Macroeconomic Dynamics, **22**, 2018, 135–152. Printed in the United States of America.
doi:10.1017/S1365100516000535

OPTIMAL INFLATION TARGETING RULE UNDER POSITIVE HAZARD FUNCTIONS FOR PRICE CHANGES

GIOVANNI DI BARTOLOMEO AND MARCO DI PIETRO
Sapienza University of Rome

This paper reconsiders optimal inflation targeting in a model where persistence is generated by rational choices of the price makers because of a time-dependent pricing mechanism. In this framework, which generalizes the traditional Calvo model, inflation persistence is intrinsic, as it is micro-founded assuming that firms' pricing decisions depend on the time elapsed from the last price reset. We use a linear–quadratic approach to study the welfare effects and optimal policies. We disentangled two distortion sources showing how welfare falls in both the average of the probability of changing prices and its distribution among different firms. Described the underlying distortions of our setup, we analyze its normative implications for optimal inflation. The issues of uncertainty and robustness are also considered: By using robust control techniques, we, in fact, consider the consequences of implementing a "wrong" monetary rule due to a misinterpretation of sources of inflation inertia.

Keywords: Time-Dependent Price Adjustments, Intrinsic Inflation Persistence, Optimal Stabilization Policy, Policy Robustness

1. INTRODUCTION

Developments of New Keynesian theory show that modern central bankers should pursue a flexible-inflation targeting to implement optimal policy. Optimal flexible-inflation targeting consists of a commitment to a targeting rule that allows short-run departures of projected inflation from the long-run target rate [Woodford (2003, Chap. 1)]. A very general feature of the optimal targeting is that optimal policies are almost always history-dependent. Hence, an optimal policy rule should react to the economy's recent history of some target variables and not simply to the set of their possible state-contingent paths. The rationale is that by committing to a history-dependent rule, central bankers can "manipulate" private sector expectations to improve the current trade-off they face.

The authors are grateful to two anonymous referees, Barbara Annichiarico, Guido Ascari, Efrem Castelnuovo, Bob Chirinko, Giuseppe Ferrero, Peter McAdam, Salvatore Nisticò, Ricardo Reis, Lorenza Rossi, Massimiliano Tancioni, Patrizio Tirelli, and seminar participants at the Universities of Rome (Sapienza CMC conference), Crete, Manchester, Mannheim (ZEW conference), and Tor Vergata for useful comments on earlier drafts. The authors also acknowledge financial support by Sapienza University of Rome. Address correspondence to: Giovanni Di Bartolomeo, Dipartimento di Economia e Diritto, Sapienza Università di Roma, via del Castro Laurenziano 9, 00161 Rome, Italy; e-mail: giovanni.dibartolomeo@uniroma1.it.

This paper reconsiders the issue of optimal inflation targeting in a model where persistence is generated by a time-dependent pricing model. By deriving the optimal inflation targeting rule in this environment, we aim to answer policy questions such as: What is the degree to which an optimal target criterion should be history-dependent? What sort of additional variables ought to matter in a target criterion in a model with inflation intrinsic (or structural) inertia? In our context, inflation persistence is obtained from the assumption that pricing decisions depend on the time spent from the last price reset. A welfare measure consistent with our framework and optimal targeting policy is obtained by using the linear–quadratic (LQ) approach proposed by Rotemberg and Woodford (1997) and further extended by, among others, Woodford (2002, 2003) and Benigno and Woodford (2005, 2012). Moreover, we use our measure to analyze the peculiar distortions stemming from intrinsic inflation persistence generated by a vintage-dependent pricing mechanism. Our approximation, in fact, makes explicit how welfare costs are generated by time-dependent pricing adjustments. These costs are distinct from those associated with relative price dispersion and consumption fluctuations that appear in the standard New Keynesian model.

We borrow from Sheedy (2007a, 2010) a log-linear economy representation that departs from the standard Calvo mechanism assuming that a firm has a higher probability of changing its price the longer it has kept it unchanged. This price assumption leads to a hybrid time-dependent Phillips curve exhibiting intrinsic inertia à la Fuhrer and Moore (1995), as it also embeds a backward term for inflation. Formally, our pricing model implies a nonnegative hazard function.

Our model presents a richer pricing mechanism, implying the existence of different distortions compared to the Calvo model. Unlike Calvo, where distortions are only related to one feature of the price-setting mechanism, i.e., the probability of resetting a price, in our environment distortions can also be associated with different distributions of the probability of updating a price, since this may be not the same among firms.

We follow Woodford's (2003) approach assuming the presence of an output or employment subsidy that offsets the distortions due to the market power of monopolistically competitive price-setters, so that the steady state under a zero-inflation policy involves an efficient level of output.[1] We consider an approximation around an efficient steady state to compare our results with those of Steinsson (2003) and Giannoni and Woodford (2005), who investigate alternative models with inflation persistence.

Steinsson (2003) considers optimal policies in the Galí and Gertler (1999) setup, where a fraction of the producers in the economy set their prices according to a rule-of-thumb, thus generating structural inflation persistence.[2] The Phillips curve is a convex combination of a forward- and a backward-looking inflation term. As long as the relative importance of the backward term relatively to the forward one increases, he finds that slumps (booms) are more persistent; commitment to an optimal inflation targeting rule implies more gradual policies; losses under optimal policies grow. By contrast, the relative gains of commitment over discretion fall in the degree of inflation inertia. Thus, the relative advantage of manipulating expectations that characterize the commitment decreases.

Giannoni and Woodford (2005) also investigate optimal inflation targeting in a New Keynesian model where inflation persistence is achieved via indexation to past inflation [see Christiano et al. (2005)]. They derive a closed form for the inflation targeting rule and show that optimal deviations from the long-run inflation target should also depend on past inflation. They find that as the degree of inflation inertia increases, optimal policies imply higher variability in inflation.

