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NOTE

NOTE ON FINITE APPROXIMATIONS OF THE ASYMPTOTICALLY IDEAL MODEL

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This note builds on recent work by Serletis and Shahmoradi [*Macroeconomic Dynamics* 9 (2005), 542–559] and estimates the AIM model at different degrees of approximation, using the same optimization procedures as in Gallant and Golub [*Journal of Econometrics* 26 (1984), 295–321]. We estimate the models subject to regularity and provide a comparison between the different versions. We argue that the AIM(3) model estimated subject to global curvature currently provides the best specification for research in semiparametric modeling of consumer demand systems.

Keywords: Flexible Functional Forms, Asymptotically Ideal Model, Global Curvature Restrictions

1. INTRODUCTION

In a recent paper, Serletis and Shahmoradi (2005) investigate the demand for money in the United States in the context of two seminonparametric flexible functional forms—the Fourier, introduced by Gallant (1981), and the Asymptotically Ideal Model (AIM), introduced by Barnett and Jonas (1983) and employed and explained in Barnett and Yue (1988). They estimate these models subject to regularity, as suggested by Barnett (2002) and Barnett and Pasupathy (2003), using methods suggested by Gallant and Golub (1984). They make a strong case, using (for the first time) parameter estimates that are consistent with global regularity, for abandoning the simple sum approach to monetary aggregation, on the basis of low elasticities of substitution among the components of the popular M2 aggregate of money.

In this paper, we build on Serletis and Shahmoradi (2005) and estimate the AIM model at different degrees of approximation, using the same optimization procedures as in Gallant and Golub (1984) and Serletis and Shahmoradi (2005).

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We estimate the models subject to regularity and provide a comparison between the different versions.

The note is organized as follows. Section 2 discusses the data and briefly sketches out the neoclassical problem facing the representative agent. Section 3 presents the AIM model at different degrees of approximation and Section 4 is devoted to computational considerations and estimation of the models. In Section 5 we explore the economic significance of the results. The final section concludes the paper.

2. THE DATA AND THE MONETARY PROBLEM

Following Serletis and Shahmoradi (2005), we assume a weakly separable monetary utility function, so that the representative money holder faces the problem

$$\max_x f(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} = m,$$

where $\boldsymbol{x}$ is the vector of monetary assets; $\boldsymbol{p}$ is the corresponding vector of nominal user costs; and m is the expenditure on the services of monetary assets. Because our demand systems are parameter-intensive, we rationalize the estimation to a small set of demand equations by using the database used by Serletis and Shahmoradi (2005). It consists of quarterly data, from 1970:1 to 2003:2 (a total of 134 observations), on three liquid asset groupings as follows:

Subaggregate A: currency, traveler's checks, and other checkable deposits including Super NOW accounts issued by commercial banks and thrifts
Subaggregate B: savings deposits issued by commercial banks and thrifts
Subaggregate C: small time deposits issued by commercial banks and thrifts

The assets in each category are aggregated using the Divisia index, defined in discrete time as

$$\log M_t - \log M_{t-1} = \sum_{j=1}^{n} s_{jt}^{*}(\log x_{jt} - \log x_{j,t-1}),$$

according to which the growth rate of each subaggregate is the weighted average of the growth rates of its components, with the Divisia weights being defined as the expenditure shares averaged over the two periods of the change, $s_{jt}^{*} = (1/2)(s_{jt} + s_{j,t-1})$ for $j = 1, \ldots, n$, where $s_{jt} = \pi_{jt}x_{jt}/\sum \pi_{kt}x_{kt}$ is the expenditure share of asset j during period t, and π_{jt} is the user cost of asset j.

3. THREE AIM MODELS

Our objective is to estimate a system of demand equations derived from an indirect utility function. The most important advantage of using the indirect utility approach is that prices enter into the estimation process as exogenous variables and the demand system is easily derived by applying Roy's identity. We follow Barnett and Yue (1988) and use the reciprocal indirect utility function for the asymptotically

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ideal model for $n = 3$,

$$h(\boldsymbol{v}) = a_0 + \sum_{k=1}^{K}\sum_{i=1}^{3} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{K}\sum_{m=1}^{K}\left[\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)}\right] + \sum_{k=1}^{K}\sum_{m=1}^{K}\sum_{g=1}^{K}\left[\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{h=1}^{3} a_{ijhkmg} v_i^{\lambda(k)} v_j^{\lambda(m)} v_h^{\lambda(g)}\right], \quad (1)$$

where $\lambda(z) = 2^{-z}$ for $z = \{k, m, g\}$ is the exponent set and a_{ik}, a_{ijkm}, and a_{ijhkmg}, for all $i, j, h = 1, 2, 3$, are the parameters to be estimated. The number of parameters is reduced by deleting the diagonal elements of the parameter arrays so that $i \neq j$, $j \neq h$, and $i \neq h$. This does not alter the span of the model's approximation.

