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The Dividend Term Structure

Jac Kragt, Frank de Jong, and Joost Driessen*

Abstract

We estimate a model for the term structure of discounted risk-adjusted dividend growth using prices of dividend futures for the Eurostoxx 50. A 2-factor model capturing short-term mean reversion within a year and a medium-term component reverting at the business-cycle horizon gives an excellent fit of these prices. Hence, investors update the valuation of dividends beyond the business cycle only to a limited degree. The 2-factor model, estimated on dividend futures data only, explains a large part of observed daily stock market returns. We also show that the 2 latent factors are related to various economic and financial variables.

I. Introduction

> Since the level of the market index must be consistent with the prices of the future dividend flows, the relation between these will serve to reveal the implicit assumptions that the market is making in arriving at its valuation. These assumptions will then be the focus of analysis and debate. (Brennan (1998), p. 14)

Dividends are a key ingredient for valuing stocks. Investors attach a present value to expected dividends and sum them to arrive at the value of a stock. As Campbell and Shiller (1988) have shown, stock prices thus vary because of changes in expected dividends, changes in interest rates, and changes in risk premiums. However, these elements may be horizon dependent. For interest rates, this is obvious because they can be readily observed. But also the expectations of dividends paid in the short run may at least partly be driven by other considerations than those of dividends paid in the distant future. Equally, risk premiums are likely to differ for various maturities (see, e.g., van Binsbergen, Brandt, and Koijen (2012)). Hence, investors will not only change the price of expected

*Kragt (corresponding author), j.c.kragt@uvt.nl; de Jong, f.dejong@uvt.nl; and Driessen, j.j.a.g.driessen@uvt.nl, Tilburg University. We thank Riddhi Prasad and Salil Aggarwal at Deutsche Bank, Christian Mueller-Glissmann at Goldman Sachs, and Stanislas Bourgois at Credit Suisse for providing over-the-counter (OTC) dividend-swap data. We thank Hendrik Bessembinder (the editor), Frederico Belo, Adlai Fisher, Andras Fulop, Peter Schotman, Mikhail Simutin (the referee), Bas Werker, and seminar participants at the 2015 Society for Financial Studies (SFS) Cavalcade, the 2016 American Finance Association (AFA) Meetings, and ESSEC Business School for their useful comments, and Andreas Rapp for a comparative analysis.

dividends from moment to moment, but they may also change them for various maturities relative to each other, similar to a term structure of interest rates. In this article, we focus on this term structure of the prices of expected dividends.

Given that the stock price is simply the sum of the present values of all dividends expected, in the late 1990s Michael Brennan called for the development of a market for derivatives referring to future dividend payments. Trading of such derivatives started at the beginning of this century. These products exchange uncertain future dividends of an underlying stock or stock index in exchange for cash at the time of expiry. As such, they are forward looking in nature because they contain price information about expected dividends corrected for their risk. More precisely, the price of a single dividend future or over-the-counter (OTC) swap is the expected dividend for a given maturity discounted at the risk premium for this maturity. Finding present values of expected dividends only requires discounting these prices at the risk-free rate.

In this article, we use data on these new dividend derivatives to study the dividend term structure for the Eurostoxx 50 index and several other markets. A key starting point of our analysis is that we show that modeling the dynamics of a single variable is sufficient to describe the entire term structure of discounted dividend derivative prices and to obtain a total value for the stock index. This single variable is equal to dividend growth minus the 1-period risk-free rate and a variable capturing the risk premium.[1] We call this variable the discounted risk-adjusted dividend growth. Hence, we do not need to separately assume processes for interest rates, risk premiums, and dividend growth rates, the simplicity of which is a major advantage of our approach.

It is important to stress that this approach is nonstandard. Existing theoretical work usually separately models dividend growth and the preferences that determine discount rates. Empirically, the present-value literature uses econometric models for the expectations about dividend growth and/or returns given past returns and dividend data. One of the earliest and best-known examples is given by Campbell and Track changes is on Shiller (1988), who use vector autoregression (VAR) methods to predict returns based on past dividends and use this to decompose returns into discount rate news and cash-flow news. Many other attempts at the decomposition of dividend growth and risk premiums have since followed (see Cochrane (2011) for an overview). Clearly, the ultimate goal of asset pricing is to understand both discount rate and cash-flow dynamics, but it has proven to be difficult to reliably separate discount rates from cash flows. We show that we can learn a great deal about how investors value dividends i) without making restrictive assumptions on preferences and dividend processes and ii) without separating dividend growth from discounting.

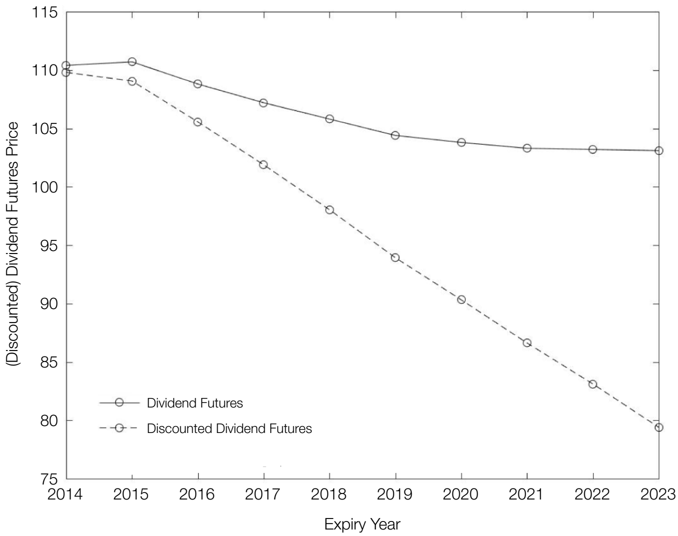
Inspired by the affine models often used for modeling the term structure of interest rates, we set up a standard affine model for discounted risk-adjusted dividend growth. Specifically, our model resembles the interest rate model of Jegadeesh and Pennacchi (1996), who use a 2-factor model, where the first factor reverts to a second factor, which in turn reverts to a long-run constant. This model

[1] In Figure 1, the prices of dividend futures and their discounted equivalents are illustrated on an arbitrary trading day.

FIGURE 1

Eurostoxx 50 Dividend Futures

Figure 1 shows the price curve of dividend futures and discounted dividend futures on a random day (Jan. 29, 2014) for the purpose of illustration. Discounted dividend futures equal the present value of future dividends. Expiries occur on the third Friday in December of each expiry year.



thus distinguishes a short-term component, a medium-term component, and constant asymptotic growth. We cast this model in state-space form and apply the Kalman filter maximum-likelihood approach to estimate it using dividend derivative prices with maturities of 1–10 years. The resulting term-structure model describes the maturity curve of dividend present values in full, including an estimate for long-term growth beyond the medium term until infinity.

In our benchmark analysis, we use daily data for Eurostoxx 50 dividend futures contracts, which extend out to horizons of up to 10 years. In a robustness check, we analyze Nikkei 225 dividend futures and OTC dividend swaps for the Financial Times Stock Exchange (FTSE) 100 and the Standard & Poor's (S&P) 500 indices.

Our key findings are as follows: First, we find evidence that our simple 2-factor affine model well describes both the term structure of dividends and the term structure of dividend volatility. It captures the dynamics of measured growth rates, and it delivers an estimate for asymptotic growth that is economically sensible. We do not need many factors, complex specifications for the factor volatilities, or drift terms to generate a good fit.

Second, we find that the factors driving this term structure have a rather strong mean reversion. The first factor has a half-life of 6 months (for reversion to the second factor) and thus captures short-term movements in expected risk-adjusted dividends. The second factor reverts to a constant at a horizon of business-cycle duration.

Third, we perform a relative pricing exercise, comparing the model-implied prices of future dividends to the observed value of the total stock index. Dividend derivatives have maturities up to 10 years, but using our term-structure model, the extrapolated growth rates beyond that are summed to arrive at a model-based estimate of the price–dividend ratio. Together with a market price for current dividends, a comparison is made to the actual stock market. This can be interpreted as an out-of-sample test of our dividend discount model because the model is estimated using dividend derivatives only and not the stock index value. At an R^2 of over 50%, we find that most of the variation in the stock market is explained by observed 1-year-ahead dividend prices and our model-implied price–dividend ratio. This demonstrates that the stock market can be understood quite well in terms of the market for dividend derivatives.

Combining the second and third findings, our results show that most of the variation in stock prices is captured by short-term and business-cycle movements in discounted risk-adjusted dividends, given the good fit to the aggregate stock market. Because the infinite growth rate is fixed, our results suggest that investors update their day-to-day valuation of dividends beyond the business-cycle horizon only to a limited degree. Apparently, depicting long-term investor expectations to be fixed is not a major impediment to capturing most of the observed stock market volatility.

Fourth, we obtain more insight into the drivers of the 2 state variables in our model. We do this by relating these 2 latent state variables, estimated using the Kalman filter, to various financial variables (interest rates, inflation swap rates, credit default swap (CDS) spreads, and implied volatilities) and economic variables (capturing economic growth and economic confidence). We find that the first state variable, with a fitted half-life of approximately 6 months, is closely related to economic confidence and the state of the economy as well as to proxies for short-term risk premiums. The second state variable, with a half-life similar to a business cycle, appears to be related to break-even inflation.

We also analyze whether liquidity issues affect our estimates. Overall, the Eurostoxx dividend futures market exhibits substantial trading volume, but particularly longer-dated contracts exhibit no volume on some days. We therefore reestimate our model on a reduced sample, only including days on which most futures have a nonzero trading volume. We find similar estimation results. To further analyze to what extent illiquidity affects this market, we analyze autocorrelations of returns and the prevalence of stale quotes. We find autocorrelations close to 0 and few days with stale quotes, which supports that illiquidity does not substantially affect our results.

This article adds to the recent literature that uses dividend derivatives in asset pricing. Our work complements that of van Binsbergen, Hueskes, Koijen, and Vrugt (2013). They introduce the concept of equity yields, which is related to our discounted risk-adjusted growth measure. However, van Binsbergen et al. (2013) do not estimate a pricing model for the term structure of discounted risk-adjusted dividend growth and do not price the stock market using this model. Instead, they focus on an empirical decomposition of dividend prices into dividend growth rates and risk premiums. They conclude that the term structure for risk premiums is pro-cyclical, whereas expected dividend growth is countercyclical.

Our work complements their study because we show that without separating dividend growth and risk premiums, one can price the entire term structure of dividends and learn about its dynamics in a formal pricing model.

In other related work, various authors (van Binsbergen et al. (2012), Cejnek and Randl (2016), and Golez (2014)) focus on realized returns of short- and long-term-horizon dividend derivatives or forward dividend prices derived from stock index futures and options and find evidence for a downward-sloping term structure of risk premiums. Wilkens and Wimschulte (2010) compare dividend derivative prices with dividend prices implied by index options. Suzuki (2014) assumes that risk premiums are proportional to dividend volatility and then models the dividend growth curve implied by derivative prices using a Nelson–Siegel approach.

