Macroeconomic Dynamics, **24**, 2020, 747–773. Printed in the United States of America.
doi:10.1017/S1365100518000470

A STICKY–DISPERSED INFORMATION PHILLIPS CURVE: A MODEL WITH PARTIAL AND DELAYED INFORMATION

Marta B. M. Areosa and Waldyr D. Areosa
Banco Central do Brasil
PUC-Rio

Vinicius Carrasco
PUC-Rio

We study the interaction between dispersed and sticky information by assuming that firms receive private noisy signals about the state in an otherwise standard model of price setting with sticky information. We compute the unique equilibrium of the game induced by the firms' pricing decisions and derive the resulting Phillips curve. The main effect of dispersion is to magnify the immediate impact of a given shock when the degree of stickiness is small. Its effect on persistence is minor: even when information is largely dispersed, a substantial amount of informational stickiness is needed to generate persistence in aggregate prices and inflation.

Keywords: Sticky Information, Dispersed Information, Phillips Curve

1. INTRODUCTION

According to Mankiw and Reis (2002), the sticky information model achieves two important goals at once: (i) it explains why prices fail to respond quickly to nominal shocks and (ii) it reconciles the backward-looking behavior needed to generate the observed persistence in aggregate prices with the assumption that agents are fully rational. One key feature they do not take into account in their paper is the fact that, in the real world, information is differential: agents get different pieces of (relevant) information when they update their information sets. What are the consequences of having firms setting prices when information is sticky and dispersed? In this paper, we build a model in which firms update their

This paper is based on the first chapter of Marta Areosa's PhD thesis (2010). We would like to thank Márcia Leon, Leonardo Rezende, Luciano Vereda, and Fábio Araújo. We are also grateful for comments from many seminar participants at PUC-Rio and Banco Central do Brasil. The financial support from Banco Central do Brasil is gratefully acknowledged. We are responsible for all errors and interpretations. The views expressed in the paper are those of the authors and do not necessarily reflect those of the Banco Central do Brasil. Address correspondence to: Marta B. M. Areosa, Research Department, Banco Central do Brasil, Av. Presidente Vargas 730, Centro, CEP. 20071-900, Rio de Janeiro, RJ, Brasil; e-mail: marta.areosa@bcb.gov.br

© 2018 Cambridge University Press 1469-8056/18

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

information set infrequently and, when information is updated, receive a noisy signal of the relevant state. Our goal is to analyze the resulting dynamics for aggregate prices and inflation rates.

In the model, firms' optimal price is a convex combination of the current state of the economy and the aggregate price level. Nevertheless, as firms do not observe the current state or other firms' pricing decisions, they have to use the available information to infer the optimal price. As in Mankiw and Reis (2002), only a fraction of firms update their information set in each period. Those that update receive two types of information: the first piece is the value of all previous periods' states, while the second piece is a noisy, idiosyncratic, private signal about the current state of the economy. Since noisy signals are idiosyncratic, the firms that update their information set will have heterogeneous information about the state [as in Morris and Shin (2002) and Angeletos and Pavan (2007)]. Hence, in our model, *heterogeneous* information disseminates *slowly* in the economy. While firms receive private signals and information about past states only infrequently, they also have access to a public information about the evolution of the state. This information is available to all firms, including those that were not selected to update their information set.

Since a firm's optimal price depends on both the current state of the economy and the price chosen by the other firms, it must form beliefs not only about the current state but also about the other firms' beliefs about the current state of the economy, and so on, so that higher-order beliefs play a key role in our model. Hence, the pricing decisions by firms induce an incomplete information game among them. In our main result, we prove that there exists a unique equilibrium of such game. The uniqueness of the equilibrium allows us to unequivocally speak about the sticky–dispersed information (henceforth, SDI) aggregate price level and Phillips curve.

The SDI aggregate price level we derive depends on all the current and past states of the economy and on the public signals. This is so for two reasons. First, there are firms for which the information set has been last updated only in the distant past. This is a direct effect of sticky information. Second, firms that have just received new information will behave, at least partly, as if they were backward-looking. This happens because of a strategic effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices. The influence of public signals on the aggregate price index emerges from these two effects. Public information helps firms to predict not only the unknown states, but also one another's actions.

From aggregate prices, we are able to derive the SDI Phillips curve and show that inflation also depends on all the current and past states of the economy and on the public signals. This dependence on current and past states is linked to the result obtained in Mankiw and Reis (2002), in which inflation depends on past expectations of current economic conditions, due to the fact that firms compute

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

expectations based on outdated information. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002). In our model, however, in addition to being sticky, information is also noisy and dispersed. The fact that information is noisy leads a firm that has just updated its information set to find it optimal to place positive weight on the states from previous periods to predict the current state. Hence, in comparison to the model economy stated in Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with noisy information. Through the complementarities in price setting, the fact that, on top of being noisy, information is dispersed magnifies such effect.

Related literature. There is a vast literature relating informational frictions and the macroeconomy.[1] To the best of our knowledge, this is the first paper to build a dynamic model of price-setting decisions where information is both sticky and dispersed. There are, however, a large number of papers that are connected to what we do. Among those papers, in addition to the ones that were already mentioned, our work follows a large literature that sheds new light into the tradition that dates back to Phelps (1968) and Lucas (1972) of considering the effects of imperfect information on price-setting decisions. Mankiw and Reis (2010) provide the most recent survey on the impact of informational frictions on pricing decisions, comparing a partial (dispersed) information model with a delayed (sticky) information model, and deriving their common implications.[2] In turn, Angeletos and La'O (2009) introduce dispersed information (and explicitly discuss the role of higher-order beliefs) in an otherwise standard setting with sticky prices *à la* Calvo (1983); throughout the paper, we compare how our dispersed information model differs from their sticky price environment. Veldkamp (2011) covers a myriad of topics related to informational asymmetries and information acquisition in macroeconomics and finance. Our paper connects to this broad literature through two strands. In our model, information is sticky [as in, e.g., Mankiw and Reis (2002)] and dispersed [as in, among others, Morris and Shin (2002)]. Also, by focusing on informational stickiness (rather than price stickiness), we complement the analysis of Angeletos and La'O (2009).

Our work is also related to the recent empirical work on belief heterogeneity. Andrade et al. (2016) document a set of facts about disagreement among professional forecasters and propose an extended version of the sticky information framework of Mankiw and Reis (2002) that can jointly explain these facts and compare the results of their sticky information model to a noisy information model, when agents observe a noisy signal of the current state of the economy. Instead of choosing one information imperfection, stickiness versus dispersion, our proposed model embeds the two in a unified framework. On the one hand, following Mankiw and Reis (2002), information spreads slowly through the economy. On the other hand, the information received by the fraction of informed firms is noisy and heterogeneous, which entails fundamental and strategic uncertainty.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

As a result, the SDI model is the natural framework to test between the two competing information frictions.

The paper that is the closest to ours is Baeriswyl and Cornand (2010). They also combine, in a single model, dispersed information and informational stickiness to study the signaling role of policy actions. There is one main difference between their paper and ours: the relevant fundamental in our economy evolves dynamically, whereas they assume that the fundamental is random noise over time. Dynamics substantially change the way agents form their beliefs. Over time, different agents have different pieces of information regarding the fundamental. In fact, since the fundamental evolves according to a process which is time-dependent, any two agents whose information sets were updated at different points in time will have different beliefs regarding the state. As a consequence, at a given point in time, there is heterogeneity in beliefs regarding the fundamental (and aggregate prices) even within the agents who have outdated information sets. Such a cross section of beliefs has nontrivial impacts on both aggregate prices and inflation. Also, by explicitly incorporating dynamics, we are also able to obtain impulse responses of aggregate prices and inflation rates to structural and informational shocks that hit the economy.

Organization. The paper is organized as follows. We describe the setup of the model in Section 2 and derive the unique equilibrium of the pricing game played by the firms in Section 3. In Section 4, we analyze which properties are satisfied by the equilibrium values of the coefficients of our model and evaluate the impulse response functions for inflation and aggregate demand. In both exercises, we compare our results for the sticky and dispersed information model with the ones obtained for the sticky prices and dispersed information model proposed by Angeletos and La'O (2009). Section 6 contains the concluding remarks. All derivations that are not in the text can be found in the Appendix.

2. THE MODEL

The model is a variation of Mankiw and Reis's (2002) sticky information model.[3] There is a continuum of firms, indexed by $z \in [0, 1]$, that set prices at every period $t \in \{1, 2, ...\}$.

Although prices can be reset at no cost at each period, information regarding the state of the economy is made available to the firms infrequently. At period t, only a fraction $(1 - \lambda) \in (0, 1)$ of firms are selected to update their information sets about the current state. For simplicity, the probability of being selected to adjust information sets is the same across firms and independent of history.

We depart from a standard sticky information model by allowing information to be *heterogeneous* and *dispersed*: a firm that updates its information set receives information regarding the past states of the economy as well as a *private* signal about the current state. There is also a *public* signal about the change in the state that is available to all firms.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

Pricing decisions. Under complete information, any given firm $z \in [0, 1]$ sets its (log-linear) price $p_t(z)$ equal to the optimal price decision p_t^* given by

$$p_t^* \equiv rP_t + (1-r)\,\theta_t, \tag{1}$$

where $P_t \equiv \int_0^1 p_t(z)\,dz$ is the aggregate price level, and θ_t is the nominal aggregate demand, the current state of the economy. This pricing rule is standard, and although we don't do it explicitly, it can be derived from a firm's profit maximization problem in a model of monopolistic competition in the spirit of Blanchard and Kiyotaki's (1987).[4]

Information. The state θ_t follows a random walk

$$\theta_t = \theta_{t-1} + \epsilon_t, \tag{2}$$

with $\epsilon_t \sim N(0, \alpha^{-1})$.

If firm $z \in [0, 1]$ is selected to update its information set in period t, it observes all *previous* periods' realizations of the state, $\left\{\theta_{t-j}, j \geq 1\right\}$. Moreover, it obtains a noisy private signal about the current state. Denoting such signal by $x_t(z)$, we follow the literature and assume

$$x_t(z) = \theta_t + \xi_t(z), \tag{3}$$

where $\xi_t(z) \sim N(0, \beta^{-1})$, β is the precision of $x_t(z)$, and the error term $\xi_t(z)$ is independent of ϵ_t for all z, t.