By applying the modified Roy's identity,

$$s_i(\boldsymbol{v}) = \frac{v_i\,(\partial h(\boldsymbol{v})/\partial v_i)}{\sum_{i=1}^{n} v_i\,(\partial h(\boldsymbol{v})/\partial v_i)}, \quad (2)$$

to (1), we obtain the AIM(K) demand system, where $s_i = p_i x_i/\boldsymbol{p}'\boldsymbol{x} = v_i x_i$. With n assets and a degree of approximation of K, the number of parameters to be estimated in the AIM(K) model is given by the formula

$$\frac{nk}{1!} + \frac{n(n-1)k^2}{2!} + \frac{n(n-1)(n-2)k^3}{3!} + \cdots.$$

In what follows, we briefly present the basic properties of three AIM models—the AIM models for $K = 1$, 2, and 3. Although there is some comparison in our presentation in this section, our purpose is basically to make clear the properties and complexities of the models. It is to be noted that we also attempted to estimate the AIM model for $K = 4$, but we were not successful, mainly because of computational difficulties in the large parameter space—for $n = 3$, the AIM(4) has 124 parameters to be estimated!

3.1. The AIM(1) Model

For $K = 1$, equation (1) becomes, because $\lambda(z) = 1/2$ for $z = \{k, m, g\}$,

$$h_{K=1}(\boldsymbol{v}) = a_0 + \sum_{i=1}^{3} a_i v_i^{1/2} + \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} v_i^{1/2} v_j^{1/2} + \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{h=1}^{3} a_{ijh} v_i^{1/2} v_j^{1/2} v_h^{1/2}. \quad (3)$$

We delete the diagonal terms (so that $i \neq j$, $j \neq h$, and $i \neq h$) and follow Barnett and Yue (1988) in reparameterizing by stacking the coefficients as they appear in (3) into a single vector of parameters, $b = (b_0, \cdots, b_7)'$, containing the eight

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coefficients in (3), as follows:

$$h_{K=1}(\boldsymbol{v}) = b_0 + b_1 v_1^{1/2} + b_2 v_2^{1/2} + b_3 v_3^{1/2} + b_4 v_1^{1/2} v_2^{1/2} + b_5 v_1^{1/2} v_3^{1/2} + b_6 v_2^{1/2} v_3^{1/2} + b_7 v_1^{1/2} v_2^{1/2} v_3^{1/2}, \qquad \textbf{(4)}$$

where $b_0 = a_0$, $b_1 = a_1$, $b_2 = a_2$, $b_3 = a_3$, $b_4 = a_{12} + a_{21}$, $b_5 = a_{13} + a_{31}$, $b_6 = a_{23} + a_{32}$, and $b_7 = a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321}$.

Applying the modified Roy's identity (2) to (4) yields the AIM(1) demand system,

$$s_1 = \left(b_1 v_1^{1/2} + b_4 v_1^{1/2} v_2^{1/2} + b_5 v_1^{1/2} v_3^{1/2} + b_7 v_1^{1/2} v_2^{1/2} v_3^{1/2}\right) / D \qquad \textbf{(5)}$$

$$s_2 = \left(b_2 v_2^{1/2} + b_4 v_1^{1/2} v_2^{1/2} + b_6 v_2^{1/2} v_3^{1/2} + b_7 v_1^{1/2} v_2^{1/2} v_3^{1/2}\right) / D \qquad \textbf{(6)}$$

$$s_3 = \left(b_3 v_3^{1/2} + b_5 v_1^{1/2} v_3^{1/2} + b_6 v_2^{1/2} v_3^{1/2} + b_7 v_1^{1/2} v_2^{1/2} v_3^{1/2}\right) / D, \qquad \textbf{(7)}$$

where D is the sum of the numerators in equations (5), (6), and (7).

3.2. The AIM(2) Model

For $K = 2$, equation (1) becomes

$$h_{K=2}(\boldsymbol{v}) = a_0 + \sum_{k=1}^{2} \sum_{i=1}^{3} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{2} \sum_{m=1}^{2} \left[\sum_{i=1}^{3} \sum_{j=1}^{3} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)} \right] + \sum_{k=1}^{2} \sum_{m=1}^{2} \sum_{g=1}^{2} \left[\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{h=1}^{3} a_{ijhkmg} v_i^{\lambda(k)} v_j^{\lambda(m)} v_h^{\lambda(g)} \right]. \qquad \textbf{(8)}$$