The remainder of the article is organized as follows: Section II deals with the theory of dividend expectations and their fit in the present-value model. It lays out the state-space model that parameterizes the dividend term structure. Dividend futures data and the treatment to prepare them for empirical tests are discussed in Section III. The empirical results are discussed in Section IV. These results are used for a reconciliation to the stock market in Section V. Section VI relates the state variables of the pricing model to observed economic and financial variables. Several robustness checks and results for other markets follow in Section VII, and the conclusions are summarized in the closing section.

II. Theory

This section starts by proposing the general framework for discounted risk-adjusted dividend growth, represented in terms of a stochastic discount factor. The section continues by laying out the state-space model for capturing time- and horizon-varying dividend growth.

A. The General Framework

To apply the present-value framework, we define g_{t+1} as the realized dividend growth rate for the period t to $t+1$ so that the dividend payable at maturity n is $D_{t+n} = D_t \exp(\sum_{i=1}^{n} g_{t+i})$. We then apply the standard asset pricing equation to price this payoff for maturity n, where its current present value $P_{t,n}$ equals the expected product of the pricing kernel and the payoff:

$$(1) \qquad P_{t,n} = E_t\left[D_t \exp\left(\sum_{i=1}^{n} m_{t+i}\right) \exp\left(\sum_{i=1}^{n} g_{t+i}\right)\right],$$

and where m_{t+1} is the log pricing kernel for the period t to $t+1$. The pricing kernel consists of the 1-period risk-free rate y_t and an additional term θ_{t+1}:

$$(2) \qquad m_{t+1} = -(y_t + \theta_{t+1}),$$

where y_t is observed at time t and reflects the risk-free return over the period t to $t+1$.[2] We aim to model a combined growth variable for the present value of

[2]To be precise, y_t is defined as the continuously compounded, 1-period risk-free rate. Using the relation $\exp(-y_t) = E_t[\exp(m_{t+1})]$, it follows that the conditional expectation of θ_{t+1} must equal half the conditional variance of θ_{t+1} if the pricing kernel follows a lognormal distribution.

future dividends and rewrite the pricing formula in equation (1) accordingly:

$$(3) \qquad P_{t,n} = D_t \left[E_t \exp\left(\sum_{i=1}^{n} \pi_{t+i} \right) \right].$$

Equation (3) shows that the basic building block of the term-structure model is what we denote as the discounted risk-adjusted dividend growth:

$$(4) \qquad \pi_{t+1} = g_{t+1} - y_t - \theta_{t+1}.$$

In our data, we observe dividend future or swap prices. The relation of dividend present values to the prices of these dividend derivatives is achieved by discounting the future prices at the n-period risk-free rate $y_{t,n}$:

$$(5) \qquad P_{t,n} = F_{t,n} \exp(-n y_{t,n}),$$

which demonstrates that dividend present values are observable directly from market data $F_{t,n}$ and $y_{t,n}$.

If the risk-adjusted growth rate π_t follows a lognormal distribution, equation (3) can be rewritten as follows:

$$(6) \qquad \ln P_{t,n} - \ln D_t = E_t \left(\sum_{i=1}^{n} \pi_{t+i} \right) + \frac{1}{2} \mathrm{Var}_t \left(\sum_{i=1}^{n} \pi_{t+i} \right).$$

The left-hand-side variable is related to the key modeling variable of van Binsbergen et al. (2013). Specifically, they refer to $-(\ln P_{t,n} - \ln D_t)/n$ as the *equity yield*.

One may ask why we choose to model π_{t+1} rather than to assume separate models for its elements of dividend growth, risk premium, and risk-free discount rates. The decomposition of stock prices into dividend growth and risk premiums knows many attempts, seminal among which is the VAR-based approach by Campbell and Shiller (1988). Information from dividend derivatives is also used in the VAR model of van Binsbergen et al. (2013). We choose to do the exact opposite of decomposition and instead amalgamate the 3 variables into 1; the proposed model variable is the growth rate of the present values of expected dividends π_{t+1}. This amalgamation facilitates a focus on the term structure of the discounted growth trajectory alone. Connecting these growth rates to the stock market via the present-value identity allows for a horizon decomposition without being sidetracked by additional assumptions on the constituent variables. In fact, because we aim to value the stock market as the sum of dividend present values, a decomposition is not needed.

Furthermore, the components of π_{t+1} are likely to be correlated. For example, Bekaert and Engstrom (2010) calculate the correlation between 10-year nominal bond yields and dividend yields in the United States over a 40-year period at no less than 0.77. Van Binsbergen et al. (2013) perform a principal-components analysis of equity yields based on dividend derivatives prices. They show that the first 2 principal components of nominal yields explain approximately 30% of $g - \theta$ movements. Taken together as a single variable π_{t+1}, it should be possible to model the dividend price dynamics with a limited number of factors because of the high correlation among its components.

B. The State-Space Model

In order to build a full term structure of discounted risk-adjusted dividend growth, we model it in state-space form. We discuss the state equations and the measurement equations.

1. State Equations

The crucial question is how to model the evolution of risk-adjusted growth rates π_{t+1}. The approach that we advocate is a decomposition of π_{t+1} *by horizon*. Our modeling approach follows Jegadeesh and Pennacchi (1996), who propose a model for estimating the term structure of London Interbank Offered Rate (LIBOR) futures. In their model, the short-term interest rate is a first latent state variable, which evolves stochastically and mean reverts to a second state variable, which in turn mean reverts to a constant. This approach falls into the set of affine term-structure models. Dai and Singleton (2000) derive the most general versions of affine term-structure models, allowing for time-varying volatilities and time-varying risk premiums. We choose a rather restrictive 2-state model with constant volatilities and show that such a simple approach already generates a very good fit of the dividend term structure.

We specify most of the model in discrete time, following the approach of Campbell, Lo, and MacKinlay (1997). Specifically, we model π_{t+1} as the sum of a time-varying conditional mean p_t and a stochastic shock:

$$\pi_{t+1} = p_t + \nu_{t+1}, \tag{7}$$

where ν_{t+1} is normal and independent and identically distributed (IID) with 0 mean. Using the definition of π_{t+1} in equation (4), we can interpret p_t as the 1-period-ahead expected dividend growth minus the expected log of the pricing kernel

$$p_t = E_t g_{t+1} - y_t - E_t \theta_{t+1}. \tag{8}$$

The stochastic shock ν_{t+1} is then composed of the unexpected dividend growth and the stochastic part of the pricing kernel:

$$\nu_{t+1} = g_{t+1} - E_t g_{t+1} - (\theta_{t+1} - E_t \theta_{t+1}). \tag{9}$$

Then, following Jegadeesh and Pennacchi (1996), the short-term factor p_t follows a mean-reverting process to a medium-term factor $\tilde{p}_t$, which itself is mean reverting to a long-term constant $\overline{p}$, where, for convenience, we first define their processes in continuous time:

$$dp_t = \varphi(\tilde{p}_t - p_t)dt + \sigma_p dW_p, \tag{10}$$

$$d\tilde{p}_t = \psi(\overline{p} - \tilde{p}_t)dt + \sigma_{\tilde{p}} dW_{\tilde{p}}, \tag{11}$$

where dW_p and $dW_{\tilde{p}}$ are Wiener processes, with σ_p and $\sigma_{\tilde{p}}$ scaling the instantaneous shocks to the factors. The horizon at which investors adjust their growth expectation from one state to the next is captured by mean-reversion parameters φ and ψ. This 2-state system results in the state equations for discrete intervals:[3]

[3]Refer to Appendix A for further details.

$$(12)\qquad \begin{pmatrix} p_{t+1} \\ \tilde{p}_{t+1} \end{pmatrix} = \begin{pmatrix} 1-e^{-\varphi} & -\dfrac{\varphi}{\varphi-\psi}(e^{-\psi}-e^{-\varphi}) \\ 0 & 1-e^{-\psi} \end{pmatrix} \begin{pmatrix} \overline{p} \\ \overline{p} \end{pmatrix} + \begin{pmatrix} e^{-\varphi} & \dfrac{\varphi}{\varphi-\psi}(e^{-\psi}-e^{-\varphi}) \\ 0 & e^{-\psi} \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \varepsilon_{t+1}.$$

Finally, we model the correlation between the innovation in the growth rate ν_{t+1} and the errors ε_{t+1} in these state equations as $\nu_{t+1}=\beta'\varepsilon_{t+1}$, where $\beta=(\beta_p,\beta_{\tilde{p}})'$ is a 2-by-1 vector. One could incorporate an independent shock to the growth rate, but this does not have an important effect on the term structure of dividend prices or the dynamics of these prices. In terms of the mathematical structure, this setup resembles the approach of Campbell et al. (1997). They derive affine term-structure models in discrete time by modeling the log pricing kernel, $m_{t+1}=-(y_t+\theta_{t+1})$, in a similar way as we model the discounted risk-adjusted growth rate $\pi_{t+1}=g_{t+1}-(y_t+\theta_{t+1})$. The key difference is that our growth variable depends on both the pricing kernel and the dividend growth rate. As discussed previously, we only model the aggregate variable π_{t+1} and do not need to make specific assumptions about its components. This is important for the interpretation of the results. For example, when modeling interest rates, Campbell et al. (1997) show that the β vector captures the risk premiums on long-term bonds. In our setup, the vector β could represent dividend risk premiums but can also be the result of the correlation of current dividend growth and the factors driving future dividend growth. Again, for pricing dividend derivatives, there is no need to specify the source of the correlation between shocks to π_{t+1} and the factors.

2. Measurement Equations

Given the dynamics of π_{t+1}, we can price dividend derivatives using equation (6). It follows that the average growth rate of dividend present values from time t to the expiry date at time n corresponds to a function of p_t and $\tilde{p}_t$. Specifically, as shown in Appendix A, filling in the dynamics of π_{t+1} in the pricing equation (6) and adding IID measurement error $\eta_{t,n}$ for each derivative's maturity n, the measurement equations for the state-space model are as follows:

$$(13)\qquad \ln P_{t,n}-\ln D_t$$
$$= n\overline{p}+\varphi_n(p_t-\overline{p})+\frac{\varphi}{\varphi-\psi}(\psi_n-\varphi_n)(\tilde{p}_t-\overline{p}) + \frac{1}{2}\sum_{i=1}^{n}\left(\sigma_p^2(\beta_p+\varphi_i)^2+\sigma_{\tilde{p}}^2\left(\beta_{\tilde{p}}+\frac{\varphi}{\varphi-\psi}(\psi_i-\varphi_i)\right)^2\right)+\eta_{t,n},$$

in which β_p and $\beta_{\tilde{p}}$ are the covariance betas of the errors of the first and second factor, and σ_p^2 and $\sigma_{\tilde{p}}^2$ are their variances. We define φ_n and ψ_n as follows:

$$\varphi_n = \frac{(1-e^{-n\varphi})}{(1-e^{-\varphi})},$$
$$\psi_n = \frac{(1-e^{-n\psi})}{(1-e^{-\psi})},$$

with $\varphi_0=0$ and $\psi_0=0$.

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For comparison, we also estimate a single-state model in the empirical analysis. This model is a special case of the 2-state model, with $\tilde{p}_t = \overline{p}$ and $\sigma_{\tilde{p}} = 0$.