There is also a public signal y_t that is available in all periods to all firms, including those that have not been selected to update their information set,

$$y_t = \theta_t - \theta_{t-1} + \eta_t,$$

where $\eta_t \sim N\left(0, \gamma^{-1}\right)$ is independent of the other shocks and over time. It is clear from equation (2) that y_t is a signal over the exogenous shock ϵ_t. In the same line, once the state θ_{t-1} is observed to those that have updated their information sets at t, $x_t(z) - \theta_{t-1}$ is a private signal over ϵ_t.

As a result, the information set of a firm z_j that was selected to update its information j periods ago is

$$\Im_t\left(z_j\right) = \left\{x_{t-j}\left(z_j\right), \Theta_{t-j-1}, Y_t\right\}. \tag{4}$$

where $\Theta_{t-j} = \{\theta_{t-k}\}_{k=j}^{\infty}$ and $Y_t = \{y_{t-k}\}_{k=0}^{\infty}$ represent the sets of all states previous to $t-j$ and of all public signals.

3. EQUILIBRIUM

Using (1), the best response for a firm z_j that was selected to update its information j periods ago—and, therefore, has $\Im_t(z_j)$ as its information set—is its forecast of p_t^*, given the available information $\Im_t(z_j)$:

$$p_t\left(z_j\right) = E\left[p_t^* \mid \Im_t\left(z_j\right)\right]. \tag{5}$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

Denoting by Λ_{t-j} the set of firms that last updated their information set at period $t-j$, we can express the aggregate price level P_t as[5]

$$\begin{aligned} P_t &= \int_{\cup_{j=0}^{\infty}\Lambda_{t-j}} p_t\left(z_j\right) dz_j \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[p_t^* \mid \Im_t\left(z_j\right)\right] dz_j. \end{aligned} \tag{6}$$

Since the optimal price p_t^* is a convex combination of the state, θ_t, and the aggregate price level, firm z needs to forecast the state of the economy *and* the pricing behavior of the other firms in the economy. The pricing behavior of each of these firms, in turn, depends on its own forecast of the other firms' aggregate behavior. It follows that firm z_j must not only forecast the state of the economy, but also predict the behavior of the other firms in the economy and make forecasts of these firms' forecasts about the state, forecasts about the forecasts of these firms' forecasts about the state, and so on. In other words, higher-order beliefs will play a key role in the derivation of equilibrium in our model.

Indeed, if one defines the average kth-order belief about the current state recursively as follows:

$$\bar{E}^k\left[\theta_t\right] = \begin{cases} \theta_t, & : k = 0, \\ \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[\bar{E}^{k-1}\left[\theta_t\right] \mid \Im_t\left(z_j\right)\right] dz_j, & : k \geq 1, \end{cases} \tag{7}$$

then in equilibrium, the aggregate price level is

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k\left[\theta_t\right]. \tag{8}$$

3.1. Computing the Equilibrium

In this section, we derive the unique equilibrium of the pricing game played by the firms. First, we obtain the higher-order beliefs. Then we compute the aggregate price level in period t as a weighted average of all higher-order beliefs about the state θ_t.[6]

3.1.1. Posterior distribution. The common posterior for a firm z_j that updated its information set in period $t-j$ given public information alone is normal with mean $w_{t-j} \equiv E\left[\theta_{t-j} | \Theta_{t-j-1}, Y_{t-j}\right] = \kappa\left(y_{t-j} + \theta_{t-j-1}\right) + (1-\kappa)\,\theta_{t-j-1}$ and variance $\sigma_z^2 \equiv (\alpha+\gamma)^{-1}$, where $\kappa \equiv \gamma\,(\alpha+\gamma)^{-1}$. Private posteriors, on the other hand, are

$$\theta_{t-j} \mid \Im_t\left(z_j\right) \sim \mathcal{N}\left(\delta x_{t-j}\left(z_j\right) + (1-\delta)\, w_{t-j}, (\alpha+\beta+\gamma)^{-1}\right), \tag{9}$$

where

$$\delta \equiv \frac{\beta}{\alpha+\beta+\gamma} \in (0,1). \tag{10}$$

Hence, a firm z_j that updated its information set in $t-j$ expects the current state to be a convex combination of the private signal $x_{t-j}\left(z_j\right)$ and a (semi-)public signal w_{t-j}—combining the public information y_{t-j} with the only relevant piece of

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

information that comes from learning all previous states $\left\{\theta_{t-j-k}\right\}_{k\geq 1}$.[7] The relative weights given to x_{t-j} (z) and w_{t-j} when the firm computes the expected value of state θ_{t-j} depend on the precision of such signals.

Using (2), one has, for $m \leq j$,

$$E\left[\theta_{t-m} \mid \Im_t\left(z_j\right)\right] = E\left[\theta_{t-j} \mid \Im_t\left(z_j\right)\right] + \sum_{i=m}^{j-1} E\left[\epsilon_{t-i} \mid \Im_t\left(z_j\right)\right]. \qquad (11)$$

As a result, a firm that last updated its information set in $t-j$ must infer about the errors $\{\epsilon_{t-i}\}_{i=m}^{j-1}$ to forecast the fundamental θ_{t-m} in any period after $t-j$. The only signal that reveals information about ϵ_{t-i} is the public signal y_{t-i} because from (2),

$$y_{t-i} = \theta_{t-i} - \theta_{t-i-1} + \eta_{t-i} = \epsilon_{t-i} + \eta_{t-i}.$$

The weight κ captures the importance of ϵ_{t-k} on the signal $y_{t-k} = \epsilon_{t-k} + \eta_{t-k}$.

Thus, the expectation of a firm z_j that last updated its information set at $t-j$ about θ is

$$E\left[\theta_{t-m} \mid \Im_t\left(z_j\right)\right] = \begin{cases} \delta x_{t-j}\left(z_j\right) + (1-\delta)\, w_{t-j} + \kappa \sum_{i=m}^{j-1} y_{t-i} & : m \leq j, \\ \theta_{t-m} & : m > j. \end{cases} \qquad (12)$$

In words, a firm that last updated its information set in period $t-j$ combines the forecast made in that period with the information from the public signals $\{y_{t-i}\}_{i=m}^{j-1}$ to forecast all future values of the fundamental θ. Moreover, since at the moment it adjusts its information set the firm observes all previous states, the firm will know for sure the value of θ_{t-m} for $m > j$.

3.1.2. Beliefs. We establish that there is a unique linear equilibrium in the game by computing the aggregate price level in period t as a weighted average of all (average) higher-order beliefs about the state θ_t, as stated in (8). In the Appendix, we use (12) and the recursion (8) to derive the following useful result:

LEMMA 1 (Higher-Order Beliefs). *The average kth-order forecast of the state is given by*

$$\bar{E}^k\left[\theta_t\right] = \sum_{m=0}^{\infty} \lambda^m \left\{(1-\lambda)\left[a_{m,k}\theta_{t-m} + b_{m,k}\theta_{t-m-1}\right] + \kappa c_{m,k} y_{t-m}\right\}, \qquad (13)$$

where the weights $\left(a_{m,k}, b_{m,k}, c_{m,k}\right)$ *are recursively defined, for* $k \geq 1$, *by*

$$\begin{bmatrix} a_{m,k+1} \\ b_{m,k+1} \\ c_{m,k+1} \end{bmatrix} = A_m \begin{bmatrix} a_{m,k} \\ b_{m,k} \\ c_{m,k} \end{bmatrix} + (1-\lambda^m)^k \begin{bmatrix} \delta \\ 1-\delta \\ \rho \end{bmatrix},$$

the initial weights are $\left(a_{m,1}, b_{m,1}, c_{m,1}\right) \equiv (\delta, 1-\delta, \rho)$, $\rho \equiv 1-\delta(1-\lambda)$, *and the matrix* A_m *is given by*

$$A_m = \begin{bmatrix} \delta\left(1-\lambda^{m+1}\right) + (1-\delta)\left(1-\lambda^m\right) & 0 & 0 \\ (1-\delta)\left[\left(1-\lambda^{m+1}\right) - \left(1-\lambda^m\right)\right] & 1-\lambda^{m+1} & 0 \\ (1-\lambda)\,\rho\lambda^m & 0 & 1 \end{bmatrix}. \qquad (14)$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

3.1.3. Equilibrium price level and SDI Phillips curve. We obtain the equilibrium aggregate price level by plugging (13) into expression (8) for P_t and the SDI Phillips curve by taking the first difference.

PROPOSITION 2 (Linear Equilibrium). *The equilibrium aggregate price level in period t, P_t, is linear in the states $\{\theta_{t-j}\}_{j=0}^{\infty}$ and in $\{y_{t-j}\}_{j=0}^{\infty}$, that is,*

$$P_t = \sum_{m=0}^{\infty} g_m \theta_{t-m} + \sum_{m=0}^{\infty} h_m y_{t-m}, \tag{15}$$

where the coefficients are given by

$$g_m \equiv \begin{cases} \frac{(1-r)(1-\rho)}{1-r(1-\rho)} & \text{if } m = 0 \\ \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r(1-\rho\lambda^m)} - \frac{1}{1-r\left(1-\rho\lambda^{m-1}\right)}\right] & \text{if } m \geq 1, \end{cases} \quad \text{and} \tag{16}$$

$$h_m \equiv \kappa \left[\frac{\rho\lambda^m}{1 - r\left(1 - \rho\lambda^m\right)}\right]. \tag{17}$$

In such equilibrium, the SDI Phillips curve is given by

$$\pi_t = \sum_{m=0}^{\infty} g_m \left(\theta_{t-m} - \theta_{t-m-1}\right) + \sum_{m=0}^{\infty} h_m \left(y_{t-m} - y_{t-m-1}\right). \tag{18}$$

The equilibrium aggregate price level P_t and consequently inflation π_t as shown in (18) depend on current and past states of the economy. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the distant past. This is a *direct* effect of sticky information and is linked to the result obtained in Mankiw and Reis (2002), in which inflation depends on past expectations of current economic conditions. In our model, as shown in (8), individual expectations about the current state *are* functions of the past states of the economy. Second, even firms that have just adjusted their information set will be, at least partly, backward-looking. This happens because of a *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices. While prices depend on δ, they reflect a nontrivial interaction between dispersion and stickiness due to strategic complementarity in pricing decisions.