Another paper closely related to our work is Sheedy (2007b), who considers a very general setup where no specific assumptions about price stickiness are imposed and attempts to derive some general principles about optimal policies, i.e., some robust insights. The aim of Sheedy (2007b) is to look at optimal policy under a general perspective; the case of positive hazard functions for price changes with no differences across sectors is used as an example. We somehow complement his work: In fact, we focus only on positive hazard functions for price changes, investigating the nature of the distortions stemming from this assumption and its implications for optimal targeting policy. Specifically, regarding optimal policies, we differ from him since we investigate the welfare properties of the optimal inflation targeting and the impact on it of different degrees of structural persistence. Furthermore, we consider the relative gains of optimal inflation targeting over alternative policy regimes.[3]

Therefore, we also study the consequences of implementing "wrong" targeting rules due to a misinterpretation of sources of inflation persistence by assuming a considerable amount of uncertainty concerning the correct specification of the model representing the economy. Inspired by Sbordone (2007), we look at the robustness of these targeting policies by using robust control techniques.

The rest of the paper is organized as follows. Section 2 presents the economic setup considered, the derivation of our welfare measure, and explores the nature of macro-distortions induced by our pricing model. Section 3 studies the implication of inflation inertia for the conduct of optimal targeting policies. We analyze how a change in the hazard slope affects the welfare gain of a commitment over discretion. Furthermore, we consider the cost for the Government of misunderstanding the sources of persistence in implementing its optimal policy. The final section concludes.

2. HAZARD FUNCTIONS, WELFARE, AND DISTORTIONS

2.1. The Price-Setting Mechanism

The core of the New Keynesian approach is price stickiness, i.e., a situation in which the nominal price is resistant to change. The most common way to introduce it is the Calvo staggered contracts model,[4] where each firm faces a sort of lottery for resetting price. In every period of time, each firm has a constant probability of being able to change its price. The alternative mechanism we consider here is instead a lottery where the probability of resetting the price, defined by a hazard function, is a function of time. In line of principle, a hazard function can be upward or downward sloping. According to the macro-empirical evidence found in Sheedy (2007a, 2010) and in Di Bartolomeo and Di Pietro (2015),[5] we assume that a firm has a higher probability of updating its price the longer it has kept it unchanged (i.e., positive slope). Now, the firm faces a probability distribution over possible probabilities of resetting conditional to the time elapsed from the last adjustment.

Formally, in order to derive the Phillips curve, we first need to specify the hazard function, which is defined as the event rate at time h conditional on survival until time h or later, where the event is "be able to reset the price." The hazard function can be written as

$$\alpha(h) = \alpha_p + \sum_{j=1}^{\min(h-1,n)} \varphi_p(j) \left\{ \prod_{k=h-j}^{h-1} [1-\alpha(k)] \right\}^{-1}, \tag{1}$$

where $\alpha(h)$ is the probability of changing a price, which was last reset h periods ago, α_p is the initial level of the hazard, $\varphi_p(j)$ is its slope, n is the number of parameters that control the slope (for $n = 1$, the slope is governed by only one parameter), and $\varphi_p(j) = \varphi_p$.

Knowing (1), representative firm i chooses its price to maximize the profit, as it enjoys monopolistic power on the goods market, constrained by the total demand for its product (i.e., $Y_t(i) = (P_t(i)/P_t)^{-\varepsilon_p} Y_t$, where $P_t(i)/P_t$ is the relative price, Y_t is the aggregate demand, and ε_p is the firm's price elasticity). We assume that a representative firm has the following production function: $Y_t(i) = A_t N_t(i)^{1-\delta}$, where labor ($N_t$) is the only factor of production, A_t denotes the technology, and $\delta \in (0, 1)$. Formally, representative firm i solves

$$\max_{P_t^*(i)} \sum_{t=0}^{\infty} \varsigma_t E_0 \beta^t \left\{ P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t - W_t \left[\frac{1}{A_t} \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \right]^{\frac{1}{1-\delta}} \right\}, \tag{2}$$

where $\varsigma_l = \prod_{h=1}^{l} (1-\alpha_h)$ with $\varsigma_0 = 1$ representing the probability that a price remains fixed for l periods (survival function) and β is the discount factor.

By assuming a competitive labor market, under very common assumptions in a New Keynesian framework, the solution of the problem (2) leads to the following log-linear inflation equation[6]:

$$\pi_t = \Psi_1 \pi_{t-1} + \beta \Psi_2 E_t \pi_{t+1} - \beta^2 \Psi_1 E_t \pi_{t+2} + \kappa x_t + u_t, \tag{3}$$

where π_t is the inflation rate, x_t is the output gap, and u_t is an i.i.d. cost push shock. If not differently stated, we indicate deviations from the steady state with

https://doi.org/10.1017/S1365100516000535 Published online by Cambridge University Press

lowercase letters. Coefficients in equation (3) are

$$\begin{cases} \Psi_1 = \dfrac{\varphi_p}{\left(1-\alpha_p\right) - \varphi_p\left[1-\beta\left(1-\alpha_p\right)\right]}, \Psi_2 = 1 + (1-\beta)\,\Psi_1, \\ \kappa = \dfrac{\left(\alpha_p+\varphi_p\right)\left[1-\beta\left(1-\alpha_p\right)+\beta^2\varphi_p\right]}{\left(1-\alpha_p\right)-\varphi_p\left[1-\beta\left(1-\alpha_p\right)\right]}\,\Xi_p, \end{cases}$$

with $\Xi_p = (\sigma + \frac{\gamma+\delta}{1-\delta})\frac{1-\delta}{1-\delta+\delta\varepsilon_p}$, where σ and γ are parameters characterizing the representative consumer's utility function [see equation (4) given later]. It is worth noting that a steeper hazard, given by a higher φ_p, entails a more persistent component of the Phillips curve as Ψ_1 increases.

Equation (3) is our Phillips curve. Coefficient κ measures its slope and also depends on the hazard function parameters (α_p, φ_p). This Phillips curve captures the intrinsic component of inflation inertia, since it has a backward term. The negative coefficient attached to π_{t+2} must be interpreted as a reduction in the total degree of its forwardness. Equation (3) nests the standard New Keynesian Phillips curve as a special case for $\varphi_p = 0$ (flat hazard): Under this case, α_p represents the expected probability of price adjustment.

The underlying intuition of equation (3) can be explained as follows. Observing a temporary shock to the real marginal cost, optimizing firms at t_0 will raise their relative prices.[7] As a result, the general level of prices will also rise. Then, at t_1, optimizing firms can be of two kinds (selection effect): those who have changed their price at t_0 and those who have not. The former need to reduce their relative prices (rollback) and the latter need to increase (catch-up)—since now the shock is vanished, but average relative prices are higher and nominal quantities should be aligned to the new general level of prices. In Calvo's setup, the two opposite effects cancel out. Under a positive hazard function, it is more likely to extract in Calvo's lottery a firm who has not adjusted its price than one who has done it. Therefore, the catch-up effect tends to prevail over the rollback, creating inflation inertia as inflation tends to continue to be positive. This explains the lagged term in equation (3). A symmetric argument explains the negative coefficient associated with $E_t\pi_{t+2}$. The inflation persistence implied by equation (3) is thus structural since we observe it, but the fundamentals explaining inflation themselves exhibit no persistence.