III. Dividend Swaps and Futures

A. The Market for Dividend Derivatives

Dividend derivatives exchange the value of a dividend index for cash at set expiry dates. The difference between the transaction price and the amount of dividends actually paid is the amount settled between the buyer and seller at expiry. The transaction price reflects the growth path expected from the current level of dividends and the premium required for the risk of the actual payment differing from what is expected. It is a risk-adjusted price and equals the present value of a dividend once the time value of money is accounted for (see equation (5)).

The dividend index measures the amount of dividends paid by the company's constituent to a stock index during a calendar year. At the end of the year, the index equals the fixing at which the dividend derivative is settled. Manley and Mueller-Glissmann (2008) provide an overview of the market for dividend derivatives and its mechanisms.

We obtain data for exchange-traded and OTC dividend derivatives. For the Eurostoxx 50 and Nikkei 225 index, we obtain data on exchange-traded dividend futures prices starting in 2008 and 2010, respectively. We also obtain data on dividend swap prices from several investment banks for the S&P 500 index and FTSE 100 index starting in 2005. Table 1 shows descriptive statistics for these markets. Maturities usually extend out to 10 years with annual intervals. For the exchange-based contracts, all maturities normally trade on a daily basis. Mixon and Onur (2017) show that OTC swaps trade infrequently; even for the S&P 500, which is the largest OTC dividend market, they trade less than daily between dealers and only once every few weeks between a dealer and a nondealer end-user.

Given these data properties, the benchmark estimation methodology uses the prices of dividend derivatives referring to the Eurostoxx 50 market. We focus on this market because it gives a relatively long sample period, its futures have maturities up to 10 years, and the futures are exchange traded and relatively liquid. Table 2 provides sample data on liquidity in Eurostoxx 50 dividend futures. It contains information about i) volume (number of contracts traded), ii) volume in euros ("notional"), iii) the number of days without trading, iv) the number of days with 0 returns ("stale prices"), and v) the autocorrelation in price changes, all for the Eurostoxx 50 index. Overall, futures with expiries up to 6 years trade nearly daily with an average daily volume in the tens of euro millions. Longer-dated futures trade less frequently, with 9- and 10-year maturities trading on average every other day.[4] However, it needs to be considered that market participants do seem to update price quotes on at least a daily basis: For most contracts, we observe few 0 daily returns. Only for the 1-year future do we observe more 0 daily returns, but this is simply because the price of this future has low volatility as a result of

[4]As a robustness check, we reestimate the model excluding trading days with reduced liquidity to investigate whether this changes the empirical results. See the Supplementary Material for further details.

TABLE 1
Summary Static Data of the Dividend Derivatives per Underlying Stock Indices

Table 1 reports the main characteristics of the dividend derivatives used in the estimation procedure.

		Eurostoxx 50	S&P 500	FTSE 100	Nikkei 225
No. of companies in the index		50	500	100	225
Currency		Euro	US$	GBP	JPY
Market capitalization ($trillions) per May 7, 2014		3.3	17.2	3.1	2.7
Data period	Dividend swaps	"NA"	Dec. 19, 2005–June 13, 2014	Dec. 19, 2005–June 13, 2014	"NA"
	Dividend futures	Aug. 4, 2008–Feb. 16, 2015	"NA"	"NA"	June 17, 2010–Feb. 16, 2015
Source of the data	Dividend swaps	"NA"	OTC	OTC	"NA"
	Dividend futures	Eurex	"NA"	"NA"	Singapore exchange
Avg. no. of trading days		256	252	253	245
Expiry horizon	Dividend swaps	"NA"	10 years	10 years	"NA"
	Dividend futures	10 years	"NA"	4 years	10 years
Expiry date		3rd Fri. of Dec.	3rd Fri. of Dec.	3rd Fri. of Dec.	Last trading day in March
Data frequency		Daily	Daily	Daily	Daily
Stock index ticker		SX5E	SPX	UKX	NKY
Dividend index ticker		DKESDPE	SPXDIV	F1DIVD	JPN225D

TABLE 2
Eurostoxx 50 Dividend Futures Liquidity

Table 2 reports various indicators of the liquidity of Eurostoxx 50 dividend futures. The numbers at the top of each column indicate the number of years remaining before the dividend future expires, where, for example, "3" means a remaining life between 2 years + 1 day and 3 years. "Volume" refers to the average daily number of contracts traded per dividend future in a given expiry year (third Friday in December to the next). "Notional" refers to the value of the average daily turnover in millions of euros, which equals the number of contracts ×100× future price. "Nontrading" refers to the number of days per year during which no trading occurred. "Stale prices" refers to the number of days per year when no price change occurred relative to the previous trading day. "Autocorrelation" refers to the autocorrelation in daily price changes.

	Daily Data (2009–2015): Years to Expiry									
	1	2	3	4	5	6	7	8	9	10
Volume	1,816	4,365	3,628	2,252	1,385	844	424	197	136	105
Notional	21	47	38	23	14	8	4	2	1	1
Nontrading	21	1	2	2	9	16	44	79	116	124
Stale prices	124	34	20	20	18	19	16	16	16	13
Autocorrelation	0.07	0.03	0.07	0.05	0.04	0.04	0.05	0.00	0.00	–0.01

its proximity to the settlement date.[5] We also include the autocorrelation of daily price changes. As suggested by Roll (1984), illiquidity would result in negative autocorrelation. Table 2 shows that there is little negative autocorrelation in price changes, including for the longest-dated expiries.

Daily dividend futures prices and zero-coupon interest rate swap data (for discounting futures prices) are sourced from Datastream. In a robustness analysis

[5]See the Supplementary Material for further explanation.

presented in the Supplementary Material, we consider Nikkei 225 dividend futures and OTC dividend swaps for the S&P 500 and FTSE 100 markets.

Dividend futures expire at a fixed date in each year, usually near the end of the calendar year.[6] Because our goal is to model growth rates for annual horizons, we use the observed futures prices to construct prices for dividend futures with constant maturities of 1 year, 2 years, and so forth, which we denote $F_{t,n}^{\text{CM}}$. In Appendix B we describe how we use interpolation to construct these prices, where we account for the seasonal pattern in dividend payments.

B. Dealing with Current Dividends

At the heart of the present-value model are the discounted values of risk-adjusted dividends. These present values $P_{t,n}$ take current dividends D_t as the starting point from which growth is projected forward at growth rate π_{t+i} (equation (3)). It is sometimes assumed that current dividends can be reasonably approximated by past realized dividends (see, e.g., van Binsbergen et al. (2013) and Cejnek and Randl (2016)). For daily data as applied in this article, however, this assumption causes issues.

The asset underlying dividend derivatives is the amount of cash dividend paid out by a stock index during the year in which the derivative expires. The index companies pay dividends throughout the calendar year, which implies that taking realized dividends as current dividends at a certain day of the year would require looking back for 12 months.[7] Clearly, a 12-month backward-looking dividend measure may not accurately reflect current dividends.[8] This problem rules out using a rolling 12-month estimate for current dividends. The first derivative to expire also does not perfectly capture current dividends. The first derivative contains investor expectations about dividends to be paid in the remaining period until the first expiry date and is not a reflection of current dividends on the observation date itself.

To avoid these data difficulties, we propose an alternative base. In lieu of an estimate for current dividends, we use dividend derivatives with 1 year of remaining life to expiry as the base from which to calculate growth rates:

$$P_{t,1}^{\text{CM}} = F_{t,1}^{\text{CM}} \exp(-y_{t,1}). \tag{14}$$

Subtracting the first-period present value gives the following measurement equation for growth rates and replaces equation (13):

[6] Eurostoxx 50 dividend futures expire on the third Friday of December.

[7] In fact, the dividend index year usually runs from the first working day following the third Friday in December until and including the third Friday in December of the following year. Dividend derivatives also apply the third Friday of December as the expiry date.

[8] To take a strong example, around the days of the Lehman bankruptcy on Sept. 15, 2008, the 12-monthdividend history of Eurostoxx 50 companies amounted to 154 euros. Due to the bankruptcy, investors would have changed their opinion strongly downward about the dividend that companies would pay if they would have had to pay on these days. After Lehman, taking a dividend history of 12 months would then overestimate current dividends as they stood in the fall of 2008. In the weeks following the default, the Eurostoxx 50 dividend future expiring in 2009 dropped from 140 to 100. Therefore, if 12-month realized dividends are used as current dividends, the shortest horizon observation for growth from 2008 to 2009 would attain a strongly negative value even though the actual growth expectation, starting from a level that would have been revised downward, could be flat or even positive.

(15)
$$\begin{aligned}\ln P_{t,n} - \ln P_{t,1} \\ = \quad & (n-1)\overline{p} + \varphi_n(p_t - \overline{p}) + \frac{\varphi}{\varphi - \psi}(\psi_n - \varphi_n)(\tilde{p}_t - \overline{p}) \\ & + \frac{1}{2}\sum_{i=1}^{n-1}\left(\sigma_p^2(\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2\left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi}(\psi_i - \varphi_i)\right)^2\right) + \eta_{t,n}.\end{aligned}$$

The state equations (equation (10)) and measurement equations (equation (15)) together form the system of which the variables are estimated by maximum likelihood. The procedure is recursive by means of a Kalman filter (Jegadeesh and Pennacchi (1996)). The error variance terms are assumed to be the same for all measurement equations (σ_η^2), except for the first one (which we denote σ_ϵ^2). This is because the definition of the first derivative to expire (set to a constant maturity of 1 year following the observation date) differs slightly from subsequent derivative prices because of an alternative weighting scheme for finding constant maturity values, as explained in Appendix B.

IV. Empirical Results

Table 3 provides the results of the 2-state model for the Eurostoxx 50 dividend market. Estimations are performed on daily data.[9] The estimation technique of the Kalman filter finds optimal solutions for various combinations of $\overline{p}$ and β_p, a fact that indicates multicollinearity. As a result, the estimates for long-term growth $\overline{p}$ as well as of both covariance betas β_p and $\beta_{\tilde{p}}$ come out unstable, with sizable standard errors. To solve this problem, we also report estimates of a model with the restriction $\beta_p = 0$ imposed while leaving $\beta_{\tilde{p}}$ as a free parameter.[10]

A. Model Fit

Before we discuss the parameters of the growth-rate model, we first establish that the 2-state model fits the data well.[11] To this end, we calculate mean absolute errors for the measurement equations (equation (15)). Given that they are specified for log prices of dividend futures, these mean absolute errors can be interpreted as relative pricing errors.[12] The first measurement equation produces a mean absolute pricing error of approximately 0.015 (1.5%), and the pricing errors of subsequent expiries are between 0.003 and 0.005 (Figure 2). The error levels are clearly small, confirming a good fit of the model to the data. A test for serial correlation in the residuals of the first measurement equation and potential impact on the parameters is conducted in the robustness section of the Supplementary Material.

[9] For robustness, we perform the same tests with monthly data (not shown here). None of the parameter estimates and test coefficients change meaningfully relative to the daily data set.

[10] The numerical values discussed in the following subsections are based on the estimates with $\beta_p = 0$ imposed.

[11] We investigate the potential impact of illiquidity by excluding trading days from the data set on which at least 2 or at least 3 (out of a total of 10) dividend futures show no trading volume. The parameter estimates are not materially affected by the reduction. The estimation results are provided in the Supplementary Material.

