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Basis Convergence and Long Memory in Volatility When Dynamic Hedging with Futures

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Abstract

When market returns follow a long memory volatility process, standard approaches to estimating dynamic minimum variance hedge ratios (MVHRs) are misspecified. Simulation results and an application to the S&P 500 index document the magnitude of the misspecification that results from failure to account for basis convergence and long memory in volatility. These results have important implications for the estimation of MVHRs in the S&P 500 example and other markets as well.

I. Introduction

The importance of managing risk exposure has elicited a voluminous literature on futures hedging over the last half-century (see Chen, Lee, and Shrestha (2003) for a review). The early literature focused mainly on estimating the minimum variance hedge ratio (MVHR) via an ordinary least squares (OLS) regression between the spot and futures (Ederington (1979), Figlewski (1986)). This approach has been extended to allow for conditional heteroskedasticity (Kroner and Sultan (1993)) and cointegration (Ghosh (1993), Lien (1996)). Bivariate error correction generalized autoregressive conditional heteroskedasticity (GARCH) models between the spot and the futures have therefore become a popular way of estimating dynamic MVHRs.[1]

These conventional approaches, however, ignore the convergence of the spot and the futures (basis convergence) over the life of the futures contract. This persists despite the early literature by Working (1953a), (1953b), (1961) that incorporated changes in the basis into the hedging decision. Since these studies, very little research has examined the importance of basis convergence when hedging.

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[1]See Park and Switzer (1995), Lien and Tse (1999), Sim and Zurbruegg (2000), Kroner and Sultan (1993), Cecchetti, Cumby, and Figlewski (1988), and Koutmos and Pericli (1998).

Castelino (1989), (1990a), (1990b), (1992) showed that once basis convergence is allowed for, the MVHR approaches unity as the hedge lifting date approaches the futures expiration date. Given that arbitrage imposes basis convergence (Brennan and Schwarz (1990), Buhler and Kempf (1995), Twite (1998), and Lim (1992)), basis convergence is likely to have a significant effect on the MVHR in many markets.

Over the last decade, an extensive literature has documented the presence of long memory in financial market volatility (Ding, Granger, and Engle (1993), Bollerslev and Mikkelsen (1996), Ding and Granger (1996), Breidt, Crato, and de Lima (1998), Dacorogna, Muller, Nagler, Olsen, and Pictet (1993), Baillie, Bollerslev, and Mikkelsen (1996), and Andersen and Bollerslev (1997a), (1997b), (1998)). If long memory in volatility is present, estimation of dynamic MVHRs via bivariate fractionally integrated volatility processes may be more appropriate.[2]

This paper therefore seeks to document the magnitude of the misspecification that results from failing to account for basis convergence and long memory in volatility when estimating dynamic MVHRs. Simulations reveal that when hedging over a five-period horizon, failing to allow for long memory in volatility does not lead to statistically significant increases in portfolio variance. However, the statistically significant increases in portfolio variance that result from failing to account for basis convergence rise as the hedge commencement date approaches the futures expiration date. For hedges $\geq$ 20 periods, failing to allow for long memory in volatility produces higher portfolio variances approximately 60% of the time, yielding statistically significant increases in variance averaging between 1% to 3%. Failing to allow for basis convergence produces higher portfolio variances approximately 70% of the time, yielding statistically significant increases in variance averaging between 2% to 4%. It is also shown that the increase in variance that results from failing to allow for basis convergence becomes larger as the correlation between the spot and the futures decreases and maturity effects increase.

An application to the S&P 500 demonstrates that as the hedge horizon increases, long memory in volatility and basis convergence become more important. For a five-day hedge, failing to allow for long memory produces higher portfolio

[2]Long memory in volatility may arise from a heavy-tailed regime switching process (Liu (2000)) or from the aggregation of multiple volatility components caused by heterogeneous information flows (Anderson and Bollerslev (1997a)) or by heterogeneous traders (Muller, Dacorogna, Dave, Olsen, Pictet, and Weizsacker (1997)). The findings of long memory in volatility may, however, be spurious due to the effect of structural breaks or aggregation (Lobato and Savin (1998)). High degrees of persistence may be explained by occasional break models that exhibit near long memory (Hyung and Franses (2001), Breidt and Hsu (2002), Granger and Hyung (2004), and Kirman and Teyssiere (2002a), (2002b)). Furthermore, when examining equity indices, the stocks comprising an index may exhibit short memory, but due to aggregation the index may exhibit long memory (Granger (1980)). Financial market volatility may in fact contain both structural breaks and long memory in volatility (Morana and Beltratti (2004)). Long memory in volatility is supported for four reasons. First, testing and estimation procedures are unable to differentiate between long memory and near long memory (Granger and Hyung (2004), Breidt and Hsu (2002), and Hsu (2001)). Second, different break tests may identify different break points (Granger and Hyung (2004), Hyung and Franses (2001), and Morana and Beltratti (2004)). Third, long memory models forecast as well as if not better than occasional break models even if the data are weakly dependent and exhibit occasional breaks (Diebold and Inoue (2001), Hyung and Franses (2001), and Morana and Beltratti (2004)). Fourth, Lobato and Savin (1998) find that stocks exhibit long memory in volatility, refuting the argument that an index exhibits long memory due to aggregation.

variances 53% of the time. As the horizon increases to 60 days, the percentage increases to 75%, yielding an average increase in portfolio variance of 1% to 2%. Similarly, failing to allow for basis convergence over a five-day horizon produces higher portfolio variances 64% of the time. As the horizon increases to 60 days, the percentage increases to 95%, yielding an average increase in variance of 6%.

The plan of the paper is as follows. Section II develops the approach and presents the simulation results. Section III illustrates the importance of long memory in volatility and basis convergence when estimating MVHRs on the S&P 500. Section IV concludes.

II. The MVHR, Basis Convergence, and Long Memory in Volatility

A. Model Development and the Single-Period MVHR

This section examines the magnitude of the misspecification that results from failing to account for basis convergence and long memory in volatility when estimating MVHRs. To do this, a model of the joint dynamics between the spot and the basis is proposed. The model is in the spirit of Chen, Duan, and Hung (1999) who model the joint dynamics using a bivariate GARCH process with maturity effects. In this paper, a bivariate fractionally integrated volatility process with maturity effects is employed. Let S_t (F_t) represent the spot (futures) at time t, and $B_t = F_t - S_t$ represent the basis at time t. The joint dynamics are as follows:

$$(1) \qquad r_{s,t} = \varepsilon_{s,t}$$

$$r_{b,t} = a_2 m_t^{\lambda_1} + \varepsilon_{b,t} m_t^{\lambda_2},$$

$$(2) \qquad \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix} \sim D\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{s,t}^2 & \sigma_{sb,t} \\ \sigma_{sb,t} & \sigma_{b,t}^2 \end{pmatrix}\right],$$

$$(3) \qquad \sigma_{i,t}^2 = \omega_i + \sum_{j=1}^{\infty} \theta_j \varepsilon_{i,t-j}^2 \quad i = s, b,$$

$$(4) \qquad \sigma_{sb,t} = \rho_{sb} \sigma_{s,t} \sigma_{b,t},$$

$$(5) \qquad \theta_j = O\left(j^{-1-d}\right),$$

where $r_{s,t}$ is $\Delta S_t/S_{t-1}$, $r_{b,t}$ (the normalized change in the basis) is $\Delta B_t/S_{t-1}$, m_t is the number of days to maturity at time t divided by 100, λ_1 and λ_2 are maturity effect parameters, D represents a distribution, and O represents the asymptotic approximation. (For simplicity, the variances have the same fractional order.) The conditional covariances at time $t-1$ can be expressed as:

$$(6) \qquad \text{Var}_{t-1}\left[r_{s,t}\right] = \sigma_{s,t}^2,$$

$$(7) \qquad \text{Var}_{t-1}\left[r_{b,t}\right] = \sigma_{b,t}^2 m_t^{2\lambda_2}, \quad \text{and}$$

$$(8) \qquad \text{Cov}_{t-1}\left[r_{s,t}, r_{b,t}\right] = \rho_{sb} \sigma_{s,t} \sigma_{b,t} m_t^{\lambda_2} = \sigma_{sb,t} m_t^{\lambda_2}.$$