2.2. A Consistent Welfare Measure

The welfare loss derives from a second-order Taylor approximation of the representative household's utility function and mainly depends on the form that takes the price dispersion. The details of the approach developed by Rotemberg and Woodford (1997, 1999) are widely discussed in Woodford (2003, Chap. 6), Galí (2008, Chap. 4), and Benigno and Woodford (2012). As we differ from them mainly for the price dispersion, the derivation of the welfare-based function is not detailed. The reader is referred, e.g., to Galí (2008, Chap. 4) for a complete derivation of the welfare function.

We assume a utility function taking the following separable form:

$$U_t = \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right), \tag{4}$$

where C_t is consumption, and σ and γ are parameters.

A second-order approximation of equation (4) is

$$U_t - U \simeq U_c Y \left(y_t + \frac{1-\sigma}{2} y_t^2 \right) + U_n N \left(n_t + \frac{1+\gamma}{2} n_t^2 \right) + \text{t.i.p.} + \mathcal{O}\left(\left\| \xi^3 \right\| \right), \tag{5}$$

where $\sigma = -\frac{U_{cc}}{U_c} Y$, $\gamma = \frac{U_{nn}}{U_n} N$, U, Y, and N are steady-state values, and t.i.p. denotes the terms independent of policy. Note that to obtain equation (5), we have used the aggregate resource constraint $C_t = Y_t$. Accordingly, the welfare loss can be written as follows:

$$W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon_p}{\Theta} var_i \left\{ p_t(i) \right\} + \left(\sigma + \frac{\gamma+\delta}{1-\delta} \right) x_t^2 \right] + \text{t.i.p.} + \mathcal{O}\left(\left\| \xi^3 \right\| \right). \tag{6}$$

In order to specify our welfare criterion, we need to find an expression that relates $var_i \left\{ p_t(i) \right\}$ to π_t. In our framework, as shown by Sheedy (2007a), the aggregate price level evolves as

$$\log P_t = \sum_{h=0}^{\infty} \theta_h \log P_{t-h}^*, \tag{7}$$

where P_t^* stands for the reset price and θ_h denotes the share of firms using a price that last changed h periods ago. Thus, the price level is an average of past reset prices weighted by the share of firms using such price at time t.

By making use of equation (7) and exploiting the properties of the variance, we can show that the discounted sum of price dispersion evolves in the following way:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\pi_t - \varphi_p \pi_{t-1}\right)^2}{\alpha_p + \varphi_p} - \pi_t^2 \right]}{1 - \beta\left(1 - \alpha_p - \varphi_p\right)} + \mathcal{O}\left(\left\| \xi^3 \right\| \right). \tag{8}$$

The proof is given in the appendix.

We substitute equation (8) into equation (6) and obtain our welfare measure:

$$W = -\Omega \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \Gamma_1 x_t^2 + \Gamma_2 \pi_{t-1}^2 + \Gamma_3 \pi_t \pi_{t-1} \right) + \text{t.i.p.} + \mathcal{O}\left(\left\| \xi^3 \right\| \right), \tag{9}$$

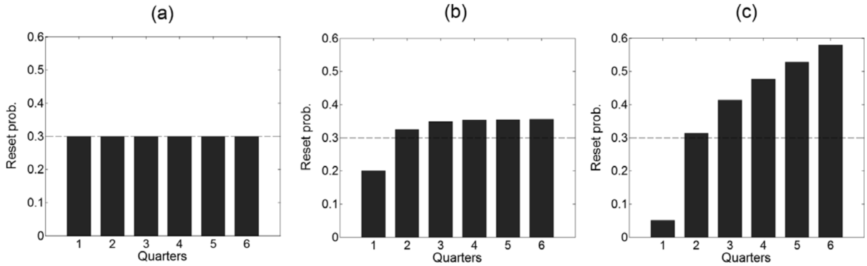

FIGURE 1. Different distributions of the reset probability with the same mean.

where $\Omega = \frac{\varepsilon_p}{\Theta}\frac{1-\alpha_p-\varphi_p}{[1-\beta(1-\alpha_p-\varphi_p)](\alpha_p+\varphi_p)}$, $\Gamma_1 = \frac{1}{\Omega}(\sigma + \frac{\gamma+\delta}{1-\delta})$, $\Gamma_2 = \frac{\varphi_p^2}{1-\alpha_p-\varphi_p}$, and $\Gamma_3 = -2\frac{\varphi_p}{1-\alpha_p-\varphi_p}$.

As for the Phillips curve, our welfare function (9) encompasses that deriving from a Calvo price setting for $\varphi_p = 0$: In such a case, the weight attached to backward inflation drops to zero.

2.3. Averages and Distributions of Reset Probability and Distortions

In the standard Calvo pricing model, the source of distortions is the (average) probability of resetting prices, which is constant among firms at each instant of time. Distortions and price duration are in fact mapped one by one to this probability. In other words, firms are ex ante homogeneous, facing the same probability of being extracted in Calvo's lottery. In our generalization of Calvo (1983), we can, instead, disentangle two sources of distortions stemming from the average and the distribution of the probability of resetting prices. Here, reset probability is not constant, unless the hazard is flat. Firms are ex ante heterogeneous, as each faces a different probability of resetting prices according to the time elapsed since the last reset.[8] Distortions are related to the average probability of resetting prices (as in the Calvo standard model), but now they also depend on how the reset probability is distributed among firms.

A given average probability of resetting prices (i.e., average reset probability, ARP) can be obtained for different slopes of the hazard function (φ_p)—we use the following relationship: ARP $= \alpha_p + \varphi_p$. For instance, we can obtain three different scenarios consistent with ARP $= 0.3$ associated with three different distributions of the reset probability across firms, as illustrated in Figure 1.

