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EXPECTATIONS-DRIVEN FLUCTUATIONS WHEN FACTOR UTILIZATION IS VARIABLE

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The present paper studies the influence of variable labor utilization on local indeterminacy and expectations-driven fluctuations, in one-sector models with (nearly) constant returns to scale. It is shown that, in comparison to the configuration of constant input utilization, considering variable utilization reduces the actual possibilities of factor substitution and, consequently, the range of input substitution elasticities that are compatible with endogenous fluctuations. In particular, local indeterminacy and expectations-driven fluctuations occur only if utilization rates are sufficiently inelastic, whereas local determinacy prevails when utilization is highly elastic. However, accounting for the fact that variable utilization reduces the *effective* elasticity of capital/labor substitution leads us to argue that expectations-driven fluctuations are more plausible because they require larger elasticities of *apparent* input substitution. In contrast with the recent literature, the analysis does not rely on significantly increasing returns to scale in production. Accordingly, the results are not at variance with recent empirical studies emphasizing the importance of variable utilization and denying the evidence of large increasing returns.

Keywords: Input Utilization, Input Substitution, Local Indeterminacy,
Expectations-Driven Fluctuations, Local Bifurcations

1. INTRODUCTION

This paper studies the influence of variable input utilization on the possibility of local indeterminacy and expectations-driven fluctuations in one-sector aggregate models. The main motivation upon which this study relies is very simple. It is

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known from the literature that if one restricts the analysis to the case in which returns to scale in capital and labor are constant, or slightly increasing, indeterminacy and endogenous fluctuations occur only when inputs are sufficiently complementary, that is, only for small enough elasticities of capital/labor substitution [see, e.g., Reichlin (1986), Woodford (1986), Benhabib and Laroque (1988), Grandmont (1993), de Vilder (1996), Grandmont et al. (1998)]. One may then cast doubt on the plausibility of this result by arguing that capital and labor are substitutable enough to rule out indeterminacy in one-sector models, presumably in view of some parameter estimates [as surveyed, for example, by Hamermesh (1993, Ch. 3) and Rowthorn (1999)].

This paper shows that these conclusions depend heavily on the questionable assumption of constant factor utilization, and outlines an unnoticed mechanism that emerges when this assumption is relaxed. More precisely, it is argued that if one considers *variable* factor utilization, input services may then be only slightly substitutable and, as a consequence, local indeterminacy and self-fulfilling fluctuations are plausible outcomes, precisely because their occurrence requires almost complementary inputs, under nearly constant returns.

Utilization variability plays an important role in the unfolding of observed fluctuations, as documented by several studies, which show, in particular, that both consumption of raw materials or electricity and the number of shifts are strongly procyclical [see, e.g., Burnside et al. (1995)]. In my model, labor utilization is procyclical, and this feature is shown to be critical for explaining the emergence of endogenous fluctuations.

To get an overview of the main argument, let us first suppose that workers' "effort" (or "quality") increases with the real wage, according for instance to usual efficiency wage considerations. Then, a declining interest rate is expected to trigger an increase both in the capital/labor ratio and in effort. This increasing "intensive" use of labor therefore counteracts the "extensive" effect increasing the capital/labor ratio when the relative rental price of capital moves down. Accordingly, actual substitution of input services is reduced and may be substantially lower, in comparison with the more usual configuration in which labor utilization does not vary.

As a variation on the same argument, assume that firms vary the intensity at which capital is operated by workers, and that the rate of capital depreciation increases with intensity. If, in addition, we assume both increasing marginal depreciation cost and diminishing returns to capital, it follows that capital utilization decreases as the capital stock moves up, so as to save on depreciation cost when capital is less productive at the margin, reducing here again actual input substitution.

In both cases, therefore, the elasticity of *effective* input substitution (when utilization is variable) may be substantially lower than the elasticity of *apparent* (or measured) substitution (when utilization is constant), when utilization rates are elastic enough. As suggested by the foregoing arguments, the key plausible mechanism is that capital utilization and labor utilization are, respectively, decreasing and increasing with the ratio of labor/capital prices: The cheaper the input, the less intensively it is used.

More importantly, my analysis emphasizes an important and different implication of this argument: By overestimating factor substitution possibilities, the assumption of constant utilization underestimates the plausibility of indeterminacy and endogenous fluctuations in a large class of dynamic models. In particular, *procyclical* input utilization appears to be a plausible source of expectations-driven fluctuations.

Although the influence of capital utilization on business cycles has been modeled in early works [see, e.g., Smith (1969), Lucas (1970), Taubman and Wilkinson (1970), Calvo (1975)] as well as in more recent studies [see the references in Black (1995), Burnside et al. (1995), and Basu (1996)], little research has been done to assess the consequence of variable utilization on the possibility of expectations-driven fluctuations, with the notable exception of that by Wen (1998). The main purpose of Wen's paper is to argue that, in the model studied by Benhabib and Farmer (1994), local indeterminacy requires lower, and more plausible, levels of increasing returns when capital utilization is elastic.

This paper departs from Wen's analysis in at least four dimensions. The main difference is that we impose mild increasing returns to scale in our model with variable labor utilization: Our results are valid when the (local) elasticity of scale is *arbitrarily* close to (but larger than) 1 (see Section 2). Moreover, the analysis does not depend, as in Wen, on the Cobb-Douglas specification. As a consequence, I am able to study the influence of variable utilization on capital/labor substitution, and I show that this influence is indeed negative, as suggested above, a result that seems to have been unnoticed in the literature and that may help to understand Wen's configuration. In addition, whereas Wen focuses on capital utilization, I study the effects of capital and labor utilization. Although labor utilization is studied in Section 2 of this paper, the reader is referred to the associated working paper for the analysis of capital utilization [see Pintus (2002, Sect. 2)].

Finally, in contrast to Wen, I formalize the previous argument in the framework presented by Woodford (1986). It is shown that considering variable utilization reduces the range of input substitution elasticities that are compatible with endogenous fluctuations. In particular, local indeterminacy and expectations-driven fluctuations occur only if utilization rates are sufficiently inelastic, when the elasticity of factor substitution is low enough, whereas local determinacy prevails and no fluctuations occur when utilization is highly elastic (see Section 2). Since the argument presented above is, by nature, related to technology, it is general enough to apply equally well to alternative business-cycle models with substitutable inputs and is expected to yield qualitatively similar results. For instance, similar conclusions are obtained in an overlapping generations model [see, again, Pintus (2002, App. C)].

In addition, a realistic feature emerges in the model with a non-Walrasian labor market (see Section 2), as firms fix the real wage so as to induce optimal labor utilization (effort), implying that unemployment may occur. A related analysis is provided by Coimbra (1999), which studies the emergence of endogenous fluctuations in an overlapping-generations model with increasing returns to scale, indivisible labor, and efficiency wages. My model, however, relies on neither significant

increasing returns nor infinitely elastic labor supply, in contrast with Coimbra's. More importantly, effort is fixed in Coimbra's model, whereas I analyze a more plausible configuration with elastic and procyclical effort (see Section 2.2).

This is also a major difference with Danthine and Donaldson (1995), who study how efficiency wages help to better account for the business cycle, in models with real shocks: Effort is, in their setting, constant in equilibrium. In that respect, closer to my study is the contribution by Uhlig and Xu (1996). The latter authors, however, focus on countercyclical effort, whereas labor utilization turns out to be procyclical in my analysis [as in the labor-hoarding model of Burnside et al. (1993)]. Moreover, Uhlig and Xu [as well as Burnside et al. (1993)] study the impact of technological shocks as the main source of fluctuations, whereas we discuss the existence of expectations-driven fluctuations.

In summary, this paper appears to complement existing studies that introduce efficiency wages in general equilibrium models, by focusing on labor utilization (effort) variability and on expectations as a potential source of business cycles.

The model on which I focus is a plausible departure from the standard optimal growth model, *à la* Ramsey, and may be a useful benchmark in the literature on local indeterminacy and expectations-driven fluctuations. The economy consists of two classes of infinitely-long-lived, heterogeneous agents who maximize their discounted utility and may save *a priori* in the form of capital or money. Workers consume and supply a variable quantity of labor, in every period. Moreover, they face a financial constraint, due to imperfect or incomplete information regarding employees' "effort" or "quality," which prevents them from borrowing against their labor income. On the other hand, capitalists consume and do not work in every period, and are assumed to be more "patient", that is, they discount future less than workers. Accordingly, capitalists hold the entire capital stock, at the steady state and thus nearby, whereas workers hold the whole money stock if it is in constant supply. Consequently, although workers have infinite horizon, they behave like two-period living agents, consuming today their money balances and saving today their wage income for consumption tomorrow.

In addition to the plausible assumption of agents' (double) heterogeneity, the model has some additional appealing features: Endogenous fluctuations occur at arbitrarily high frequencies, since the period may be interpreted as short, and the capital market imperfection is explicit.