In our model, however, besides being sticky, information is also *dispersed*. The effect of private information is captured by the positive weight given to the state in period θ_{t-m-1} by a firm that has its information set updated in $t-m$. If, instead of having a private signal of θ_{t-m}, firms knew the state, they would ignore the information given by θ_{t-m-1}. But, since the private signal the firm observes is noisy, it is always optimal to place some weight on past states to forecast the current state. Hence, in comparison to an economy *à la* Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with dispersed information.

The expression (18) also shows that public information has long-lasting effects on inflation. This result emerges from the fact that, in contrast with Mankiw and Reis (2002), in our model agents that do not update their information set are partially informed about the state through the public signal. Comparatively

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

to this result, private signals have a relatively mild influence on prices insofar as idiosyncratic shocks die out with aggregation. The only effect that remains from private signals comes from the modification on the strategic interaction that occurs when firms compute the equilibrium. In contrast, shocks that come from public signals last forever.

The model with i.i.d. disturbances. We argue that we are able to obtain impulse responses to various shocks because, in contrast to Baeriswyl and Cornand (2010), we explicitly incorporate dynamics. In order to formalize this statement, consider that, instead of following a random walk as in equation (2), the state θ_t is simply an i.i.d. disturbance:

$$\theta_t = \epsilon_t, \text{ with } \epsilon_t \sim N\left(0, \alpha^{-1}\right).$$

In this case, if firm $z \in [0, 1]$ is selected to update its information set in period t, it continues to observe all previous periods' realizations of the state, $\left\{\theta_{t-j}, j \geq 1\right\}$, but now they are uninformative about θ_t. Moreover, the noisy private signal about the current state is no longer useful to forecast the state in any future period:

$$\begin{aligned} x_t(z) &= \theta_t + \xi_t(z), \quad \text{with } \xi_t(z) \sim N\left(0, \beta^{-1}\right), \\ &= \epsilon_t + \xi_t(z). \end{aligned}$$

If, in order to be more directly comparable to Baeriswyl and Cornand (2010), we ignore the public signal y, we are able to show that the higher-order beliefs, and consequently the price level, become functions of the current state:

$$\begin{aligned} \bar{E}^k[\theta_t] &= \left(\frac{\beta(1-\lambda)}{\beta+\alpha}\right)\theta_t, \\ P_t &= (1-r)\textstyle\sum_{k=1}^{\infty} r^{k-1}\bar{E}^k[\theta_t] \\ &= \frac{\beta(1-r)(1-\lambda)}{\alpha+\beta[1-r(1-\lambda)]}\theta_t. \end{aligned}$$

As a result, the only relevant shock is the current fundamental $\theta_t = \epsilon_t$ itself, which will have only an immediate and temporary effect on the price level.

4. PROPERTIES OF THE MODEL

In this section, we analyze the properties of our model using the work of Angeletos and La'O (2009) as our benchmark. Perhaps the most prevalent form of introducing nominal rigidity in macro models comes from the seminal paper by Calvo (1983), in which, in each period, randomly selected firms set their prices optimally, while the remaining firms keep their prices unchanged. Angeletos and La'O (2009) introduce dispersed information in an otherwise standard Calvo model. In contrast, we introduce dispersed information in a sticky information model *à la* Mankiw and Reis (2002). One might wonder whether there are meaningful differences in considering the effect of dispersed information in those two

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

models and, therefore, whether there are relevant differences in Angeletos and La'O (2009) and our paper.

To compare the models, we ignore public information, which is absent in Angeletos and La'O (2009), and use the fact that the aggregate price level in both models can be written as

$$P_t = \phi_0\theta_t + \phi_1\theta_{t-1} + \sum_{m=2}^{\infty} \phi_m\theta_{t-m},$$

where $\sum_{m=2}^{\infty} \phi_m = 1 - \phi_0 - \phi_1$.[8]

The coefficients $\phi' s$ of the two models are functions of three parameters representing (i) the degree of stickiness, λ, (ii) the degree of strategic complementarity in pricing decisions, r, and (iii) the ratio of the precision of private signals to the precision of the prior, β/α.[9] The main difference is the parameter λ, which represents the degree of Calvo (1983) price rigidity in Angeletos and La'O (2009) while, in our model, represents the degree of Mankiw and Reis (2002) information rigidity.

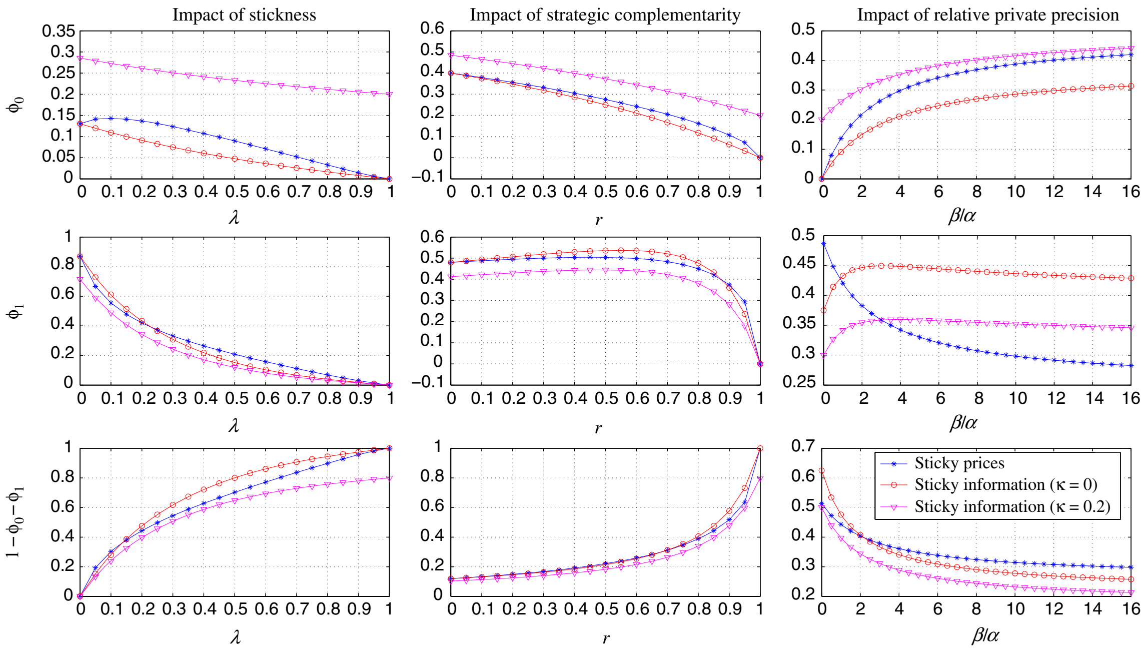
First, we analyze which properties are satisfied by the equilibrium values of the coefficients $\phi' s$ in the two models. Then, we evaluate the impulse response functions of the two models for inflation and aggregate demand.

4.1. Comparative Statics

The comparative statics is illustrated in Figure 1. For ease of comparison, we use the same baseline parameterization of Angeletos and La'O (2009): we identify the length of a period as 1 year, a probability of price/information change equal to $1/3$ per quarter ($\lambda = 0.20$) , and strong complementarity in pricing decisions ($r = 0.85$). These values are consistent with standard calibrations of the Calvo model, but not necessarily with sticky information models. We, however, postpone the analysis of a calibration more in line with Mankiw and Reis (2002) to the evaluation of impulse response functions. We also follow Angeletos and La'O (2009) and set $\beta/\alpha = 1$, meaning that the variance of the forecast error of the typical firm about the current innovation in nominal demand is one half the variance of the innovation itself.

The first column of Figure 1 plots the coefficients ϕ_0, ϕ_1, and $1 - \phi_0 - \phi_1$ as functions of λ, the relevant degree of (price vs. informational) stickiness for each model. For both models, ϕ_1 is decreasing in λ while $1 - \phi_0 - \phi_1$ is increasing in λ. The most relevant difference is in the coefficient attached to the current state: ϕ_0 is decreasing in the degree of information stickiness but non-monotonic in the degree of price stickiness (it increases for low values but decreases for high values). As in the standard sticky price and sticky information models, the aggregate price level becomes more and more persistent as the number of firms that cannot adjust prices or update their information sets increases (higher values of λ). In both models, this pattern is characterized by an increasing weight $1 - \phi_0 - \phi_1$ attached to past fundamentals as λ increases. Clearly, the incorporation of public signals, characterized by $\kappa \neq 0$ (or $\gamma \neq 0$), does not

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press



The first column presents the impact of the relevant rigidity parameter λ (prices or information) on the elasticities of the equilibrium price level of both our model (sticky information), with and without the incorporation of public signals, and Angeletos and La'O (2009) model (sticky prices) with respect to the current nominal shock ϕ_0, the most recent past nominal shock ϕ_1 and all other past nominal shocks $(1 - \phi_0 - \phi_1)$. The second and third columns present analogous impacts of the degree of strategic complementarity in pricing decisions (r) and of the ratio of the precision of private signals to the precision of the prior (β/α).

FIGURE 1. Price-level coefficients as functions of the structural parameters of the model.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

change the shape described for the sticky information model. However, it shifts upward the curve associated with the coefficient ϕ_0. As all coefficients must sum to one, an increase in ϕ_0 is associated with a decline in the other coefficients. The rationale behind the shift of the curve associated with ϕ_0 is easy to understand: when the public signal becomes more precise, firms are better informed about the errors that drive the fundamental in all periods (ϵ_{t-i}, for all $i \geq 0$), meaning that their pricing decision will generate an equilibrium price level that is closer to the one that would be produced if there were complete information. At this limit case (complete information), firms ignore all past states and set $P_t = \theta_t$. Past states are only relevant to firms' pricing decisions because of outdated information. Not only are there firms in the economy for which the information set has been last updated in the far past, but also firms that have just received new information will behave, at least partly, as if they were backward-looking due to a strategic effect. Public information helps firms to predict not only the unknown states, but also one another's actions. This shift of ϕ_0 also appears in the analyses presented for the other two parameters—r and (β/α)—since firms can always use public signals to better infer the fundamental θ_t.