If $\lambda_1 > 0$ $(\lambda_2 > 0)$, the normalized change in the basis (basis volatility and covariance) converges to zero over the life of the futures contract. Following

Chen et al. (1999), the single-period MVHR between time $t-1$ and t (Φ_{t-1}) can be expressed as:[3]

$$(9)\quad \Phi_{t-1} = \frac{\mathrm{Var}_{t-1}[\Delta S_t] + \mathrm{Cov}_{t-1}[\Delta S_t, \Delta B_t]}{\mathrm{Var}_{t-1}[\Delta S_t] + \mathrm{Var}_{t-1}[\Delta B_t] + 2\mathrm{Cov}_{t-1}[\Delta S_t, \Delta B_t]}$$

$$= \frac{\mathrm{Var}_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}\right] + \mathrm{Cov}_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}, \frac{\Delta B_t}{S_{t-1}}\right]}{\mathrm{Var}_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}\right] + \mathrm{Var}_{t-1}\left[\frac{\Delta B_t}{S_{t-1}}\right] + 2\mathrm{Cov}_{t-1}\left[\frac{\Delta S_t}{S_{t-1}}, \frac{\Delta B_t}{S_{t-1}}\right]}$$

$$= \frac{\sigma^2_{s,t} + \sigma_{sb,t} m_t^{\lambda_2}}{\sigma^2_{s,t} + \sigma^2_{b,t} m_t^{2\lambda_2} + 2\sigma_{sb,t} m_t^{\lambda_2}}.$$

Figure 1, Graphs A and B display Φ_{t-1} as the futures contract approaches expiration for different values of λ_2. Both graphs assume $\sigma^2_s = 1$ and $\sigma^2_f = 1.2$. Graph A assumes $\rho_{sf} = 0.55$, while Graph B assumes $\rho_{sf} = 0.85$.[4] When $\lambda_2 = 0$, the conventional MVHR without maturity effects applies and the MVHR remains constant. As maturity effects increase (λ_2 increases), the distance between the conventional MVHR and the MVHR allowing for basis convergence increases. Graph B of Figure 1 illustrates that for a given λ_2, as ρ_{sf} increases, the difference between the conventional MVHR and the MVHR allowing for basis convergence decreases. These preliminary investigations indicate that the misspecification that results from failing to account for basis convergence increases as maturity effects increase and the correlation between the spot and the futures decreases.

To consider the effect on the MVHR from incorrectly fitting a bivariate short memory volatility process, a short memory process is defined as one where

$$(10)\quad \theta_j = O(p^{-j}),$$

where p represents the short memory parameter.[5] Let $(\sigma^e_{s,t})^2$ represent the bias in the estimate of $\sigma^2_{s,t}$ when incorrectly fitting a short memory volatility process to the data generating process (DGP). A similar interpretation can be placed on $(\sigma^e_{b,t})^2$ and $\sigma^e_{sb,t}$ for the basis and the covariance. Asymptotic approximations for these biases are

$$(11)\quad \begin{aligned} \left(\sigma^e_{s,t}\right)^2 &= \sum_{j=1}^{\infty} \left(j^{-1-d} - p^{-j}\right) \varepsilon^2_{s,t-j}, \\ \left(\sigma^e_{b,t}\right)^2 &= \sum_{j=1}^{\infty} \left(j^{-1-d} - p^{-j}\right) \varepsilon^2_{b,t-j}, \\ \sigma^e_{sb,t} &= \rho_{sb}\sigma^e_{s,t}\sigma^e_{b,t}. \end{aligned}$$

[3]The MVHR is derived as follows. The hedged profit from time $t-1$ to t (π_t), is $\pi_t = (1-\Phi_{t-1})\Delta S_t - \Phi_{t-1}\Delta B_t$. The variance of π_t conditional on the information available at $t-1$ is $\mathrm{Var}_{t-1}(\pi_t) = (1-\Phi_{t-1})^2\mathrm{Var}_{t-1}[\Delta S_t] + \Phi^2_{t-1}\mathrm{Var}_{t-1}[\Delta B_t] + 2(1-\Phi_{t-1})(-\Phi_{t-1})\mathrm{Cov}_{t-1}[\Delta S_t, \Delta B_t]$. The first-order condition with respect to Φ_{t-1} yields the single-period MVHR.

[4]Note that $\sigma^2_b = \sigma^2_s + \sigma^2_f - 2\rho_{sf}\sigma_s\sigma_f$ and $\sigma_{sb} = \rho_{sf}\sigma_s\sigma_f - \sigma^2_s$.

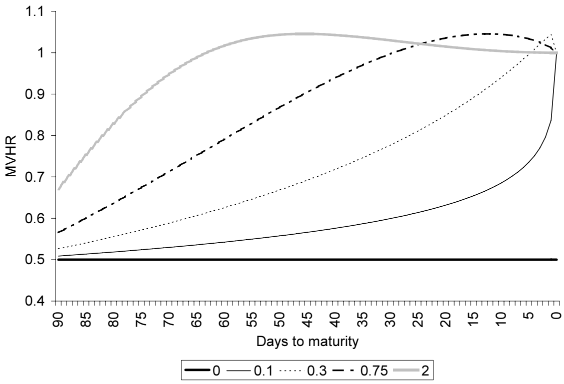
[5]For example, in the GARCH(1, 1) model ($\sigma^2_t = \omega + \alpha\varepsilon^2_{t-1} + \beta\sigma^2_{t-1}$), $p = \beta^{-1}$.

FIGURE 1

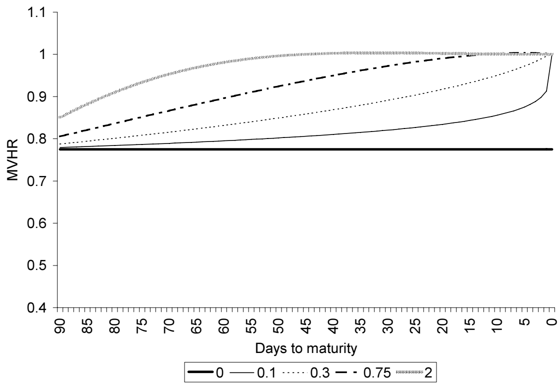
The Evolution of the MVHR for Different Values of λ_2

Figure 1, Graphs A and B display the single-period MVHR (equation (9)) as the futures contract approaches expiration for increasing values of λ_2. Both graphs assume $\sigma_s^2 = 1$ and $\sigma_f^2 = 1.2$.