All the panels of Figure 1 imply ARP $= 0.3$, but Panel (a) is built by assuming $\varphi_p = 0$ (Calvo price model); in Panel (b), $\varphi_p = 0.1$; in Panel (c), $\varphi_p = 0.25$.

The figure shows that as long as the hazard function becomes steeper, there is more cross-sectional dispersion that implies a low probability of resetting prices for firms that have recently adjusted their prices and a high probability for the firms that set their prices in the far past. It is clear that in Panel (c) there is a greater dispersion in the probability to adjust a price, i.e., there is heterogeneity

https://doi.org/10.1017/S1365100516000535 Published online by Cambridge University Press

TABLE 1. Welfare loss for different price-setting schemes

	Hazard function slope (φ_p)		
Average reset probability (ARP)	0	0.1	0.25
0.4	1.000	1.230	1.647
0.3	1.348	1.685	2.338
0.25	1.509	1.897	2.686

in the resetting probability across firms. Similar figures can be drawn for different average probabilities.

Given the average reset probability, we can refer to scenario (a) as Calvo, scenario (b) as the mid-dispersion case, and scenario (c) as the high-dispersion case. It is worth noting that given the ARP level, Panel (a) shows that the probability of being able to post a new price is constant and equal among all the firms in the Calvo scenario. By contrast, if the dispersion in the reset probability is not zero, given the ARP level, more dispersion implies a lower individual probability for firms that have recently changed their prices and a higher probability for those that did it in the far past to compensate and keep the ARP constant.

Table 1[9] reports the welfare effects associated with several average probabilities with different dispersions. For the sake of comparison, we consider the same monetary policy rule in all cases.[10] We express the welfare effects in the more common form of welfare loss, which are further normalized to the Calvo scenario with a reset probability equal to 0.4 (i.e., our baseline). The table is built on a standard calibration of model parameters. We set the discount factor (β) equal to 0.99. The labor share is $2/3$ so we set $\delta = 0.33$. We assume a net price mark-up of 20% setting $\varepsilon_p = 6$; we consider a log-linear utility function calibrating $\sigma = 1$ and $\gamma = 2$. The process governing the cost push shock is a white noise. Our qualitative results are independent of the calibration.

The table shows that a decrease in the ARP induces more distortions as in the traditional setup. Moreover, here, distortions also increase in the dispersion of the reset probability.

It is well known, in Calvo's setup, that the welfare losses associated with the lower values for ARP are related to the price dispersion. In a flexible-price environment, the optimal pricing rule is always to apply a mark-up over the marginal cost. However, when prices are sticky, firms do not know when they will be able to reoptimize; thus, they must balance the cost to apply a mark-up over the current marginal cost against the benefit of staying close to the profit maximizing mark-up over time following their inflation expectations about the future and reset probabilities. This balance is costly and induces distortions because it implies an ex post heterogeneity and a difference in the relative prices that induces misallocation of resources. Distortions vanish in the zero-inflation long run when all relative prices are equal.

Positive hazard functions imply a sort of price race that enhances the distortions due to price dispersion associated with stickiness described above. By definition, everything equal (i.e., the ARP), positive hazard functions lead to more cross-sectional dispersion in the reset probability. As long as the probability of firms that have recently adjusted falls, the probability of the other firms must rise. After a period of a temporary positive shock to the real marginal cost, firms extracted from Calvo's lottery adjust their relative price—some will lower their prices due to the rollback, whereas others will increase them due to the catch-up. If the rollback and catch-up do not compensate each other, the general level of price will move and some firms will be forced to follow it. Specifically, average price will increase. In the next period, rollbackers need to reduce less their price but catch-up firms need to rise it more because of the increase in the price level. Since the latter are more likely to be extracted, relative prices will take more time to be equalized at the long-run value; therefore, welfare costs are magnified.

The next subsection investigates how these two features of the price-setting process affect the conduct of monetary policy under different assumptions about the policy regime (discretion and commitment), and how they affect the relative gains of the latter over the former.

3. INFLATION TARGETING AND INTRINSIC INERTIA

3.1. Optimal Targeting Rule

The general Government's problem consists of maximizing (9) subject to (3), conditional to the policy regime. Formally, the Government's problem at some point of time t can be expressed as a minimization of the following Lagrangian expression:

$$\min_{\{\pi_{t+j}\}_{j=0}^{+\infty},\,\{x_{t+j}\}_{j=0}^{+\infty}} E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{2}\left(\pi_{t+j}^2 + \Gamma_1 x_{t+j}^2 + \Gamma_2 \pi_{t+j-1}^2 + \Gamma_3 \pi_{t+j}\pi_{t+j-1}\right) + \lambda_{t+j}\left(\pi_{t+j} - \Psi_1 \pi_{t+j-1} - \beta\Psi_2 \pi_{t+j+1} + \beta^2 \Psi_1 \pi_{t+j+2} - \kappa x_{t+j} - u_{t+j}\right)\right], \tag{10}$$

where λ_{t+j} is the Lagrangian multiplier.

By solving the above problem for the "timeless perspective" regime [Woodford (2003)], at a generic time t, we obtain the following optimal flexible targeting rule:

$$\pi_t = -\frac{\Gamma_1}{\kappa(1+\Gamma_2\beta)} \times \left[x_t - \Psi_1 \beta E_t x_{t+1} - \Psi_2 x_{t-1} + \Psi_1 x_{t-2} + \frac{\Gamma_3}{2}\frac{\kappa}{\Gamma_1}\left(\pi_{t-1} + \beta E_t \pi_{t+1}\right)\right]. \tag{11}$$

https://doi.org/10.1017/S1365100516000535 Published online by Cambridge University Press

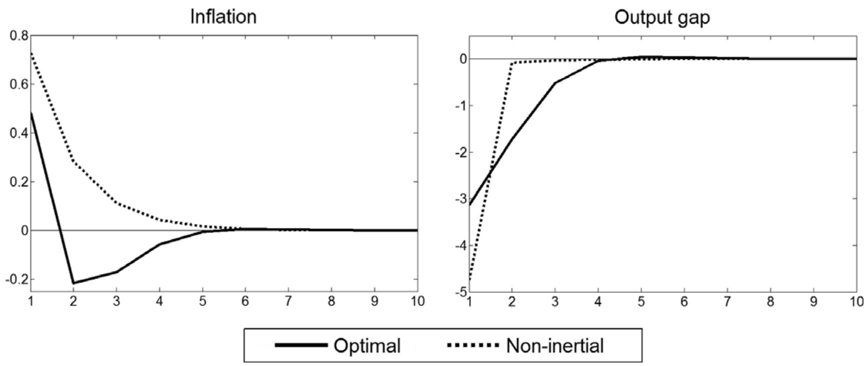

FIGURE 2. Optimal response to a cost push shock (high-dispersion scenario).