Again, to avoid the extensive multiple subscripting in the coefficients a_{ijhkmg}, we follow Barnett and Yue (1988) and reparameterize by stacking the coefficients as they appear in (8) into a single vector of parameters, $\boldsymbol{b} = (b_0, \cdots, b_{26})'$, containing the 27 coefficients in (8), as follows [because $z = 1, 2$, so that $\lambda(1) = 1/2$ and $\lambda(2) = 1/4$, for $z = \{k, m, g\}$]:

$$\begin{aligned} h_{K=2}(\boldsymbol{v}) &= b_0 + b_1 v_1^{1/2} + b_2 v_2^{1/2} + b_3 v_3^{1/2} + b_4 v_1^{1/4} + b_5 v_2^{1/4} + b_6 v_3^{1/4} + b_7 v_1^{1/2} v_2^{1/2} \\ &+ b_8 v_1^{1/2} v_2^{1/4} + b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} + b_{11} v_1^{1/2} v_3^{1/2} + b_{12} v_1^{1/2} v_3^{1/4} \\ &+ b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} + b_{15} v_2^{1/2} v_3^{1/2} + b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} \\ &+ b_{18} v_2^{1/4} v_3^{1/4} + b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} + b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} \\ &+ b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} \\ &+ b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}. \end{aligned} \qquad \textbf{(9)}$$

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Applying the modified version of Roy's identity, (2), to (9), we obtain the AIM(2) demand system,

$$\begin{aligned} s_1 = \big(&2b_1 v_1^{1/2} + b_4 v_1^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + 2b_8 v_1^{1/2} v_2^{1/4} + b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} \\ &+ 2b_{11} v_1^{1/2} v_3^{1/2} + 2b_{12} v_1^{1/2} v_3^{1/4} + b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\ &+ b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 2b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + 2b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\ &+ b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}\big)/D \end{aligned} \qquad (\mathbf{10})$$

$$\begin{aligned} s_2 = \big(&2b_2 v_2^{1/2} + b_5 v_2^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + b_8 v_1^{1/2} v_2^{1/4} + 2b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} \\ &+ 2b_{15} v_2^{1/2} v_3^{1/2} + 2b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\ &+ 2b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 2b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\ &+ 2b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}\big)/D \end{aligned} \qquad (\mathbf{11})$$

$$\begin{aligned} s_3 = \big(&2b_3 v_3^{1/2} + b_6 v_4^{1/4} + 2b_{11} v_1^{1/2} v_3^{1/2} + b_{12} v_1^{1/2} v_3^{1/4} + 2b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_2^{1/4} \\ &+ 2b_{15} v_1^{1/2} v_3^{1/2} + b_{16} v_1^{1/2} v_3^{1/4} + 2b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\ &+ 2b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\ &+ b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + 2b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}\big)/D, \end{aligned} \qquad (\mathbf{12})$$

where now D is the sum of the numerators in equations (11), (12), and (12).

3.3. The AIM(3) Model

For $K = 3$ and $\lambda(z) = 2^{-z}$ for $z = \{k, m, g\}$, equation (1) becomes, after reparameterization by stacking the coefficients as they appear in (1) for $K = 3$ into a single vector of parameters $\boldsymbol{b} = (b_0, \cdots, b_{63})'$ containing the 64 coefficients in (1) for $K = 3$ and $n = 3$,

$$\begin{aligned} &h_{K=3}(\boldsymbol{v}) \\ &= b_0 + b_1 v_1^{1/2} + b_2 v_2^{1/2} + b_3 v_3^{1/2} + b_4 v_1^{1/4} + b_5 v_2^{1/4} + b_6 v_3^{1/4} + b_7 v_1^{1/8} + b_8 v_2^{1/8} + b_9 v_3^{1/8} \\ &\quad + b_{10} v_1^{1/2} v_2^{1/2} + b_{11} v_1^{1/2} v_2^{1/4} + b_{12} v_1^{1/2} v_2^{1/8} + b_{13} v_1^{1/2} v_3^{1/2} + b_{14} v_1^{1/2} v_3^{1/4} + b_{15} v_1^{1/2} v_3^{1/8} \\ &\quad + b_{16} v_1^{1/4} v_3^{1/2} + b_{17} v_1^{1/4} v_2^{1/4} + b_{18} v_1^{1/4} v_2^{1/8} + b_{19} v_1^{1/4} v_3^{1/2} + b_{20} v_1^{1/4} v_3^{1/4} + b_{21} v_1^{1/4} v_3^{1/8} \\ &\quad + b_{22} v_1^{1/8} v_2^{1/2} + b_{23} v_1^{1/8} v_2^{1/4} + b_{24} v_1^{1/8} v_2^{1/8} + b_{25} v_1^{1/8} v_3^{1/2} + b_{26} v_1^{1/8} v_3^{1/4} + b_{27} v_1^{1/8} v_3^{1/8} \\ &\quad + b_{28} v_2^{1/2} v_3^{1/2} + b_{29} v_2^{1/2} v_3^{1/4} + b_{30} v_2^{1/2} v_3^{1/8} + b_{31} v_2^{1/4} v_3^{1/2} + b_{32} v_2^{1/4} v_3^{1/4} + b_{33} v_2^{1/4} v_3^{1/8} \\ &\quad + b_{34} v_2^{1/8} v_3^{1/2} + b_{35} v_2^{1/8} v_3^{1/4} + b_{36} v_2^{1/8} v_3^{1/8} + b_{37} v_1^{1/2} v_2^{1/2} v_3^{1/2} + b_{38} v_1^{1/2} v_2^{1/4} v_3^{1/2} \\ &\quad + b_{39} v_1^{1/2} v_2^{1/8} v_3^{1/2} + b_{40} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{41} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{42} v_1^{1/2} v_2^{1/8} v_3^{1/4} + b_{43} v_1^{1/2} v_2^{1/2} v_3^{1/8} \\ &\quad + b_{44} v_1^{1/2} v_2^{1/4} v_3^{1/8} + b_{45} v_1^{1/2} v_2^{1/8} v_3^{1/8} + b_{46} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{47} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{48} v_1^{1/4} v_2^{1/8} v_3^{1/2} \\ &\quad + b_{49} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{50} v_1^{1/4} v_2^{1/4} v_3^{1/4} + b_{51} v_1^{1/4} v_2^{1/8} v_3^{1/4} + b_{52} v_1^{1/4} v_2^{1/2} v_3^{1/8} + b_{53} v_1^{1/4} v_2^{1/4} v_3^{1/8} \end{aligned}$$