Graph A. $\rho_{sf} = 0.55$



Graph B. $\rho_{sf} = 0.85$



The bias in the estimated dynamic MVHR (Φ_{t-1}^e) will be

$$\Phi_{t-1}^e = \frac{\left(\sigma_{s,t}^e\right)^2 + \left(\sigma_{sb,t}^e\right) m_t^{\lambda_2}}{\left(\sigma_{s,t}^e\right)^2 + \left(\sigma_{b,t}^e\right)^2 m_t^{2\lambda_2} + 2\left(\sigma_{sb,t}^e\right) m_t^{\lambda_2}}. \tag{12}$$

The direction and magnitude of the bias depend on: i) the ability of the short memory process to approximate the memory decay of the long memory process; ii) the resulting dominance of one lag structure over the other;[6] iii) the evolution of the stochastic processes $\varepsilon_{s,t}$ and $\varepsilon_{b,t}$; and iv) maturity effects captured via m_t and λ_2. Because an analytical solution is mathematically intractable, simulations will examine whether the bias has a significant effect on risk reduction.[7]

[6]It is clear only that for higher order lags, the long memory lags will dominate the short memory lags.

[7]Further, if the DGP is a FIGARCH$(1, d, 1)$ process, a fitted GARCH$(1, 1)$ process will exhibit IGARCH$(1, 1)$ or near IGARCH$(1, 1)$ behavior (Baillie et al. (1996)). If unconditional expectations

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B. Extension to a Multi-Period MVHR

The dynamic MVHR above only seeks to minimize the variability in returns period by period and fails to take into account any interperiod dependencies. A dynamic multi-period MVHR that synthesizes Lee (1999) and Chen et al. (1999) is therefore proposed. Consider a hedger who seeks to minimize the variability in portfolio returns over r periods. The end-of-period wealth W_{t+r-1} is

$$W_{t+r-1} = W_{t-1} + \left(w'r_t + w'r_{t+1} + \ldots + w'r_{t+r-1}\right), \tag{13}$$

where $w' = (1-\Phi, -\Phi)$ and $r_t' = (\Delta S_t, \Delta B_t)$. The hedger seeks to minimize the variability in wealth over the life of the hedge conditional on the information at time $t-1$. The dynamic multi-period MVHR at time $t-1$ equals

$$\Phi_{t-1} = \frac{\sigma^*_{sb,t} + \sigma^{2^*}_{s,t}}{\sigma^{2^*}_{s,t} + \sigma^{2^*}_{b,t} + 2\sigma^*_{sb,t}}, \tag{14}$$

where $\sigma^*_{ij,t}$ is the appropriate element in $\left[\text{vech}(H_t)^{\#} + \ldots + \text{vech}(H_{t+r-1})^{\#}\right]$ and

$$\text{vech}\left(H_t\right)^{\#} = \begin{pmatrix} \text{Var}_{t-1}\left[\dfrac{\Delta S_t}{S_{t-1}}\right] \\ \text{Cov}_{t-1}\left[\dfrac{\Delta S_t}{S_{t-1}}, \dfrac{\Delta B_t}{S_{t-1}}\right] \\ \text{Var}_{t-1}\left[\dfrac{\Delta B_t}{S_{t-1}}\right] \end{pmatrix} = \begin{pmatrix} \sigma^2_{s,t} \\ \sigma_{sb,t} m_t^{\lambda_2} \\ \sigma^2_{b,t} m_t^{2\lambda_2} \end{pmatrix}. \tag{15}$$

The bias in the forecasts from a short memory process is likely to increase as the hedge horizon increases. However, like the single-period MVHR, an analytical solution for the bias in the multi-period MVHR is mathematically intractable.

C. Simulation

To determine whether failing to allow for basis convergence and long memory in volatility has a significant effect on risk reduction, the following DGP is considered

$$\begin{aligned} r_{s,t} &= \varepsilon_{s,t}, \\ r_{b,t} &= a_2 m_t^{\lambda_1} + \varepsilon_{b,t} m_t^{\lambda_2}, \\ \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix} &\sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{s,t} & \sigma_{sb,t} \\ \sigma_{sb,t} & \sigma^2_{b,t} \end{pmatrix}\right], \\ \sigma^2_{i,t} &= \frac{\omega_i}{1-\beta_i} + \left(1 - \frac{(1-L)^{d_i}}{1-\beta_i L}\right)\varepsilon^2_{i,t} \quad i = s, b, \\ \sigma_{sb,t} &= \rho_{sb}\sigma_{s,t}\sigma_{b,t}. \end{aligned} \tag{16}$$

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are taken, $\sum_{j=1}^{\infty}\theta_j = 1$ for both processes. This also says nothing about the dominance of one lag structure over the other (for lower order lags).

A linear convergence in $r_{b,t}$ is also considered (with the remaining components of the model unchanged),

$$r_{b,t} = a_2 + \lambda_1 m_t + \varepsilon_{b,t} m_t^{\lambda_2}. \tag{17}$$

Eleven simulations are performed, each consisting of 200 bivariate samples of 3,000 observations. On rollover, the number of periods to expiration is 75, with rollover to the next contract occurring 10 periods prior to expiration.[8] The first 2,940 observations are used for estimation, the remaining 60 for dynamic ex ante MVHR estimation. The following models are used to forecast single- (equation (9)) and multi-period (equation (14)) ex ante MVHRs:

Model 1: constant correlation (CC) FIGARCH$(1, d, 0)$ with maturity effects (the DGP)—hereafter FIGARCH-mat.[9]

Model 2: CC GARCH(1,1) with maturity effects—hereafter GARCH-mat.[10]

Model 3: CC GARCH(1,1) without maturity effects ($\lambda_1 = \lambda_2 = 0$)—hereafter GARCH.

MVHRs are estimated over 5-, 20-, 40-, and 60-period horizons.[11] All hedges commence 35 periods prior to expiration, except for simulation 10, which commences 13 periods prior to expiration. To consider the impact on risk reduction from failing to allow for long memory in volatility, the portfolio variances from Models 1 and 2 will be compared. To consider the impact on risk reduction from failing to account for basis convergence, the portfolio variances from Models 2 and 3 will be compared. Table 1 provides details of the simulations.

The simulations examine the effects of: i) changes in d (compare simulations 1, 2, 3 or 4, 5, 6); ii) changes in λ_2 (compare simulations 1 and 4, 2 and 5, 3 and 6); iii) nonlinear versus linear convergence (the first six simulations impose nonlinear convergence, the remaining five linear convergence); iv) changes in ρ_{sb} (compare simulations 8 and 9);[12] v) fitting an incorrectly specified long memory volatility model (the DGP for simulation 11 is the HYGARCH$(1, d, 1)$ model);[13] and vi) changing the number of periods between the hedge commencement date and the futures expiration date (compare simulations 8 and 10).[14]

[8]To avoid startup problems, the first 7,000 observations are removed from each replication. The normal random variables $z_t = \varepsilon_t/\sigma_t$ are generated from Ox version 3.2.