Optimal policy imposes to deviate from the long-run inflation target by considering past values of output and inflation and expected inflation.

As usual, optimal targeting under the timeless perspective regime is characterized by the fact that the Government, by reacting to the past output gap, affects current inflation expectations and obtains a more favorable trade-off between inflation and output at the cost of a longer stabilization (solving the so-called stabilization bias problem). Compared to the standard textbook case, our Phillips curve (3) implies a trade-off between inflation and output gap, which also depends on lagged inflation (π_{t-1}) and an additional forward term ($E_t\pi_{t+2}$). Thus, in order to manipulate expectations, the Government now finds it optimal to react also to the expected future output gap ($E_t x_{t+1}$) and its past value for an additional lag (x_{t-2}). Moreover, our optimal inflation targeting rule calls for responding also to both a backward and an expected value of inflation. If $\varphi_p = 0$, Ψ_1 drops to zero and the textbook case is obtained—as both lagged inflation and the additional forward term no longer affect the Phillips curve trade-off.

In order to implement optimal policy, as argued by Svensson and Woodford (2005), a history-dependent target criterion is necessary. In fact, no target criterion that involves only the projected paths of the target variables from the present time onward is able to determine an equilibrium with the optimal responses to disturbances.

3.2. The Gains of Commitment

We study the effects of optimal targeting by numerical methods. Our simulation is performed by using the calibration reported in Section 2.3.

We illustrate the optimal targeting policy in Figure 2, which plots the impulse response functions to a one-standard-deviation cost push innovation in the high-dispersion scenario, assuming an average reset probability equal to 0.3.[11] We plot the dynamic responses for both the optimal targeting rule (11) and a discretionary policy (noninertial rule).

https://doi.org/10.1017/S1365100516000535 Published online by Cambridge University Press

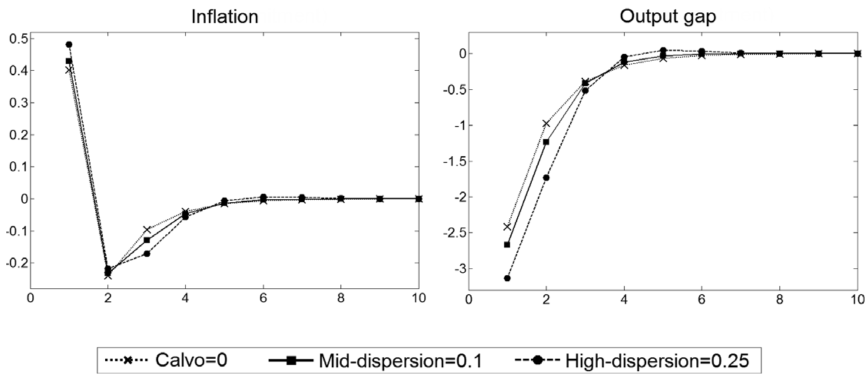

FIGURE 3. Inflation and output gap equilibrium path under an optimal inflation targeting rule.

It is clear from Figure 2 that when an optimal inflation targeting rule is implemented (solid line), the variability of both inflation and output gap is reduced with respect to the case in which monetary policy is conducted according to a noninertial policy rule (dotted line): This entails smaller welfare losses. Under an optimal targeting rule, inflation initially jumps and then undershoots converging progressively to its steady state. Looking at the output gap equilibrium path, it remains below zero for a time and exhibits a gradual convergence to its long-run value. What we observe is that an optimal targeting policy involves a history-dependent behavior: The optimal rule should consider both current and past values of the target variables (in our case inflation and output gap).

In Figure 3, we consider the impulse response functions associated with optimal inflation targeting for different degrees of inertia. Specifically, we consider the three scenarios discussed in Section 2.3 (i.e., Calvo, mid-dispersion, and high dispersion under a common average reset probability equal to 0.3).

The path followed by inflation and output gap is the same in each scenario. They differ only on a quantitative point of view. A greater dispersion in the reset probability leads to a larger variability in both inflation and output gap, and, as a consequence, higher losses. As inflation becomes more persistent, the deflation experienced by the economy is harder and involves a larger fall in the output gap that returns to its steady state more slowly. In other words, in the presence of a greater dispersion in the reset probability, it is more difficult to stabilize the economy. The intuition of the result is twofold. As explained, this is due to the existence of inflation persistence, which makes a disinflation more costly. Moreover, in our setup, a dispersion in the reset probability also leads to additional stabilization costs, since manipulation of expectations becomes more difficult when dispersion increases because of the presence of two expectation terms in the Phillips curve (with opposite signs).

Following Steinsson (2003), it is interesting to compare the relative performance of an optimal inflation targeting with respect to a noninertial discretionary

TABLE 2. Gains from commitment and inflation inertia

Hazard function slope (φ_p)	% gain from commitment
0	31.2
0.05	34.4
0.10	38.0
0.15	42.0
0.20	46.5
0.25	51.5
0.30	55.8

rule for different degrees of inertia. Table 2 shows the relative gains of an optimal commitment on discretion—computed according to the utility-based welfare maximization. We consider different slopes for the hazard function starting from Calvo ($\varphi_p = 0$) to the high-dispersion scenario ($\varphi_p = 0.3$). In all cases, we keep fixed the average probability of changing prices (ARP) at 0.3. It is worth remembering that the inflation intrinsic persistence grows with φ_p. Thus, the table reports the percentage gains of an optimal inflation targeting for different degrees of inflation inertia.

The greater the φ_p, the higher the relative gain of commitment. This result differs from Steinsson (2003), who finds that the gain falls in the degree of inflation persistence. Indeed, he finds some increasing gains for low degrees of inertia. Gains of commitment are mainly related to the Government ability to manipulate expectations in order to improve the inflation–output gap trade-off.