Since endogenous fluctuations may occur in this model for sufficiently low elasticities of factor substitution, when returns to scale are almost constant, these phenomena may be interpreted as being more plausible, in the following sense: Actual input substitution possibilities may indeed be substantially reduced when utilization rates are variable. However, most available measures of input substitution rely on the assumption of constant utilization. In fact, there does not exist, to my knowledge, direct measures of effective input substitution that take into account this variability and with which one may confront the results of our analysis.

In my model, it turns out that the range of elasticities compatible with indeterminacy and endogenous cycles overlaps the range of reported estimates of the

(measured or apparent) substitution elasticity. More importantly, it turns out that indeterminacy and endogenous cycles occur for *higher* values of the apparent elasticity (see Section 3). Moreover, stochastic equilibria driven by self-fulfilling beliefs (sunspots) are shown to be more plausible than deterministic cycles, in agreement with a large strand of the literature [as surveyed, for instance, by Benhabib and Farmer (1999)].

As suggested earlier, variable utilization is shown to imply that effective input substitution is reduced in comparison to measured input substitution, when labor intensity may vary [Pintus (2002, Sect. 2) presents similar results when capital utilization is variable]. This technology-related result depends on the assumption that effort not only increases labor productivity but also decreases the rate of capital depreciation. The latter feature seems natural if we interpret labor intensity as effort: Increased effort implies better maintenance (or lower wear and tear) and lower capital depreciation. However, this assumption is not critical to my analysis: Appendix A shows that the elasticity of the depreciation rate with respect to effort can be taken to be arbitrarily close to zero. Moreover, it has two plausible implications: First, the real wage is no longer constant [in contrast to Solow (1979)]; second, effort is then quite inelastic, when profits are maximized [see equation (2)] because the elasticity of effort with respect to the real wage, assumed to be positive, is then smaller than 1 [in agreement with Akerlof and Yellen (1986, p. 14)].

On the other hand, I also assume, more traditionally, that the economy with variable labor utilization exhibits imperfect competition in the labor market (only) because the real wage is chosen by firms so as to induce optimal effort. Moreover, firms are assumed to benefit from increasing returns to scale arising, to fix ideas, from externalities. Here again, this assumption is not critical and is needed to rule out a degenerate situation in which the economy is stationary: If returns are constant in the economy with labor utilization, the real wage and effort are constant, in which case input prices are fixed and, consequently, the absence of any opportunity to substitute consumption over time leads to all feasible intertemporal equilibria being stationary [see Pintus (2002, Sect. 3)]. A possible way to rule out this degeneracy is, as shown below, to assume the presence of increasing returns to scale, however *arbitrarily small*: In that case, input prices and intertemporal equilibria are no longer necessarily stationary, as shown in Section 2. Therefore, the analysis relies on the presence of negligible increasing returns.

In summary, my analysis is carried in terms of our key parameter—the elasticity of effort to the real wage—and shows that the dynamics of the model differ drastically when it is different from zero, even for values that are far below 1. For instance, local determinacy prevails for values greater than 1/2, when other parameter values are set to reasonable levels (see Section 3).

It is worth noting that our results are not at variance with some recent convergent studies emphasizing the importance of assuming variable utilization when measuring aggregate returns to scale and productivity [see, e.g., Burnside et al. (1995), Burnside (1996), and Basu (1996)]. In particular, these studies conclude that empirical evidence does not favor actual business-cycle models uniquely based

on either technological shocks (because of a much lower variance when utilization varies), or shocks to expectations (because of necessary levels of increasing returns not found in data) as, for example, in Kiyotaki (1988), Matsuyama (1991), Boldrin (1992), Benhabib and Farmer (1994), and Cazzavillan et al. (1998).

The paper is organized as follows. The following section introduces and studies the technology and the dynamic model with variable labor utilization, while Section 3 shows that local indeterminacy and endogenous fluctuations are more plausible in the context of variable labor utilization.

2. VARIABLE LABOR UTILIZATION AND ENDOGENOUS FLUCTUATIONS

This section shows that considering endogenous labor utilization and capital depreciation significantly alters the local dynamics of the Woodford (1986) infinite-horizon model. In particular, it turns out that local indeterminacy and expectations-driven fluctuations occur only if effort is not too elastic to the real wage. These results essentially arise from the fact that a more “intensive” use of labor reduces the elasticity of effective input substitution. Moreover, deterministic cycles appear to be less robust than stochastic equilibria (sunspots).

2.1. Technology

In each period $t \in N$, labor hours $h_t \geq 0$ and the capital stock $k_{t-1} \geq 0$ resulting from the previous period are combined in variable proportion to produce a consumption good under constant returns to scale at the firms level. Labor intensity $e(\omega)$ is interpreted as effort and is an increasing function of the real wage ω .

I make the simplest assumption that capitalists/entrepreneurs set the real wage and, thereby, workers’ labor intensity. More precisely, we adopt the specification for technology suggested by Negishi (1979) and Solow (1979); that is, $F[k, e(\omega)h] \stackrel{\text{def}}{=} e(\omega)hAf[a/e(\omega)]$, where $A > 0$ is a scaling factor and $a = k/h$ denotes the capital/labor(hours) ratio.

These assumptions are usual in the efficiency wage literature, according to which several informal arguments account for the fact that firms have the possibility to stimulate labor productivity by means of pecuniary incentives [see Akerlof and Yellen (1986), Weiss (1990)].¹ On the other hand, labor services are defined as the product of effort and hours so as to discuss local stability and indeterminacy in terms of interpretable parameters, most notably the elasticity of factor substitution. This assumption arises naturally if capital and labor services are separable inputs and if returns to scale are constant. In the case of Cobb-Douglas or CES production functions, for instance, it is equivalent to assuming that capital services are a general function of both the utilization ratio and the capital stock: It is easily shown that this function is then linear with respect to labor hours.

In addition, I assume that capital depreciation depends on effort: The depreciation function $\delta[e(\omega)]$ varies with effort and, consequently, with the real wage. Here we view effort as measuring the quality of labor and, moreover, assume that

capital depreciation decreases with effort. As shown below, the latter assumption implies that effort is (realistically) quite inelastic to the real wage, the corresponding elasticity being less than 1 at the optimum [see equation (2)], in contrast with the analysis by Solow (1979), which implies a unitary elasticity at the optimum. However, our analysis will be shown to be valid when depreciation is fairly inelastic to effort: $\delta'[e]$ is assumed to be arbitrarily close to zero. In that sense, this assumption is not critical to our analysis.

Finally, I assume that the economy benefits from external economies of scale and, more precisely, that total factor productivity $A = A[k, e(\omega)h]$, while given to any individual producer, is increasing in average capital and (efficient) labor. The following analysis does not rely on either any particular external effect (capital or labor) or the form of increasing returns (internal or external). In fact, as it clearly appears later, it is the presence of (negligible) increasing returns that is needed here to avoid that intertemporal equilibria degenerate, being stationary when returns to scale are constant [see also Pintus (2002, Sect. 3)]. Therefore, this second assumption is not critical either.

Assumption 1. The intensive production function $f(x) \geq 0$ is continuous for $x \geq 0$, C^r for $x > 0$ and r large enough, with $f'(x) > 0$ and $f''(x) < 0$. The depreciation function $0 \leq \delta(e) \leq 1$ is continuous for $e \geq 0$, C^r for $e > 0$ and r large enough, with $\delta'(e) < 0$ and $\delta''(e) > 0$. The effort function $e(\omega) \geq 0$ is continuous for $\omega \geq 0$, C^r for $\omega > 0$ and r large enough, with $e'(\omega) > 0$ and $e''(\omega) < 0$. Therefore, the function $\delta[e(\omega)]$ is decreasing and convex.

The decision program solved by a typical entrepreneur/capitalist consists in maximizing profits given by $he(\omega)Af[a/e(\omega)] - \omega h - \{r + \delta[e(\omega)]\}k$ over $k \geq 0$, $h \geq 0$, and $\omega \geq 0$, given the real interest rate r and total productivity A , and it follows that interior optima $k > 0$, $h > 0$, $\omega > 0$ satisfy

$$\begin{aligned} Af'(x) &= r + \delta[e(\omega)], \\ A(f(x) - xf'(x)) &= \omega/e(\omega), \\ e'(\omega)A(f(x) - xf'(x)) &= 1 + a\delta'[e(\omega)]e'(\omega), \end{aligned} \tag{1}$$

if $x \stackrel{\text{def}}{=} a/e(\omega)$.

In particular, it follows from the two last equalities in equations (1) that

$$\varepsilon_e(\omega) = 1 + a\delta'[e(\omega)]e'(\omega), \tag{2}$$

where $\varepsilon_e(\omega)$ is the elasticity of effort to the real wage.

Accordingly, one derives from equation (2), under appropriate assumptions, that the real wage depends, in an open neighborhood I of a steady state at which $a = \bar{a}$ to be defined below, on the capital/labor ratio. Moreover, the elasticity of real wage is then given by

$$\varepsilon_\omega(a) = [\varepsilon_e(\omega) - 1]/(\varepsilon_{e'}(\omega) - \varepsilon_e(\omega)[\varepsilon_e(\omega) - 1]\{\varepsilon_\delta[e(\omega)] + 1\}), \tag{3}$$

independently of the level of increasing returns.