[12] These errors are thus not annualized. Transformed into annual growth rates, the errors are even smaller.

TABLE 3
Base Model of Discounted Risk-Adjusted Dividend Growth

Table 3 reports the maximum-likelihood estimates of a 2-state-space and a single-state-space model applied to the growth rate of discounted Eurostoxx 50 dividend futures. Estimates are based on daily prices of dividend futures and interest rates. Measurement equations capture discounted dividend growth starting 1 year following the observation date. The estimates include 8 measurement equations, from 1 to 8 years, except for the period from the start of the dataset until May 13, 2009, in which the number is 5 because of a lack of data. σ_η measures the standard deviations of the second until the eighth measurement equations, σ_ε of the first. This distinction is made to reflect that the base from which growth rates are determined is calculated by applying an alternative weighting scheme between the first and second derivatives to expire. See Appendix B. Standard errors are in parentheses. Parameter $\overline{p}$ is estimated in the Kalman filter; $\overline{p}^*$ equals $\overline{p}$ plus the convexity term (equation (21)).

	Estimates Using Listed Dividend Futures of the Eurostoxx 50 Index (Sample Period: Aug. 4, 2008–Feb. 16, 2015)			
	Two State $dp_t=\varphi(\tilde{p}_t-p_t)\,dt+\sigma_p\,dW_p$, $d\tilde{p}_t=\psi(\overline{p}-\tilde{p}_t)\,dt+\sigma_{\tilde{p}}\,dW_{\tilde{p}}$		Single State $dp_t=\varphi(\overline{p}-p_t)\,dt+\sigma_p\,dW_p$	
Variable	1	2	3	4
$\overline{p}$	−0.0586 (9.5339)	−0.0404 (0.0197)	−0.2067 (28.7770)	−0.0435 (0.0144)
φ	1.5130 (0.3160)	1.5132 (0.3158)	1.7297 (0.4894)	1.7292 (0.4894)
ψ	0.2433 (0.1089)	0.2434 (0.1088)		
β_p	0.1553 (67.007)	Set to 0	0.6246 (69.2935)	Set to 0
$\beta_{\tilde{p}}$	−2.6695 (6.2539)	−2.6693 (6.2523)		
σ_p	0.5701 (0.7876)	0.5704 (0.7870)	0.7033 (1.2245)	0.7033 (1.2245)
$\sigma_{\tilde{p}}$	0.0437 (0.0947)	0.0437 (0.0946)		
σ_ε	0.0219 (0.0295)	0.0219 (0.0294)	0.0177 (0.0071)	0.0177 (0.0071)
σ_η	0.0063 (0.0025)	0.0063 (0.0025)	0.0441 (0.0806)	0.0441 (0.0806)
$\overline{p}^*$	−0.0258	−0.0258	−0.0320	−0.0320
Log likelihood per contribution	24.57	24.57	18.35	18.35

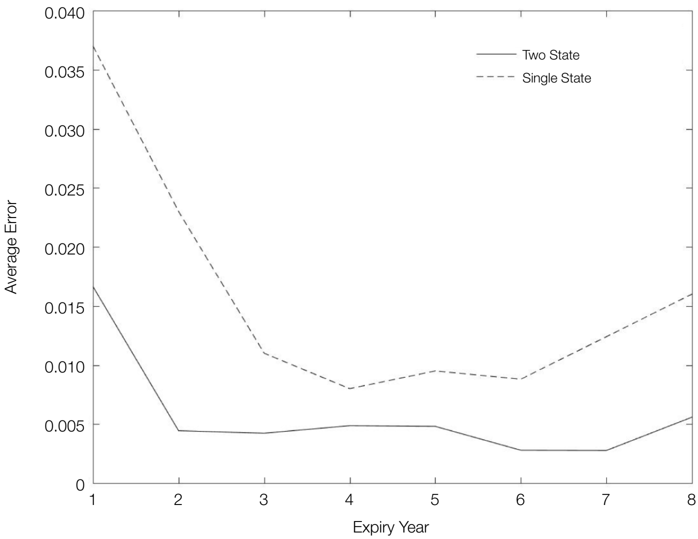
For comparison, Figure 2 also presents pricing errors for a single-state model, in which the medium-term factor is set to a long-term constant. Although still not substantial, the single-state estimation errors are larger by a factor of 2–3, showing that it is economically important to allow for 2-state variables driving the dividend term structure.

We then assess how well the model fits the volatility term structure of dividend futures. Specifically, we calculate the annualized variance of changes in the log dividend price for each maturity n, both as observed in the data and as implied by the 2-state model:

$$\begin{aligned}(16)\qquad \sigma_t^2(\ln P_{t+1,n}-\ln P_{t,n}) &= \sigma_t^2(\Delta\ln P_{t,1})+\sigma_p^2\left(\sum_{i=1}^{n-1}\exp(-\varphi i)\right)^2 \\ &\quad +\sigma_{\tilde{p}}^2\left(\frac{\varphi}{\varphi-\psi}\left(\sum_{i=1}^{n-1}\exp(-\varphi i)-\sum_{i=1}^{n-1}\exp(-\psi i)\right)\right)^2.\end{aligned}$$

FIGURE 2
Mean Absolute Estimation Errors

Figure 2 shows the average of the absolute estimation error of the 2-state and the single-state base model for the Eurostoxx 50 index. The measurement variables are discounted dividend risk-adjusted growth rates of 1–8 years.



We note that we do not construct a model for the volatility of current dividend changes $\sigma_t^2(\Delta \ln P_{t,1})$; instead, we use the 1-year observed volatility. Figure 3 shows that the Eurostoxx 50 dividend market portrays an increasing but concave volatility curve as maturities increase and that our model fits this pattern quite accurately. The concavity of the volatility term structure is consistent with the fast mean reversion of the 2-state variables in our model. The volatility of the long-maturity dividend futures prices converges to a value of more than 20%, consistent with typical values for the volatility of the stock market return. In the Supplementary Material, we provide further results on volatility term structure for the U.S. market, where we discuss the relation with macro asset pricing models calibrated to U.S. data.

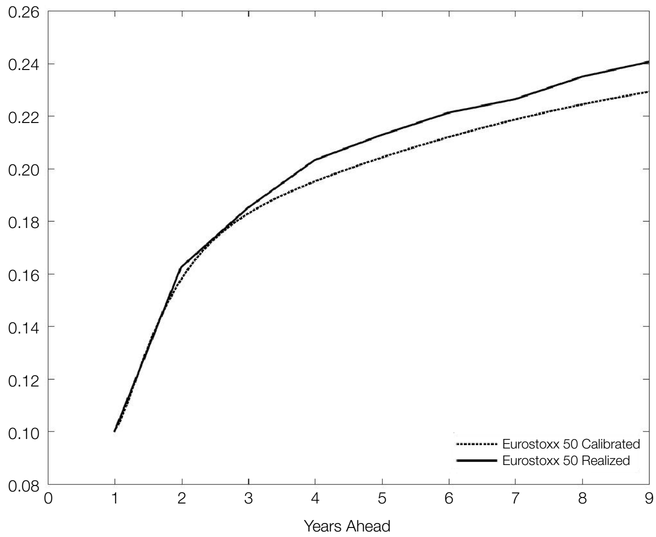
B. Mean-Reversion Estimates

As shown in Table 3, the mean reversion toward medium-term growth φ attains a level that translates to a half-life of approximately half a year. Mean reversion toward the long-run constant ψ is broadly measured in a half-life of approximately 3 years, a space of time that comes close to that of a business cycle. All mean-reversion parameters are significant at the 1% level (Table 3). The estimates for φ and ψ are positive, which implies that the growth rate is stationary and thus tends to a long-term constant.

A benchmark for the speed of reversion cannot be provided because there are no other attempts in the literature to fit the dividend term structure. Jegadeesh and Pennacchi (1996) apply the 2-factor model to interest rates and find the opposite pattern; at a half-life of 4.5 years, short mean reversion is slower than

FIGURE 3
Volatility of Dividend Returns

Figure 3 shows the volatilities of dividend returns $\sigma_t(\ln P_{t+1,n} - \ln P_{t,n})$. Volatilities are calculated both by the 2-state model and as observed in the data.



medium-term mean reversion at 2.3 years in interest rates. The first factor thus mean reverts much faster in dividends than in bonds, whereas the second factors are comparable.

C. Discounted Risk-Adjusted Dividend Growth Rates

Given the mean-reversion estimates, the first factor reflects short-term movements in risk-adjusted growth, the medium-term factor reflects an assessment of the business cycle, and $\overline{p}$ depicts a structural level that can be linked closely to the average dividend yield. Graphs A and B in Figure 4 provide estimates of expected growth rates by recalculating the factors by means of the measurement equations (equation (15)) into the 1-year growth and the 1-year-forward 4-year growth of discounted risk-adjusted dividends. Forward growth rates describe the level of growth expected after the 1-year growth rate has materialized.

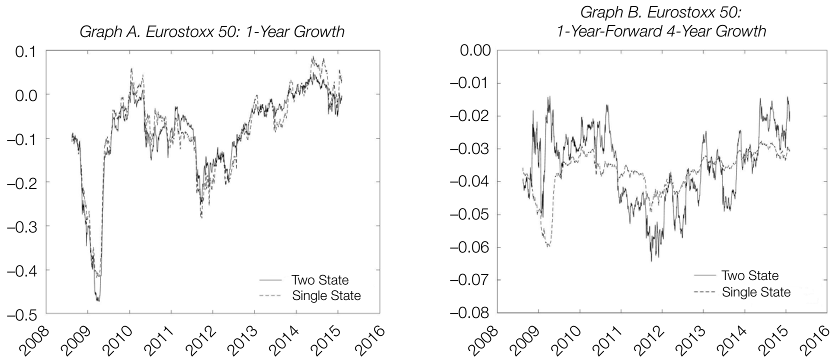
The 1-year growth is mostly determined by the first factor. Graph A of Figure 4 shows that it is highly volatile for the Eurostoxx 50, with the global credit crisis in 2008/2009 showing a decline by nearly half and during the Eurozone sovereign debt crisis in 2011 by a quarter. Outside these periods, it moves between broadly -10% and $+5\%$.

Given the values found for the mean-reversion parameters, the medium-term factor largely determines the 1-year-forward 4-year growth depicted in Graph B of Figure 4. Forward growth circles around the long-run constant between -2% and -6%. The sovereign debt crisis in 2011 shows a somewhat more negative rate than the global credit crisis. Investors apparently expected that the serious short-term blow to dividends in 2008/2009 would not be corrected or reversed

FIGURE 4

Calibrated Risk-Adjusted Dividend Growth Rates

Figure 4 shows the calibrated risk-adjusted growth rates of Eurostoxx 50 dividends. Graph A shows the 1-year growth rate $\pi_{t,1}$, and Graph B shows the average annual growth rates of the 4 years following the first year of growth: $\pi_{t,t+1\rightarrow t+5}$.



(by positive growth) afterward. However, the less negative blow in 2011 would be followed by a period more negative than the long-run constant (Graph B), implying that investors expected that the European sovereign debt crisis would bear consequences for the business cycle.