$$+ b_{54} v_1^{1/4} v_2^{1/8} v_3^{1/8} + b_{55} v_1^{1/8} v_2^{1/2} v_3^{1/2} + b_{56} v_1^{1/8} v_2^{1/4} v_3^{1/2} + b_{57} v_1^{1/8} v_2^{1/8} v_3^{1/2} + b_{58} v_1^{1/8} v_2^{1/2} v_3^{1/4}$$
$$+ b_{59} v_1^{1/8} v_2^{1/4} v_3^{1/4} + b_{60} v_1^{1/8} v_2^{1/8} v_3^{1/4} + b_{61} v_1^{1/8} v_2^{1/2} v_3^{1/8} + b_{62} v_1^{1/8} v_2^{1/4} v_3^{1/8}$$
$$+ b_{63} v_1^{1/8} v_2^{1/8} v_3^{1/8}. \quad (\mathbf{13})$$

Applying the modified Roy's identity to (13) yields the AIM(3) demand system

$$s_1$$
$$= \big(4b_1 v_1^{1/2} + 2b_4 v_1^{1/4} + b_7 v_1^{1/8} + 4b_{10} v_1^{1/2} v_2^{1/2} + 4b_{11} v_1^{1/2} v_2^{1/4} + 4b_{12} v_1^{1/2} v_2^{1/8}$$
$$+ 2b_{13} v_1^{1/4} v_2^{1/2} + 2b_{14} v_1^{1/4} v_2^{1/4} + 2b_{15} v_1^{1/4} v_2^{1/8} + 4b_{16} v_1^{1/2} v_3^{1/2} + 4b_{17} v_1^{1/2} v_3^{1/4}$$
$$+ 4b_{18} v_1^{1/2} v_3^{1/8} + 2b_{19} v_1^{1/4} v_3^{1/2} + 2b_{20} v_1^{1/4} v_3^{1/4} + 2b_{21} v_1^{1/4} v_3^{1/8} + b_{22} v_1^{1/8} v_2^{1/2}$$
$$+ b_{23} v_1^{1/8} v_2^{1/4} + b_{24} v_1^{1/8} v_2^{1/8} + b_{25} v_1^{1/8} v_3^{1/2} + b_{26} v_1^{1/8} v_3^{1/4} + b_{27} v_1^{1/8} v_3^{1/8}$$
$$+ 4b_{37} v_1^{1/2} v_2^{1/2} v_3^{1/2} + 4b_{38} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 4b_{39} v_1^{1/2} v_2^{1/8} v_3^{1/2} + 4b_{40} v_1^{1/2} v_2^{1/2} v_3^{1/4}$$
$$+ 4b_{41} v_1^{1/2} v_2^{1/4} v_3^{1/4} + 4b_{42} v_1^{1/2} v_2^{1/8} v_3^{1/4} + 4b_{43} v_1^{1/2} v_2^{1/2} v_3^{1/8} + 4b_{44} v_1^{1/2} v_2^{1/4} v_3^{1/8}$$
$$+ 4b_{45} v_1^{1/2} v_2^{1/8} v_3^{1/8} + 2b_{46} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{47} v_1^{1/4} v_2^{1/4} v_3^{1/2} + 2b_{48} v_1^{1/4} v_2^{1/8} v_3^{1/2}$$
$$+ 2b_{49} v_1^{1/4} v_2^{1/2} v_3^{1/4} + 2b_{50} v_1^{1/4} v_2^{1/4} v_3^{1/4} + 2b_{51} v_1^{1/4} v_2^{1/8} v_3^{1/4} + 2b_{52} v_1^{1/4} v_2^{1/2} v_3^{1/8}$$
$$+ 2b_{53} v_1^{1/4} v_2^{1/4} v_3^{1/8} + 2b_{54} v_1^{1/4} v_2^{1/8} v_3^{1/8} + b_{55} v_1^{1/8} v_2^{1/2} v_3^{1/2} + b_{56} v_1^{1/8} v_2^{1/4} v_3^{1/2}$$
$$+ b_{57} v_1^{1/8} v_2^{1/8} v_3^{1/2} + b_{58} v_1^{1/8} v_2^{1/2} v_3^{1/4} + b_{59} v_1^{1/8} v_2^{1/4} v_3^{1/4} + b_{60} v_1^{1/8} v_2^{1/8} v_3^{1/4}$$
$$+ b_{61} v_1^{1/8} v_2^{1/2} v_3^{1/8} + b_{62} v_1^{1/8} v_2^{1/4} v_3^{1/8} + b_{63} v_1^{1/8} v_2^{1/8} v_3^{1/8}\big)/D \quad (\mathbf{14})$$