[9]The FIGARCH model was proposed by Baillie et al. (1996). Other long memory volatility models include FIEGARCH (Bollerslev and Mikkelsen (1996)), long memory ARCH (Ding and Granger (1996)), and FIAPARCH (Tse (1998)).

[10]This model imposes $\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$ for $i = s, b$ with the remaining components of the model correctly specified.

[11]All models are estimated via the quasi-maximum likelihood estimator (QMLE). See Section III for estimation procedure.

[12]The correlation between the spot and the basis is related to the correlation between the spot and the futures as follows: $\rho_{sb} = (\rho_{sf}\sigma_f - \sigma_s)(\sigma_s^2 + \sigma_f^2 - 2\rho_{sf}\sigma_s\sigma_f)^{-1/2}$.

[13]Davidson (2004) proposes the HYGARCH$(1, d, 1)$ as $\sigma_t^2 = \omega(1-\beta)^{-1} + (1 - (1 - \phi L)[1 + \gamma((1-L)^d - 1)](1-\beta L)^{-1})\varepsilon_t^2$, which equals the FIGARCH$(1, d, 0)$ model if $\gamma = 1$ and $\phi = 0$. The simulation assumes $\phi_s = 0.4$, $\gamma_s = 0.95$, $\phi_b = 0.2$, and $\gamma_b = 0.9$.

[14]The bias and mean square error (MSE) for the parameter estimates indicate that the estimates of λ_1 are unreliable when employing a nonlinear rate of convergence. The estimates of λ_1 for the linear specification are more reliable. For the remaining parameters, the bias and MSE are small and

TABLE 1

Parameter Values Used in Simulations

Table 1 presents the parameter values for the DGP specified in equations (16) to (17). Each simulation generated 200 bivariate samples of 3,000 observations. The first 2,940 observations are used for model estimation, with the remaining 60 observations used for ex ante MVHR estimation. All hedges commence 35 periods prior to expiration, except for simulation 10, which commences 13 periods prior to expiration. The simulations assumed: $a_2 = 0.05$, $\omega_s = \omega_b = 0.1$, and rollover 10 periods prior to expiration.

Simulation	λ_1	λ_2	d_s	β_s	d_b	β_b	ρ_{sb}
Nonlinear Convergence							
1	0.5	0.5	0.25	0.2	0.2	0.15	0.4
2	0.5	0.5	0.45	0.2	0.5	0.3	0.4
3	0.5	0.5	0.7	0.5	0.65	0.4	0.4
4	0.5	0.1	0.25	0.2	0.2	0.15	0.4
5	0.5	0.1	0.45	0.2	0.5	0.3	0.4
6	0.5	0.1	0.7	0.5	0.65	0.4	0.4
Linear Convergence							
7	0.05	0.1	0.7	0.5	0.65	0.4	0.4
8	0.2	0.5	0.45	0.2	0.5	0.3	0.4
9	0.2	0.5	0.45	0.2	0.5	0.3	0.1
Linear Convergence with Hedge Commencement Date Closer to Futures Expiration Date							
10	0.2	0.5	0.45	0.2	0.5	0.3	0.4
Linear Convergence with DGP a Bivariate HYGARCH(1, *d*, 1)							
11	0.2	0.5	0.45	0.3	0.5	0.4	0.4

Figure 2 presents Wilcoxon matched-pairs signed-rank test statistics for differences in portfolio variances between two models.[15] Figure 2, Graphs A and B test the importance of basis convergence when estimating the single- (Graph A) and multi-period (Graph B) MVHRs. Let M^{sp}_{g-gmat} and M^{mp}_{g-gmat} represent the population median difference of the portfolio variance from the GARCH model minus the GARCH-mat model for the single- and multi-period MVHRs, respectively. Figure 2, Graphs A and B present the statistics for a test of $H_0\!:\!M^{sp}_{g-gmat} \leq 0$ against $H_a\!:\!M^{sp}_{g-gmat} > 0$ (Graph A) and $H_0\!:\!M^{mp}_{g-gmat} \leq 0$ against $H_a\!:\!M^{mp}_{g-gmat} > 0$ (Graph B). Positive and significant statistics indicate that failing to allow for basis convergence results in higher portfolio variances. Figure 2, Graphs C and D examine the importance of long memory in volatility when estimating single- (Graph C) and multi-period (Graph D) MVHRs. Let $M^{sp}_{gmat-fmat}$ and $M^{mp}_{gmat-fmat}$ represent the population median difference of the portfolio variance from the GARCH-mat model minus the FIGARCH-mat model for the single- and multi-period MVHRs. Figure 2, Graphs C and D present the results for a test of $H_0\!:\!M^{sp}_{gmat-fmat} \leq 0$ against $H_a\!:\!M^{sp}_{gmat-fmat} > 0$ (Graph C), and $H_0\!:\!M^{mp}_{gmat-fmat} \leq 0$ against $H_a\!:\!M^{mp}_{gmat-fmat} > 0$ (Graph D). Similarly, positive and significant statistics indicate that failing to allow for long memory in volatility results in higher portfolio variances. A further test examines whether the portfolio variances from the

insensitive to the use of a linear or a nonlinear rate of convergence. (Results are available on request.) Further, the in-sample results suggest that as λ_2 and d increase, the importance of capturing basis convergence and long memory in volatility increases. When $\lambda_2 = 0.1$, the GARCH-mat model has a lower SIC than the GARCH model 28% of the time. When $\lambda_2 = 0.5$, the percentage increases to 100%. When $d = 0.25$, the FIGARCH-mat model has a lower SIC than the GARCH-mat model 91% of the time. For higher values of d, this percentage increases to 100%.

[15]The test is whether the population median difference between two matched samples is zero. Here each sample consists of the 200 portfolio variances achieved using a given model. For samples greater than 25, the critical values can be approximated by the normal distribution.

multi-period MVHR are less than the single-period MVHR when using Model 1. These results are available on request.[16]

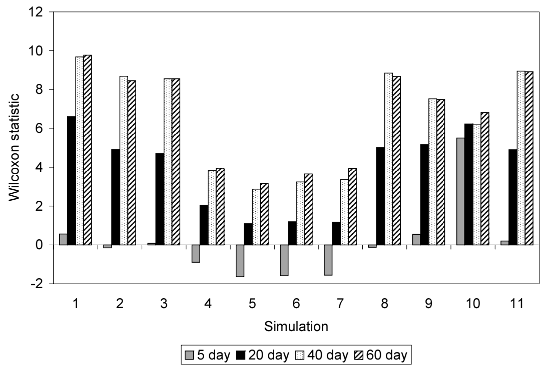
Table 2, Panel A examines the effect of ignoring basis convergence when estimating MVHRs. For illustrative purposes, Table 2 shows simulations 4 and 8 as well as the average across all simulations. The first two columns for each

FIGURE 2

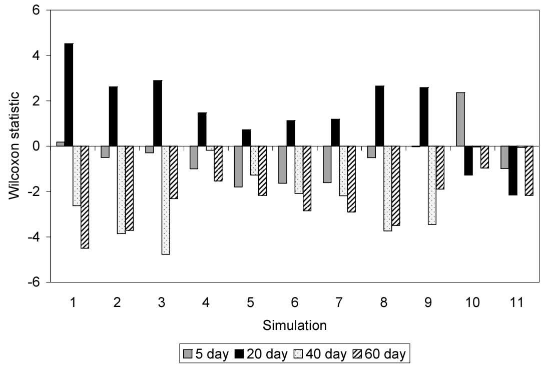
Wilcoxon Statistics for Differences in Portfolio Variances

Figure 2 presents Wilcoxon matched-pairs signed-rank test statistics for differences in portfolio variances between two models. Figure 2, Graphs A and B test for the importance of basis convergence when estimating the single- (Graph A) and multi-period (Graph B) MVHRs. Graphs C and D examine the importance of long memory in volatility when estimating single- (Graph C) and multi-period (Graph D) MVHRs. Each sample consists of 200 portfolio variances achieved using a particular model. Critical values can be approximated via the normal distribution. A positive and statistically significant value supports the alternative hypothesis.