The model imposes the long-run growth rate to be constant, whereas the speed at which medium-term growth adjusts to it is estimated from the data. Structural factors such as population growth and technological progress determine how investors perceive the long run, extending from the business-cycle horizon into the infinite future. Structural developments should be slow moving, if at all, and are approximated by imposing asymptotic constancy. Thus, at horizons extending well beyond business cycles, investors may have time-varying opinions of economic and financial variables, but they do not change them once taken together. This means that any rise in long-maturity interest rates is offset by a rise in long-term dividend growth or a fall in long-term risk premiums. Mean reversion toward such a constant therefore implies that a horizon exists at which investors effectively do not change their opinion about present-value growth.

D. Long-Term Growth and Dividend Yields

Equipped with model estimates for the growth parameters, a dividend term structure (DTS) can be calibrated. It depicts the present values that investors attach to expected dividends per horizon n expressed as a proportion of the total present value:

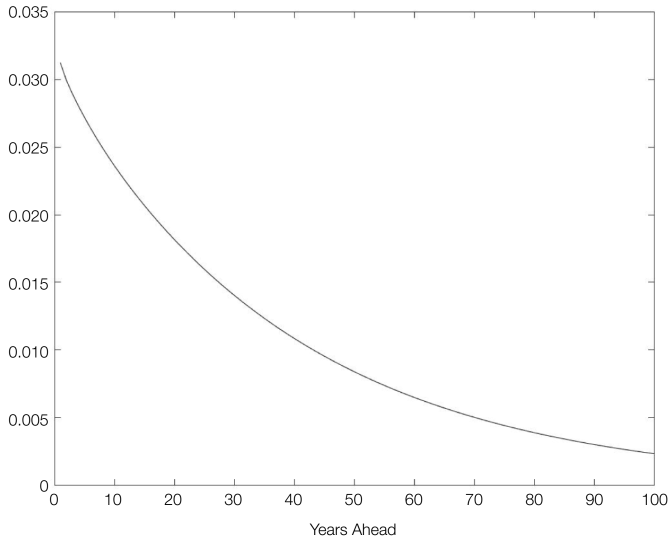
$$(17) \qquad \text{DTS}_n = \frac{\hat{P}_{t,n}}{\sum_1^{\infty} \hat{P}_{t,n}}.$$

The value for $\hat{P}_{t,1}$ is the observed discounted price of the derivative expiring 1 year from t. The values for subsequent expiries $n \geq 2$ are implied by the estimated 2-state model. Figure 5 shows that the DTS is strongly negatively sloping at the outset but adjusts to the long-term growth path rather quickly. The first

FIGURE 5

Calibrated Average Dividend Term Structure of the Eurostoxx 50 Index

Figure 5 shows the average of the calibrated present values of dividends per expiry year $\overline{P}_{t,n}$ divided by the sum of the averages. This represents the average dividend yield per expiry year in present-value terms.



dividend point on the Eurostoxx 50 DTS is therefore high, which translates into an equally high current dividend yield. The surface below the calibrated DTS equals 1 by definition. This dividend term structure indicates that the fundamental value of the European stock market is front loaded to a substantial degree, relative to other markets such as the S&P 500.

Next we focus on the long-term growth estimates in more detail. At long maturities, the dividend term structure is mainly determined by the long-term mean of discounted risk-adjusted dividend growth. To see this, recall the present-value identity for stock prices S_t:

$$(18) \qquad S_t = \sum_{n=1}^{\infty} P_{t,n} = D_t \sum_{n=1}^{\infty} \exp(n\pi_{t,n}),$$

where $\pi_{t,n}$ is the observed annualized discounted risk-adjusted growth rate of dividends payable at maturity n, and $\pi_{t,n} = (\ln P_{t,n} - \ln D_t)/n$, which is the negative of what van Binsbergen et al. (2013) call the equity yield. The dividend–price ratio is found by rearranging identity (18) to

$$(19) \qquad \frac{D_t}{S_t} = \frac{1}{\sum_{n=1}^{\infty} \exp(n\pi_{t,n})},$$

where $\pi_{t,n}$ is the discounted risk-adjusted growth rate of dividends payable at maturity n. For the sake of interpretation, if both factors in the 2-state model are equal to the mean, $\pi_{t,n}$ is a horizon-invariant constant and identity (19) simplifies to

$$(20)\qquad \frac{D_t}{S_t} \quad = \quad -\overline{p}^*.$$

The dividend yield equals the negative of long-term growth for which the state-space approach thus provides an estimate. Combined with the constant convexity term, the estimate for $\overline{p}$ constitutes a measure of long-term growth $\overline{p}^*$. On an annual basis, this value is given by

$$(21)\quad \overline{p}^* \quad = \quad \overline{p}+\frac{1}{2}\left(\sigma_p^2(\beta_p+\varphi_{i\to\infty})^2+\sigma_{\tilde{p}}^2\left(\beta_{\tilde{p}}+\frac{\varphi}{\varphi-\psi}(\psi_{i\to\infty}-\varphi_{i\to\infty})\right)^2\right),$$

in which the values for $\varphi_{i\to\infty}$ and $\psi_{i\to\infty}$ are set for i approaching infinity. Calculations (not reported) show that the interdependence between the estimates of β_p and $\overline{p}$ is such that different combinations render very little influence on the value of $\overline{p}^*$.

The estimated long-term growth constant $\overline{p}^*$ equals -2.6%. This value appears reasonable relative to dividend yields (equation (20)). The average dividend yield in Europe was 4.3% during our short data period. The average 1-year-forward 4-year growth rate was -3.7%, which deviates less than 1% from the average dividend yield. In contrast, the average short-term growth rate p_t was -9.8%, which deviates substantially more from the long-term growth. A tentative conclusion is that the business cycle stood close to the long-term average during the data period, but the sentiment was somewhat negative in Europe. Overall, the estimates for long-term growth seem a fair assessment of the long-term cash-flow run rate of the stock market. It is noteworthy that the estimates are produced without input from the stock market itself. It is also important to observe that the state-space model estimates discounted long-term growth to be negative because present-value theory requires stock valuations to be finite. The flexibility of the model would allow for positive values, but the estimates correctly imply that dividend present values decline at a horizon that is sufficiently long.

There are very few studies that estimate such long-term growth rates. One example is Giglio, Maggiori, and Stroebel (2015), who compare prices of houses of different contractual ownership to arrive at a very long-term discount rate. Leased housing reverts to the owner of the land after the lease expires, whereas freehold housing remains with the owner of the house indefinitely. The difference in price between the two for comparable properties equals today's present value put to ownership once the lease has expired. At lease expiries of over 100 years, this provides an interesting comparison to the estimates for the long-term discounted risk-adjusted dividend growth. The discounts Giglio et al. (2015) find in the data equate to a value for infinite growth of approximately -2% for periods of 100 years and more. This level makes sense economically and is also reasonably close to the long-term discounted risk-adjusted dividend growth estimates.

V. Reconciliation to the Stock Market

The second part of our research agenda is to analyze the implications of the model for the value of the stock market. Given that we estimate the model using dividend derivative data only, this constitutes an out-of-sample test of the model. Alternatively, if one takes the model assumptions for granted, it can be seen as a relative pricing exercise of the dividend derivative prices versus stock market levels.

A. The Empirical Approach

The present-value model incorporates expected index dividends, which can be extrapolated from the estimated dividend term structure. This provides the following estimate for the stock market:

$$\text{(22)} \qquad \hat{S}_t = D_t \sum_{n=1}^{\infty} \exp(n\widehat{\pi}_{t,n}) = D_t \widehat{\text{PD}}_t,$$

with the summation of fitted growth rates $\widehat{\pi}_{t,n}$ equal to the estimated dynamic price–dividend ratio $\widehat{PD}_t$ and where the fitted growth rates satisfy

$$\text{(23)} \qquad n\widehat{\pi}_{t,n} = n\overline{p} + \varphi_n(p_t - \overline{p}) + \frac{\varphi}{\varphi - \psi}(\psi_n - \varphi_n)(\tilde{p}_t - \overline{p}) + \frac{1}{2}\sum_{i=1}^{n}\left(\sigma_p^2(\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2\left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi}(\psi_i - \varphi_i)\right)^2\right).$$

Successful reconciliation of dividend derivative price information to the stock market is uncommon in the literature. For example, Suzuki (2014) builds a Nelson–Siegel model of the Eurostoxx 50 dividend growth term structure and makes assumptions about the level for longer-dated values. These include a fixed level imposed at 4% for discounted growth after 25 years. Under these conditions, Eurostoxx 50 dividends reconcile well with the stock market dynamically.

In contrast to Suzuki (2014), we do not impose a fixed level because the state-space model itself renders an estimate for the long-term growth path of the present value of dividends independent from stock market information, and it captures the shape and the dynamics of the term structure up to the medium term at the same time. The entirety of the present value term structure is thus described by a handful of variables from two markets (the interest rate swap market and the dividend derivative market) in a single estimation procedure. The fit of the reconciliation to the observed stock market acts as a joint check on the validity and the robustness of the 2-state model and the present-value identity. To that end, equation (23) is used to calculate the fitted dividend growth rates and present values as implied by the estimated state-space model.

All variables are taken as estimated by the state-space model applied to dividend derivative data. The fact that current dividends are unobservable in equation (22) is resolved by starting with the value of the first constant-maturity derivative $F_{t,1}$, discounted at the risk-free rate. We thus get the following for the model implied stock market level:

$$\text{(24)} \qquad \hat{S}_t = F_{t,1}\exp(-y_{t,1})\left(1 + \sum_{n=2}^{\infty}\exp(n\widehat{\pi}_{t,n} - \widehat{\pi}_{t,1})\right) = F_{t,1}\exp(-y_{t,1})(1 + \widehat{PD}_t^1),$$

in which $n\widehat{\pi}_{t,n} - \widehat{\pi}_{t,1}$ are the fitted values, estimated as a single variable, of the measurement variables in equation (15), and $\widehat{PD}_t^1$ represents the estimate for the price–dividend ratio as implied by the sum of exponential growth rates, where growth starts from the present value of the dividend derivative that expires in 1 year.[13]

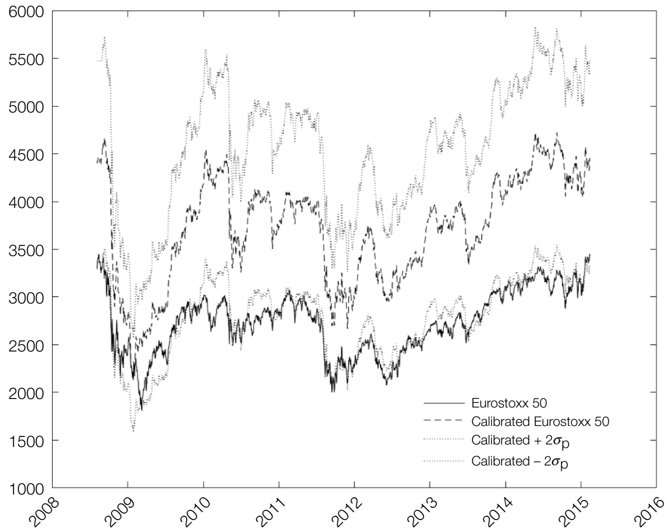
B. Stock-Market-Level Reconciliation

We first discuss the empirical results of the reconciliation with stock market levels.[14] The 2-state model estimates applied to equation (24) cause the stock index to be overestimated at a reasonably constant level distance to the actual stock index for most of the data period (Figure 6). There is no clear trend among the factors driving the estimated valuation away or toward the stock index. The historical dividend yield (4.3%) is somewhat higher than the negative of the long-term estimate (−2.6%), and the index is overestimated at some 20% to 30%, except during the outbreak of the global credit crisis. The level estimate of the stock index is

FIGURE 6

Present-Value Model Estimates for the Level of the Eurostoxx 50 Index

Figure 6 shows the model estimates of the level of the Eurostoxx 50 index. Calculations are described in $\hat{S}_t = F_{t,1}\exp(-y_{t,1})(1+\sum_{n=2}^{\infty}\exp(n\widehat{\pi}_{t,n}-\widehat{\pi}_{t,1}))$ (equation (24)), including their ranges, with an estimate for $\overline{p}$ of $-2\sigma_{\overline{p}}$ to $+2\sigma_{\overline{p}}$ and stock market observations S_t.