$$s_2$$
$$= \big(4b_2 v_2^{1/2} + 2b_5 v_2^{1/4} + b_8 v_2^{1/8} + 4b_{10} v_1^{1/2} v_2^{1/2} + 2b_{11} v_1^{1/2} v_2^{1/4} + b_{12} v_1^{1/2} v_2^{1/8}$$
$$+ 4b_{13} v_1^{1/4} v_2^{1/2} + 2b_{14} v_1^{1/4} v_2^{1/4} + b_{15} v_1^{1/4} v_2^{1/8} + 4b_{22} v_1^{1/8} v_2^{1/2} + 2b_{23} v_1^{1/8} v_2^{1/4}$$
$$+ b_{24} v_1^{1/8} v_2^{1/8} + 4b_{28} v_2^{1/2} v_3^{1/2} + 4b_{29} v_2^{1/2} v_3^{1/4} + 4b_{30} v_2^{1/2} v_3^{1/8} + 2b_{31} v_2^{1/4} v_3^{1/2}$$
$$+ 2b_{32} v_2^{1/4} v_3^{1/4} + 2b_{33} v_2^{1/4} v_3^{1/8} + b_{34} v_2^{1/8} v_3^{1/2} + b_{35} v_2^{1/8} v_3^{1/4} + b_{36} v_2^{1/8} v_3^{1/8}$$
$$+ 4b_{37} v_1^{1/2} v_2^{1/2} v_3^{1/2} + 2b_{38} v_1^{1/2} v_2^{1/4} v_3^{1/2} + b_{39} v_1^{1/2} v_2^{1/8} v_3^{1/2} + 4b_{40} v_1^{1/2} v_2^{1/2} v_3^{1/4}$$
$$+ 2b_{41} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{42} v_1^{1/2} v_2^{1/8} v_3^{1/4} + 4b_{43} v_1^{1/2} v_2^{1/2} v_3^{1/8} + 2b_{44} v_1^{1/2} v_2^{1/4} v_3^{1/8}$$
$$+ b_{45} v_1^{1/2} v_2^{1/8} v_3^{1/8} + 4b_{46} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{47} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{48} v_1^{1/4} v_2^{1/8} v_3^{1/2}$$
$$+ 4b_{49} v_1^{1/4} v_2^{1/2} v_3^{1/4} + 2b_{50} v_1^{1/4} v_2^{1/4} v_3^{1/4} + b_{51} v_1^{1/4} v_2^{1/8} v_3^{1/4} + 4b_{52} v_1^{1/4} v_2^{1/2} v_3^{1/8}$$
$$+ 2b_{53} v_1^{1/4} v_2^{1/4} v_3^{1/8} + b_{54} v_1^{1/4} v_2^{1/8} v_3^{1/8} + 4b_{55} v_1^{1/8} v_2^{1/2} v_3^{1/2} + 2b_{56} v_1^{1/8} v_2^{1/4} v_3^{1/2}$$
$$+ b_{57} v_1^{1/8} v_2^{1/8} v_3^{1/2} + 4b_{58} v_1^{1/8} v_2^{1/2} v_3^{1/4} + 2b_{59} v_1^{1/8} v_2^{1/4} v_3^{1/4} + b_{60} v_1^{1/8} v_2^{1/8} v_3^{1/4}$$
$$+ 4b_{61} v_1^{1/8} v_2^{1/2} v_3^{1/8} + 2b_{62} v_1^{1/8} v_2^{1/4} v_3^{1/8} + b_{63} v_1^{1/8} v_2^{1/8} v_3^{1/8}\big)/D \quad (\mathbf{15})$$