In the absence of externalities, that is, when A is constant, equations (1) determine, under appropriate conditions, a , ω , and r : In that case, the dynamics of the model degenerate. Therefore, the assumption of increasing returns, that is, the assumption that $A = A[k, e(\omega)h]$ is increasing in k and eh , is intended, in this context, to allow the capital/labor ratio a , as well as the real wage and the rental price of capital, to be variable along nonstationary intertemporal equilibria. Consequently, the following analysis is carried over under the assumption that the level of increasing returns is arbitrarily small. To simplify the presentation, we may, therefore, abstract from the explicit presence of externalities and omit the arguments of A .

Moreover, as seen from equations (2), the configuration where the depreciation rate is constant— $\delta'[e] = 0$ —leads to $\varepsilon_e(\omega) = 1$, that is, to wage rigidity and, consequently, to stationary intertemporal equilibria or degenerate dynamics with fixed labor [see Pintus (2002, Sect. 3) for a study of this limiting case]. In contrast, endogenous depreciation—*i.e.* $\delta'[e] < 0$ —allows for real wage variability, that is, $\varepsilon_\omega < 1$ under Assumption 1, in view of the previous equation, and, accordingly, for intertemporal substitution. Moreover, the assumption that depreciation decreases with effort implies, as seen from equations (2), that the elasticity of effort is smaller than one.²

Appendix B shows that the necessary and sufficient conditions for concavity of the Hessian of the profit function are met if, in addition to Assumption 1, $\delta[e]$ is sufficiently inelastic at the steady state to be defined later.

Finally, marginal productivities of (efficient) labor and capital are defined respectively, as

$$\begin{aligned} \omega/e(\omega) &= A[f(x) - xf'(x)] \stackrel{\text{def}}{=} \hat{\omega}(x) = \omega(a)/\hat{e}(a), \\ R &= Af'(x) + 1 - \delta[e] \stackrel{\text{def}}{=} \rho(x) + 1 - \delta[\hat{e}(a)] \stackrel{\text{def}}{=} R(a), \end{aligned} \tag{4}$$

where $\hat{e}(a) \stackrel{\text{def}}{=} e[\omega(a)]$ and $x = a/\hat{e}(a)$, while real wage $\omega(a)$ is implicitly defined under Assumption 1 by equations (2). Therefore, $R(a)$ denotes the net interest factor and $\hat{\omega}(x)$ denotes marginal productivity of efficient labor.

2.2. Variable Labor Utilization and Factor Substitution

The purpose of this section is to analyze the influence of a variable labor utilization on capital/labor substitution, a mechanism that is central to the occurrence of indeterminacy and endogenous fluctuations.

A natural and usual measure of input substitution possibilities is the (local) elasticity of capital over labor *services* with respect to the ratio of marginal productivities, that is, $d \ln(a/e)/d \ln(\hat{\omega}/\rho)$. When labor utilization (e) is constant, this measure reduces to $d \ln a/d \ln(\hat{\omega}/\rho)$. As a consequence, $d \ln(a/e)/d \ln(\hat{\omega}/\rho) \leq d \ln a/d \ln(\hat{\omega}/\rho)$ if and only if $d \ln e/d \ln(\hat{\omega}/\rho) \geq 0$. Therefore, labor *services* (eh) are less substitutable to capital than labor hours (h) if and only if labor utilization increases with the relative rental price of labor: the more productive (or more expensive) labor is, the more intensively it is used.

In our framework, this general argument takes the following form. Define $\sigma(a) = d \ln[a/\hat{e}(a)]/d \ln(\hat{\omega}/\rho)$ as the elasticity of *effective* input substitution and $\hat{\sigma}(a) = d \ln a/d \ln(\hat{\omega}/\rho)$ as the elasticity of *apparent* (or measured) input substitution. It follows that $\sigma(a) = [1 - \varepsilon_e(\omega)\varepsilon_\omega(a)]\hat{\sigma}(a)$, where ε_e and ε_ω denote, respectively, the elasticities of the functions $e(\omega)$ and $\omega(a)$. In view of equation (2), $\sigma(a) < \hat{\sigma}(a)$ if $\varepsilon_e(\omega(a))$ is close to zero, for all a in I , since $\varepsilon_\omega(a)$ is then close to zero [see equation (3)].

In that case, input substitution is reduced when effort is variable because both the wage and labor utilization increase when the capital/hours ratio moves up, under diminishing returns to labor hours, implying that the capital-efficient labor increases less than the capital/hours ratio: Labor is used more intensively when it is more productive. More precisely, efficient labor is optimal when its marginal productivity $A\{f[a/e(\omega)] - af'[a/e(\omega)]/e(\omega)\}$ equals its cost $\omega/e(\omega)$ [see the second equality in equations (1)]. As a consequence, when the capital/labor ratio increases from a_0 to $a_1 > a_0$, in Figure 1, that is, when capital is substituted for labor, the real wage increases from $\omega(a_0)$ to $\omega(a_1) > \omega(a_0)$, in Figure 1, and, accordingly, effort $e(\omega)$ moves up. It follows, quite reasonably, as seen in Figure 1, that labor utilization is procyclical [as in the different model of Burnside et al. (1993)].

The above discussion is summarized in the following lemma.

LEMMA 1 (Variable Labor Utilization and Input Substitution). *Let $\sigma(a) \stackrel{\text{def}}{=} d \ln\{a/e[\omega(a)]\}/d \ln(\hat{\omega}/\rho) > 0$ be the elasticity of effective capital/labor substitution and $\hat{\sigma}(a) \stackrel{\text{def}}{=} d \ln a/d \ln(\hat{\omega}/\rho) > 0$ be the elasticity of apparent (or measured) capital/labor substitution. It follows that $\sigma(a) = \{1 - \varepsilon_e[\omega(a)]\varepsilon_\omega(a)\}\hat{\sigma}(a)$, where $\varepsilon_e(\omega) \stackrel{\text{def}}{=} \omega e'(\omega)/e(\omega) > 0$ and $\varepsilon_\omega(a) \stackrel{\text{def}}{=} a\omega'(a)/\omega(a) > 0$ denote, respectively, the elasticities of utilization $e(\omega)$ and of real wage $\omega(a)$, whereas the latter function*

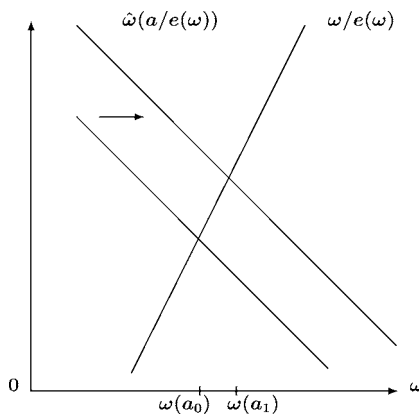


FIGURE 1. The optimal real wage $\omega(a)$ and the optimal labor utilization rate $e(\omega)$ increase with the capital/labor ratio a .

is implicitly defined, under Assumption 1, by equations (2). Then, one has $\varepsilon_\omega(a) = s(a)/(\sigma(a)\{1 - \varepsilon_e[\omega(a)]\} + s(a)\varepsilon_e[\omega(a)]) > 0$, where $0 < s(a) \stackrel{\text{def}}{=} x\rho(x)/[Af(x)] < 1$ is the capital share in total income and $x \stackrel{\text{def}}{=} a/e[\omega(a)]$. As a consequence, the elasticity of effective factor substitution $\sigma(a) = \hat{\sigma}(a) - s(a)\varepsilon_e[\omega(a)]/\{1 - \varepsilon_e[\omega(a)]\}$ is lower than the elasticity of measured factor substitution $\hat{\sigma}(a)$, when labor utilization is variable, that is when $\varepsilon_e(\omega) > 0$. On the contrary, both definitions coincide, that is, $\hat{\sigma}(a) \equiv \sigma(a)$, when labor utilization is constant, that is, when $\varepsilon_e(\omega) = 0$.