The second column of Figure 1 plots the same coefficients as functions of the degree of strategic complementarity in pricing decisions. Once again, the coefficients of both models present similar dynamics: ϕ_0 is a decreasing function in r, $1 - \phi_0 - \phi_1$ is an increasing function in r, and ϕ_1 is non-monotonic in r, first increasing and then decreasing in r. The pattern here is also similar to the one observed in a model without dispersed information. Under sticky prices, after a monetary shock, firms that can adjust their price following a monetary shock will find it optimal to stay closer to the past price level the higher the degree of strategic complementarity between them and the firms that cannot adjust (and are thus stuck to the past price level). Under sticky information, firms that updated their information sets will find it optimal to attach more weight to the information that is available for most of the firms that cannot update. As a result, both models present an increasing weight $1 - \phi_0 - \phi_1$ attached to past fundamentals as r increases.

Finally, the third column of Figure 1 plots these coefficients as functions of β/α, the ratio of the precision of private signals to the precision of the prior. Clearly, ϕ_0 is increasing in this ratio, while $1 - \phi_0 - \phi_1$ is decreasing for both models. The major difference between the two models is in the coefficient ϕ_1, which decreases under sticky prices but is non-monotone under sticky information, first increasing and after that slowly decreasing. Consider the coefficients ϕ_0 and ϕ_1, which characterize, respectively, the price impact of the current and the past shocks. To understand how the precision of information affects these coefficients, first consider the choice of the price-setting firm. Under sticky prices, each firm chooses a price that is a linear combination of past prices and past nominal shocks (which are common knowledge among all firms) and the firm's own expectation of current nominal demand (which is unknown in the current period). Aggregating across firms gives the aggregate price level as a linear combination

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

of past price levels and past nominal shocks and of the average expectation of θ_t. As in any static incomplete information model with Gaussian signals, the firm's own expectation of the fundamental is merely a weighted combination of its private signal and the common prior, which here coincides with θ_{t-1}. If firms have less precise private information relative to the prior, that is, lower β/α, they place less weight on their private signals than on their prior when forming their expectations of θ_t. As a result, the average expectation is less sensitive to the current shock θ_t and more anchored to the past shock θ_{t-1}. This explains why less precise information (a lower β/α) implies a lower ϕ_0 and a higher ϕ_1.

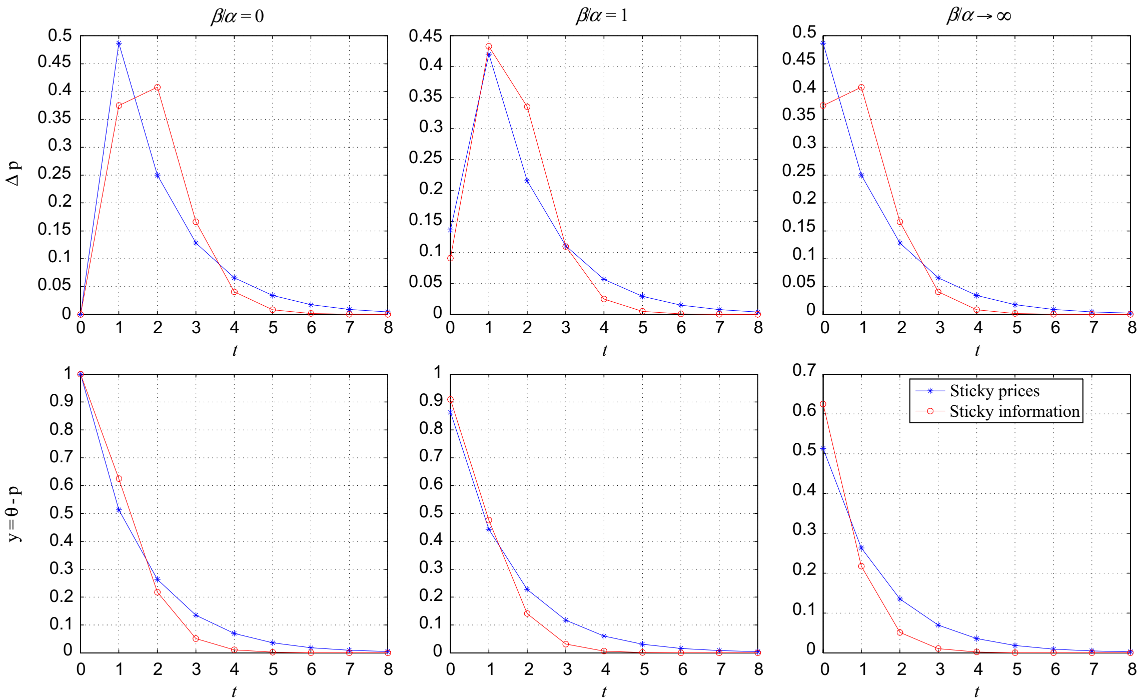
Now consider the case of sticky information. Firms updating their information sets in t will also attach more weight to the private signal x_t to the detriment of the past observed fundamental θ_{t-1}. As the weight of the current fundamental θ_t in the aggregate price level depends only on the private information of period t, an increase in private precision that increases the weight that firms updating in t attach to x_t translates into an increase in the weight of θ_t, that is, a higher ϕ_0. Now consider how an increase in private precision affects θ_{t-1}. Following the same logic, firms that updated their information sets in $t-1$ will increase the weight of x_{t-1} and, consequently, of $\theta_{t-1.}$ However, as we have already noticed, firms that updated in t will attach less weight to θ_{t-1}, which they now observe, and more to x_t. The former effect dominates for lower values of private relative precision β/α while the latter takes place as β/α increases.

4.2. Impulse Responses

We now study how the precision of information affects the impulse responses of the inflation rate and real output to an innovation in nominal demand.

Figure 2 plots these impulse responses for inflation (Δp_t) and real output ($y_t = \theta_t - p_t$). We identify the period as a year and set $\lambda = 0.20$ and $r = 0.85$. Once again, we follow Angeletos and La'O (2009) and consider three alternative values for the precision of information: (i) $\beta/\alpha = 1$ (baseline case); (ii) $\beta/\alpha \to \infty$ (standard sticky price/information model); and (iii) $\beta/\alpha = 0$ (alternative extreme: no information about the current shock other than the past shock). The first line of Figure 2 illustrates how the incompleteness of information affects the dynamics of inflation for Cases (i)–(iii) for both sticky prices and sticky information models. Two observations are common to both models. First, the instantaneous impact effect of a monetary shock on inflation is increasing in β/α. As the precision of private information increases, prices initially react more to a nominal disturbance. Second, as the precision of private information decreases, the second-period inflation becomes higher and higher. As the past nominal demand now becomes common knowledge, prices with low sensitivity to the monetary shock in the last period greatly increase in the second period to reflect this new information. Note, however, that prices react less under sticky information than under sticky prices. Besides this, inflation continues to rise and returns more gradually under sticky information. This is so because, for $r < 1$, firms also care about

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press



The first line presents the impulse responses of the inflation rate to an innovation in nominal demand, for three different values of the relative precision of information—$\alpha = \beta$, $\beta/\alpha = \infty$, and $\beta/\alpha = 0$—and the same calibration as Angeletos and La'O (2009). The second line presents the corresponding impulse responses of real output.

FIGURE 2. Impulse responses to an innovation in nominal demand—Angeletos and La.O's (2009) calibration.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

the overall price level and, therefore, need to consider what information other firms have. For small values of r, even an informed firm will not adjust its price much to the change in aggregate demand until many other firms have also learned of it. From the second line of Figure 2, the instantaneous impact effect of a monetary shock on inflation is decreasing in β/α for both models. Furthermore, the sticky information model seams to converge more quickly to the steady-state level than the model with sticky prices.

At first sight, the differences between the two models (sticky prices vs. sticky information) under dispersed information seem quite modest. However, if we consider a calibration close to the standard sticky information model of Mankiw and Reis (2002), the results differ dramatically. We increase the degree of strategic complementarity ($r = 0.90$) and consider that firms on average make adjustments once a year ($\lambda = 0.25$). As becomes clear in Figure 3, inflation under sticky information becomes much more inertial than under sticky prices. The sticky price model with dispersed information could also present a more inertial pattern if we change the model so that the shock becomes common knowledge after a number of periods different from one.

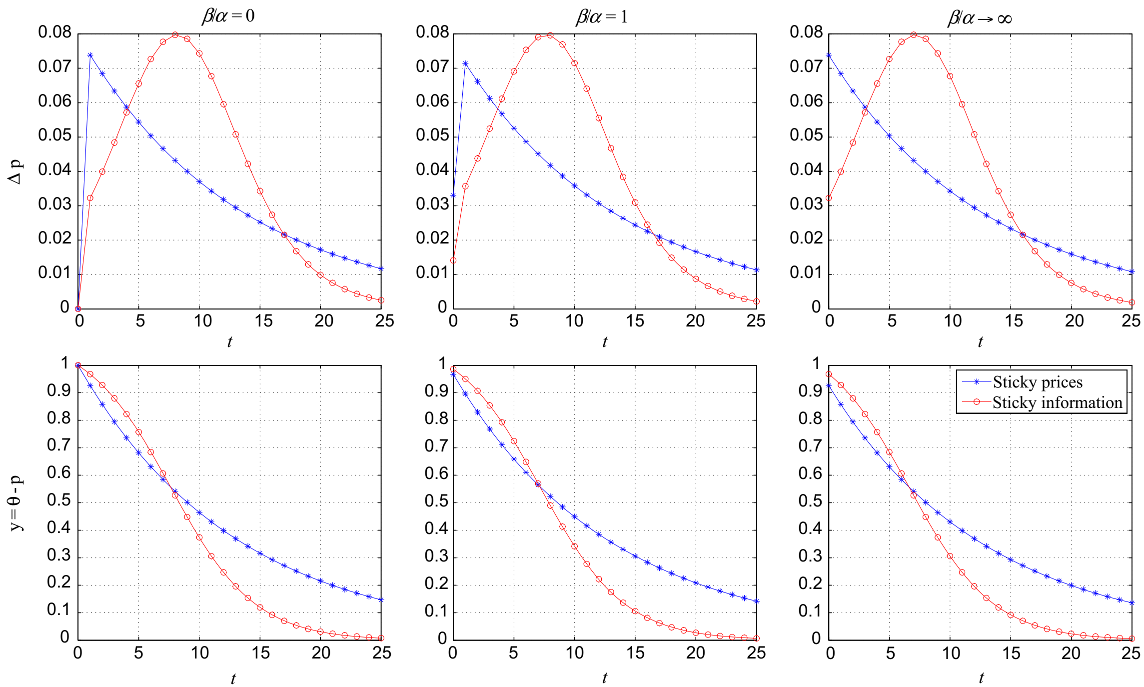
As pointed out by Angeletos and La'O (2009), inflation can start low if firms initially have little information about the innovation and can rise in the early phases of learning, but once firms have accumulated enough information about the shock then inflation will begin to fall. This is true for both models (sticky prices and sticky information). The major difference is that the standard sticky information model seems to capture all the relevant inertia.

