BOOK REVIEWS

Mark Steiner, ***The Applicability of Mathematics as a Philosophical Problem***. Cambridge, MA and London: Harvard University Press (cloth 1998, paper 2002), viii + 215 pp., $45.00 (cloth), 19.95 (paper).

This book is a comprehensive presentation of Mark Steiner's views on the philosophical problems of the applicability of mathematics, a topic on which he has been extensively publishing for more than a decade. The result is a study of singular importance to students of history and philosophy of mathematics. The book is full of brilliant technical details that make it interesting to historians and philosophers of physics, too. Though the text is full of formulae, they are so profoundly and elegantly explained that the book will surely attract students of theoretical physics as well.

Steiner's book has **two distinct objectives.** The first is to analyze the different ways in which mathematics is applicable to the physical sciences. The second consists in exploring the implications of that applicability for our view of the universe and the place in it of the human mind (2).

In realizing the **first** goal Mark Steiner distinguishes among the *semantic* problems that arise from the use of mathematics in logical deduction; the *metaphysical* problems that arise from the gap between the abstract objects of mathematics and the real world; and the *descriptive* and *epistemological* problems that arise from the use of mathematics to describe nature. The structure of the book clearly reflects this diversity of issues.

In the first chapter, "The Semantic Applicability of Mathematics: Frege's Achievements," the author reveals how Frege completely solved the semantic and metaphysical problems of applicability. The key question confronting Frege was the following: how can the abstract entities of mathematics be relevant to the empirical world? Frege's answer was: they aren't. They are related, not to empirical objects, but to empirical concepts of natural science. It is the empirical concepts that are used to describe the real world. Abstract objects are only used to characterize those descriptions.

And the second chapter, "The Descriptive Applicability of Mathematics," deals with the appropriateness of specific mathematical concepts in describing physical phenomena. The author's main problem consists now in the question: why are the specific concepts and even formalisms of

Philosophy of Science, 70 (July 2003) pp. 628–646. 0031-8248/2003/7003-0010$10.00

mathematics useful in describing empirical reality? Steiner's answer is: the problem must be solved "piecemeal for each concept" (47). To eliminate the mystery of a particular mathematical concept describing a particular physical phenomenon, one should relate the concept to a nonmathematical property. For instance, linearity is applicable to the extent that the Principle of Superposition holds; and the latter is applicable to the extent that "nature operates in a smooth manner"(32). There is no mystery concerning the applicability of linearity. This mathematical property can be reduced to physical properties which nature may exhibit.

In the third chapter, "Mathematics, Analogies, and Discovery in Physics," and in the fourth one bearing the title "Pythagorean Analogies in Physics," Mark Steiner's aim is to disclose how extensively the famous discoveries of contemporary physics exploited mathematical analogies. At the end of the nineteenth century, physics was in crisis. Scientists were attempting to describe the unseen world of the very small, obeying different laws than those governing the macroscopic world. How, then, did scientists arrive at the atomic laws of nature? Steiner's answer is: by mathematical analogy. Of course, not *only* by mathematical analogy and not only the atomic problem. Yet scientists looked for laws bearing a similar mathematical form to the laws they were trying to replace. Often these analogies were "Pythagorean," by which Steiner means that the analogies were inexpressible in any other language but that of mathematics. Mathematics itself thus provided the conceptual basis for making guesses about the laws of the atomic and subatomic world.

In some remarkable cases, even mathematical notation (rather than structures) provided the analogies used in physics. So, the analogies were to the forms of equations, and not to their mathematical meaning. This is a special case of Pythagorean analogies which Steiner calls "formalist" ones. Thus in Chapters 3–4 Steiner's aim is to demonstrate that the strategy physicists pursued to guess at the laws of nature was a Pythagorean one: they used the relations and even the notation of mathematics to frame analogies and formulate guesses according to them. This does not mean that every guess, or even a large number of them, was correct. "What succeeded was the global strategy" (5).

The author specially points out that the book should not create an impression that bold mathematical speculations, rather than scrupulous empirical inquiry, was what formed twentieth century physics. No scientist described in the book could have formulated valuable theories without scrutinizing empirical data and prior modeling. Steiner's point is that the empirical information was brought to bear on new cases through the medium of mathematical classification.

Thus, Steiner's aims in the two chapters consist in analyzing the actual strategies employed by physicists to make those discoveries. He carefully describes and exemplifies six kinds.