Proof. Define $\sigma(a) = d \ln[a/\hat{e}(a)]/d \ln(\hat{\omega}/\rho)$ as the elasticity of effective input substitution and $\hat{\sigma}(a) = d \ln a/d \ln(\hat{\omega}/\rho)$ as the elasticity of apparent input substitution, with $\hat{e}(a) = e[\omega(a)]$. It follows that $\sigma(a)/\hat{\sigma}(a) = [1 - \varepsilon_e(\omega)\varepsilon_\omega(a)]$, where ε_e and ε_ω denote, respectively, the elasticities of the functions $e(\omega)$ and $\omega(a)$, while real wage $\omega(a)$ is implicitly defined, under Assumption 1, by equations (2). I next derive the rental prices elasticities as functions of our underlying parameters, most notably the elasticity of effective input substitution. From $1/\sigma(a) \stackrel{\text{def}}{=} \varepsilon_{\hat{\omega}}(x) - \varepsilon_\rho(x)$, where $x = a/\hat{e}(a)$, and from the derivative of the identity $Af(x) = x\rho(x) + \hat{\omega}(x)$ [see equations (4)] with respect to x , one derives $\varepsilon_{\hat{\omega}}(x) = s(a)/\sigma(a)$ and $\varepsilon_\rho(x) = -[1 - s(a)]/\sigma(a)$, where $0 < s(a) = x\rho(x)/[Af(x)] < 1$ denotes the capital share in total income while we denote $\varepsilon_\phi(y)$ the elasticity of a given function $\phi(y)$ with respect to y . Moreover, the second coordinate in equations (1) and the first definition in equations (4), that is, $\hat{\omega}(x) = \omega(a)/\hat{e}(a)$, yield $\varepsilon_{\hat{\omega}}(x) = \varepsilon_\omega(a)[1 - \varepsilon_e(\omega)]/[1 - \varepsilon_e(\omega)\varepsilon_\omega(a)]$, where the dependence of ε_e on a is, for brevity, omitted. Therefore, we get

$$\varepsilon_\omega(a) = s(a)/\{\sigma(a)[1 - \varepsilon_e(\omega)] + s(a)\varepsilon_e(\omega)\}, \tag{5}$$

and

$$\sigma(a) = \hat{\sigma}(a) - s(a)\varepsilon_e(\omega)/[1 - \varepsilon_e(\omega)]. \tag{6}$$

Finally, differentiating the definition of net interest factor $R(a) = A\rho(a/\hat{e}(a)) + 1 - \delta[\hat{e}(a)]$ [see equations (4)] with respect to a leads to the following expression, to be used in the next section:

$$\begin{aligned} \varepsilon_R(a) &= \{\rho(x)\varepsilon_\rho(x)[1 - \varepsilon_e(\omega)\varepsilon_\omega(a)] - \delta(e)\varepsilon_\delta(e)\varepsilon_e(\omega)\varepsilon_\omega(a)\}/R(a), \text{ or} \\ \varepsilon_R(a) &= -\{\rho(x)[1 - s(a)][1 - \varepsilon_e(\omega)] + s(a)\delta(e)\varepsilon_\delta(e)\varepsilon_e(\omega)\}/ \\ &\quad (R(a)\{\sigma(a)[1 - \varepsilon_e(\omega)] + s(a)\varepsilon_e(\omega)\}), \end{aligned} \tag{7}$$

if we use equation (5). ■

To illustrate the main result of Lemma 1, I now turn to a simple example. If the production function is of the Cobb-Douglas type and both effort and capital depreciation are isoelastic functions, the profit is given by $Ak^s [e(\omega)h]^{1-s} - \omega h - \{r + \delta[e(\omega)]\}k$, with $e(\omega) = \omega^\epsilon$, $\delta(e) = -e^\alpha/\alpha$. The share of capital is given

by $0 < s < 1$, whereas $0 < \epsilon < 1$ and $\alpha < 0$ determine the convexity of labor utilization and capital depreciation, respectively.

At the unique optimum, the derivative of the profit function with respect to h vanishes, that is, $A(1 - s)\{k/[he(\omega)]\}^s e(\omega) = \omega$, with $e(\omega) = \omega^\epsilon$. I derive from the latter equality the optimal real wage $\omega(a) = [A(1 - s)a^s]^{1/[1 - \epsilon(1 - s)]}$, whose elasticity is given by $s/[1 - \epsilon(1 - s)]$. It is then easily checked that $\hat{\omega}(x)/\rho(x) = (1 - s)a/[se(\omega)]$: Accordingly, the elasticity of effective input substitution $\sigma = d \ln[a/\hat{e}(a)]/d \ln(\hat{\omega}/\rho)$ is equal to one while the elasticity of measured input substitution $\hat{\sigma} = d \ln a/d \ln(\hat{\omega}/\rho) = 1 + s\epsilon/(1 - \epsilon)$, from equation (6), so that $\hat{\sigma} = [1 - \epsilon(1 - s)]/(1 - \epsilon) > 1$. In particular, $\hat{\sigma}$ tends to $+\infty$ when $\epsilon_e = \epsilon$ tends to 1 from below: the possibilities to substituting one factor for another are reduced when the elasticity of labor utilization with respect to the real wage is high. On the contrary, σ and $\hat{\sigma}$ coincide when both labor utilization and capital depreciation rates are constant, that is, when $\epsilon = 0$.

More generally, $\epsilon_e(\omega) = 0$ corresponds to the case where the utilization rate of labor e as well as the depreciation rate for capital δ are constant, that is, where $\sigma(a) \equiv \hat{\sigma}(a)$ in view of equation (6). Therefore, a moderate elasticity of effort may imply low input substitution. In other words, the assumption of constant utilization rate for labor overestimates input substitution: When the relative rental price of capital goes down, the “extensive” effect increases capital over labor, but an “intensive” effect goes in opposite direction by triggering an increase in labor utilization, implying that the elasticity of effective capital/labor substitution is lower than that considered when the utilization rate of labor is assumed to be fixed.

2.3. Intertemporal Equilibria

I now complete the description of the model, the detailed structure of which is presented by Woodford (1986). Specifically, a representative worker solves the following problem:

$$\text{maximize } [V_2(c_{t+1}^w/B) - V_1(h_t)] \quad \text{such that } p_{t+1}c_{t+1}^w = w_t h_t, c_{t+1}^w \geq 0, h_t \geq 0, \tag{8}$$

where $B > 0$ is a scaling factor, c_{t+1}^w is next-period consumption, h_t is labor supply, $e(\omega_t)$ is effort, $p_{t+1} > 0$ is next-period price of consumption, assumed to be perfectly foreseen, and $w_t > 0$ is nominal wage rate; that is, $w_t = p_t \omega_t$. I consider the case where leisure and consumption are *gross substitutes* and therefore assume the following.

Assumption 2. The utility functions $V_1(h)$ and $V_2(c)$ are continuous for $0 \leq h \leq h^*$ and $c \geq 0$, where $h^* > 0$ is the (maybe infinite) workers’ endowment of labor. They are C^r for, respectively, $0 < h < h^*$ and $c > 0$, and r large enough, with $V_1'(h) > 0$, $V_1''(h) > 0$, $\lim_{h \rightarrow h^*} V_1'(h) = +\infty$, and $V_2'(c) > 0$, $V_2''(c) < 0$. Moreover, consumption and leisure are gross substitutes, that is $-cV_2''(c) < V_2'(c)$.

Since workers do not choose effort and are induced to provide the level of work intensity that is optimal for producers, it is natural to simplify as possible labor supply decisions. Therefore, we abstract from nonseparability issues and simply assume that labor disutility does not depend on effort. Then, the necessary conditions of the program in equations (8) describe the intertemporal substitution in terms of the functions V_1 and V_2 only and, therefore, we do not need more than what is stated in Assumption 2.³

It is easily shown that intertemporal competitive equilibria with perfect foresight (intertemporal equilibria thereafter) are summarized by the dynamics of two variables. Moreover, the discussion is conveniently amenable in terms of relevant parameters if the two variables are in fact chosen to be the capital/labor ratio $a = k/h$ (a nonpredetermined variable) and the capital stock k (a predetermined variable).⁴

DEFINITION 1. *An intertemporal competitive equilibrium with perfect foresight of the Woodford model with capital/labor substitution is a sequence (a_t, k_{t-1}) of R^2_{++} , $t = 0, 1, \dots$, such that*

$$\begin{cases} \omega(a_{t+1})/a_{t+1} = \gamma(k_{t-1}/a_t)/k_t, \\ k_t = \beta R(a_t)k_{t-1}. \end{cases} \tag{9}$$

The first equality in equations (9) is the first-order condition of workers' optimization program (8), where γ is the function whose graph is the offer curve, that is, the locus described by workers' optimal choices in the (h_t, c_{t+1}) plane when the relative price w_t/p_{t+1} varies. Under the assumption of gross substitutability, the elasticity of γ is larger than one or, equivalently, the labor supply elasticity with respect to the real wage is (given the expected inflation factor p_{t+1}/p_t) is $\varepsilon_h(\omega_t p_t/p_{t+1}) = 1/[\varepsilon_\gamma(h_t) - 1]$ and is positive. From equation (2), $\omega(a)$ denotes the equilibrium real wage.

The second equality in equations (9) summarizes capitalists' savings, under the assumption, made only for convenience since capitalists' preferences do not matter qualitatively here, of a logarithmic instantaneous utility function. The parameter $0 < \beta < 1$ represents capitalists' discount factor while, in view of equations (4), $R(a)$ denotes equilibrium gross returns to capital.

2.4. Robustness of Sunspots and Cycles

To ensure the existence of a (monetary) steady state, we appropriately scale the two parameters A and B . Moreover, the steady state is shown to be unique and is normalized, without loss of generality, at $(\bar{a}, \bar{k}) = (1, 1)$.