Graph A. Single-Period: $H_0 : M^{sp}_{g-gmat} \leq 0$, $H_a : M^{sp}_{g-gmat} > 0$



Graph B. Multi-Period: $H_0 : M^{mp}_{g-gmat} \leq 0$, $H_a : M^{mp}_{g-gmat} > 0$



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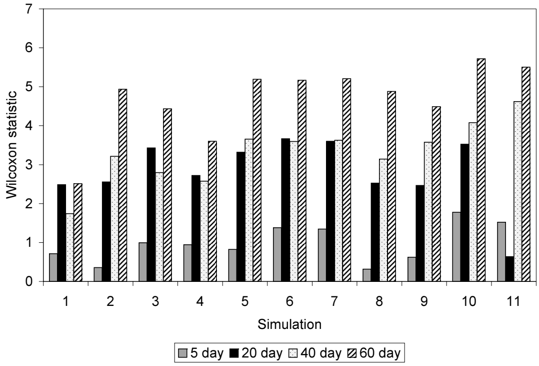
[16]For simulation 11, the FIGARCH$(1, d, 0)$ estimates of β_s are on the lower boundary of zero for virtually all models. The spot equation is therefore changed to FIGARCH$(1, d, 1)$. The results for this specification produce 30 models with ϕ_i or β_i estimates on the lower boundary. The reported statistics are based on the sample of portfolio variances where these results are removed. Exclusion of these results has no influence on the conclusions drawn.

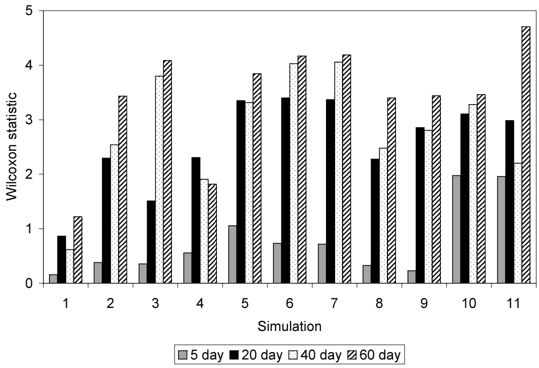
FIGURE 2 (continued)

Wilcoxon Statistics for Differences in Portfolio Variances

Graph C. Single-Period: $H_0 : M^{sp}_{gmat-fmat} \leq 0$, $H_a : M^{sp}_{gmat-fmat} > 0$



Graph D. Multi-Period: $H_0 : M^{mp}_{gmat-fmat} \leq 0$, $H_a : M^{mp}_{gmat-fmat} > 0$



simulation represent the mean and median percentage increase in portfolio variance from failing to allow for basis convergence. The mean/median for each horizon is based on a 200 × 1 vector, where each element is calculated via:

$$\left[\left(\sigma_t^g - \sigma_t^{gmat} \right) / \, \sigma_t^{gmat} \right] \times 100 \quad \text{for} \quad t = 1, \ldots, 200, \tag{18}$$

where $\sigma^g_{(200\times1)} = [\sigma^g_1, \ldots, \sigma^g_{200}]'$ and $\sigma^{gmat}_{(200\times1)} = [\sigma^{gmat}_1, \ldots, \sigma^{gmat}_{200}]'$ represent the portfolio variances for the GARCH and GARCH-mat models, respectively. A positive mean/median that is statistically significant indicates that the GARCH-mat model produces smaller variances on average than the GARCH model. (The statistical significance is determined via the Wilcoxon test statistics in Figure 2, Graphs A and B.) The third column (count %) indicates the percentage of times that higher portfolio variances are achieved if basis convergence is ignored (i.e., the percentage of times that $\sigma_t^{gmat} < \sigma_t^g$ for $t = 1, \ldots, 200$).

Panel B of Table 2 examines the impact of failing to allow for long memory in volatility. The mean/median for each horizon is based on a 200 × 1 vector, where each element is calculated via:

$$\text{(19)} \qquad \left[\left(\sigma_t^{gmat} - \sigma_t^{fmat}\right) \Big/ \sigma_t^{fmat}\right] \times 100 \quad \text{for} \quad t = 1, \ldots, 200,$$

where $\sigma_{(200\times1)}^{fmat} = [\sigma_1^{fmat}, \ldots, \sigma_{200}^{fmat}]'$ represents the portfolio variances for the FIGARCH-mat model. Similarly, a positive mean/median that is statistically significant indicates that the FIGARCH-mat model on average produced a smaller portfolio variance than the GARCH-mat model. The third column (count %) is the percentage of times that higher portfolio variances are achieved if long memory is ignored (i.e., the percentage of times that $\sigma_t^{fmat} < \sigma_t^{gmat}$ for $t = 1, \ldots, 200$).

TABLE 2

Selected Simulation Results

In Panel A, the first two columns for each simulation represent the mean/median percentage increases in portfolio variance from failing to allow for basis convergence (calculated via equation (18)). A positive mean/median indicates that the GARCH-mat model produced on average a smaller portfolio variance than the GARCH model. The third column indicates the % of times that the GARCH-mat model produced a smaller portfolio variance than the GARCH model. In Panel B, the first two columns for each simulation represent the mean/median percentage increases in portfolio variance from failing to allow for long memory in volatility (calculated via equation (19)). A positive mean/median indicates that the FIGARCH-mat model on average produced a smaller portfolio variance than the GARCH-mat model. The third column indicates the % of times that the FIGARCH-mat model produced a smaller portfolio variance than the GARCH-mat model. ** and * indicate that the differences are significant at the 1% and 5% levels, respectively (where the Wilcoxon matched-pairs signed-rank test is employed). # excludes the results from simulation 10. The mean, median, and count % are 44.52%, 20.82%, and 66%, respectively, for the single-period MVHR and 60.44%, 4.64%, and 54.5% for the multi-period MVHR. The remaining results for simulation 10 are comparable to simulation 8 and have been included in the averages.