In Figure 4, we extend the analysis previously made by incorporating public signals in the sticky information model. In contrast to private signals that just give information about the state of each firm's last update, we assume that public signals are available every period. This difference in our modeling strategy strengthens the importance of those signals. Clearly, when the precision of the public signal increases, the impulse responses of both aggregate demand and inflation to an innovation in nominal demand become attenuated. The occurrence of this phenomenon is easy to understand. When agents do not observe a state, they use the information conveyed in public signals to make inferences about it. As the precision of public signals grows, $\gamma \to \infty$, the informational shocks vanish ($\eta_{t-j} \xrightarrow{p} 0, \forall j$), meaning that $y_{t-i} \xrightarrow{p} \theta_{t-i} - \theta_{t-i-1}$. Therefore, no matter how long it has been since firm z_j last updated its information set, it will always be able to assess the state θ_t through

$$\theta_{t-j-1} + \textstyle\sum_{i=0}^{j} y_{t-i} \xrightarrow{p} \theta_t.$$

Not surprisingly, when $\gamma \to \infty$ (and consequently $\kappa \to 1$), inflation becomes $\pi_t = \theta_t - \theta_{t-1}$, which is the inflation rate that would prevail if firms had complete information about the fundamental. Alternatively, we can express inflation as $\pi_t = \epsilon_t$. This simple expression shows that ϵ_t has an immediate impact on inflation when information is complete, meaning that the impulse response of inflation is concentrated at the point $t = 0$ and zero afterwards. When the precision of

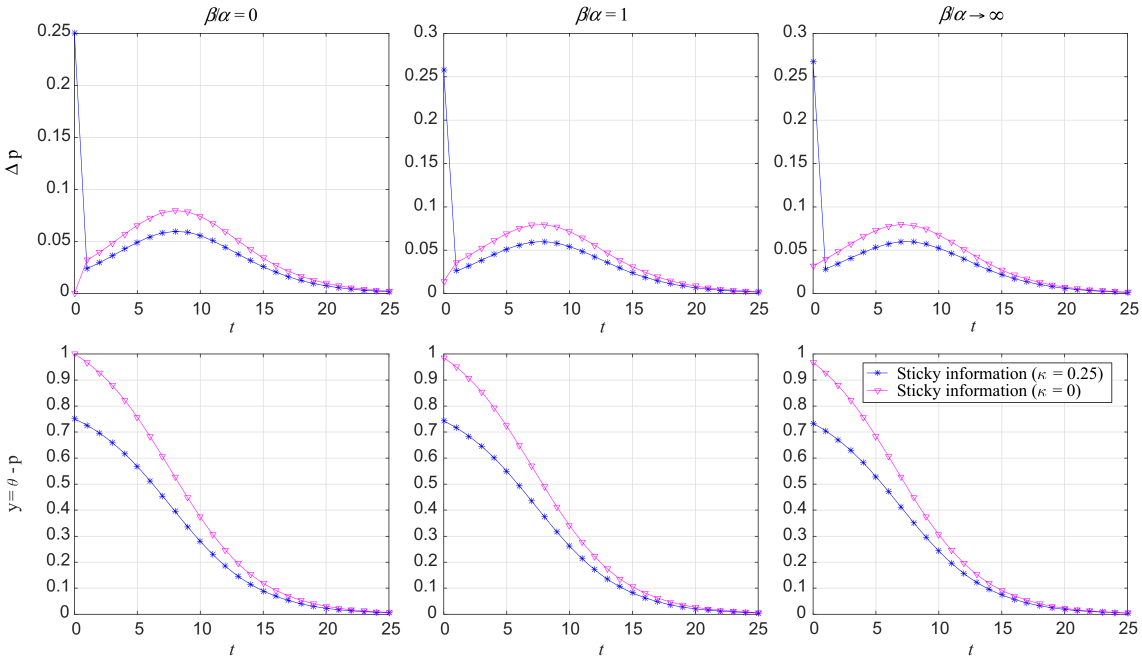
https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press



The first line presents the impulse responses of the inflation rate to an innovation in nominal demand, for three different values of the relative precision of information—$\alpha = \beta$, $\beta/\alpha = \infty$, and $\beta/\alpha = 0$—and the same calibration as Mankiw and Reis (2002). The second line presents the corresponding impulse responses of real output.

FIGURE 3. Impulse responses to an innovation in nominal demand—Mankiw and Reis' (2002) calibration.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press



The first line presents the impulse responses of the inflation rate to an innovation in nominal demand, for three different values of the relative precision of information—$\alpha = \beta$, $\beta/\alpha = \infty$, and $\beta/\alpha = 0$—and the same calibration as Mankiw and Reis (2002) but considering inclusion of public information. The second line presents the corresponding impulse responses of real output.

FIGURE 4. Impulse responses to an innovation in nominal demand—Mankiw and Reis' (2002) calibration and including public signals.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

the public signal grows without approaching infinity, as shown in Figure 4, the impulse response keeps the same shape, but, for any $t > 0$, it is partially attenuated by a greater influence of public signals (since it is closer to the complete information case) and becomes concentrated at the point $t = 0$.

The incorporation of public signals clearly attenuates the movements of the real output. This observation is just an extension of the analysis just made: when the informational friction vanishes, the impulse responses converge to the complete information case. Since the model does not incorporate any nominal rigidity, under complete information, shocks do not move the real output.

4.3. Efficiency Criterion

We use an efficiency benchmark that addresses whether higher welfare could be obtained if agents were to use their available information in a different way than they do in equilibrium. Following Angeletos and Pavan (2007), we adopt as our efficiency benchmark the strategy that maximizes *ex ante* utility subject to the sole constraint that information cannot be transferred from one agent to another. We modify their efficiency criterion to nest the assumption that information is sticky. The Lagrangian for our problem is

$$E\Pi = -(1-\lambda)\int_{(\Theta_t, Y_t)} \left[\sum_{j=0}^{\infty} \lambda^j \int_{x_{t-j}} u\left(x_{t-j}, \Theta_t, Y_t\right) dF(x_{t-j} \mid \Theta_t, Y_t)\right] dF(\Theta_t, Y_t) + \int_{(\Theta_t, Y_t)} \eta\left(\Theta_t, Y_t\right) h\left(\Theta_t, Y_t\right) dF(\Theta_t, Y_t),$$

where $u\left(x_{t-j}, \Theta_t, Y_t\right)$ is the "utility" function of the firm, and $\eta\left(\Theta_t, Y_t\right)$ is the Lagrange multiplier associated to the constraint

$$h\left(\Theta_t, Y_t\right) \equiv P_t(\Theta_t, Y_t) - \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \int_{x_{t-j}} p_t\left(x_{t-j}, \Theta_{t-j-1}, Y_t\right) dF(x_{t-j} \mid \Theta_t, Y_t).$$

This criterion may be understood as measure of social welfare, if "welfare" is now evaluated from the perspective of firms. We know that p_t^* is obtained as the first-order condition of $u\left(x_{t-j}, \Theta_t, Y_t\right)$. But since many different functions can generate the same first-order condition, this social welfare measure can vary. For instance, in Morris and Shin (2002), (1) appears as the first-order condition of a beauty-context utility function. Using this function, Morris and Shin (2002) showed that the provision of public information may diminish social welfare. Nevertheless, Woodford (2003) shows that

$$u\left(x_{t-j}, \Theta_t, Y_t\right) \equiv -\left(p_t\left(z\right) - p_t^*\right)^2$$

guarantees profit maximization in a way that is consistent with the approach presented in Blanchard and Kiyotaki (1987). We consider an extension of this function.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

As a result, any direct comparison regarding the impact of communication on welfare between our work and the related literature, specifically Baeriswyl and Cornand (2010) and Tang (2013), should be misleading as we are applying a diverse welfare criterion, Angeletos and Pavan (2007), on a diverse group of agents, firms instead of households.

5. COMMUNICATION POLICY

We now use the model to discuss communication policy by a benevolent central banker. Assume the central banker has access to information about the fundamental growth of the economy; namely, it observes

$$g_t = \theta_t - \theta_{t-1} + u_t,$$

where u_t reflects the fact that the central banker's information is a noisy signal of the fundamental growth.

The central banker's communication policy takes a very simple form: it commits to disclosing a garbled transformation of g_t,

$$y_t = g_t + v_t = \theta_t - \theta_{t-1} + u_t + v_t,$$

where v_t is a noise the central banker adds to information it has at the time of disclosure.

The central banker controls the precision of v_t, given by σ_v^{-2}. Hence, a fully transparent communication policy corresponds to the choice of $\sigma_v^{-2} \to \infty$, whereas a policy of not communicating *any* information corresponds to $\sigma_v^{-2} = 0$.[10]

Before proceeding, we notice that by making $\eta_t \equiv u_t + v_t$, all the results we have derived in previous sections hold true for a *given* communication policy and by taking the information disclosed by the central banker as the relevant public information in the economy. As before, we denote by γ the precision of such public signal.