PROPOSITION 1 (Existence and Uniqueness of the Steady State). *Under Assumptions 1, 2, and $\lim_{c \rightarrow 0} cV'_2(c) < V'_1(1) < \lim_{c \rightarrow +\infty} cV'_2(c)$, $(\bar{a}, \bar{k}) = (1, 1)$ is the unique steady state of the dynamical system in equations (9) if and only if $A = (1/\beta - 1 + \delta'\{e[\omega(1)]\})/f'\{1/e[\omega(1)]\}$, and B is the unique solution of $\omega(1)V'_2[\omega(1)/B]/B = V'_1(1)$.*

Proof. In view of equations (9), the monetary steady states are the solutions (\bar{a}, \bar{h}) in R_{++}^2 of $\omega(\bar{a})\bar{k}/\bar{a} = \gamma(\bar{k}/\bar{a})$ and $\beta R(\bar{a}) = 1$. By definition of $R(a)$, in equations (4), the latter equation yields $Af'[\bar{a}/\hat{e}(\bar{a})] + 1 - \delta[\hat{e}(\bar{a})] = 1/\beta$, with $\hat{e}(a) = e[\omega(a)]$. We set $A = \{1/\beta - 1 + \delta[\hat{e}(1)]\}/f'[1/\hat{e}(1)]$ to ensure that $\bar{a} = 1$. Moreover, $\omega(1) = \gamma(1)$ or, equivalently, $\omega(1)V_2'[\omega(1)/B]/B = V_1'(1)$ is achieved by scaling the unique solution $B > 0$, under Assumption 2, implying that $cV_2'(c)$ is continuous and increasing and $\lim_{c \rightarrow 0} cV_2'(c) < V_1'(1) < \lim_{c \rightarrow +\infty} cV_2'(c)$. ■

In the context of a non-Walrasian labor market in which the real wage is decided by entrepreneurs so as to provide the correct incentives, it is possible that labor demand is lower than labor supply, at the steady state. In other terms, it is possible that the real efficiency wage is larger than the market-clearing wage of the corresponding perfectly competitive economy and, accordingly, that the steady state is characterized by unemployment.

The introduction of efficiency wage is, as noticed above, natural in the present model and, more importantly, is consistent with an observed feature of modern labor markets. Under various forms, institutions or social norms—for example, minimum wage, unemployment benefits, or labor unions—have emerged and allow, to some extent, agents who have limited access to capital markets to reduce the volatility of labor income. The above formulation is consistent with this observation: As shown in equation (5), the wage elasticity is, *ceteris paribus*, lower when labor utilization varies with the real wage, that is, when $\varepsilon_e(\omega) > 0$, provided that the elasticity of input substitution is smaller than the capital share, that is, that $\sigma(a) < s(a)$. Proposition 2 show that this condition is in fact necessary to the occurrence of endogenous fluctuations when labor utilization and capital depreciation are variable.

Accordingly, we study in the sequel the dynamics in the neighborhood of a steady state of the model with an efficiency wage fixed by capitalists and assume that a fraction of workers is unemployed and do not receive any compensation for that state. In particular, unemployed agents are chosen randomly and do not have the possibility of insuring themselves against unemployment risk.

Equations (9) define, near the steady state (\bar{a}, \bar{k}) , a dynamical system of the form $(a_{t+1}, k_t) = G(a_t, k_{t-1})$, provided that $\varepsilon_\omega(\bar{a}) \neq 1$. We study the local dynamics as a function of the following parameters: The depreciation rate for capital $0 < \delta \stackrel{\text{def}}{=} \delta[\hat{e}(\bar{a})] \leq 1$, the elasticity of depreciation $\varepsilon_\delta \stackrel{\text{def}}{=} \hat{e}(\bar{a})\delta'[\hat{e}(\bar{a})]/\delta[\hat{e}(\bar{a})] < 0$, the share of capital $0 < s \stackrel{\text{def}}{=} \bar{a}\rho[\bar{a}/\hat{e}(\bar{a})]/\{A\hat{e}(\bar{a})f'[\bar{a}/\hat{e}(\bar{a})]\} < 1$, the elasticity of input substitution $\sigma \stackrel{\text{def}}{=} \sigma(\bar{a}) > 0$, the capitalist's discount factor $0 < \beta < 1$, the elasticity of effort $\varepsilon_e \stackrel{\text{def}}{=} \varepsilon_e[\omega(\bar{a})] > 0$, and the elasticity of the function whose graph is the offer curve $\varepsilon_\gamma \stackrel{\text{def}}{=} \varepsilon_\gamma(\bar{h}) > 1$, all evaluated at the steady state.

Straightforward algebra yields the following results.

LEMMA 2 (Dynamics near the Steady State). *Under Assumptions 2 and 1, let $\varepsilon_R = \bar{a}R'(\bar{a})/R(\bar{a}) < 0$, $\varepsilon_\omega = \bar{a}\omega'(\bar{a})/\omega(\bar{a}) > 0$, $\varepsilon_\gamma = \bar{h}\gamma'(\bar{h})/\gamma(\bar{h}) < 1$, and $\varepsilon_e = \omega(\bar{a})e'[\omega(\bar{a})]/e[\omega(\bar{a})] > 0$ be the elasticities of the functions $R(a)$, $\omega(\bar{a})$, $\gamma(h)$,*

and $e(\omega)$, evaluated at the steady state, and suppose $\varepsilon_\omega \neq 1$. The Jacobian matrix of the map in equations (9), evaluated at the steady state, has trace T and determinant D given by

$$T = T_1 - (\varepsilon_\gamma - 1)/(\varepsilon_\omega - 1), \quad \text{with} \quad T_1 = 1 + D_1,$$

$$D = \varepsilon_\gamma D_1, \quad \text{with} \quad D_1 = (1 + \varepsilon_R)/(1 - \varepsilon_\omega)$$

$$= [\sigma - \theta(1 - s) + s\varepsilon_e(1 - \delta\beta\varepsilon_\delta)/$$

$$(1 - \varepsilon_e)]/(\sigma - s), \quad (10)$$

where $\theta = 1 - \beta(1 - \delta)$.

Moreover, slope $\Delta_\sigma = 1 + \varepsilon_R = 1 - [\theta(1 - s) + s\delta\beta\varepsilon_\delta\varepsilon_e/(1 - \varepsilon_e)]/[\sigma + s\varepsilon_e/(1 - \varepsilon_e)]$.

Direct inspection of Lemma 2 shows that a half-line Δ_σ is generated from (T_1, D_1) in the (T, D) plane, when ε_γ increases from 1, while all other parameters, that is, \bar{a} , ε_R , and ε_ω are held fixed.⁵ Its slope is given by $1 + \varepsilon_R$, that is, slope Δ_σ in Lemma 2. Moreover, it is easily seen from equations (10) that the origin $[T_1(\sigma), D_1(\sigma)]$ of Δ_σ is located on the line (AC) (of equation $T = 1 + D$, when 1 is an eigenvalue), as shown in Figure 2.

In the extreme case of constant utilization and depreciation ($\varepsilon_e = 0$), the configuration in Figure 2 arises when σ (which equals $\hat{\sigma}$) is made to vary, under appropriate

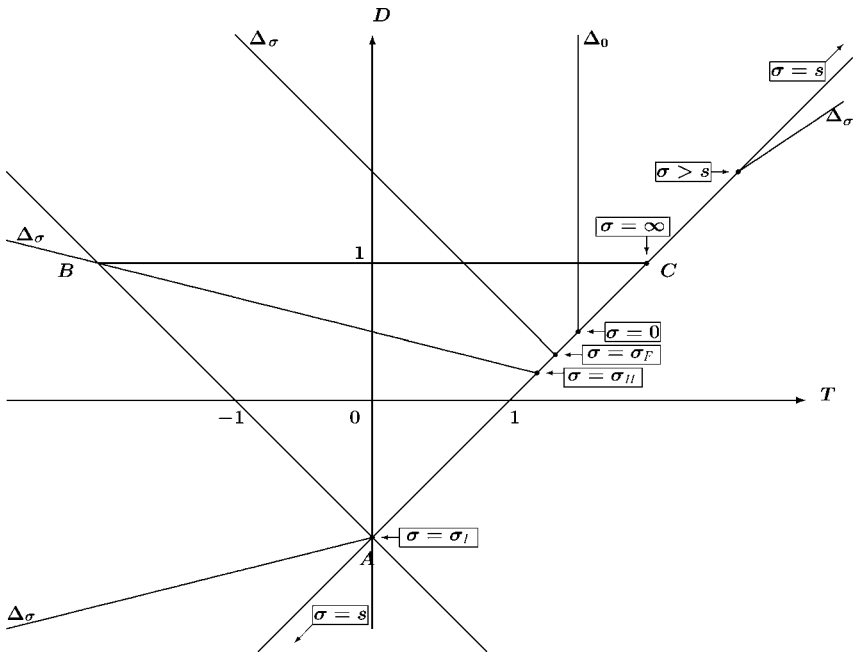


FIGURE 2. Local stability and bifurcations with constant factor utilization.

assumptions on the parameters [see Grandmont et al. (1998)]: $D_1(\sigma)$ is a decreasing function, while the slope of Δ_σ increases with σ . It follows that local indeterminacy (when two eigenvalues are inside the unit circle) and endogenous fluctuations emerge only for low values of σ , that is, for $\sigma < \sigma_I = [\theta(1 - s) + s]/2$, σ_I being defined as the value of σ such that $[T_1(\sigma), D_1(\sigma)]$ coincides with point A [or equivalently $D_1(\sigma) = -1$] in Figure 2, and indeed for σ 's that are significantly less than the share of capital s .