(1) Strategies which presume that all the solutions of the equation E are akin. This is not by itself a Pythagorean strategy, because often an equation expresses a physical trait which all its solutions exhibit. However, there are cases—Maxwell (77–79) and Schrödinger (79–82)—in which there is evidence that the solutions of a common equation are not analogous. A physicist who ignores this evidence, and relies instead on the common equation, pursues a Pythagorean strategy. For instance, Maxwell had a mathematical structure which described many different phenomena of electricity and magnetism. The mathematical structure itself defined the analogy between the different phenomena and the analogy suggested the existence of electromagnetic radiation as an experimental phenomenon.

Schrödinger's discovery of wave mechanics also illustrates this strategy. He began with a vector of fixed frequency, based on an analogy to an optical wave, where the frequency is given by the fixed energy. In writing down the wave equation by taking derivatives, Schrödinger completely abstracted away from this intuition, ending with an equation having no parallel in classical optics.

(2) One looks for solutions in nature even where there is reason to doubt their very possibility. One of the most vivid examples: Dirac's equation in relativistic quantum mechanics. The other one: Schwarzchild solution for the equations of General Relativity.

(3) Suppose we have successfully classified a family of objects by a mathematical structure S. Then we project that this structure, or some related mathematical structure T, should classify other families of objects, even if, given present knowledge, S is not reducible to a physical property. This reasoning has been rampant in elementary particle physics, where symmetries have led to some remarkable discoveries (the history of spin).

(4) One formulates equations by analogy to the mathematical form of other equations, even if little or no physical motivation exists for the analogy. One case is Einstein's derivation of the field equations of General Relativity with extensive use of Poisson's equation. Another example of this type of induction (the derivation of an equation from another one, using a Pythagorean mathematical analogy) is the procedure Heisenberg (with Born and Jordan) used to derive matrix mechanics. Another equation produced by this kind of strategy is the Klein-Gordon equation.

(5) A refuted law is used to test new laws: the "old" law is stipulated to be a special or limiting case of any "new" law.

(6) A refuted law—false by definition—is nevertheless used to arrive at new laws.

In the fifth chapter, "Formalisms and Formalist Reasoning in Quantum Mechanics," the author presents the extension of the quantum mechanical formalism to configuration spaces with "deviant" topologies. In quantum mechanics, formalist analogies often take the form of pseudodeductions: formalist reasoning shows that the extension of the formalism to new situations is constrained by the formalism itself. Here Steiner's examples are of the most nontrivial kind, revealing his skill in explaining the most obscure notions of modern physics.

And in the last chapter, "Formalist Reasoning: The Mystery of Quantization," the author describes the attempts by physicists to "guess" the laws of quantum systems using a strategy known as "quantization." The strategy begins by assuming that the system obeys the classical laws—a false assumption, of course. Then the classical description is converted (by syntactic transformation) into what is hoped is a true quantum description of the same system. The examples given include Dirac's quantization of the electromagnetic field and his relativistic equation for the electron, which led to the discovery of the positron. And finally Steiner refers to the program in physics known as "gauge field theories," inaugurated by Yang and Mills.

Efforts to realize the first goal help the author to realize the **second** one and to consider epistemological and ontological problems of mathematics in more detail. Indeed, the stories of quantization support the main thesis of the book. The founders of quantum mechanics spoke of a "correspondence principle" relating classical and quantum mechanics. Yet many of the examples given by the author show that the correspondence principle was deeply anthropocentric due to formalist reasoning. "Hence, the true 'correspondence' was between the human brain and the physical world as a whole. The world, in other words, looks 'user friendly'" (176).

The author not only gathers piles of facts from history of physics to establish his thesis but provides an excellent theoretical explanation as well. Given the nature of contemporary mathematics, a Pythagorean strategy cannot avoid being an "anthropocentric strategy." This is because the concept of mathematics itself is "species-specific." There is no objective criterion for a structure to be mathematics. Today mathematicians have adopted internal criteria to decide whether to study a structure as mathematical. Two of these are "beauty" and "convenience." Yet what we call beautiful is species-specific. The same is true for the second criterion, too. Thus, relying on mathematics in guessing the laws of nature is relying on human standards of beauty and convenience.

All this, enthusiastically concludes Mark Steiner, enables mathematics to become a modern panacea in science; it could provide a bridge between the "two cultures," mitigate the "science wars," and do lots and lots of other good things. May it be so.

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