To evaluate the optimal communication policy, we need to establish what the objective function of the central banker is. We first assume that each firm incurs a quadratic cost for setting prices that are different than the target ones. In fact, firms' payoffs in period t are

$$\Pi_t\left(z\right) = E\left[-\left(p_t\left(z\right) - \left(\left(1-r\right)\theta_t + rP_t\right)\right)^2 - \tau\left(p_t\left(z\right)\right) \mid \Im_{t-j}\left(z\right)\right].$$

We take the central banker as maximizing—by choice of the precision v_t—firms' *ex ante* (i.e. period zero) total profits, which we denote by $E\Pi$. Simple computations allow us to write such total profits as a function of γ as

$$E\Pi\left(\gamma\right) = -\left(\frac{1-\lambda}{\alpha+\beta+\gamma} + \frac{\lambda}{\alpha+\gamma}\right)\sum_{j=0}^{\infty}\lambda^j\Omega_j^2,$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

where

$$\Omega_j\left(\rho\left(\gamma\right)\right)=\frac{1-r}{1-r\left(1-\rho\lambda^j\right)}.$$

It is easy to verify that the first derivative of this expression with respect to γ is *always* positive. Hence, full transparency is always optimal.

PROPOSITION 3. *The optimal communication policy by the central banker entails full transparency, that is,* $\sigma_v^{-2}\to\infty$.

In a setting like ours, one could think that, much as Morris and Shin (2002), public information could harm social welfare. The reason is that, besides revealing information about the state, public information is, due to strategic complementarities, indicative of what other firms will do, so each firm responds excessively to public signals. Proposition 3 shows, however, that the positive effect of more precise public signals in reducing the relative importance of past shocks vis à vis current shocks—which, in turn, reduces the persistence of aggregate prices in our sticky information setting—outweighs any costs that might emerge from the strategic effects of public information. Full transparency is always optimal.

5.1. Taxation

In the above section, we assumed there existed a central banker with access to a piece of information that could be made public so as to help pricing decisions and, as a consequence, improve welfare. We could give a step and answer the following questions: (i) if a benevolent planner who observed the information set of all firms could choose the pricing rule to be adopted by those firms, which rule would he choose and (ii) could such rule be implemented through taxes?

We start by answering the first question. The planner's problem would be to choose pricing rules for the firms to maximize *ex ante* total profits:

$$\max_{\{p_t(z)\}_{z\in[0,1]}} E\Pi.$$

Computation shows that the set of individual price schedules that solve this program is given by

$$p_t\left(z\right)=E\left[\left(1-r^*\right)\theta_t+r^*P_t\left(\Theta_t,Y_t\right)\mid\Im_{t-j}\left(z\right)\right],\tag{19}$$

where

$$\left(1-r^*\right)\equiv\left(1-r\right)^2,$$

so that $r>r^*$.

Hence, from a social point of view, price setters place too much weight on aggregate prices relative to the true state of the economy θ_t. Of course, this would be inconsequential if all agents knew θ_t, for they would necessarily choose the same (and "correct") price θ_t. In our sticky and dispersed information model, such

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

"externality" has consequences. A natural way to remedy the bad consequences of such externality is through taxes. But how should this be done?

Assume that a benevolent planner sets linear taxes $\tau\,(p_t\,(z))$, which, of course, might depend on the price the firm chooses. Given those taxes, the profit function of firm z becomes

$$\Pi_t(z) = E\left[-\,(p_t(z) - ((1-r)\,\theta_t + rP_t))^2 - \tau(p_t(z))\mid \Im_{t-j}(z)\right].$$

Now, if a tax scheme successfully fixes the externality firms impose on each other, while solving

$$\max_{p_t(z)} E\left[-(p_t(z) - ((1-r)\,\theta_t + rP_t))^2 - \tau(p_t(z))\mid \Im_{t-j}(z)\right],$$

firms choose prices that lead to (19). Combining the firms' first-order condition with (19), we get

$$\frac{d\tau(p_t(z))}{dp_t(z)} = 2r(1-r)\,(\theta_t - P_t).$$

Therefore,

$$\tau(p_t\,(z)) = 2r(1-r)\,(\theta_t - P_t)\,p_t(z).$$

Of course, the difficulty is that neither θ_t nor P_t is observable by all firms (in fact, θ_t is only observed for a given firm when its selected to update its information set). However, one could solve the problem by charging

$$\tau(p_t\,(z)) = 2r(1-r)E\left[\theta_t - P_t \mid \Im_{t-j}(z)\right]p_t(z).$$

PROPOSITION 4. *By charging taxes*

$$\tau(p_t(z)) = 2r(1-r)E\left[\theta_t - P_t \mid \Im_{t-j}\,(z)\right]p_t(z),$$

a benevolent planner will induce pricing rules by the firms which are socially optimal.

6. CONCLUSION

In this paper, we considered the impact of sticky and dispersed information on individual price-setting decisions, and the resulting effect on the aggregate price level and the inflation rate. We analyzed which properties are satisfied by the equilibrium values of the coefficients of the model, which are functions of three parameters representing (i) the degree of information stickiness, (ii) the degree of strategic complementarity in pricing decisions, and (iii) the ratio of the precision of private signals to the precision of the prior. Then we evaluated the impulse response functions of the two models for inflation and aggregate demand. In both exercises, we compared our results from the sticky and dispersed information

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

model with the ones obtained by a model with dispersed information and sticky prices.

The model we propose nests the dispersed information model and the sticky information model as special cases, and can be extended in many directions. One possibility is to analyze how communication interacts with other policy instruments (e.g. interest rates) available to a central banker. We believe these extensions/applications are interesting avenues for future research.

NOTES

1. For example, Hahn (2014) studies the impact of central bank transparency under heterogeneous information, Giuli (2010) analyzes the rationale for delegating monetary policy to a central banker more conservative than the society if information is sticky, and Carrera and Ramírez-Rondán (2017) provide evidence consistent with agents that update information faster when inflation is higher.
2. The theories of "rational inattention" proposed by Sims (2003, 2010) and "inattentiveness" proposed by Reis (2006a,b) have been used to justify models of dispersed information and sticky information.
3. Subsequent refinements of the sticky information models can be found in Mankiw and Reis (2006, 2007, 2010) and Reis (2006a,b, 2009).
4. See Woodford (2003) for details.
5. In a slight notational abuse, we use $\int_{\Lambda_j} [\bullet]\, dz_j$ to denote the Lebesgue integral over the subset Λ_j.
6. We could also follow Morris and Shin (2002) and first derive an equilibrium for which the aggregate price level is a linear function of fundamentals. We could then establish, using (8), that this linear equilibrium is the unique equilibrium of our game.
7. θ_{t-j-1} is the only piece of information in $\Theta_{t-j} = \left\{\theta_{t-j-k}\right\}_{k=1}^{\infty}$ the firm needs to use because the state's process is Markovian.
8. On request, we can send a sketch of a proof on the impossibility of writing the price index obtained from the SDI model as a function of the past price indexes, as presented in Angeletos and La'O (2009).
9. Using the notation introduced in Angeletos and La'O (2009), $\phi_0 = c_2$ and $\phi_1 = c_1 c_2 + c_3$.
10. We are implicitly assuming that the central banker is forced to choose a communication policy which is i.i.d. over time. This, however, is without loss given its objective function and the fact that the environment is stationary.

REFERENCES

Andrade, P., R. K. Crump, S. Eusepi and E. Moench (2016) Fundamental disagreement. *Journal of Monetary Economics* 83(C), 106–128.

Angeletos, G.-M. and J. La'O (2009) Incomplete information, higher-order beliefs and prices inertia. *Journal of Monetary Economics* 56(Suplement 1), S19–S37.

Angeletos, G.-M. and A. Pavan (2007) Efficient use of information and social value of information. *Econometrica* 75(4), 1103–1142.

Baeriswyl, R. and C. Cornand (2010) The signaling role of policy actions. *Journal of Monetary Economics* 57(6), 682–695.

Blanchard, O. J. and N. Kiyotaki (1987) Monopolistic competition and the effects of aggregate demand. *American Economic Review* 77(4), 647–666.

Calvo, G. A. (1983) Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

Carrera, C. and N. Ramírez-Rondán (2017) Inflation, information rigidity, and the sticky information Phillips curve. *Macroeconomic Dynamics*, 1–19.

Giuli, F. (2010) Robust policies in a sticky information economy. *Macroeconomic Dynamics* 14(3), 311–342.

Hahn, V. (2014) Transparency in monetary policy, signaling, and heterogeneous information. *Macroeconomic Dynamics* 18(2), 369–394.

Lucas, R. E. (1972) Expectations and the neutrality of money. *Journal of Economic Theory* 4(2), 103–124.

Mankiw, N. G. and R. Reis (2002) Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics* 117(4), 1295–1328.

Mankiw, N. G. and R. Reis (2006) Pervasive stickiness. *American Economic Review: Papers and Proceedings* 96(2), 164–169.

Mankiw, N. G. and R. Reis (2007) Sticky information in general equilibrium. *Journal of the European Economic Association* 5(2–3), 603–613.

Mankiw, N. G. and R. Reis (2010) Imperfect information and aggregate supply. In: B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics* 1st ed., Chapter 5, Vol. 3A, pp. 183–229. Amsterdam: Elsevier/North-Holland.

Morris, S. and H. S. Shin (2002) The social value of public information. *American Economic Review* 92(5), 1521–1534.

Phelps, E. S. (1968) Money-wage dynamics and labor market equilibrium. *Journal of Political Economy* 76(4), 678–711.

Reis, R. (2006a) Inattentive consumers. *Journal of Monetary Economics* 53(8), 1761–1800.

Reis, R. (2006b) Inattentive producers. *Review of Economic Studies* 73(3), 793–821.

Reis, R. (2009) Optimal monetary policy rules in an estimated sticky-information model. *American Economic Journal: Macroeconomics* 1(2), 1–28.

Sims, C. A. (2003) Implications of rational inattention. *Journal of Monetary Economics* 50(3), 665–690.

Sims, C. A. (2010) Rational inattention and monetary economics. In: B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics* 1st ed., Chapter 4, Vol. 3A, pp. 155–181. Amsterdam: Elsevier/North-Holland.