In Steinsson (2003), inflation inertia is introduced by a simple rule-of-thumb, and results come out from the fact that as the backward component of the Phillips curve increases, the forward-looking one falls, reducing the gain from commitment. By contrast, in our model, inflation inertia is driven by the slope of the hazard function and grows with the dispersion of the reset probability among firms. Optimizing firms know that, in the period immediately after their resets, they will be very unlikely to change their prices again [see Figure 1(c)]. Thus, in choosing a new price, each firm gives much importance to the future state of the economy, when the price will probably be allowed to reset again. As a consequence, commitment gains are higher in a world with high dispersion as expectations on the future state of the economy matter more. Thus, here, implementing a policy rule that is able to influence private sector expectations may lead to an improvement of the inflation–output gap trade-off.

The relative effects of inflation inertia on the optimal policy regimes are thus not general, but they depend on the way intrinsic inflation persistence is introduced.

3.3. Targeting and Misinterpretation of Persistence Sources

Now, we assume model uncertainty and consider the cost of misunderstanding the true sources of inflation persistence. In the spirit of Sbordone (2007), we compare the dynamics of implementing a stabilization policy when the policy maker overestimates the degree of intrinsic persistence with those arising from ignoring actual structural persistence. At the end of the section, by using robust control techniques (specifically, Savage's *minimax* regret criterion), we look at robust policies.[12]

We consider two specifications, in both setting ARP = 0.3. The first specification is a purely forward-looking New Keynesian Phillips curve ($\varphi_p = 0$), in which persistence is extrinsic, i.e., it depends on the autocorrelation of the cost push shock. The second is the high-dispersion scenario ($\varphi_p = 0.25$) that endogenously generates (intrinsic) inflation persistence by way of a lagged inflation term in the Phillips curve.

In each specification, we compute the two welfare losses associated with a conduct for monetary policy, where the Government attributes the observed persistence to an extrinsic or intrinsic nature. Of course, for each specification, only one policy rule is "correct," whereas the other is based on a misinterpretation of the source of inflation persistence. Formally, the two rules are as follows:

$$\pi_t = -\frac{\Gamma_1}{\kappa}\left(x_t - \Psi_2 x_{t-1}\right), \tag{12}$$

$$\pi_t = -\frac{\Gamma_1}{\kappa(1+\Gamma_2\beta)} \times \left[x_t - \Psi_1\beta E_t x_{t+1} - \Psi_2 x_{t-1} + \Psi_1 x_{t-2} + \frac{\kappa\Gamma_3}{2\Gamma_1}\left(\pi_{t-1} + \beta E_t \pi_{t+1}\right)\right], \tag{13}$$

where equation (12) is the optimal commitment for the standard Calvo case (extrinsic persistence, i.e., $\varphi_p = 0$) and equation (13) denotes the optimal inflation targeting rule followed when inertia is intrinsic, i.e., $\varphi_p = 0.25$. Formally, we want to investigate whether there are welfare losses in acting following equation (13) when inflation is characterized by intrinsic inertia, and we repeat the same exercise by implementing (12) in a context where inertia is driven by the serial correlation of the shock (i.e., it is extrinsic).

Losses in the two specifications under different policy rules are presented in Table 3. In the last row of the table, we also compute the percentage loss deriving from the implementation of an optimal stabilization policy under wrong policymaker beliefs about the nature of inertia.[13]

Central bankers' misunderstandings are always costly. Conducting a stabilization policy under a wrong belief about the true source of inertia always entails welfare worsening due to an inefficient management of expectations.[15] Table 3 illustrates how misunderstanding the true source of persistence implies an additional loss in both specifications. In percent terms, the additional losses are about 0.3% when the true model is a purely forward-looking Phillips (i.e., extrinsic inertia) and 0.2% when inflation persistence is intrinsic. Ignoring intrinsic inflation inertia leads to a smaller additional loss.

TABLE 3. Welfare loss for different targeting rules[14]

	True source of inflation persistence	
	Extrinsic ($\varphi_p = 0$)	Intrinsic ($\varphi_p = 0.25$)
Targeting policy:		
Rule (12)	1.000	1.259
Rule (13)	1.003	1.256
Loss difference (%)	0.3	0.2

TABLE 4. Regret for different targeting rules

	True source of inflation persistence	
	Extrinsic($\varphi_p = 0$)	Intrinsic ($\varphi_p = 0.25$)
Targeting policy:		
Rule (12)	0	0.0025
Rule (13)	0.0029	0

Both misunderstandings lead to some costs, so in a context of model uncertainty, it is interesting to consider policy robustness. In a framework of our kind, as emphasized by Brock et al. (2007) and Sbordone (2007), a good robustness criterion is to select from among different policy rules by using the *minimax* regret criterion developed by Savage (1951).[16] The regret is the difference between the loss incurred by implementing certain policy in a certain given specification and the loss obtained by using the optimal policy in that given specification. Robust policy is derived from Table 4, where we report the regret for the two different rules (policies) in our two alternative specifications of the economy. The robust policy is then chosen by applying Savage's criterion.

The table confirms the results found in Table 3. Making a mistake about the origin of inflation inertia always involves incremental costs: A regret of 0.0029 arises when persistence is entirely generated by a serially correlated shock, whereas the regret is 0.0025 when the persistence is due to a backward component in the Phillips curve. The regret is minimized by the rule (12). Thus, under model uncertainty, the policy maker should always conduct monetary policy by implementing a targeting rule derived ignoring intrinsic inertia.

The intuition of our result is as follows. Now, the central bank faces uncertainty on the effects of targeting on the economy since the true model is unknown. If the true source of persistence is extrinsic, but the central bank assumes that it is intrinsic, too loose policies compared to the optimal one will be implemented. By contrast, when the central bank wrongly assumes that the true source of persistence is structural, monetary policy is too aggressive to inflation. In an LQ context of model uncertainty,[17] the effects of "wrong" policies are not symmetric: Too aggressive policies are more costly; hence, robust policy actions should be more cautious [Brainard (1967)]. In our case, when the central bank misunderstands the true model of the economy, the overreaction in the case of intrinsic persistence costs more than the underreaction in the case of extrinsic persistence. As a result, if persistence is driven by structural factors, the costs of a wrong policy are higher. Thus, under model uncertainty, robust policy implies to ignore extrinsic persistence.

