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Linking very short arcs from large database of asteroid observations

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With the improvements of the observational technology for the new surveys the number of asteroid detections is rapidly increasing. For this reason we must use very efficient methods to compute orbits with these data. We have to identify observations taken in different nights as belonging to the same asteroid. If we do not have an efficient algorithm for that, the unidentified observation database can increase without control, and we risk to detect the same objects multiple times.

For this problem we deal with very short arcs (VSAs) of asteroid observations. A VSA is usually not enough to compute an orbit; however, by linear or quadratic interpolation, we can compute an attributable $\mathcal{A} = (\alpha, \delta, \dot{\alpha}, \dot{\delta})$ at the mean epoch of the observations, that is a vector whose components are the topocentric angular position and velocity of the asteroid (Milani and Gronchi 2010). To define an orbit we only need to know the topocentric radial distance ρ and radial velocity $\dot{\rho}$ of the observed body at that epoch.

The linkage problem consists of computing one or more preliminary orbits by using the information encoded in two attributables. Among the different ways to deal with this problem, we can consider the first integrals of Kepler's motion to derive equations for this purpose (see Taff & Hall 1977). An alternative procedure is the classical orbit determination method by Gauss (1809), which uses three angular positions (α, δ) of the asteroid, typically belonging to three different VSAs. In this case the preliminary orbits are computed by solving a polynomial equation of degree 8.

With modern telescopes the number N of observations per night is very large. This yields a huge number of computations in the identification procedure. Using Gauss' method for the identification problem we have to solve $O(N^3)$ polynomial equations. If instead we use a linkage algorithm, we have to apply it $O(N^2)$ times. Therefore, if we could find a polynomial equation for the linkage problem with low degree, we would significantly decrease the computational complexity of the problem.

With the first integrals of the two-body motion we can indeed write polynomial equations for the linkage. Gronchi *et al.* (2010) used the conservation of angular momentum and energy to write equations of degree 48. Subsequently, Gronchi *et al.* (2011) considered the Laplace-Lenz vector projected along a suitable direction in place of energy, thus reducing to 20 the degree.

We present here a recent achievement with the first integrals approach (see Gronchi *et al.* 2015). By a combination of all the integrals we derive a polynomial equation of degree 9 in the topocentric radial distance of the asteroid at the mean epoch of one of the two attributables.

More precisely, given two attributables $\mathcal{A}_1, \mathcal{A}_2$ at times $\bar{t}_1, \bar{t}_2$, we consider the algebraic system

$$\mathbf{c}_1 = \mathbf{c}_2, \quad \mathbf{L}_1 = \mathbf{L}_2, \quad \mathcal{E}_1 = \mathcal{E}_2, \tag{0.1}$$

where $\mathbf{c}, \mathbf{L}, \mathcal{E}$ are the expressions of the angular momentum, the Laplace-Lenz vector and the energy. The indexes 1,2 refer to the epochs. System (0.1) is composed by 7 equations in the 4 unknowns $(\rho_1, \rho_2, \dot{\rho}_1, \dot{\rho}_2)$. We search for a polynomial system, consequence of (0.1), which leads to a univariate polynomial equation with low degree.

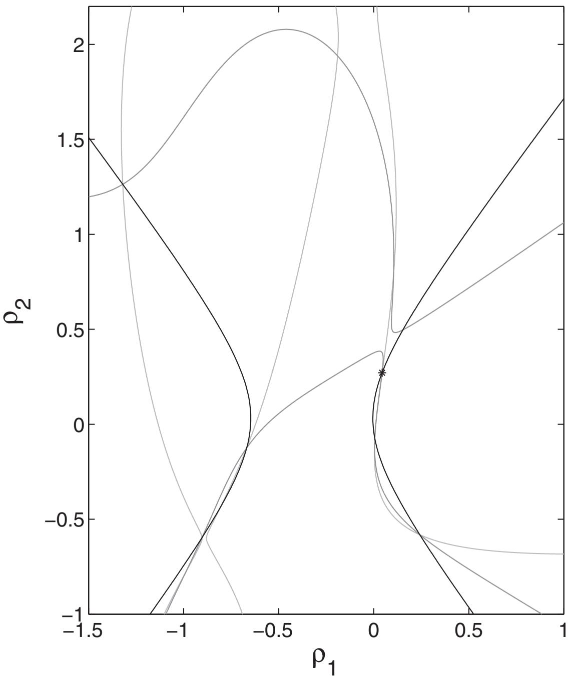

Figure 1. Intersection of the curves defined by $q = 0$ (black), $p_1 = 0$ (light grey) and $p_2 = 0$ (dark grey). The point $(\rho_1, \rho_2) = (0.0438, 0.2713)$, marked by an asterisk, represents the right solution.

After elimination of $\dot{\rho}_1$, $\dot{\rho}_2$ and some algebraic manipulations we are left with a system of 3 bivariate polynomials $q = p_1 = p_2 = 0$, which can be reduced to a system of two univariate polynomials of degree 10, whose greatest common divisor has degree 9.

As a test case we consider the asteroid (101955) Bennu. We link 5 observations made on September 11, 1999 together with 11 observations made on March 30, 2000. In Figure 1 we show the curves $q = p_1 = p_2 = 0$ and the computed solution, after discarding non-real and non-positive solutions.

We conclude by comparing the new method with the methods presented in Gronchi *et al.* (2010, 2011) in terms of computational time. These algorithms were run 1000 times for a test case with asteroid (99942) Apophis, using observations made in June and December 2004. We found that the new algorithm, with a polynomial of degree 9, improves the performance by one and two orders of magnitude with respect to the other methods, with polynomials of degree 20 and 48.

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