	Simulation 4			Simulation 8			Average (all simulations)		
Horizon	Mean %	Median %	Count %	Mean %	Median %	Count %	Mean %	Median %	Count %
Panel A. The Impact on Portfolio Variance from Failing to Allow for Basis Convergence									
Single-Period MVHR									
5	−0.10	−0.22	45.00	0.38	−0.07	49.50	0.33#	0.00#	47.85#
20	0.34	0.19*	55.00	3.26	2.69**	67.50	2.44	2.10	62.50
40	0.54	0.45**	59.50	6.20	5.61**	77.50	3.58	3.17	71.18
60	0.35	0.31**	62.00	3.63	3.10**	76.00	2.20	1.86	71.41
Multi-Period MVHR									
5	−0.10	−0.24	46.00	0.74	−0.63	47.50	0.45#	−0.41#	46.30#
20	0.49	0.17	53.50	6.50	4.27**	59.00	3.08	1.58	55.27
40	−0.04	−0.04	48.50	−1.40	−1.96**	38.50	−0.68	−0.90	43.14
60	−0.07	−0.12	44.50	−0.96	−0.77**	38.00	−0.38	−0.54	40.55
Panel B. The Impact on Portfolio Variance from Failing to Allow for Long Memory									
Single-Period MVHR									
5	0.49	0.14	53.00	2.23	−0.05	49.50	1.87	0.29	52.27
20	0.75	0.40**	59.50	1.53	0.87**	60.00	1.40	0.70	59.18
40	0.43	0.24**	58.50	1.09	0.59**	59.50	1.00	0.63	60.41
60	0.42	0.32**	60.00	1.14	0.88**	64.00	0.99	0.76	65.00
Multi-Period MVHR									
5	0.23	0.20	54.00	2.35	−0.31	48.50	2.08	0.16	51.50
20	0.57	0.34*	56.50	1.88	0.91*	57.00	1.78	0.85	59.73
40	0.40	0.25*	59.50	2.88	0.87**	55.00	2.39	0.89	58.36
60	0.49	0.07*	52.50	3.52	1.04**	62.00	3.05	1.01	60.23

When employing the single-period MVHR, the following conclusions with respect to basis convergence are drawn. First, basis convergence becomes more important as the horizon increases. Table 2 shows that for hedges ≥ 20 periods, failing to allow for basis convergence increases the variance on average 63% to

71% of the time, yielding increases in variance averaging between 2% to 4%.[17] Second, basis convergence becomes more important as λ_2 increases. To illustrate, when $\lambda_2 = 0.1$ (simulation 4), for hedges over 60 periods, failing to allow for basis convergence increases the portfolio variance 62% of the time, yielding on average a statistically significant increase in variance of 0.3%. When $\lambda_2 = 0.5$ (simulation 8), both measures increase. Failing to allow for basis convergence increases the portfolio variance 76% of the time, yielding on average a statistically significant increase in variance of 3% to 4%.[18] Third, for hedges over five periods, as the hedge commencement date nears the futures expiration date, basis convergence becomes more important. For the five-period hedges commencing 35 periods prior to expiration, failing to allow for basis convergence does not produce statistically significant differences in portfolio variance. However for the five-period hedge that commenced 13 periods prior to expiration (simulation 10), failing to allow for basis convergence increases the portfolio variance 66% of the time, yielding on average a statistically significant increase in variance of 45%.[19]

When employing the multi-period MVHR, an approach that allows for basis convergence suffers due to a smoothing effect. This effect is problematic when contract rollover occurs during the hedge, and its effect increases as the hedging horizon is lengthened. To illustrate, Figure 3 displays 60-day MVHRs for the FIGARCH-mat model (simulation 2—the 50th sample). The single-period MVHR displays convergence to unity as the futures contract approaches rollover (the 25th period of the hedge). On rollover, the MVHR decreases and converges to unity over the remainder of the hedge. The multi-period MVHR smoothes the impact of basis convergence on the MVHR.

All simulations except for simulation 10 commence 25 periods prior to rollover. Rollover therefore occurs only during the 40- and 60-period hedges (except for simulation 10 where it occurs during all hedges). When hedging over 40 and 60 periods using the multi-period MVHR, allowing for basis convergence increases the portfolio variance on average approximately 60% of the time, yielding an average increase in variance of approximately 1%.[20] This effect is more pronounced when maturity effects are strong. To illustrate, when hedging over 40 periods, with $\lambda_2 = 0.1$ (simulation 4), allowing for basis convergence increases the portfolio variance 51.5% of the time, yielding no statistically significant change in variance. When $\lambda_2 = 0.5$ (simulation 8), both measures increase. Allowing for basis convergence increases the portfolio variance 61.5% of the time, yielding on average a statistically significant increase in portfolio variance of 2%.[21]

[17]Figure 2 reveals that most of these increases are statistically significant.

[18]The test statistics for simulations 4 to 7 (where $\lambda_2 = 0.1$) are also lower than the test statistics for the remaining simulations (where $\lambda_2 = 0.5$). These results are consistent with Figure 1, which shows that the differences between the conventional MVHR and the MVHR allowing for basis convergence increase as λ_2 increases.

[19]Note that 45% is the mean increase, and the median increase is 21%. The mean, median, and count percentages for the remaining horizons are similar to simulation 8. Figure 2, Graph A reveals that all the test statistics for simulation 10 are significant. Further details are available on request.

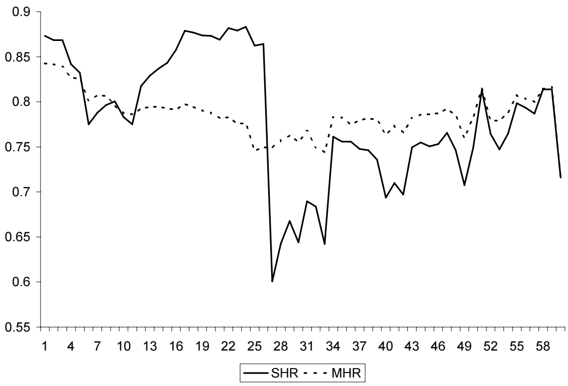
[20]Figure 2 reveals that the test statistics are negative for most hedges ≥ 40 periods.

[21]It should be noted that when employing the multi-period approach over the 20-period horizon, allowing for basis convergence on average produces decreases in portfolio variance (see Figure 2, Graph B and Table 2, Panel A). This is because no 20-period simulations (except simulation 10) experience a contract rollover.

FIGURE 3

60-Period Single- versus Multi-Period MVHR for the FIGARCH-mat Model

Figure 3 presents 60-period single- and multi-period MVHRs for simulation 2 (the 50th sample). Rollover of the futures contract occurs at period 25.



For both the single- and multi-period MVHR, when hedging over five periods, failing to allow for long memory does not produce statistically significant changes in portfolio variance. For hedges $\geq$ 20 periods, failing to allow for long memory increases the variance approximately 60% of the time, yielding statistically significant increases in variance ranging on average between 1% to 3%. These results are insensitive to a linear or nonlinear rate of convergence, the length of the memory, or the fitting of an incorrectly specified long memory volatility model.

III. Application

This section examines the consequences of failing to allow for long memory in volatility and basis convergence when estimating MVHRs on the S&P 500. The data consist of 3,238 daily observations from January 5, 1988 to October 19, 2000. The index data were obtained from IRESS, and the floor settle price for the futures was obtained from the Chicago Mercantile Exchange. To create the price series, only those days where trading occurred in both markets were included. Once all mismatched price observations had been removed, the index return was created as $\Delta S_t/S_{t-1}$ and the normalized change in the basis as $\Delta B_t/S_{t-1}$. (Both series were then multiplied by 100.)[22] The nearby futures contract was employed, with rollover 15 trading days prior to expiration.[23]

[22]On November 3, 1997, the futures multiplier was reduced from $500 to $250 and the minimum tick increased from 0.05 to 0.10. The respecification produced a decrease in the average transaction size but no significant change in volatility or other liquidity and market measures (Karagozoglu, Martell, and Wang (2003)). Nyblom (1989) tests will be used to determine parameter stability.