4. CONCLUSIONS

In this paper, we reconsider optimal inflation targeting in a model with intrinsic inflation persistence by using Rotemberg and Woodford's (1997) LQ approach. Inflation inertia was micro-founded assuming that the probability to update a price for each firm is a function of time; specifically, in our model, firms' pricing decisions depend on the time spent from the last price reset. By our welfare approximation, we explored the nature of macro-distortions induced by our pricing model and its implications for the optimal monetary policy.

By generalizing standard Calvo's price mechanism, we disentangled two sources of distortions by considering both the average (as in Calvo) and the cross-distribution across firms of the probability of resetting prices. We showed that, in addition to the welfare costs due to a higher average, further welfare losses are generated by increases in the dispersion of the probability of resetting prices.

After computing optimal inflation targeting policy, we found that the effects of inflation inertia on the optimal policy are different from those stemming from alternative models of inflation persistence based on rule-of-thumb assumptions or indexation mechanisms. In our setup, the relative gain of commitment over discretion increases in the degree of the inflation persistence as—once intrinsic inflation is introduced by a positive hazard function—the interaction between the different components of the Phillips curve becomes complex. Therefore, relative effects of inflation inertia on the optimal policy regimes are at least not general, but they depend on the way intrinsic inflation persistence is introduced.

Assuming model uncertainty, we finally analyzed the effects on optimal monetary policy of misinterpreting the sources of inflation inertia, which can be either intrinsic (driven by a backward component of the Phillips curve) or extrinsic (driven by the shock autocorrelation). Implementing the wrong rule is always costly, whatever the right model is. Moreover, using robust control techniques, we found that robust policy requires to ignore intrinsic persistence and to implement a simple textbook timeless perspective policy. The result comes from general Brainard's cautious principle.

NOTES

1. The approach can be, however, generalized to the case of a distorted steady state [Benigno and Woodford (2012)].
2. See also Amato and Laubach (2003).

3. It is worth noticing that, although Sheedy (2007b) does not provide welfare comparison under different regimes, he derives alternative policy rules. Among these, he computes a sort of discretionary optimal policy. This rule, labeled "Markovian," is obtained by ignoring the effects of the policy on lagged terms (see example 2 in his paper). We instead compare the effects of optimal inflation targeting to the traditional discretionary policy obtained by the solution of the Bellman equation as in Walsh (2003, Section 8.4.5). The two discretionary policies do not lead to the same results (results are available upon request).

4. Common alternatives include the Rotemberg (1982) price model, sticky information, and state-dependent models. See Walsh's (2010, Chap. 6) textbook for details.

5. Di Bartolomeo and Di Pietro (2015) also check the stability of the slope with respect to policy regime changes. They conclude that the positive hazard slope is regime invariant. See Benati (2008, 2009) for a general discussion on the point.

6. The reader is referred to Sheedy (2007a) for the complete derivation. For the sake of simplicity, we assume $n = 1$. An alternative, but equivalent, derivation is described in Sheedy (2010).

7. Although they will not fully exploit the mark-up over the marginal cost since they expect that tomorrow the shock will vanish, but they do not known if they will be able to reset the price again.

8. Of course, two firms who have reset at the same time have the same probability to readjust.

9. The table is built by considering a transitory cost push shock. Thus, it reports conditional welfare.

10. We consider a policy consistent with a Taylor rule, which only responds to current inflation by a coefficient equal to 1.5. The same results hold if optimal policies are instead considered. More details are available upon request.

11. Simulations are performed by using Söderlind's (1999) toolkit.

12. For a review of the issue, see Hansen and Sargent (2011) and references therein.

13. In the online Appendix, we provide the inflation and output gap IRFs to a cost push shock under model uncertainty.

14. Losses are normalized to the case of correct policy without inflation structural persistence (where the welfare loss is the smallest). An alternative consists of expressing all of them in consumption equivalent terms. The loss difference measures the additional cost of implementing the wrong policy.

15. It is worth noting that the result is not trivial since optimal timeless rules can be dominated by other inertial rules. See Blake (2001) or Jensen and McCallum (2002) for a discussion.

16. See Stoye (2011) for its axiomatization. A brief explanation of this criterion is provided in the online appendix.

17. Effects can be modified in more complicated models.

REFERENCES

Amato, Jeffery D. and Thomas Laubach (2003) Rule-of-thumb behavior and monetary policy. *European Economic Review* 47(5), 791–831.

Benati, Luca (2008) Investigating inflation persistence across monetary regimes. *Quarterly Journal of Economics* 123(3), 1005–1060.

Benati, L. (2009) Are "Intrinsic Inflation Persistence" Models Structural in the Sense of Lucas (1976)? ECB working paper series no. 1038.

Benigno, Pierpaolo and Michael Woodford (2005) Inflation stabilization and welfare: The case of a distorted steady state. *Journal of the European Economic Association* 3(6), 1185–1236.

Benigno, Pierpaolo and Michael Woodford (2012) Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory* 147(1), 1–42.

Blake, Andrew P. (2001) A "Timeless Perspective" on Optimality in Forward-Looking Rational Expectations Models. NIESR discussion papers no. 188.

Brainard, William C. (1967) Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411–425.

Brock, William A., Steven N. Durlauf, James M. Nason, and Giacomo Rondina (2007) Simple versus optimal rules as guides to policy. *Journal of Monetary Economics* 54(5), 1372–1396.

Calvo, Guillermo A. (1983) Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.

Di Bartolomeo, G. and M. Di Pietro (2015) Intrinsic Persistence of Wage Inflation in New Keynesian Models of the Business Cycles. Mimeo, Department of Economics and Law, Sapienza University of Rome.

Fuhrer, Jeffrey C. and George Moore (1995) Inflation persistence. *Quarterly Journal of Economics* 109(2), 127–159.

Galí, Jordi (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton, NJ: Princeton University Press.

Galí, Jordi and Mark Gertler (1999) Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44(2), 195–222.

Giannoni, Marc P. and Michael Woodford (2005) Optimal inflation targeting rules. In B. S. Bernanke and M. Woodford (eds.), *The Inflation Targeting Debate*, pp. 93–172. Chicago, IL: University of Chicago Press.

Hansen, Lars P. and Thomas J. Sargent (2011) Wanting robustness in macroeconomics. In B.M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics*, vol. 3B, pp. 1097–1157. Amsterdam: Elsevier.

Jensen, Christian and Bennett T. McCallum (2002) The non-optimality of proposed monetary policy rules under timeless perspective commitment. *Economics Letters* 77(2), 163–168.