Tang, J. (2013) Uncertainty and the Signaling Channel of Monetary Policy. Federal Reserve Bank of Boston Working Papers: 15-8.

Veldkamp, L. (2011) *Information Choice in Macroeconomics and Finance*. Princeton: Princeton University Press.

Woodford, M. (2003) Imperfect common knowledge and the effects of monetary policy. In P. Aghion, R. Frydman, J. Stiglitz and M. Woodford (eds.), *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, pp. 25–58. Princeton and Oxford: Princeton University Press.

APPENDIX: HIGHER-ORDER BELIEFS

In this appendix we derive the general formula of the kth-order average expectation

$$\bar{E}^k\left[\theta_t\right] = (1-\lambda)\sum_{m=0}^{\infty}\lambda^m\left[a_{m,k}\theta_{t-m} + b_{m,k}\theta_{t-m-1}\right] + \kappa\sum_{m=0}^{\infty}\lambda^m c_{m,k}\, y_{t-m},$$

considering that the weights $\left(a_{m,k}, b_{m,k}, c_{m,k}\right)$ are recursively defined, for $k \geq 1$, by

$$\begin{bmatrix} a_{m,k+1} \\ b_{m,k+1} \\ b_{m,k+1} \end{bmatrix} = \begin{bmatrix} \delta \\ (1-\delta) \\ \rho \end{bmatrix} (1-\lambda^m)^k + A_m \begin{bmatrix} a_{m,k} \\ b_{m,k} \\ c_{m,k} \end{bmatrix},$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \delta\left(1-\lambda^{m+1}\right)+(1-\delta)\left(1-\lambda^{m}\right) & 0 & 0 \\ (1-\delta)\left[\left(1-\lambda^{m+1}\right)-\left(1-\lambda^{m}\right)\right] & 1-\lambda^{m+1} & 0 \\ (1-\lambda)\,\rho\lambda^{m} & 0 & 1 \end{bmatrix}, \tag{A1}$$

the initial weights are $\left(a_{m,1}, b_{m,1}, c_{m,1}\right) \equiv (\delta, 1-\delta, \rho)$, and $\rho \equiv 1-\delta(1-\lambda)$.

Proof (Higher-Order Beliefs). We start by computing $\bar{E}^1[\theta_t]$ as

$$\begin{aligned} \bar{E}^1[\theta_t] &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\bar{E}^0[\theta_t] \mid \Im_t(z_m)\right] dz_m \\ &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\theta_t \mid \Im_t(z_m)\right] dz_m \\ &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\left[\delta x_{t-m}(z_m)+(1-\delta)\,\theta_{t-m-1}+(1-\delta)\,\kappa y_{t-m}+\kappa\sum_{i=0}^{m-1} y_{t-i}\right] dz_m \\ &= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\lambda^m\left[\delta\theta_{t-m}+(1-\delta)\,\theta_{t-m-1}+(1-\delta)\,\kappa y_{t-m}+\kappa\sum_{i=0}^{m-1} y_{t-i}\right] \\ &= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\lambda^m\left[\delta\theta_{t-m}+(1-\delta)\,\theta_{t-m-1}\right]+\rho\kappa\displaystyle\sum_{i=0}^{\infty}\lambda^i y_{t-i}. \end{aligned}$$

The first two equalities use the definition of $\bar{E}^k[\bullet]$, for $k\in\{0,1\}$, as stated in (7). The third equality uses (12) to compute firms' expectations. The forth equality results from the computation of the integral, considering that the Lebesgue measure of Λ_m is $(1-\lambda)\,\lambda^m$ and idiosyncratic shocks die out with aggregation, $\int_{\Lambda_m} x_{t-m}(z_m)\,dz_m = \int_{\Lambda_m}\theta_{t-m}+\xi_{t-m}(z_m)\,dz = (1-\lambda)\,\lambda^m\theta_{t-m}$. The last equality just rearranges the terms. This expression shows that $\left(a_{m,1}, b_{m,1}, c_{m,1}\right)\equiv(\delta, 1-\delta, \rho)$. We can use this result to obtain $\bar{E}^2[\theta_t]$ as

$$\begin{aligned} \bar{E}^2[\theta_t] &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\bar{E}^1[\theta_t] \mid \Im_t(z_m)\right] dz_m \\ &= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{\infty}\lambda^j E\left[\delta\theta_{t-j}+(1-\delta)\,\theta_{t-j-1} \mid \Im_t(z_m)\right] dz_m \\ &\quad +\rho\kappa\textstyle\sum_{k=0}^{\infty}\lambda^k y_{t-k}. \end{aligned}$$

This last equality holds because y_{t-k} belongs to all firms' information sets. We use (12) to rewrite $\bar{E}^2[\theta_t]$ considering three different cases: (i) $j<m$, when firm z_m does not observe either θ_{t-j} or θ_{t-j-1}, (ii) $j=m$, when firm z_m observes θ_{t-j-1}, but not θ_{t-j}, and (iii) $j>m$, when firm z_m observes both θ_{t-j} and θ_{t-j-1}. Thereafter,

$$\begin{aligned} \bar{E}^2[\theta_t] &= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left\{\delta E\left[\theta_{t-j} \mid \Im_t(z_m)\right]+(1-\delta)E\left[\theta_{t-j-1} \mid \Im_t(z_m)\right]\right\} dz_m \\ &\quad +(1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\lambda^m\left\{\delta E\left[\theta_{t-m} \mid \Im_t(z_m)\right]+(1-\delta)\,\theta_{t-m-1}\right\} dz_m \\ &\quad +(1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}\lambda^j\left[\delta\theta_{t-j}+(1-\delta)\,\theta_{t-j-1}\right] dz_j+\rho\kappa\sum_{k=0}^{\infty}\lambda^k y_{t-k}. \end{aligned}$$

Using (12) to compute expectations, we get

$$\begin{aligned} \bar{E}^2[\theta_t] &= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left\{\delta x_{t-m}(z)+(1-\delta)\,\theta_{t-m-1}\right\} dz_m \\ &\quad +(1-\lambda)\,\kappa\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left[\sum_{i=j}^{m-1}\left[\delta y_{t-i}+(1-\delta)\,y_{t-i-1}\right]\right] dz_m \\ &\quad +(1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\lambda^m\left[\delta\left[\delta x_{t-m}(z)+(1-\delta)\,\theta_{t-m-1}+(1-\delta)\,\kappa y_{t-m}\right]\right. \\ &\qquad \left.+(1-\delta)\,\theta_{t-m-1}\right] dz_m \\ &\quad +(1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}\lambda^j\left[\delta\theta_{t-j}+(1-\delta)\,\theta_{t-j-1}\right] dz_m+\rho\kappa\sum_{k=0}^{\infty}\lambda^k y_{t-k}. \end{aligned}$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

Evaluating the integral and rearranging the terms, we find

$$\begin{aligned}\bar{E}^2[\theta_t] &= (1-\lambda)^2 \sum_{m=0}^{\infty} \lambda^m \left[\delta\theta_{t-m} + (1-\delta)\,\theta_{t-m-1}\right] \sum_{j=0}^{m-1} \lambda^j \\ &\quad + \kappa\,(1-\lambda)^2 \sum_{m=0}^{\infty} \lambda^m \sum_{i=0}^{m-1} \left[\delta y_{t-i} + (1-\delta)\,y_{t-i-1}\right] \sum_{j=0}^{i} \lambda^j \\ &\quad + (1-\lambda)^2 \sum_{m=0}^{\infty} \lambda^{2m} \left[\delta^2\theta_{t-m} + \left[1-\delta^2\right]\theta_{t-m-1} + (1-\delta)\,\delta\kappa y_{t-m}\right] \\ &\quad + (1-\lambda)^2 \sum_{j=1}^{\infty} \lambda^j \left[\delta\theta_{t-j} + (1-\delta)\,\theta_{t-j-1}\right] \sum_{m=0}^{j-1} \lambda^m + \rho\kappa \sum_{k=0}^{\infty} \lambda^k y_{t-k} \\ &= (1-\lambda) \sum_{m=0}^{\infty} \lambda^m (1-\lambda^m) \left[\delta\theta_{t-m} + (1-\delta)\,\theta_{t-m-1}\right] \\ &\quad + \kappa\,(1-\lambda) \sum_{i=0}^{\infty} \left[\delta y_{t-i} + (1-\delta)\,y_{t-i-1}\right] \left(1-\lambda^{i+1}\right) \sum_{m=i+1}^{\infty} \lambda^m \\ &\quad + (1-\lambda)^2 \sum_{m=0}^{\infty} \lambda^{2m} \left[\delta^2\theta_{t-m} + \left(1-\delta^2\right)\theta_{t-m-1} + (1-\delta)\,\delta\kappa y_{t-m}\right] \\ &\quad + (1-\lambda) \sum_{j=1}^{\infty} \lambda^j \left(1-\lambda^j\right) \left[\delta\theta_{t-j} + (1-\delta)\,\theta_{t-j-1}\right] + \rho\kappa \sum_{k=0}^{\infty} \lambda^k y_{t-k} \\ &= (1-\lambda) \sum_{m=0}^{\infty} 2\lambda^m (1-\lambda^m) \left[\delta\theta_{t-m} + (1-\delta)\,\theta_{t-m-1}\right] \\ &\quad + (1-\lambda)^2 \sum_{m=0}^{\infty} \lambda^{2m} \left[\delta^2\theta_{t-m} + \left(1-\delta^2\right)\theta_{t-m-1}\right] \\ &\quad + \kappa \sum_{m=0}^{\infty} \lambda^m y_{t-m} \left\{(1-\delta)\,(1-\rho\lambda^m) + \delta\left(1-\rho\lambda^{m+1}\right) + 2\rho - 1\right\}.\end{aligned}$$