[23]The conclusions are insensitive to the use of a 10-day rollover.

A. Methodology and Estimation Results

The models in this section extend those in the simulations to allow for varying correlations and dynamics in the conditional mean.[24] The following represents the estimated FIGARCH-mat model (Model 1):

$$(20)\qquad \begin{aligned} \frac{\Delta S_t}{S_{t-1}} &= a_1 + b_1 \frac{\Delta S_{t-1}}{S_{t-2}} + \varepsilon_{s,t} + b_2 \varepsilon_{s,t-1} + b_3 \varepsilon_{s,t-2} + b_4 \varepsilon_{s,t-3} \\ \frac{\Delta B_t}{S_{t-1}} &= a_2 + DI_t + \lambda_1 (m_t) + b_5 \frac{\Delta B_{t-1}}{S_{t-2}} + b_6 \frac{\Delta B_{t-2}}{S_{t-3}} + \varepsilon_{b,t} (m_t)^{\lambda_2} \\ &\quad + b_7 \varepsilon_{b,t-1} (m_{t-1})^{\lambda_2}, \end{aligned}$$

where $I_t = 1$ on contract rollover, 0 otherwise, with[25]

$$(21)\qquad \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{s,t} & \sigma_{sb,t} \\ \sigma_{sb,t} & \sigma^2_{b,t} \end{pmatrix} \right],$$

$$(22)\qquad \sigma^2_{i,t} = \frac{\omega_i}{1-\beta_i} + \left(1 - \frac{(1-\phi_i L)(1-L)^{d_i}}{1-\beta_i L} \right) \varepsilon^2_{i,t} \qquad i = s, b,$$

$$(23)\qquad \rho_{sb,t} = (1 - \theta_1 - \theta_2)\, \rho_{sb} + \theta_1 \rho_{sb,t-1} + \theta_2 \psi_{t-1},$$

$$(24)\qquad \psi_{t-1} = \frac{\sum_{h=1}^{M} z_{s,t-h} z_{b,t-h}}{\sqrt{\left(\sum_{h=1}^{M} z^2_{s,t-h}\right)\left(\sum_{h=1}^{M} z^2_{b,t-h}\right)}},$$

where $\varepsilon_{i,t} = z_{i,t}\sigma_{i,t}$ for $i = s, b$ and $M \geq 2$.[26] The GARCH-mat model (Model 2) replaces equation (22) with GARCH$(1,1)$ conditional variances, and the GARCH model (Model 3) imposes the further restriction $\lambda_1 = \lambda_2 = 0$.[27] All models are

[24]The impact of ρ_{sf} on the importance of basis convergence is therefore dynamic. Nonetheless, the insights from Section II still hold.

[25]To determine the mean specification for the index, univariate ARMA-GARCH$(1,1)$ equations were initially estimated. The autocorrelation function (ACF) and partial autocorrelation function (PACF) for returns, and Box Pierce statistics for the standardized residuals, as well as the SIC were used to determine the most appropriate ARMA specification. The chosen mean specification was then incorporated into the bivariate models estimated in the second stage. In the second stage, to determine the mean specification for the basis, three alternatives were considered. The first employed the nonlinear specification of equation (16), the second employed the linear specification of equation (17), and the third augmented equation (17) with a dummy variable equal to one on rollover and zero otherwise (the selected specification). Similarly, the ACF and Box-Pierce statistics for the standardized residuals as well as the SIC (now for the bivariate model) were used to determine the mean specification for the basis.

[26]The varying correlation approach of Tse and Tsui (2002) is adopted and modified, given that positive definiteness can be imposed analytically. The analytical conditions for positive definiteness in the multivariate time varying correlation FIGARCH specifications of previous research (Teyssiere (1997), (1998), Pafka and Matyas (2001)) have remained elusive. (Brunetti and Gilbert (2000) is the only other paper that has estimated multivariate FIGARCH models; they assume CC.) The use of numerical procedures is inadequate, because there is no guarantee that positive definiteness will hold when forecasting out of sample. To impose positive definiteness, the non-negativity conditions for the FIGARCH$(1,d,1)$ process in Bollerslev and Mikkelsen (1996) are combined with the restrictions $0 \leq \theta_1, \theta_2 \leq 1$, $\theta_1 + \theta_2 \leq 1$ and $-1 \leq \rho \leq 1$ (Tse and Tsui (2002)). To estimate ψ_{t-1}, $M = 3$. When $M = 2$, the GARCH models sometimes failed to achieve strong convergence.

[27]The diagonal GARCH$(1,1)$ model of Bollerslev, Engle, and Wooldridge (1988) was also estimated (with and without maturity effects). Positive definiteness was imposed via the conditions in Silberberg and Pafka (2001). The conclusions are insensitive to the use of this model.

estimated via QMLE. To estimate the FIGARCH-mat model, a truncation lag of 1,000 observations is used, with the pre-sample values equal to the unconditional variance estimate.[28]

The simulations show that when hedging over ≥ 20 periods, failure to allow for basis convergence and long memory in volatility will on average result in higher portfolio variances. Table 2, however, indicates that higher variances will not necessarily apply all the time. The following procedure is therefore adopted: i) estimate Models 1 to 3 from January 5, 1988 to October 12, 1999 and use the models to estimate single- and multi-period dynamic ex ante MVHRs and their portfolio variances over 5-, 20-, 40-, and 60-day horizons; ii) increase the estimation window by 1 day and repeat part (i); and iii) repeat part (ii) another 198 times.

The increase in the estimation window will have little effect on the parameter estimates but will change the distance between the hedge commencement date and the futures expiration date. This will affect the bias in the MVHR (according to the evolution of $\varepsilon_{s,t}$, $\varepsilon_{b,t}$, and m_t—see equation (12)). It will also induce dependence between the 200 portfolio variances for each model, invalidating the Wilcoxon matched-pairs signed-rank test.

Table 3 presents the average parameter estimates and t values for the 200 FIGARCH-mat models. To conserve space, the ARMA and ω_s/ω_b parameter estimates are not presented.[29] The mean estimates for the FIGARCH-mat and GARCH-mat models are comparable and all models exhibit non-normality. For Models 1 and 2, Box Pierce diagnostics for $z_{i,t}$ and $z_{i,t}^2$ are satisfactory (for $i{=}s, b$). Model 3, however, suffers from serial correlation in $z_{b,t}$. Engle and Ng (1993) tests suggest that the index may suffer from asymmetries. Nyblom (1989) tests show that the dummy variable has some evidence of parameter instability, with the remaining parameters for all models generally stable.

Basis convergence is important given that the SIC and likelihood ratio test strongly reject the restriction $\lambda_1 = \lambda_2 = D = 0$. There is an increase in the basis on rollover (captured via D), which decreases over the life of the futures contract (given that $\lambda_1 > 0$). The volatility of the basis and the covariance also decrease as the contract approaches maturity ($\lambda_2 > 0$).[30] The estimates of d are significant, and the SIC is lower for the FIGARCH-mat model than the GARCH-mat model. The correlation parameters are significant and indicate that correlations are time varying and persistent. In summary, SIC statistics, diagnostics, and the significance of d all suggest that the FIGARCH-mat model is the best model in sample.