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$$
\begin{aligned}
s_3 \\
&= \big(4b_3 v_3^{1/2} + 2b_6 v_3^{1/4} + b_9 v_3^{1/8} + 4b_{16} v_1^{1/2} v_3^{1/2} + 2b_{17} v_1^{1/2} v_3^{1/4} + b_{18} v_1^{1/2} v_3^{1/8} \\
&\quad + 4b_{19} v_1^{1/4} v_3^{1/2} + 2b_{20} v_1^{1/4} v_3^{1/4} + 4b_{25} v_1^{1/8} v_3^{1/2} + 2b_{26} v_1^{1/8} v_3^{1/4} + b_{27} v_1^{1/8} v_3^{1/8} \\
&\quad + 4b_{28} v_2^{1/2} v_3^{1/2} + 2b_{29} v_2^{1/2} v_3^{1/4} + b_{30} v_2^{1/2} v_3^{1/8} + 4b_{31} v_2^{1/4} v_3^{1/2} + 2b_{32} v_2^{1/4} v_3^{1/4} \\
&\quad + b_{33} v_2^{1/4} v_3^{1/8} + 4b_{34} v_2^{1/8} v_3^{1/2} + 2b_{35} v_2^{1/8} v_3^{1/4} + b_{36} v_2^{1/8} v_3^{1/8} + 4b_{37} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\
&\quad + 4b_{38} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 4b_{39} v_1^{1/2} v_2^{1/8} v_3^{1/2} + 2b_{40} v_1^{1/2} v_2^{1/2} v_3^{1/4} + 2b_{41} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\
&\quad + 4b_{42} v_1^{1/2} v_2^{1/8} v_3^{1/4} + b_{43} v_1^{1/2} v_2^{1/2} v_3^{1/8} + b_{44} v_1^{1/2} v_2^{1/4} v_3^{1/8} + b_{45} v_1^{1/2} v_2^{1/8} v_3^{1/8} \\
&\quad + 4b_{46} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 4b_{47} v_1^{1/4} v_2^{1/4} v_3^{1/2} + 4b_{48} v_1^{1/4} v_2^{1/8} v_3^{1/2} + 2b_{49} v_1^{1/4} v_2^{1/2} v_3^{1/4} \\
&\quad + 2b_{50} v_1^{1/4} v_2^{1/4} v_3^{1/4} + 2b_{51} v_1^{1/4} v_2^{1/8} v_3^{1/4} + b_{52} v_1^{1/4} v_2^{1/2} v_3^{1/8} + b_{53} v_1^{1/4} v_2^{1/4} v_3^{1/8} \\
&\quad + b_{54} v_1^{1/4} v_2^{1/8} v_3^{1/8} + 4b_{55} v_1^{1/8} v_2^{1/2} v_3^{1/2} + 4b_{56} v_1^{1/8} v_2^{1/4} v_3^{1/2} + 4b_{57} v_1^{1/8} v_2^{1/8} v_3^{1/2} \\
&\quad + 2b_{58} v_1^{1/8} v_2^{1/2} v_3^{1/4} + 2b_{59} v_1^{1/8} v_2^{1/4} v_3^{1/4} + 2b_{60} v_1^{1/8} v_2^{1/8} v_3^{1/4} \\
&\quad + 2b_{61} v_1^{1/8} v_2^{1/2} v_3^{1/8} + b_{62} v_1^{1/8} v_2^{1/4} v_3^{1/8} + b_{63} v_1^{1/8} v_2^{1/8} v_3^{1/8}\big)/D, \qquad \textbf{(16)}
\end{aligned}
$$

where now D is the sum of the numerators in equations (14)–(16).

4. COMPUTATIONAL CONSIDERATIONS AND ESTIMATION

Demand systems (5)–(7), (10)–(12), and (14)–(16) can be written as

$$
s_t = \psi(\boldsymbol{v}_t, \boldsymbol{\theta}) + \epsilon_t \qquad \textbf{(17)}
$$

with an error term appended. In (17), $\boldsymbol{s} = (s_1, \cdots, s_n)'$, $\boldsymbol{\psi}(\boldsymbol{v}, \boldsymbol{\theta}) = [\psi_1(\boldsymbol{v}, \boldsymbol{\theta}), \cdots, \psi_n(\boldsymbol{v}, \boldsymbol{\theta})]'$, and $\psi_i(\boldsymbol{v}, \boldsymbol{\theta})$ is given by the right-hand side of each of (5)–(7), (10)–(12), and (14)–(16).

As Gallant and Golub (1984, p. 298) put it,

> all statistical estimation procedures that are commonly used in econometric research can be formulated as an optimization problem of the following type [Burguete, Gallant and Souza (1982)]:
>
> $$\widehat{\boldsymbol{\theta}} \text{ minimizes } \varphi(\boldsymbol{\theta}) \text{ over } \Theta$$
>
> with $\varphi(\boldsymbol{\theta})$ twice continuously differentiable in $\boldsymbol{\theta}$.