[13]The stock index estimate is approached by numeric summation, which is approximated by the following:

$$(25) \qquad \hat{S}_t \approx F_{t,1}^{\mathrm{CM}}\exp(-y_{t,1})\left[1+\sum_{n=2}^{\overline{n}}\exp(n\widehat{\pi}_{t,n}-\widehat{\pi}_{t,1})+\frac{\exp(\overline{n}\widehat{\pi}_{t,\overline{n}})}{-\widehat{\overline{p}}^{*}}\right].$$

In the estimations, $\overline{n}$ is set at 50 years. The number of years that $\overline{n}$ is set to is not material to the stock index estimates, unless reduced to less than 10 years.

[14]The state-space model estimations are produced by setting the short-term beta to 0.

highly sensitive to the long-term growth parameter. For the mean squared errors of this level comparison to be minimized, the estimate for long-term discounted growth would have to be closer to the historical dividend yield, approximately 0.7% higher.

C. Dynamic Reconciliation

Following the present-value model, *stock returns* are a consequence of investors changing their valuation of future dividends. The dynamics of stock indices can be retrieved from the present-value model estimate as provided in equation (24). The present value of the first dividend amount to be paid over the coming year is the starting point of the growth term structure. The first dividend value is observable, and the growth path of discounted risk-adjusted dividends starting after it is a model-implied estimate. The dynamic fit and the relative importance to the stock returns of the first derivative on the one hand and the growth path on the other require testing. For this reason, the estimated return of the stock market is split into its drivers. Equation (22) is repeated with logs denoted in lowercase as a regression equation:

$$(26) \qquad \Delta s_t \quad = \quad \alpha + \beta_f \Delta f_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{pd}} \Delta \widehat{pd}_t + \varepsilon_t .$$

Stock index log returns are regressed by ordinary least squares (OLS) on the log return of the first constant-maturity derivative Δf_t; changes in the 1-year risk-free rate Δy_t; and the log returns of the estimated price–dividend ratio $\widehat{pd}_t^1$, which is the sum of the normalized dividend present values of the state-space model. The betas of the returns of the first dividend and the price–dividend ratio are predicted to be close to $+1$, whereas the beta of the risk-free rate is expected to equal -1. Data are daily.

The stock index returns respond well to the prediction of the present-value model, shown in Table 4. The model is quite capable of explaining variation in stock returns, reaching an R^2 of above 50%. Although we cannot benchmark this explanatory power, it appears substantial given that the model does not incorporate any direct information on the stock market. Each of the regressors adds considerably to the explanatory power, and the constant is close to 0. The stock market return appears highly sensitive to changes in the first constant-maturity derivative, with a daily beta of almost 0.90. The beta of the price–dividend ratio equals 0.66. Hence, the explanatory power is quite evenly divided between short-term dividends and the price–dividend ratio.

The 1-year zero-coupon interest rate brings the price of the first derivative to its present value. Its relevance seems limited. Although the expected beta is -1, the estimated beta is significant but economically small at 0.15.[15]

The interpretation of the assumption that the long-run discounted risk-adjusted dividend growth is constant is not that investors do not change their opinion about what value to attach to dividend present values far into the future. The value ascribed to dividends expected 10 years and, for example, 20 years from

[15]The impact of short-term dividends and the price–dividend ratio is mitigated by negative coefficients found once lags are added to the set of regressors (not shown here). This suggests that either the stock market overreacts to shocks to dividends, which is corrected on the following day, or that dividend prices may partly follow stock prices by at least a 1-day lag.

TABLE 4

Reconciliation of the Base Present-Value Model (2 State) Constituent Returns to Stock Market Returns

Table 4 reports the reconciliation of the returns of the value of the modeled Eurostoxx 50 to actual returns. The ordinary least squares (OLS) regression estimates equation (26):

$$\Delta s_t = \alpha + \beta_f \Delta f_t + \beta_{\Delta y} \Delta y_t + \beta_{\widehat{pd}} \Delta \widehat{pd}_t + \varepsilon_t,$$

where Δs_t is stock index log returns, Δf_t is the log return of the first constant-maturity dividend derivative, Δy_t is the change in the 1-year zero-coupon swap rate, and $\Delta \widehat{pd}_t$ is the first differenced log of the sum of the normalized present value of dividends as estimated in the 2-state-space model. β is fixed at 0. The modeled present values of dividends are tested for their explanatory power of the dynamics of the stock market. Daily data are used. Standard errors are in parentheses.

	Sample Period: Aug. 4, 2008–Feb. 16, 2015			
Variable	1	2	3	4
Constant	0.0005 (0.0003)	0.0002 (0.0003)	0.0006 (0.0004)	−0.0001 (0.0003)
f_t	0.8978 (0.0337)	1.0009 (0.0426)		
Δy_t	0.1446 (0.0127)		0.2022 (0.0178)	
$\widehat{pd}_t$	0.6587 (0.0216)			0.6893 (0.027)
Adj. R^2	0.540	0.248	0.071	0.280

today is influenced by the estimate of present values in the near term and medium term. But the value of the 20-year dividend does not change relative to that of the 10-year dividend regardless of changes in near- and medium-term expectations; the relationship between them is (approximately) fixed. Therefore, long-run constancy excludes mean reversion to levels. The dividend levels attained in the past are not a target for investors to project their long-term expectations onto. Only long-term growth is.

VI. The Relation to Economic Variables

In this section, we aim to understand the drivers of the 2-state variables in the dividend term structure. To that end, we relate the state variables to two classes of variables. The first set of variables captures economic growth and economic confidence. We include economic confidence variables because existing work has shown that these are related to stock returns (Lemmon and Portniaguina (2006)) and might capture investor sentiment (Baker and Wurgler (2006)). The second set of variables relates to the term structure of several financial products, such as interest rates, inflation swap rates, CDS spreads, and implied volatility. In the following discussion, we will see that the first state variable, with a fitted half-life of approximately 6 months, is closely related to economic confidence and the state of the economy as well as to proxies for short-term risk premiums. We will also argue that the second state variable, with a half-life similar to a business cycle, appears to be related to expected inflation.

A. Data and Methodology

The data for all variables refer to the Eurozone, and the sample period runs from Aug. 2008 to June 2015. The economic variables are all sourced from Eurostat: producer price index (PPI), retail sales, industrial production, and unemployment. We obtain data for consumer confidence and business confidence from Ecofin.

Price data for all financial products are collected via Datastream and include interest rate swaps (IRSs), Harmonised Index of Consumer Prices inflation-linked swaps (ILSs), CDSs, and option implied volatilities (IVs), all denominated in euros. IRS, ILS, and CDS data are zero-coupon rates for 1- to 30-year maturities. The CDS rates refer to the credit spreads of the basket of companies residing in the Eurostoxx 50 index, weighted by their respective share in the index. The composition of the Eurostoxx 50 index changes over time, and not all of these 50 firms have CDSs traded on their senior debt. On average, the CDS index we build contains 97% of the total weight of the Eurostoxx 50 share price index. Data are available for 1- to 30-year maturities. The implied volatility of the Eurostoxx 50 index is calculated from at-the-money index options, as calculated by Datastream, and is used for expiries from 1 month to 5 years.

The methodology in this section is straightforward: We regress the first or proportional differences of the variables mentioned on the first differences of the fitted state variables and the returns of the first discount future and the stock market.[16] The univariate regression equations are as follows:

$$\Delta y_{n,t} \quad = \quad \alpha_{i,n} + \beta_{i,n} \Delta p_{i,t} + \epsilon_t, \tag{27}$$

where t is the month, and $p_{i,t}$ for $i=1,2$ indicates the first and second state variables (p and $\tilde{p}$, respectively). $\Delta y_{n,t}$ represents both the economic variables and the financial products.

B. Regression Results

1. The Relation to the Economy

Table 5 reports the results of the regressions of the economic and confidence variables on the state variables. The results show that in particular, the first state variable is strongly correlated with business and consumer confidence. At an R^2 of 24% and with highly significant coefficients, confidence in the economy and near-term risk-neutral growth in dividends appear to have the same driver. The intuition is clear because both higher dividend expectations and a lower risk premium fit in with confidence in a strengthening economy. Note that higher confidence could relate to larger risk appetite among market participants.

Industrial production, PPI, and unemployment are indicators of economic activity. Albeit weaker than for the confidence variables, their relationship to the first state variable is also significant. These variables are a direct reflection of the actual state of the economy and not a direct link to the risk premium, although they might impact the risk premium indirectly. Changes in retail sales are not well explained by any of the state variables.

The relation between the economic and confidence variables and the second state variable p_2 is less strong, although there is a negative correlation with the

[16] We use proportional differences for PPI, retail sales, and industrial production and first differences for all other variables. Data for the financial products and the state variables are available on a daily basis. We nonetheless choose to perform regressions on monthly averages of the data. This approach reduces the impact from measurement error in the state variables as well as the potential for misaligned timing of daily closing prices of the variables that we compare.

TABLE 5

Regressions of Economic and Sentiment Variables on Eurostoxx 50 Variables

Table 5 reports estimated parameters of regressions of economic and sentiment variables on Eurostoxx 50 modeled and index variables. The explained variables $\Delta y_{n,t}$ in equation (27) are relative changes in the producer price index (PPI), retail sales, and industrial production and first-order changes in business confidence, consumer confidence, and unemployment. The regressors $\Delta p_{i,t}$ are first-order changes in the first and second state variables and relative changes in the discounted 1-year constant-maturity dividend future and the stock market, all of the Eurostoxx 50 index. t-statistics are in parentheses.