We can write this expression as

$$\bar{E}^2[\theta_t] = (1-\lambda) \sum_{j=0}^{\infty} \lambda^j \left[a_{j,2}\theta_{t-j} + b_{j,2}\theta_{t-j-1}\right] + \kappa \sum_{j=0}^{\infty} \lambda^j c_{j,2} y_{t-j},$$

where

$$\begin{aligned}\alpha_{j,2} &= \left[1-(1-\delta)^2\right]\left(1-\lambda^j\right) + \delta^2\left(1-\lambda^{j+1}\right), \\ b_{j,2} &= (1-\delta)^2\left(1-\lambda^j\right) + \left(1-\delta^2\right)\left(1-\lambda^{j+1}\right), \\ c_{j,2} &= (1-\delta)\left(1-\rho\lambda^j\right) + \delta\left(1-\rho\lambda^{j+1}\right) + 2\rho - 1.\end{aligned}$$

This result shows that (13) holds for $k=2$. Note that

$$a_{j,k} + b_{j,k} = \sum_{n=0}^{k-1} \left(1-\lambda^j\right)^n \left(1-\lambda^{j+1}\right)^{k-1-n} \tag{A2}$$

for $k=2$. We use induction to obtain to prove that (**A2**) holds for a generic k and to compute $\bar{E}^k[\theta_t]$. Suppose that (13) holds for $k-1$. Then

$$\begin{aligned}a_{j,k} + b_{j,k} &= \begin{bmatrix}1 & 1 & 0\end{bmatrix} \begin{bmatrix}\delta \\ (1-\delta) \\ \rho\end{bmatrix} \left(1-\lambda^j\right)^{k-1} + \begin{bmatrix}1 & 1 & 0\end{bmatrix} A_j \begin{bmatrix}a_{j,k-1} \\ b_{j,k-1} \\ c_{j,k-1}\end{bmatrix} \\ &= \left(1-\lambda^j\right)^{k-1} + \left(1-\lambda^{j+1}\right)\left(a_{j,k-1} + b_{j,k-1}\right) \\ &= \left[1-\lambda^j\right]^{k-1} + \sum_{n=0}^{k-2} \left[1-\lambda^j\right]^n \left[1-\lambda^{j+1}\right]^{k-1-n} \\ &= \sum_{n=0}^{k-1} \left[1-\lambda^j\right]^n \left[1-\lambda^{j+1}\right]^{k-1-n}.\end{aligned}$$

Based on this result, we prove that

$$\begin{aligned}\sum_{j=0}^{m-1} \lambda^j \left(a_{j,k-1} + b_{j,k-1}\right) &= \sum_{j=0}^{m-1} \lambda^j \left[1-\lambda^{j+1}\right]^{k-2} \sum_{n=0}^{k-2} \left[\frac{1-\lambda^j}{1-\lambda^{j+1}}\right]^n \\ &= \frac{(1-\lambda^m)^{k-1}}{1-\lambda}.\end{aligned}$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

To obtain $\bar{E}^k[\theta_t]$, we follow the same steps we used before: we decompose $\bar{E}^k[\theta_t]$ into three different cases ($j<m$, $j=m$, $j>m$), we use (12) to compute expectations, and, finally, we evaluate the Lebesgue integral:

$$
\begin{aligned}
\bar{E}^k[\theta_t] &= \textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m} E\left[\bar{E}^{k-1}[\theta_t] \mid \Im_t(z_m)\right] dz_m \\
&= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j a_{j,k-1}E\left[\theta_{t-j}\mid \Im_t(z_m)\right] dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j b_{j,k-1}E\left[\theta_{t-j-1}\mid \Im_t(z_m)\right] dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\lambda^m\left\{a_{m,k-1}E\left[\theta_{t-m}\mid \Im_t(z_m)\right]+b_{m,k-1}\theta_{t-m-1}\right\} dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}\lambda^j\left[a_{j,k-1}\theta_{t-j}+b_{j,k-1}\theta_{t-j-1}\right] dz_m + \kappa\displaystyle\sum_{j=0}^{\infty}\lambda^j c_{j,k}y_{t-j} \\
&= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left(a_{j,k-1}+b_{j,k-1}\right)\left[\delta x_{t-m}(z_m)+(1-\delta)\theta_{t-m-1}\right] dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left(a_{j,k-1}+b_{j,k-1}\right)(1-\delta)\kappa y_{t-m} dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=0}^{m-1}\lambda^j\left\{\left(a_{j,k-1}+b_{j,k-1}\right)\kappa\sum_{i=j+1}^{m-1}y_{t-i}+\kappa a_{j,k}y_{t-j}\right\} dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\lambda^m a_{m,k-1}\left[\delta x_{t-m}(z_m)+(1-\delta)\theta_{t-m-1}+(1-\delta)\kappa y_{t-m}\right] dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\lambda^m b_{m,k-1}\theta_{t-m-1} dz_m \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\int_{\Lambda_m}\sum_{j=m+1}^{\infty}\lambda^j\left[a_{j,k-1}\theta_{t-j}+b_{j,k-1}\theta_{t-j-1}\right] dz_m + \kappa\sum_{j=0}^{\infty}\lambda^j c_{j,k}y_{t-j} \\
&= (1-\lambda)^2\textstyle\sum_{m=0}^{\infty}\lambda^m\left[\delta\theta_{t-m}+(1-\delta)\theta_{t-m-1}+(1-\delta)\kappa y_{t-m}\right] \\
&\qquad \textstyle\sum_{j=0}^{m-1}\lambda^j\left(a_{j,k-1}+b_{j,k-1}\right) \\
&\quad + \kappa(1-\lambda)^2\textstyle\sum_{m=0}^{\infty}\lambda^m\sum_{i=1}^{m-1}y_{t-i}\sum_{j=0}^{i-1}\lambda^j\left(a_{j,k-1}+b_{j,k-1}\right) \\
&\quad + \kappa(1-\lambda)^2\textstyle\sum_{j=0}^{\infty}\lambda^j a_{j,k-1}y_{t-j}\sum_{m=j+1}^{\infty}\lambda^m \\
&\quad + (1-\lambda)^2\textstyle\sum_{m=0}^{\infty}\lambda^{2m}\left[a_{m,k-1}\left[\delta\theta_{t-m}+(1-\delta)\kappa y_{t-m}\right]\right. \\
&\qquad \left.+\left(a_{m,k-1}(1-\delta)+b_{m,k-1}\right)\theta_{t-m-1}\right] \\
&\quad + (1-\lambda)^2\textstyle\sum_{j=1}^{\infty}\lambda^j\left[a_{j,k-1}\theta_{t-j}+b_{j,k-1}\theta_{t-j-1}\right]\sum_{m=0}^{j-1}\lambda^m+\kappa\sum_{j=0}^{\infty}\lambda^j c_{j,k-1}y_{t-j} \\
&= (1-\lambda)\textstyle\sum_{m=0}^{\infty}\lambda^m(1-\lambda^m)^{k-1}\left[\delta\theta_{t-m}+(1-\delta)\theta_{t-m-1}\right] \\
&\quad + (1-\lambda)^2\textstyle\sum_{m=0}^{\infty}\lambda^{2m}\left[a_{m,k-1}\delta\theta_{t-m}+\left(a_{m,k-1}(1-\delta)+b_{m,k-1}\right)\theta_{t-m-1}\right] \\
&\quad + (1-\lambda)\textstyle\sum_{m=0}^{\infty}\lambda^m\left[a_{m,k-1}\theta_{t-m}+b_{m,k-1}\theta_{t-m-1}\right](1-\lambda^m) \\
&\quad + \kappa\textstyle\sum_{m=0}^{\infty}\lambda^m y_{t-m}\left\{\rho(1-\lambda^m)^{k-1}+(1-\lambda)\rho\lambda^m a_{m,k-1}+c_{m,k-1}\right\}
\end{aligned}
$$

Since $(1-\lambda)\lambda^m=\left(1-\lambda^{m+1}\right)-(1-\lambda^m)$, we can rewrite the last four lines as

$$\bar{E}^k[\theta_t]=\textstyle\sum_{m=0}^{\infty}\lambda^m\left\{(1-\lambda)\left[a_{m,k}\theta_{t-m}+b_{m,k}\theta_{t-m-1}\right]+\kappa c_{m,k}y_{t-j}\right\},$$

where

$$
\begin{aligned}
a_{m,k} &\equiv \delta(1-\lambda^m)^{k-1}+\left[\delta(1-\lambda)\lambda^m+(1-\lambda^m)\right]a_{m,k} \\
&= \delta(1-\lambda^m)^{k-1} \\
&\quad + \left[\delta\left(1-\lambda^{m+1}\right)+(1-\delta)(1-\lambda^m)\right]a_{m,k}, \\
b_{m,k} &\equiv (1-\delta)(1-\lambda^m)^{k-1}+(1-\delta)(1-\lambda)\lambda^m a_{m,k-1}+\left[(1-\lambda)\lambda^m+(1-\lambda^m)\right]b_{m,k-1}
\end{aligned}
$$

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press

$$= (1-\delta)\,(1-\lambda^m)^{k-1}\,,$$
$$+ (1-\delta)\left[\left(1-\lambda^{m+1}\right)-(1-\lambda^m)\right]a_{m,k-1}+\left(1-\lambda^{m+1}\right)b_{m,k-1}$$
$$c_{m,k} = \rho\,(1-\lambda^m)^{k-1}+(1-\lambda)\,\rho\lambda^m a_{m,k}+c_{m,k}.$$

Rewriting these weights in matrix format, we obtain

$$\begin{bmatrix} a_{m,k} \\ b_{m,k} \\ c_{m,k} \end{bmatrix} = A_m \begin{bmatrix} a_{m,k-1} \\ b_{m,k-1} \\ c_{m,k-1} \end{bmatrix} + (1-\lambda^m)^{k-1} \begin{bmatrix} \delta \\ 1-\delta \\ \rho \end{bmatrix},$$

where the matrix A_m is given by

$$A_m = \begin{bmatrix} \delta\left(1-\lambda^{m+1}\right)+(1-\delta)\,(1-\lambda^m) & 0 & 0 \\ (1-\delta)\left[\left(1-\lambda^{m+1}\right)-(1-\lambda^m)\right] & 1-\lambda^{m+1} & 0 \\ (1-\lambda)\,\rho\lambda^m & 0 & 1 \end{bmatrix}. \quad \textbf{(A3)}$$

which is exactly our result. ■

https://doi.org/10.1017/S1365100518000470 Published online by Cambridge University Press