We follow Gallant and Golub (1984) and use Zellner's (1962) seemingly unrelated regression method to estimate $\boldsymbol{\theta}$. Hence, $\varphi(\boldsymbol{\theta})$ has the form

$$
\varphi(\boldsymbol{\theta}) = \frac{1}{T}\epsilon_t'\epsilon_t = \frac{1}{T}\sum_{t=1}^{T}[\boldsymbol{s}_t - \boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\theta})]'\,\widehat{\Sigma}^{-1}\,[\boldsymbol{s}_t - \boldsymbol{\psi}(\boldsymbol{v}_t, \boldsymbol{\theta})], \qquad \textbf{(18)}
$$

TABLE 1. AIM(1), AIM(2), and AIM(3) parameter estimates[a]

AIM(1)		AIM(2)		AIM(3)					
b_1	1.220	b_1	−6.926	b_1	23.639	b_{27}	13.481	b_{52}	−37.896
b_2	4.631	b_2	−1.935	b_2	−15.531	b_{28}	37.097	b_{53}	−24.348
b_4	9.783	b_4	−2.977	b_4	37.152	b_{29}	15.006	b_{54}	−32.874
b_5	31.940	b_5	−14.185	b_5	25.708	b_{30}	27.463	b_{55}	−32.210
b_6	28.92	b_6	−4.432	b_6	28.984	b_{31}	10.649	b_{56}	−23.317
b_7	−25.146	b_7	−3.326	b_7	1.909	b_{32}	−15.161	b_{57}	−32.610
		b_8	−11.115	b_8	−26.289	b_{33}	18.378	b_{58}	−13.859
		b_9	14.818	b_9	−32.888	b_{34}	32.645	b_{59}	10.234
		b_{10}	2.416	b_{10}	29.205	b_{35}	−31.968	b_{60}	−29.066
		b_{11}	−12.377	b_{11}	35.927	b_{36}	−9.359	b_{61}	−19.955
		b_{12}	12.813	b_{12}	29.789	b_{37}	−33.307	b_{62}	−33.444
		b_{13}	−10.896	b_{13}	14.989	b_{38}	19.922	b_{63}	−32.505
		b_{14}	4.425	b_{14}	−7.987	b_{39}	38.002		
		b_{15}	3.568	b_{15}	2.704	b_{40}	−18.815		
		b_{16}	−13.164	b_{16}	18.594	b_{41}	−0.300		
		b_{17}	−4.136	b_{17}	−32.112	b_{42}	21.614		
		b_{18}	5.035	b_{18}	−1.365	b_{43}	12.656		
		b_{19}	−7.425	b_{19}	27.250	b_{44}	11.074		
		b_{20}	2.626	b_{20}	12.357	b_{45}	29.289		
		b_{21}	13.912	b_{21}	27.581	b_{46}	−27.686		
		b_{22}	−0.721	b_{22}	36.119	b_{47}	−24.265		
		b_{23}	1.980	b_{23}	36.892	b_{48}	−14.171		
		b_{24}	5.727	b_{24}	35.297	b_{49}	37.909		
		b_{25}	−7.050	b_{25}	−33.044	b_{50}	36.103		
		b_{26}	5.627	b_{26}	16.543	b_{51}	29.294		
Value of the objective function:									
0.216		0.236		0.225					

[a] Sample period, quarterly data 1970:1–2003:2. The normalization $b_3 = 1 - b_1 - b_2$ is used.

where T is the number of observations and $\widehat{\Sigma}$ is an estimate of the variance–covariance matrix of (17).

In minimizing (18), as in Serletis and Shahmoradi (2005), we use the TOMLAB/NPSOL tool box with MATLAB. We also follow Serletis and Shahmoradi (2005) and impose the curvature restriction globally, using methods suggested by Gallant and Golub (1984)—see Serletis and Shahmoradi (2005) for details regarding the method for imposing the curvature restriction.

Using NPSOL, we perform the computations and report the parameter estimates in Table 1 for each of the three models, together with the minimized value of the objective for each model, in the last row of the table. It is to be noted that the imposition of curvature globally does not produce spurious violations of monotonicity, mentioned by Barnett and Pasupathy (2003), thereby ensuring true theoretical

TABLE 2. Income elasticities at the mean[a]

Asset	AIM(1)	AIM(2)	AIM(3)
A	1.112	0.988	0.905
B	1.208	1.821	1.838
C	0.620	0.115	0.286

[a] Sample period, quarterly data 1970:1–2003:2.

regularity. Hence, in what follows we discuss the income and price elasticities as well as the elasticities of substitution based on the three models that (with our data set) satisfy both the neoclassical monotonicity and curvature conditions.

5. ELASTICITIES

We present the income elasticities in Table 2, evaluated at the mean of the data, for the three subaggregates (and the three models)—all elasticities in this paper have been acquired using numerical differentiation and the formulas presented in Serletis and Shahmoradi (2005). The elasticities η_{Am}, η_{Bm}, and η_{Cm} are all positive (suggesting that assets A, B, and C are all normal goods), which is consistent with economic theory. However, there are differences between the models, with the AIM(1) model shown to be inconsistent with the other two models.