In particular, Hopf bifurcations are expected, implying the occurrence of an invariant closed curve surrounding the steady state, on which the dynamics is either periodic or quasiperiodic, when $0 < \sigma < \sigma_H$: At the intersection of Δ_σ and the interior of the segment $[BC]$, the two complex eigenvalues have modulus one. Moreover, a flip bifurcation generally occurs when $\sigma_F < \sigma < \sigma_I$: An eigenvalue is equal to -1 when Δ_σ intersects the line (AB) , that is, $T = -1 - D$.

On the contrary, local determinacy is bound to prevail, that is, there exists a neighborhood in which no endogenous fluctuations occur, for larger values of σ , that is, when $\sigma > \sigma_I$ (see Figure 2).

Most importantly, we are now going to show that when $\varepsilon_e > 0$ is close to zero—when labor utilization varies with the real wage—local indeterminacy and expectations-driven fluctuations occur for even smaller elasticities of input substitution σ .

Starting from the configuration in Figure 2 for which labor utilization and capital depreciation are constant (for $\varepsilon_e = 0$), the origin and the (absolute value of the) slope of the half-line Δ_0 (for $\sigma = 0$) as well as the critical value σ_I decrease with ε_e . Even though indeterminacy still occurs for low σ 's, the effect of variable utilization is, therefore, *to reduce the range of input substitution elasticities that are consistent with local indeterminacy and endogenous fluctuations*. However, if we fix ε_e at a positive value and vary σ , starting at a given Δ_0 , we get the same qualitative picture: the origin and the (absolute value of the) slope of the half-line Δ_σ decrease with σ . Therefore, a convenient way to summarize the influence of ε_e is to represent how Δ_0 (for $\sigma = 0$) moves with our key parameter, ε_e : this is the purpose of Figure 3.

More precisely, it is seen from Lemma 2 that $D_1(\sigma)$ decreases, along the line (AC) , and that $\text{slope}_{\Delta_\sigma}(\sigma)$ increases with both σ and ε_e , whenever $\theta(1 - s) < s$. In particular, $D_1(0) = \theta(1 - s)/s - \varepsilon_e(1 - \delta\beta\varepsilon_\delta)/(1 - \varepsilon_e)$ decreases from $\theta(1 - s)/s < 1$ to -1 , that is, $\sigma_I = [s + \theta(1 - s) - s\varepsilon_e(1 - \delta\beta\varepsilon_\delta)/(1 - \varepsilon_e)]/2$, defined by $D_1(\sigma_I) = -1$, decreases from $[\theta(1 - s) + s]/2$ to zero, while $\text{slope}_{\Delta_0}(0)$ increases from $-\infty$ to a positive value, when ε_e increases from zero to $\varepsilon_{eI} = [s + \theta(1 - s)]/[\theta(1 - s) + s(2 - \delta\beta\varepsilon_\delta)]$, as summarized in Figure 3.

Accordingly, the intersection between the half-plane generated by the line Δ_σ , when σ moves up, and the indeterminacy triangle ABC gets smaller and smaller, in Figure 3, as ε_e is increased: The half-line Δ_0 (for $\sigma = 0$) goes toward the negative orthant of the plane. This intersection may even become empty when ε_e is sufficiently high, that is, if $\varepsilon_e > \varepsilon_{eI} = [s + \theta(1 - s)]/[\theta(1 - s) + s(2 - \delta\beta\varepsilon_\delta)]$. In that case, $D_1(\sigma) < -1$ and $0 < \text{slope}_{\Delta_\sigma}(\sigma) < 1$ when $\sigma < \sigma_0$, while $D_1(\sigma) > 1$ and

However, the existence of stochastic equilibria driven by self-fulfilling beliefs (sunspots) is still compatible with values of ε_e between ε_{eH} and ε_{eI} .⁶ For instance, if $s = 1/3$, $\beta = 0.988$, $\delta = 0.025$, and $\varepsilon_\delta = -0.1$ (resp. $\varepsilon_\delta = -10$), $\varepsilon_{eI} \approx 0.52$ (resp. $\varepsilon_{eI} \approx 0.46$), and the half-line Δ_σ intersects both the triangle ABC and the line (AB) , if $\sigma < \sigma_I$ (see Figures 3). Accordingly, the steady state is asymptotically stable (locally indeterminate) when $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$, and becomes, through a flip bifurcation, a saddle (locally determinate), as an eigenvalue decreases and goes through -1 when ε_γ goes through $\varepsilon_{\gamma F}$. It follows that stochastic equilibria do exist, in general, around the steady state if $\varepsilon_\gamma < \varepsilon_{\gamma F}$, and possibly ε_γ larger but close to $\varepsilon_{\gamma F}$ in the presence of an attracting period-2 cycle. Moreover, a period-2 cycle is expected around the steady state when if ε_γ is sufficiently close to $\varepsilon_{\gamma F}$.

Without additional information on higher-order derivatives of the Jacobian matrix, one cannot establish whether local deterministic cycles (originated through flip or Hopf bifurcations) are stable or unstable. However, for sake of brevity, I do not develop this analysis and focus on sunspots when discussing the plausibility of expectations-driven fluctuations.

Therefore, we have established the following results.

PROPOSITION 2 (Local Stability and Bifurcations of the Steady State). *Consider the monetary steady state (\bar{a}, \bar{k}) , normalized by the procedure in Proposition 1, and suppose $\theta(1 - s) < s$, where $\theta = 1 - \beta(1 - \delta)$, $0 < \beta < 1$ is the capitalists' discount factor, $0 \leq \delta \stackrel{\text{def}}{=} \delta\{e[\omega(\bar{a})]\} \leq 1$ is the depreciation rate for capital and $0 < s = \bar{a}\rho\{\bar{a}/e[\omega(\bar{a})]\}/(Ae[\omega(\bar{a})]f\{\bar{a}/e[\omega(\bar{a})]\}) < 1$ denotes the capital share in total income.*

*Under the assumptions of Lemma 1 and Proposition 1, the following results are generic.*⁷

- (i) if $\varepsilon_{eF} < \varepsilon_e < \varepsilon_{eH}$, then
 - (a) $0 < \sigma < \sigma_H$: The steady state is a sink (locally indeterminate) when $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$, where $\varepsilon_{\gamma H}$ is the value of ε_γ for which Δ_σ crosses $[BC]$. The steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at $\varepsilon_\gamma = \varepsilon_{\gamma H}$ and is a source when $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$, where $\varepsilon_{\gamma F}$ is the value of ε_γ for which Δ_σ crosses the line (AB) . A flip bifurcation occurs (one characteristic root goes through -1) at $\varepsilon_\gamma = \varepsilon_{\gamma F}$ and the steady state is a saddle (locally determinate) if $\varepsilon_\gamma > \varepsilon_{\gamma F}$.
 - (b) $\sigma_H < \sigma < \sigma_I$: The steady state is a sink when $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$. A flip bifurcation occurs at $\varepsilon_\gamma = \varepsilon_{\gamma F}$ and the steady state is a saddle if $\varepsilon_\gamma > \varepsilon_{\gamma F}$.
 - (c) $\sigma_I < \sigma$ and $\sigma \neq \sigma_0$: The steady state is a saddle for all $\varepsilon_\gamma > 1$.
- (ii) if $\varepsilon_{eH} < \varepsilon_e < \varepsilon_{eI}$, then
 - (a) $0 < \sigma < \sigma_I$: The steady state is a sink when $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$. A flip bifurcation occurs at $\varepsilon_\gamma = \varepsilon_{\gamma F}$ and the steady state is a saddle if $\varepsilon_\gamma > \varepsilon_{\gamma F}$.
 - (b) $\sigma_I < \sigma$ and $\sigma \neq \sigma_0$: The steady state is a saddle for all $\varepsilon_\gamma > 1$.
- (iii) if $\varepsilon_{eI} < \varepsilon_e$, then the steady state is a saddle, independently of $\sigma > 0$ and $\varepsilon_\gamma > 1$.