[28] Baillie et al. (1996) suggest that FIGARCH estimation using QMLE will produce consistent and asymptotically normal estimates. For finite samples, QMLE provides suitable estimates of univariate FIGARCH (Baillie et al. (1996)), multivariate FIGARCH (Pafka and Matyas (2001)) and VC-MGARCH (Tse and Tsui (2002)). It is therefore likely that the QMLE procedure will provide satisfactory estimates.

[29] The model was initially estimated with a FIGARCH$(1, d, 1)$ specification in both equations. A FIGARCH$(1, d, 0)$ specification for the basis was employed, given that ϕ_b was insignificant.

[30] These results are consistent with Castelino and Franses (1982), as well as Chen et al. (1999) who estimate λ_2 for the Nikkei 225 as 0.06.

TABLE 3

Average Parameter Estimates and *t*-Values for the FIGARCH-mat Model

Table 3 presents selected average parameter estimates and *t*-values for the 200 FIGARCH-mat models estimated for the S&P 500 index (see equations (20) to (24)). The first model is estimated using daily data from January 5, 1988 to December 22, 1999. The data set is then increased by one observation and the estimation reperformed. This procedure occurs another 198 times. ** and *indicate significance at the 1% and 5% levels, respectively.

	D	λ_1	λ_2
Mean	0.72**	0.01	0.18**
	(16.73)	(1.46)	(3.65)
	d_s	ϕ_s	β_s
Variance–Index	0.32**	0.31*	0.57**
	(7.38)	(2.47)	(4.13)
	d_b	ϕ_b	β_b
Variance–Basis	0.24**	—	0.10*
	(5.98)		(2.03)
	ρ	θ_1	θ_2
Correlation	0.40**	0.98**	0.01**
	(6.75)	(194.69)	(2.97)

B. Hedging Outcomes

All dynamic MVHRs produce portfolio variances less than the unhedged position and that obtained via the conventional OLS approach. Table 4 examines the effect of failing to allow for basis convergence and long memory in volatility, and is similar to Table 2. The first two columns for Panel A present the mean/median percentage increase in portfolio variance when failing to allow for basis convergence (calculated using equation (18)). The third column (count %) indicates the percentage of times that higher portfolio variances are achieved if basis convergence is ignored. Panel B presents the mean/median percentage increase in portfolio variance when failing to allow for long memory (calculated via equation (19)). The third column indicates the percentage of times that higher variances are achieved if long memory is ignored.

The results are consistent with the simulations. First, basis convergence becomes more important as the horizon is extended. For a five-day single-period MVHR, failing to allow for basis convergence increases the portfolio variance 64% of the time, yielding an average increase in variance of 4%. As the horizon increases to 60 days, failing to allow for basis convergence increases the portfolio variance 95% of the time, yielding an average increase in variance of 6%. A similar result holds for the multi-period MVHR.[31]

Second, for short-term hedges, basis convergence is more likely to be important when the hedge commencement date is close to the futures expiration date. The portfolio variance from the GARCH model minus the portfolio variance from the GARCH-mat model for the single-period MVHR over a five-day horizon re-

[31]For the multi-period MVHR over a five-day horizon, failing to allow for basis convergence increases the portfolio variance 66% of the time. As the horizon increases to 60 days, this percentage increases to 82%, yielding a 1% to 2% increase in portfolio variance. The lower percentage gains and count percentages for the multi-period MVHR are consistent with the smoothing effect discussed above.

TABLE 4

The Impact on Portfolio Variance from Failing to Allow for Basis Convergence and Long Memory in Volatility: S&P 500

In Panel A, the first two columns represent the mean/median percentage increases in portfolio variance from failing to allow for basis convergence (calculated via equation (18)). A positive mean/median indicates that the GARCH-mat model produces on average a smaller portfolio variance than the GARCH model. The third column indicates the % of times that the GARCH-mat model produces a smaller portfolio variance than the GARCH model. In Panel B, the first two columns represent the mean/median percentage increases in portfolio variance from failing to allow for long memory in volatility (calculated via equation (19)). A positive mean/median indicates that the FIGARCH-mat model on average produces a smaller portfolio variance than the GARCH-mat model. The third column indicates the % of times that the FIGARCH-mat model produces a smaller portfolio variance than the GARCH model.

	Panel A. Basis Convergence			*Panel B. Long Memory*		
Horizon	Mean %	Median %	Count %	Mean %	Median %	Count %
Single-Period MVHR						
5	4.44	2.39	63.50	0.87	1.62	53.00
20	4.26	3.14	76.00	0.99	1.35	66.00
40	5.97	5.44	90.00	1.10	1.34	71.50
60	5.56	5.59	94.50	0.76	1.46	74.50
Multi-Period MVHR						
5	3.79	3.01	65.50	0.66	1.20	53.00
20	2.76	2.37	67.50	0.21	0.94	63.50
40	2.22	1.80	73.50	0.23	0.61	59.00
60	1.50	1.31	81.50	0.07	0.55	56.00

veals that basis convergence is important near the rollover dates of November 29, 1999, February 28, 2000, and May 26, 2000.

Third, for the single-period MVHR, the costs from failing to allow for long memory increase with the hedge horizon. For a five-day hedge, failing to allow for long memory increases the portfolio variance 53% of the time. As the horizon increases to 60 days, failing to allow for long memory increases the portfolio variance 75% of the time, yielding on average a 1% to 2% increase in portfolio variance.

Fourth, when implementing the multi-period MVHR, long memory does not appear as important. For hedges over 20 to 60 days, failing to allow for long memory in volatility increases the portfolio variance 56% to 64% of the time, yielding on average an increase in portfolio variance of approximately 0.5% to 1%.[32]

Fifth, the multi-period MVHR suffers from the smoothing of the maturity effect. When using the FIGARCH-mat model, the single-period MVHR outperforms the multi-period MVHR 65% of the time for the 20-day horizons. As the horizon increases to 60 days, the percentage increases to 85% of the time (results available on request). An alternative multi-period MVHR (for example, Lien and Luo (1994)) may be more appropriate and demonstrate even greater benefits from allowing for basis convergence and long memory. This is an area for further research.

Finally, an application to the Australian All Ordinaries Index provides similar results. The FIGARCH-mat model provides the best fit in sample, and long

[32]These results are similar to simulation 4 where long memory is important when employing the single-period MVHR for hedges $\geq$ 20 days, but less important when employing the multi-period MVHR.

memory in volatility and basis convergence become more important as the hedge horizon increases.[33]

IV. Conclusion

This paper shows that when basis convergence occurs over the life of a futures contract and returns follow a long memory volatility process, the standard approaches to estimating dynamic MVHRs are misspecified. Simulations and applications to the S&P 500 and the Australian All Ordinaries Index reveal the magnitude of this misspecification and show that basis convergence and long memory in volatility are important, particularly when hedging over long-term horizons. The results also illustrate that the increases in portfolio variance that result from failing to allow for basis convergence rise as i) the correlation between the spot and the futures decreases; ii) maturity effects increase; and iii) the hedge commencement date approaches the futures expiration date (particularly over short-term horizons). Given the presence of arbitrage and the ubiquitous findings of long memory in financial market volatility, these findings are likely to be of interest in many other markets.

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