	Monthly Data (Aug. 2008–June 2015)											
	1st State Variable			2nd State Variable			Discounted 1st Future			Stock Market		
	a	b	R^2	a	b	R^2	a	b	R^2	a	b	R^2
PPI	0.00	0.01	6.3%	0.00	−0.07	3.2%	0.00	0.03	5.3%	0.00	0.03	6.8%
	(0.18)	(2.34)		(−0.12)	(−1.62)		(−0.26)	(2.12)		(0.29)	(2.43)	
RETAIL_SALES	0.00	0.00	0.0%	0.00	−0.01	0.0%	0.00	0.02	1.0%	0.00	0.02	4.0%
	(0.24)	(−0.05)		(−0.16)	(−0.1)		(−0.25)	(0.91)		(0.24)	(1.85)	
INDUSTRIAL_ PRODUCTION	0.00	0.02	14.2%	0.00	−0.13	3.2%	0.00	0.05	3.2%	0.00	0.07	9.9%
	(0.42)	(3.65)		(−0.25)	(−1.62)		(−0.13)	(1.62)		(0.5)	(2.98)	
BUSINESS_ CONFIDENCE	0.00	0.04	23.7%	0.00	−0.56	13.5%	0.00	0.21	14.8%	0.00	0.23	27.8%
	(0.16)	(5.00)		(−0.1)	(−3.55)		(−0.3)	(3.75)		(0.28)	(5.57)	
CONSUMER_ CONFIDENCE	0.00	0.04	23.5%	0.00	−0.39	9.2%	0.00	0.16	11.8%	0.00	0.19	28.0%
	(−0.2)	(4.98)		(0.09)	(−2.86)		(−0.52)	(3.28)		(−0.12)	(5.60)	
UNEMPLOYMENT	0.00	0.00	3.8%	0.00	0.01	1.7%	0.00	0.00	0.7%	0.00	−0.01	5.5%
	(2.69)	(−1.80)		(2.63)	(1.03)		(2.59)	(−0.77)		(2.77)	(−2.29)	

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confidence variables. Note that the first and second state variable have a correlation of −0.65, so this opposite sign is not surprising. The economic interpretation of this negative correlation is discussed in Section VI.C.

We also relate the return on the 1-year dividend future to the same set of variables and find that the discounted first dividend and the economic and confidence variables are positively related. The strongest regression results are found for the confidence variables, as is also the case for the first state variable. In multivariate regressions (not shown here) their significance remains strong and is of similar magnitude across the explained variables, but the significance of the second state variable vanishes. As shown in the previous section, the stock market index is well explained by the combination of the state variables and the first discounted future. The results in this section therefore show that the interpretation of the economic drivers of the index components runs mostly through the first discounted future and the first state variable and less so through the second state variable.

2. The Relation to Financial Markets

The second set of results concerns the financial products. The regression slope coefficients of the IRSs and ILSs (reported in Tables 6 and 7) are typically significantly different from 0 for both state variables (either positive or negative), and the R^2s are on the order of magnitude of 10%–20%. The responsiveness of various points on the term structure of IRSs is consistent at coefficients between 0.20 and 0.26. Such level consistency highlights that the IRS slope is not influenced by changes in the first state variable. The coefficients of the term structure of ILSs are less consistent between 0.14 and 0.32 for the 1-year maturity. Longer-dated maturities have smaller coefficients, but their significance is higher than those of shorter maturities.

The second state variable shows a negative sign across the term structure of both IRSs and ILSs. Regardless of the exact point on the term structure, the coefficients are significant, and for IRSs, often more so than the equivalent coefficients of the first state variable. The picture for the slope of the coefficients is the same as for the first state variable: consistent for the first state variable and decreasing for the second because maturities are longer.

The CDS spreads and IVs of the Eurostoxx 50 firms, reported in Tables 8 and 9, are negatively related to the first state variable with a fairly high R^2. The intuition here is that both measures indicate a price of risk, which appears to drive the risk premium of shares. There isn't a strong relation to the second state variable or the first dividend future, so the strong relation between the stock market and CDS and IV is mainly driven by the first state variable. Note that the regression coefficients of the CDS spreads gradually decrease as maturities extend, while remaining significant. The IV coefficients are only significant for maturities of up to 2 years. The latter again indicates that the first state variable is related to the risk premium at a horizon shorter than a typical business cycle.

C. Interpreting the Second State Variable

Based on the regression results, we now provide a potential interpretation of the second state variable. Table 10 summarizes some relevant results from Tables 6 and 7. It reports the regression coefficients (betas) of nominal yields (y_n^{NOM}),

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TABLE 6

Regressions of Interest Rate Swaps on Eurostoxx 50 Variables

Table 6 reports estimated parameters of regressions of interest rate swaps (IRSs) on Eurostoxx 50 modeled and index variables. The explained variables $\Delta y_{n,t}$ in equation (27) are first-order changes in the zero-coupon euro IRS rates for various maturities. The regressors $\Delta p_{i,t}$ are first-order changes in the first and second state variables and relative changes in the discounted 1-year constant-maturity dividend future and the stock market. t-statistics are in parentheses.

	Monthly Data: Aug. 2008–June 2015											
	1st State Variable			2nd State Variable			Discounted 1st Future			Stock Market		
	a	b	R^2	a	b	R^2	a	b	R^2	a	b	R^2
1	−0.06	0.25	13.6%	−0.06	−3.49	9.0%	−0.06	1.78	18.7%	−0.06	1.16	12.5%
	(−3.58)	(3.57)		(−3.44)	(−2.82)		(−3.50)	(4.32)		(−3.59)	(3.40)	
2	−0.06	0.26	16.4%	−0.06	−3.77	11.0%	−0.05	1.96	23.9%	−0.06	1.46	20.8%
	(−3.48)	(3.98)		(−3.33)	(−3.17)		(−3.43)	(5.04)		(−3.64)	(4.60)	
3	−0.06	0.27	18.1%	−0.05	−4.15	14.4%	−0.05	1.68	18.8%	−0.06	1.39	20.1%
	(−3.50)	(4.23)		(−3.38)	(−3.68)		(−3.30)	(4.33)		(−3.60)	(4.51)	
4	−0.05	0.26	18.7%	−0.05	−4.49	17.9%	−0.05	1.39	13.7%	−0.05	1.25	17.3%
	(−3.48)	(4.31)		(−3.42)	(−4.19)		(−3.18)	(3.58)		(−3.49)	(4.12)	
5	−0.05	0.26	18.6%	−0.05	−4.72	20.6%	−0.05	1.16	10.0%	−0.05	1.13	14.8%
	(−3.40)	(4.30)		(−3.42)	(−4.58)		(−3.06)	(2.99)		(−3.36)	(3.75)	
7	−0.05	0.25	17.6%	−0.05	−4.78	22.1%	−0.05	0.86	5.7%	−0.05	1.00	12.2%
	(−3.20)	(4.15)		(−3.27)	(−4.78)		(−2.84)	(2.20)		(−3.12)	(3.34)	
10	−0.04	0.23	14.7%	−0.04	−4.61	19.8%	−0.04	0.65	3.2%	−0.04	0.92	9.9%
	(−2.87)	(3.74)		(−2.94)	(−4.47)		(−2.56)	(1.62)		(−2.81)	(2.97)	
20	−0.04	0.20	8.9%	−0.04	−3.99	11.7%	−0.04	0.95	5.3%	−0.04	1.07	10.7%
	(−2.26)	(2.81)		(−2.28)	(−3.28)		(−2.09)	(2.12)		(−2.32)	(3.11)	
30	−0.04	0.26	12.2%	−0.04	−4.34	11.7%	−0.04	1.31	8.6%	−0.04	1.49	17.4%
	(−2.03)	(3.36)		(−2.00)	(−3.27)		(−1.84)	(2.75)		(−2.16)	(4.13)	

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TABLE 7
Regressions of Inflation-Linked Swaps on Eurostoxx 50 Variables

Table 7 reports estimated parameters of regressions of inflation-linked swaps (ILSs) on Eurostoxx 50 modeled and index variables. The explained variables $\Delta y_{n,t}$ in equation (27) are first-order changes in the zero-coupon euro ILS rates for various maturities. The regressors $\Delta p_{i,t}$ are first-order changes in the first and second state variables and relative changes in the discounted 1-year constant-maturity dividend future and the stock market. t-statistics are in parentheses.

Monthly Data: Aug. 2008–June 2015

	1st State Variable			2nd State Variable			Discounted 1st Future			Stock Market		
	a	b	R^2	a	b	R^2	a	b	R^2	a	b	R^2
1	−0.02	0.32	8.0%	−0.02	−5.20	7.1%	−0.01	2.02	8.6%	−0.02	2.13	14.9%
	(−0.55)	(2.65)		(−0.53)	(−2.49)		(−0.43)	(2.75)		(−0.65)	(3.76)	
2	−0.02	0.27	9.8%	−0.02	−4.76	9.8%	−0.01	1.89	12.4%	−0.02	1.84	18.5%
	(−0.78)	(2.97)		(−0.75)	(−2.96)		(−0.63)	(3.38)		(−0.90)	(4.28)	
3	−0.02	0.23	8.4%	−0.02	−4.26	9.5%	−0.01	1.65	11.4%	−0.02	1.62	17.2%
	(−0.83)	(2.72)		(−0.82)	(−2.91)		(−0.70)	(3.23)		(−0.96)	(4.10)	
4	−0.02	0.24	12.0%	−0.02	−4.25	12.2%	−0.02	1.27	8.7%	−0.02	1.54	20.1%
	(−0.99)	(3.32)		(−0.96)	(−3.35)		(−0.82)	(2.78)		(−1.12)	(4.51)	
5	−0.02	0.23	12.9%	−0.02	−3.97	12.8%	−0.01	1.16	8.7%	−0.02	1.50	23.1%
	(−1.03)	(3.46)		(−1.01)	(−3.45)		(−0.86)	(2.78)		(−1.19)	(4.92)	
7	−0.02	0.18	12.2%	−0.02	−3.14	11.7%	−0.01	0.93	8.1%	−0.02	1.23	22.6%
	(−1.17)	(3.34)		(−1.14)	(−3.26)		(−1.00)	(2.67)		(−1.34)	(4.86)	
10	−0.01	0.15	12.3%	−0.01	−2.51	11.9%	−0.01	0.77	8.9%	−0.02	0.99	23.2%
	(−1.33)	(3.37)		(−1.30)	(−3.30)		(−1.16)	(2.81)		(−1.52)	(4.94)	
20	−0.01	0.16	17.1%	−0.01	−2.14	10.8%	−0.01	0.48	4.3%	−0.01	0.91	24.6%
	(−1.31)	(4.09)		(−1.22)	(−3.13)		(−1.08)	(1.90)		(−1.46)	(5.14)	
30	−0.01	0.18	20.0%	−0.01	−2.13	9.7%	−0.01	0.44	3.3%	−0.01	1.00	26.6%
	(−1.22)	(4.49)		(−1.09)	(−2.94)		(−0.97)	(1.66)		(−1.36)	(5.41)	

TABLE 8
Regressions of Credit Default Swaps on Eurostoxx 50 Variables

Table 8 reports estimated parameters of regressions of credit default swaps (CDSs) on Eurostoxx 50 modeled and index variables. The explained variables $\Delta y_{n,t}$ in equation (27) are first-order changes in a basket of zero-coupon euro CDS rates for various maturities. The regressors $\Delta p_{i,t}$ are first-order changes in the first and second state variables and relative changes in the discounted 1-year constant-maturity dividend future and the stock market. t-statistics are in parentheses.