The most important result of the above analysis is that variable labor utilization and capital depreciation rates reduce the range of *effective* input substitution elasticities $0 < \sigma < \sigma_I$ for which deterministic and stochastic endogenous fluctuations occur. In particular, this range is small if effort is quite elastic to the real wage, that is, if $\varepsilon_e(\omega) = \omega e'(\omega)/e(\omega)$ is large enough at the steady state and nearby. In fact, the critical elasticity σ_I decreases with ε_e and indeed becomes negative if $\varepsilon_e > \varepsilon_{eI} = [s + \theta(1 - s)]/[\theta(1 - s) + s(2 - \delta\beta\varepsilon_\delta)]$. Other things being equal, the existence of endogenous fluctuations depends on more stringent conditions, in terms of parameter values involving the elasticity of effective substitution.

In the next section, however, I will argue that the plausibility of endogenous fluctuations, in the context of an “intensive” labor utilization reducing factor substitution, is indeed improved if the discussion is cast in terms of the *apparent* (or measured) elasticity.

Remark. In the preceding subsection, I made σ increase from zero, given ε_e and ε_δ . Moreover, we have derived in equation (3) that $\varepsilon_\omega = (\varepsilon_e - 1)/[\varepsilon_{e'} - \varepsilon_e(\varepsilon_e - 1)(1 + \varepsilon_\delta)]$. In view of equation (5), therefore, the relation between all technology parameters $(\varepsilon_e - 1)/[\varepsilon_{e'} - \varepsilon_e(\varepsilon_e - 1)(1 + \varepsilon_\delta)] = s/[\sigma(1 - \varepsilon_e) + s\varepsilon_e]$ has to hold at the steady state. Given $\varepsilon_e < 1$ and $\sigma > 0$, we have therefore implicitly assumed that, for instance, $\varepsilon_{e'}$ is set at $(\varepsilon_e - 1)[\sigma(1 - \varepsilon_e)/s + \varepsilon_e(2 + \varepsilon_\delta)]$, which is negative if, in addition, $2 + \varepsilon_\delta > 0$. In any case, these assumptions are not critical for the above qualitative results.

3. PLAUSIBILITY OF ENDOGENOUS FLUCTUATIONS WHEN LABOR UTILIZATION IS VARIABLE

Even though, as shown by Proposition 2 (and Figure 3), considering variable labor utilization and capital depreciation rates reduces the scope for endogenous fluctuations, in parameter space, it is argued in this section that their *plausibility* is, on the contrary, improved when variable intensity is taken into account. Section 2.2 has established that effective substitution is reduced when utilization (and depreciation) are variable. As a consequence, this has led, in Section 2.4, to conditions for local indeterminacy and endogenous fluctuations that are more restrictive.

The analysis also suggests that the elasticity of effective input substitution is, in this context, reduced and may be substantially lower than the elasticity of measured input substitution, which is in fact obtained under the assumption of constant utilization and depreciation. In summary, the more elastic utilization, the lower the elasticities of capital/labor substitution that are compatible with endogenous fluctuations, but, at the same time, the lower the elasticity of effective input substitution. Therefore, one has to assess more carefully, in this context, the plausibility of self-fulfilling cycles, which is the purpose of this section. I illustrate that a rather different conclusion is reached if the discussion is cast in terms of apparent substitution possibilities: Endogenous fluctuations are indeed

compatible with *higher* elasticities of apparent substitution, when labor utilization is variable.

To simplify the discussion, suppose that the length of the period is short enough, so that we can neglect $\theta = 1 - \beta(1 - \delta) \approx 0$ (as $\beta \approx 1$ and $\delta \approx 0$), and fix $s = 1/3$. Proposition 2 (see also Figure 3) shows that endogenous fluctuations are possible only if $\varepsilon_e < \varepsilon_{eI} \approx 1/2$ and $\sigma < \sigma_I \approx s(1/2 - \varepsilon_e)/(1 - \varepsilon_e)$. For instance, σ_I is, respectively, equal to 15%, 13% and 3% when $\varepsilon_e = 0.1, 0.2,$ and 0.45 . Because direct estimates of the elasticity of effective input substitution are not available, we are forced to compare the range of measured substitution elasticities $\hat{\sigma}$ with the corresponding estimates. By using Lemma 1, which provides the relation $\sigma = \hat{\sigma} - s\varepsilon_e/(1 - \varepsilon_e)$, the condition $\sigma < \sigma_I \approx s(1/2 - \varepsilon_e)/(1 - \varepsilon_e)$ is rewritten as $\hat{\sigma} < s/[2(1 - \varepsilon_e)]$. Accordingly, the elasticity $\hat{\sigma}$ has to be lower than, respectively, 19%, 21%, and 30% when $\varepsilon_e = 0.1, 0.2,$ and 0.45 , to be consistent with the existence of expectations-driven cycles. On the contrary, when labor utilization is constant, that is, when $\varepsilon_e = 0(\sigma \equiv \hat{\sigma})$, the upper bound σ_I is approximatively equal to 17%.

Accordingly, *the occurrence of local indeterminacy and endogenous fluctuations implies a less stringent condition—in terms of the elasticity of apparent factor substitution $\hat{\sigma}$ —when labor utilization and capital depreciation are variable*, in contrast to the model in which capital utilization is variable: The higher ε_e , the less restrictive the condition $\hat{\sigma} < s/[2(1 - \varepsilon_e)]$. In particular, the values of the apparent elasticity that are consistent with endogenous fluctuations, that is, $\hat{\sigma} < s$ (when $\varepsilon_e = 1/2 \approx \varepsilon_{eI}$), fall within the lowest end of the range of estimates [see, e.g., Hamermesh (1993, Ch. 3) or Rowthorn (1999, Table 2)].

We now examine the range of labor supply elasticities with respect to the real wage—that is, $1/(\varepsilon_\gamma - 1)$ —that are consistent with local indeterminacy and bifurcations. The second part of Proposition 2 and Appendix B show that the steady state is locally indeterminate and, therefore, that stochastic equilibria do exist in its neighborhood, if $\varepsilon_\gamma < \varepsilon_{\gamma F} \approx (s - \sigma)/[\sigma + s\varepsilon_e/(1 - \varepsilon_e)]$, when $\sigma < \sigma_I$. Imposing to the labor supply elasticity $\varepsilon_l = 1/(\varepsilon_\gamma - 1)$ to be less than unity—that is, $2 < \varepsilon_\gamma < \varepsilon_{\gamma F}$ —implies then $\sigma < s(1/3 - \varepsilon_e)/(1 - \varepsilon_e)$, which is a stronger condition than $\sigma < \sigma_I \approx s(1/2 - \varepsilon_e)/(1 - \varepsilon_e)$; σ now has to be lower than 9% and 6%, respectively, if $\varepsilon_e = 0.1, 0.2$. Equivalently, $\hat{\sigma} < s/[3(1 - \varepsilon_e)]$ has to hold: The measured elasticity $\hat{\sigma}$ has to be smaller than 12%, 14%, and 20% when $\varepsilon_e = 0.1, 0.2,$ and 0.45 , respectively, to be compatible with endogenous fluctuations when labor supply elasticity is less than unity (probably a strong requirement in view of most empirical studies). The existence of sunspots is therefore in agreement with the estimates of labor supply elasticity, provided that the elasticity of capital/labor substitution is low enough.

In summary, because the analysis of the model with variable utilization gives a precise framework to understand why input substitution possibilities may be low, the existence of endogenous fluctuations for small elasticities of capital/labor substitution may therefore be considered in this context as more plausible, even though direct estimates would allow us to assess more carefully this conclusion.

In fact, the model with variable effort is compatible with larger elasticities of apparent input substitution, and it predicts that sunspots are more plausible than in the extreme case of constant labor utilization.

4. CONCLUSION

The most important result of the paper is that the scope of local indeterminacy and expectations-driven fluctuations is reduced by an *intensive* (utilization) effect that goes against factor substitution, when returns to scale are nearly constant. More precisely, it is shown that under the plausible assumption that labor utilization (effort) is not too elastic, local indeterminacy and expectations-driven fluctuations occur for even lower elasticities of effective input substitution. However, it is also shown that endogenous fluctuations may be viewed as more plausible in the context of variable utilization: These phenomena happen to occur for higher elasticities of apparent substitution, and indeed for values that fall within the range of available estimates. Given the “technological” nature of the argument, the same qualitative results can be shown to hold in alternative business-cycle models with substitutable inputs [see Pintus (2002, App. C) for an illustration in the overlapping generations framework]. Moreover, the negative influence of utilization on input substitution, which seems to have been unnoticed in the literature, may help to understand, and possibly to generalize, Wen’s (1998) results.

In the model that we study, periodic or quasiperiodic cycles, originated through Hopf bifurcations, typically occur when utilization is constant but no longer emerge when one relaxes this latter assumption only slightly. However, stochastic equilibria are more likely to occur. Therefore, it would be interesting to simulate a calibrated version of the model in which fluctuations are driven by self-fulfilling beliefs, so as to evaluate its relative performance in explaining aggregate data comovements and volatility, in comparison with other models in the literature [see, e.g., Benhabib and Farmer (1999)].

As another extension of my analysis, it would also be fruitful to further study the interesting setting proposed by Uhlig and Xu (1996), in which efficiency wages are set in the context of imperfect monitoring. In particular, the configuration of procyclical effort seems to lead to endogenous cycles, as simply mentioned by these authors.