In Table 3 we show the uncompensated (Cournot) own- and cross-price elasticities, evaluated at the mean of the data, for the three models and the three assets. The own-price elasticities are all negative, as predicted by the theory. For the cross-price elasticities, economic theory does not predict any signs, but we note that most of the off-diagonal terms are negative, indicating that the assets, taken as a whole, are gross complements. This is a frequent finding in this literature. It is also apparent that the AIM(1) price elasticities are typically different from those of the AIM(2) and AIM(3) models.

TABLE 3. Price elasticities at the mean[a]

Asset	Model	η_{iA}	η_{iB}	η_{iC}
	AIM(1)	−0.616	−0.367	−0.128
A	AIM(2)	−0.551	−0.225	−0.211
	AIM(3)	−0.502	−0.252	−0.150
	AIM(1)	−0.558	−0.555	−0.086
B	AIM(2)	−0.750	−0.751	−0.322
	AIM(3)	−0.756	−0.758	−0.323
	AIM(1)	0.015	0.081	−0.716
C	AIM(2)	0.025	0.130	−0.270
	AIM(3)	0.039	0.123	−0.453

[a] Sample period, quarterly data 1970:1–2003:2.

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TABLE 4. Allen elasticities at the mean[a]

Asset	Model	σ_{iA}	σ_{iB}	σ_{iC}
	AIM(1)	−0.351	−0.125	0.656
A	AIM(2)	−0.212	0.190	0.170
	AIM(3)	−0.290	0.039	0.383
	AIM(1)		−0.670	0.894
B	AIM(2)		−0.833	0.575
	AIM(3)		−0.764	0.714
	AIM(1)			−1.920
C	AIM(2)			−0.934
	AIM(3)			−1.282

[a] Sample period, quarterly data 1970:1–2003:2.

From the point of view of monetary policy, the measurement of the elasticities of substitution among the three monetary assets is of prime importance, and there are currently two methods for calculating the partial elasticity of substitution between two variables, the Allen and the Morishima—see Serletis and Shahmoradi (2005) for a discussion and the formulas that we use in this paper. In Table 4 we show estimates of the Allen elasticities of substitution, evaluated at the means of the data. We expect the nine diagonal terms, representing the own-elasticities of substitution for the three assets and the three models, to be negative. This expectation is clearly achieved. However, because the Allen elasticity of substitution produces ambiguous results off-diagonal, we will use the Morishima elasticity of substitution to investigate the substitutability/complementarity relation between assets. Based on the Morishima elasticities of substitution, the assets are all Morishima substitutes, as documented in Table 5. Moreover, all Morishima elasticities of substitution are less than unity, irrespective of the model used. This clearly indicates difficulties for a simple sum–based monetary policy, consistent with the conclusions reached by Serletis and Shahmoradi (2005).

TABLE 5. Morishima elasticities at the mean[a]

Asset	Model	σ^m_{iA}	σ^m_{iB}	σ^m_{iC}
	AIM(1)		0.094	0.424
A	AIM(2)		0.185	0.176
	AIM(3)		0.138	0.283
	AIM(1)	0.161		0.836
B	AIM(2)	0.289		0.427
	AIM(3)	0.234		0.582
	AIM(1)	0.727	0.441	
C	AIM(2)	0.285	0.363	
	AIM(3)	0.480	0.426	

[a] Sample period, quarterly data 1970:1–2003:2.

6. CONCLUSIONS

As Barnett, Geweke, and Wolf (1991) show in the context of production functions, the AIM(1) model is exactly Diewert's (1971) generalized Leontief model. Although the demand version of the AIM(1) model is a little different from its production version, it is obvious that the AIM(1) with only 8 structural parameters does not exhibit anything like the same cyclical sensitivity as the AIM(2) with 27 structural parameters and the AIM(3) model with 64 parameters.

As the AIM model belongs to the class of seminonparametric models, there is no unique rule for selecting the optimal order of approximation. In this regard, Barnett, Geweke and Yue (1991) mention that the most systematic approaches currently available are those of Eastwood and Gallant (1991), who show that Fourier functions produce consistent and asymptotically normal parameter estimates when the number of parameters to be estimated equals the number of effective observations raised to the power of 2/3—this result follows from Huber (1981) and is similar to optimal bandwidth results in many nonparametric models. In our case, with 3 assets and 134 observations, we should estimate (approximately) 42 parameters.

If we follow the Eastwood and Gallant (1987) approach to selecting the order of approximation in the AIM model, we should set $K = 3$, in which case we end up with 64 parameters. Although in general it is possible to overfit the data, as Barnett and Yue (1988) argue, overfitting is impossible if one imposes global regularity, as we did in this paper.

We believe that the AIM(3) model estimated subject to global curvature currently provides the best specification for research in semiparametric modeling of consumer demand systems.

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