Rotemberg, Julio J. (1982) Sticky prices in the United States. *Journal of Political Economy* 90(6), 1187–1211.

Rotemberg, Julio J. and Michael Woodford (1997) An optimization-based econometric framework for the evaluation of monetary policy. *NBER Macroeconomic Annual* 1997, 297–361.

Rotemberg, Julio J. and Michael Woodford (1999) Interest rate rules in an estimated sticky price model. In J.B. Taylor (ed.), *Monetary Policy Rules*, pp. 57–119. Chicago, IL: The University of Chicago Press.

Savage, Leonard J. (1951) The theory of statistical decision. *Journal of the American Statistical Association* 46(253), 55–67.

Sbordone, Argia (2007) Inflation persistence: Alternative interpretations and policy implications. *Journal of Monetary Economics* 54(5), 1311–1339.

Sheedy, Kevin D. (2007a) Intrinsic Inflation Persistence. Centre for Economic Performance discussion paper no. 837.

Sheedy, Kevin D. (2007b) Robustly Optimal Monetary Policy. Centre for Economic Performance discussion paper no. 840.

Sheedy, Kevin D. (2010) Intrinsic inflation persistence. *Journal of Monetary Economics* 57(8), 1049–1061.

Söderlind, Paul (1999) Solution and estimation of RE macromodels with optimal policy. *European Economic Review* 43(4–6), 813–823

Steinsson, Jon (2003) Optimal monetary policy in an economy with inflation persistence. *Journal of Monetary Economics* 50, 1425–1456.

Stoye, Jörg (2011) Statistical decisions under ambiguity. *Theory and Decision* 70(2), 129–148.

Svensson, Lars E. O. and Michael Woodford (2005) Implementing optimal policy through inflation-forecast targeting. In B. S. Bernanke and M. Woodford (eds.), *The Inflation-Targeting Debate*, pp. 19–92. Chicago, IL: University of Chicago Press.

Walsh, C. E. (2010), *Monetary Theory and Policy*. Cambridge, MA: MIT Press.

Woodford, Michael (1999) Commentary: How should monetary policy be conducted in an era of price stability? New Challenges for Monetary Policy, Federal Reserve Bank of Kansas City, 277–316.

Woodford, Michael (2002) Inflation stabilization and welfare. *B.E. Journal of Macroeconomics* 2(1), 1–53.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

APPENDIX

The welfare function can be derived in three steps. We can write the cross-sectional mean and variance as $\bar{P}_t \equiv E_i p_t(i)$ and $\Delta_t^p = var_i [p_t(i)]$, where we define $p_t(i) = \log P_t(i)$.

1. As shown by Sheedy (2007a), the aggregate price level evolves according to equation (7) and thus the cross-sectional mean can be written as

$$\bar{P}_t = E_i [p_t(i)] = \left(1-\alpha_p\right)[E_i p_{t-1}(i)] - \varphi_p [E_i p_{t-2}(i)] + \left(\alpha_p + \varphi_p\right) p_t^*, \qquad \textbf{(A.1)}$$

from which we obtain

$$p_t^* - \bar{P}_{t-1} = \frac{\pi_t - \varphi_p \pi_{t-1}}{\alpha_p + \varphi_p}. \qquad \textbf{(A.2)}$$

2. The cross-sectional variance is

$$\begin{aligned} \Delta_t &= var_i \left[p_t(i) - \bar{P}_{t-1}\right] \\ &= E_i \left\{\left[p_t(i) - \bar{P}_{t-1}\right]^2\right\} - \left\{E_i \left[p_t(i) - \bar{P}_{t-1}\right]\right\}^2 \\ &= E_i \left\{\left[p_t(i) - \bar{P}_{t-1}\right]^2\right\} - \pi_t^2, \end{aligned} \qquad \textbf{(A.3)}$$

where, by equation (A.1), the first right-hand-side term is

$$\begin{aligned} E_i \left\{\left[p_t(i) - \bar{P}_{t-1}\right]^2\right\} &= \left(1-\alpha_p\right) E_i \left\{\left[p_{t-1}(i) - \bar{P}_{t-1}\right]^2\right\} - \varphi_p E_i \left\{\left[p_{t-2}(i) - \bar{P}_{t-1}\right]^2\right\} \\ &+ \left(\alpha_p + \varphi_p\right)\left[p_t^* - \bar{P}_{t-1}\right]^2. \end{aligned} \qquad \textbf{(A.4)}$$

By using $\Delta_{t-1} = E_i \left\{\left[p_{t-1}(i) - \bar{P}_{t-1}\right]^2\right\}$, $\Delta_{t-2} = E_i \left\{\left[p_{t-2}(i) - \bar{P}_{t-1}\right]^2\right\}$, and plugging (A.2) and (A.4) into (A.3), we obtain

$$\Delta_t = \left(1-\alpha_p\right)\Delta_{t-1} - \varphi_p \Delta_{t-2} + \frac{\left(\pi_t - \varphi_p \pi_{t-1}\right)^2}{\alpha_p + \varphi_p} - \pi_t^2 + \mathcal{O}\left(\left\|\xi^3\right\|\right). \qquad \textbf{(A.5)}$$

3. Then, integrating (A.5) and by discounting over all periods $t \geq 0$, we obtain equation (8), i.e.,

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{1}{1-\beta\left(1-\alpha_p-\varphi_p\right)} \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\pi_t - \varphi_p \pi_{t-1}\right)^2}{\alpha_p + \varphi_p} - \pi_t^2\right] + \mathcal{O}\left(\left\|\xi^3\right\|\right). \qquad \textbf{(A.6)}$$

Finally, substituting equation (A.6) into equation (6) gives an expression for the welfare

$$\begin{aligned} W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t &\left[\frac{\varepsilon_p}{\Theta} \frac{\left(\alpha_p + \varphi_p\right)^{-1}\left(\pi_t - \varphi_p \pi_{t-1}\right)^2 - \pi_t^2}{1-\beta\left(1-\alpha_p-\varphi_p\right)} + \left(\sigma + \frac{\gamma+\delta}{1-\delta}\right) x_t^2\right] \\ &+ \text{t.i.p.} + \mathcal{O}\left(\left\|\xi^3\right\|\right). \end{aligned} \qquad \textbf{(A.7)}$$

https://doi.org/10.1017/S1365100516000535 Published online by Cambridge University Press