Finally, some remarks follow from the foregoing analysis. First, it turns out that endogenous fluctuations are, in the model with labor utilization, more plausible than those arising in the model with capital utilization [see Pintus (2002, Sect. 2)]. However, as we did in Section 3, one may object to quantitatively assessing the results of the model by using *direct* estimates of the relevant parameters. To our knowledge, the issue has not been covered by empirical studies, although it probably could be. In fact, most recent estimates of capital/labor substitution ignore variable factor utilization [see, e.g., Hamermesh (1993, Ch. 3) and Rowthorn (1999)]. On the other hand, the Cobb-Douglas specification is often assumed in most empirical work studying the impact of capital utilization, as in, for instance,

Burnside et al. (1995). The analysis of the present paper suggests a possible explanation for the coexistence of two groups of estimates that provide, on the one hand, low values and, on the other hand, significantly higher values [see also Rowthorn (1999) for another explanation of this difference]. It may be the case that the first group of studies relies on data that incorporate partly intensive effects when measuring the productive services of capital and labor by using, for example, electricity or raw materials consumptions. It remains to be seen how empirical estimates of input substitution would be modified by the introduction of variable factor utilization.

NOTES

1. As noted in the introduction of the paper, the financial constraint imposed on workers may be interpreted as resulting from incomplete or imperfect information regarding workers' effort or abilities. Therefore, this may justify as well the efficiency wage hypothesis.

2. This formulation is therefore compatible with the argument of Akerlof and Yellen (1986, p. 14) suggesting that effort may be quite inelastic when the influence of effort on capital depreciation is taken into account.

3. The local dynamics of the model with the alternative assumption of disutility derived from efficient labor eh are shown, although at some additional cost, to be qualitatively similar to those established later.

4. See Grandmont et al. (1998, App. A) for more detail on the derivation of intertemporal equilibria with finance-constrained workers.

5. Direct inspection of equations (9) shows that \bar{a} does not depend on ε_γ . See the proof of Proposition 1.

6. See Grandmont et al. (1998) on how to construct such sunspot equilibria when the steady state is indeterminate or surrounded by deterministic cycles.

7. See Appendix B for the expressions of the critical values ε_{eH} , ρ_H , $\varepsilon_{\gamma H}$, $\varepsilon_{\gamma F}$, and ρ_0 . The expressions $\varepsilon_{eF} = \theta(1-s)/[\theta(1-s) + s(2-\delta\beta\varepsilon_\delta)]$, $\varepsilon_{eI} = [s + \theta(1-s)]/[\theta(1-s) + s(2-\delta\beta\varepsilon_\delta)]$, $\sigma_I = (s + \theta(1-s) - s\varepsilon_e(1-\delta\beta\varepsilon_\delta)/(1-\varepsilon_e))/2$ are given earlier in the text.

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APPENDIX A. CONCAVITY OF THE PROFIT FUNCTION

This section shows that, under Assumption 1, the profit function with variable labor utilization and capital depreciation, whose Jacobian appears implicitly in equations (1), is concave at an optimum if, in addition, the depreciation function is not too elastic with respect to effort. To that effect, it is shown that the Hessian matrix satisfies the necessary and sufficient conditions on *all* principal minors of a negative semi-definite matrix, as presented by Gantmacher (1959, Ch. X, Para. 4, Theorem. 6). In fact, the assumption of constant returns to scale implies that the first principal minor of order two vanishes and, therefore, one cannot ensure that the Hessian matrix is negative semi-definite by assuming only that the three *successive* principal minors have the appropriate sign.

One directly checks that the (symmetric) Hessian matrix, evaluated at an optimum satisfying equations (1), is given by

$$H_l = \begin{bmatrix} Af''/(eh) & -kAf''/(eh^2) & -e'kAf''/(e^2h) - \delta'e' \\ & k^2Af''/(eh^3) & e'\hat{\omega} - 1 - e' Af''k^2/(eh)^2 \\ & & e''h\hat{\omega} + k^2(e')^2 Af''/(he^3) \\ & & & -k(\delta'(e')^2 + \delta'e'') \end{bmatrix},$$

where the dependence of the functions $Af''(a/e)$, $\delta(e)$, $e(\omega)$ and their derivatives is omitted for brevity.

First-order minors: The three diagonal terms of H_l are nonpositive under Assumption 1.

Second-order minors: The minor formed from the first two rows and columns of H_l is shown to vanish. Moreover, the two other minors are nonnegative if and only if $Af''(e''h\hat{\omega} - k[\delta''(e')^2 + \delta'e''])/(eh) - (\delta'e')^2 - 2\delta'k(e')^2 Af''/(he^2) \geq 0$ at an optimum. Whereas the first term of the left-hand side is positive under Assumption 1, the last two positive terms have to be subtracted. However, these two negative terms are small if e' or δ' are small. The local analysis of Proposition 2 is therefore valid under the assumption that δ' is small enough at the steady state to ensure the concavity of the profit maximization problem.

Third-order minors: Finally, it is directly checked that the third-order minor of H_l vanishes.

APPENDIX B. CRITICAL VALUES OF PROPOSITION 2

B.1. HOPF BIFURCATION

The critical value $\varepsilon_{\gamma H}$ is defined by $D = \varepsilon_{\gamma H} D_1 = 1$, that is, $\varepsilon_{\gamma H} = (\sigma - s)/[\sigma - \theta(1 - s) + s\varepsilon_e(1 - \delta\beta\varepsilon_\delta)/(1 - \varepsilon_e)]$, in view of equations (10).

Although the (complicated) expression of ε_{eH} can be computed as the solution of $T(\varepsilon_{\gamma H}) = -2$ when $\sigma = 0$ (see Figure 3), we give here upper and lower bounds,

because they are, for practical purpose, quite close when θ is close to zero. As deduced from Figure 3, a lower bound for ε_{eH} is given by $\varepsilon_{eF} \stackrel{\text{def}}{=} \theta(1-s)/[\theta(1-s) + s(2 - \delta\beta\varepsilon_\delta)]$ (see below). An upper bound is given by the solution of $\text{slope}_\Delta(0) = D_1(0) = 0$ (see Figure 3), that is $\theta(1-s)/[\theta(1-s) + s(1 - \delta\beta\varepsilon_\delta)]$. In summary, $\varepsilon_{eF} = \theta(1-s)/[\theta(1-s) + s(2 - \delta\beta\varepsilon_\delta)] < \varepsilon_{eH} < \theta(1-s)/[\theta(1-s) + s(1 - \delta\beta\varepsilon_\delta)]$.

Moreover, σ_H is the smallest root of the polynomial $Q_H(\sigma) \stackrel{\text{def}}{=} a\sigma^2 + b\sigma + c$ deduced from $T(\varepsilon_{\gamma H}) = -2$ (see Figure 2). Its coefficients are shown, from equations (10), to have the following expressions:

$$\begin{aligned} a &= 4(1 - \varepsilon_e)^2, \\ b &= 4(1 - \varepsilon_e)(2s\varepsilon_e - \alpha - s), \\ c &= 4(s\varepsilon_e)^2 - 4s\varepsilon_e(\alpha + s) + \alpha(\alpha + 3s), \end{aligned}$$

where $\alpha \stackrel{\text{def}}{=} \theta(1-s) - \varepsilon_e[\theta(1-s) - s\delta\beta\varepsilon_\delta]$.

B.2. FLIP BIFURCATION

In view of Figure 3, the critical values ε_{eF} and $\varepsilon_{\gamma F}$ are, respectively, defined by $\text{slope}_\Delta(0) = -1$ and $1 + T(\varepsilon_\gamma) + D(\varepsilon_\gamma) = 0$, that is, $\varepsilon_{eF} = \theta(1-s)/[\theta(1-s) + s(2 - \delta\beta\varepsilon_\delta)]$ and $\varepsilon_{\gamma F} = [\theta(1-s) + 2(s - \sigma) + s\varepsilon_e\delta\beta\varepsilon_\delta/(1 - \varepsilon_e)]/[2\sigma - \theta(1-s) + s\varepsilon_e(2 - \delta\beta\varepsilon_\delta)/(1 - \varepsilon_e)]$, in view of equations (10).

B.3. LOCAL INDETERMINACY

In view of Figure 3, ε_{eI} is obtained from $D_1(0) = -1$, that is $\varepsilon_{eI} = [\theta(1-s) + s]/[\theta(1-s) + s(2 - \delta\beta\varepsilon_\delta)]$, from equations (10). From Figure 2, σ_I is defined by $D_1(\sigma_I) = -1$, that is, $\sigma_I = [\theta(1-s) + s - s\varepsilon_e(1 - \delta\beta\varepsilon_\delta)/(1 - \varepsilon_e)]/2$, in view of equations (10).

Finally, σ_0 is derived from $\varepsilon_\omega(\bar{a}) = 1$, using equations (5); that is, $\sigma_0 = s$.