of the work. In many of the chapter introductions lucid explanations are given of the motivation for the work which follows. A comprehensive survey, with references, of the algebra required in the book is given in Appendix A.