	Monthly Data: Aug. 2008–June 2015											
	1st State Variable			2nd State Variable			Discounted 1st Future			Stock Market		
	a	b	R^2	a	b	R^2	a	b	R^2	a	b	R^2
1	0.00	−0.43	39.4%	0.00	2.91	5.9%	0.00	−1.28	9.1%	0.00	−2.40	50.4%
	(−0.06)	(−7.25)		(−0.16)	(2.24)		(−0.27)	(−2.84)		(0.07)	(−9.07)	
2	0.00	−0.42	40.2%	0.00	2.76	5.7%	−0.01	−1.21	8.7%	0.00	−2.41	54.5%
	(−0.10)	(−7.37)		(−0.20)	(2.20)		(−0.30)	(−2.78)		(0.04)	(−9.84)	
3	0.00	−0.35	32.8%	0.00	1.66	2.4%	−0.01	−1.05	7.6%	0.00	−2.16	50.6%
	(−0.19)	(−6.29)		(−0.27)	(1.41)		(−0.36)	(−2.58)		(−0.07)	(−9.11)	
4	0.00	−0.34	33.7%	0.00	1.53	2.2%	−0.01	−0.94	6.8%	0.00	−2.12	54.2%
	(−0.20)	(−6.40)		(−0.28)	(1.36)		(−0.36)	(−2.43)		(−0.07)	(−9.78)	
5	0.00	−0.29	26.8%	0.00	0.78	0.6%	−0.01	−0.84	5.9%	0.00	−1.94	50.4%
	(−0.21)	(−5.43)		(−0.29)	(0.72)		(−0.37)	(−2.26)		(−0.08)	(−9.06)	
7	0.00	−0.26	23.9%	0.00	0.49	0.3%	0.00	−0.74	5.2%	0.00	−1.84	50.2%
	(−0.12)	(−5.04)		(−0.21)	(0.48)		(−0.28)	(−2.11)		(0.03)	(−9.02)	
10	0.00	−0.23	20.6%	0.00	0.19	0.0%	0.00	−0.66	4.5%	0.00	−1.75	49.0%
	(−0.06)	(−4.58)		(−0.16)	(0.19)		(−0.21)	(−1.94)		(0.11)	(−8.81)	
20	0.00	−0.24	23.0%	0.00	0.38	0.2%	0.00	−0.63	4.3%	0.00	−1.75	51.7%
	(0.02)	(−4.92)		(−0.09)	(0.40)		(−0.14)	(−1.91)		(0.22)	(−9.30)	
30	0.00	−0.24	23.1%	0.00	0.37	0.2%	0.00	−0.62	4.2%	0.00	−1.75	51.6%
	(0.04)	(−4.92)		(−0.08)	(0.38)		(−0.13)	(−1.87)		(0.24)	(−9.29)	

TABLE 9
Regressions of Option Implied Volatility on Eurostoxx 50 Variables

Table 9 reports estimated parameters of regressions of option implied volatility on Eurostoxx 50 modeled and index variables. The explained variables $\Delta y_{n,t}$ in equation (27) are first-order changes in the option implied volatility of the Eurostoxx 50 index for various option maturities. The regressors $\Delta p_{i,t}$ are first-order changes in the first and second state variables and relative changes in the discounted 1-year constant-maturity dividend future and the stock market. t-statistics are in parentheses.

	Monthly Data: Aug. 2008–June 2015											
	1st State Variable			2nd State Variable			Discounted 1st Future			Stock Market		
	a	b	R^2	a	b	R^2	a	b	R^2	a	b	R^2
1M	0.00	−0.07	10.6%	0.00	0.16	0.2%	0.00	−0.56	20.8%	0.00	−0.73	55.6%
	(0.16)	(−3.10)		(0.08)	(0.40)		(−0.02)	(−4.60)		(0.50)	(−10.05)	
2M	0.00	−0.15	38.1%	0.00	−0.59	4.3%	0.00	−0.54	24.1%	0.00	−0.57	68.5%
	(0.46)	(−6.63)		(−0.49)	(−1.80)		(0.40)	(−4.76)		(1.16)	(−12.45)	
3M	0.00	−0.06	16.4%	0.00	0.30	1.4%	0.00	−0.47	27.4%	0.00	−0.54	58.2%
	(0.10)	(−3.98)		(0.01)	(1.06)		(−0.13)	(−5.52)		(0.40)	(−10.61)	
6M	0.00	−0.04	15.9%	0.00	0.20	1.1%	0.00	−0.39	33.2%	0.00	−0.40	54.0%
	(0.01)	(−3.90)		(−0.07)	(0.94)		(−0.24)	(−6.34)		(0.25)	(−9.74)	
9M	0.00	−0.09	44.7%	0.00	−0.24	2.4%	0.00	−0.33	30.1%	0.00	−0.29	58.0%
	(−0.04)	(−7.59)		(−0.90)	(−1.32)		(−0.02)	(−5.55)		(0.20)	(−9.92)	
1Y	0.00	−0.08	43.7%	0.00	−0.16	1.3%	0.00	−0.27	25.0%	0.00	−0.23	47.0%
	(−0.18)	(−7.44)		(−0.96)	(−0.97)		(−0.20)	(−4.89)		(−0.07)	(−7.96)	
2Y	0.00	−0.05	12.9%	0.00	0.08	0.2%	0.00	−0.17	6.6%	0.00	−0.09	4.6%
	(−0.36)	(−3.25)		(−0.69)	(0.40)		(−0.39)	(−2.24)		(−0.48)	(−1.85)	
3Y	0.00	−0.04	3.3%	0.00	0.25	1.0%	0.00	−0.12	1.4%	0.00	−0.01	0.0%
	(−0.37)	(−1.55)		(−0.49)	(0.83)		(−0.40)	(−1.02)		(−0.54)	(−0.11)	
4Y	0.00	−0.02	0.4%	0.00	0.42	1.0%	0.00	−0.03	0.0%	0.00	0.09	0.8%
	(−0.30)	(−0.54)		(−0.30)	(0.83)		(−0.35)	(−0.14)		(−0.48)	(0.77)	
5Y	0.00	−0.02	0.1%	0.00	0.46	0.8%	0.00	−0.02	0.0%	0.00	0.11	0.7%
	(−0.29)	(−0.31)		(−0.27)	(0.75)		(−0.32)	(−0.07)		(−0.43)	(0.72)	

TABLE 10
Regression Slope Coefficients on (minus) the Second Factor

Table 10 reports the slope coefficients of the regression $y_{n,t}=\alpha_n+\beta_n(-\tilde{p}_t)+\epsilon_{n,t}$, where y_n is either the nominal yield y_n^{NOM}, the break-even inflation $\hat{i}_n$, or the real interest rate y_n^{REAL}, and n denotes the time to maturity in years.

n	y_n^{NOM}	i_n	y_n^{REAL}
1	3.49	5.20	−1.81
2	3.77	4.76	−1.01
5	4.72	3.97	0.75
10	4.61	2.52	2.09
30	4.34	2.13	2.21

break-even inflation (i_n), and the implied real yields ($y_n^{\mathrm{REAL}}=y_n^{\mathrm{NOM}}-i_n$) on the second state variable $\tilde{p}$. For ease of interpretation, Table 10 reports the regression coefficients with the sign flipped, so these are the slope coefficients of the regression on $-\tilde{p}$.

The betas (regression slope coefficients) of the break-even inflation are positive and decline with maturity. The betas of the short-term real yield are negative, whereas the beta of the slope of the real term structure is positive and increasing with maturity. We argue that these patterns can be explained if $-\tilde{p}$ is proportional to the instantaneous risk-neutral expected (i.e., break-even) inflation i:

$$(28) \qquad -\tilde{p} \quad \propto \quad i.$$

This interpretation implies that the regression coefficients for the break-even inflation are as follows:

$$(29) \qquad \beta^i \quad = \quad \frac{\mathrm{cov}(i_n,-\tilde{p})}{\mathrm{var}(\tilde{p})} \quad \propto \quad \mathrm{cov}(i_n,i),$$

with $1/(\sigma(i)\sigma(\tilde{p}))$ as the constant of proportionality. In Table 10, the regression coefficients of the break-even inflation are all positive and gradually decline with maturity. This declining pattern is quite natural because shocks to the risk-neutral expected inflation die out because of the mean reversion in the factor.

To understand the betas on real interest rates, we first decompose the break-even inflation as the sum of expected inflation and an inflation risk premium, $i=i^e+\theta^i$. The real yield can be decomposed into a real interest rate and real risk premium $y_n^{\mathrm{REAL}}=r+\theta_n^{\mathrm{REAL}}$. The betas for the real yields then are

$$(30) \qquad \beta_n^{\mathrm{real}} \quad \propto \quad \mathrm{cov}(r,i)+\mathrm{cov}(\theta_n^{\mathrm{REAL}},i).$$

For the 1-year maturity, where θ_1^{REAL} is likely to be small, the real rate beta is negative. This can be explained by a negative correlation between the short-term real interest rate and the inflation risk premium, as argued by Chernov and Mueller (2012). Betas of maturities longer than 2 years are positive, suggesting that the real interest rate risk premium covaries stronger with risk-neutral expected inflation for longer maturities than for short maturities.

Our findings are in line with the definition of our state variable as $p=g-y-\theta$. The interpretation of the second state variable as the expected inflation suggests that either $y-g$ or θ is positively correlated with expected inflation. A positive correlation between expected inflation and the discount rate $y-g$ is predicted by the inflation illusion models of Modigliani and Cohn (1979) and Campbell and Vuolteenaho (2004). A positive correlation between expected inflation and the equity risk premium is documented by Bekaert and Engstrom (2010). Both these explanations imply that real discount rates for stocks are positively correlated with expected inflation.

D. Further Interpretation

In summary, we can interpret the first state variable as driven by short-term risk premiums, economic confidence, and macro news and the second state variable as (minus) expected inflation. These 2 variables are negatively correlated in the sample (the correlation coefficient is −0.65). This is consistent with Hasseltoft and Burkhardt (2012), who document that since the turn of the century, the correlation between growth and inflation has become positive. The relation of stock prices to the economy then works as follows: A good state of the economy, evidenced by strong economic data and rising confidence, improves the immediate outlook for dividends and reduces the risk premium shown in the first state variable. A strong economy also feeds through into higher nominal interest rates and break-even inflation, which therefore correlate positively with the first state variable, and into lower CDS spreads and IVs, which indeed correlate negatively. A strong economy decreases the second state variable, which is particularly visible in the confidence betas. Higher interest rates and break-even inflation are the likely causes of the reduction of the second state variable because it follows from the relatively strong discounting effect of interest rates on the present value of dividends that are further ahead in the future. The element of risk is less important for such longer-dated dividends. This interpretation is consistent with Ang and Ulrich ((2012), p. 23), who find that "The term structure of equity returns is downward sloping due to the risk premium associated with expected inflation decreasing with horizon." In an improving economy, for near-term dividends, the discounting effect is outweighed by an improving dividend outlook and lower risk premium, as captured by the first state variable. Because this effect is stronger than the negative consequences for the second state variable, overall, this leads to the result that the stock market, interest rates, and inflation are positively related.

VII. Other Markets and Robustness Checks

In the Supplementary Material we provide a range of robustness checks on our empirical results and also present results for 3 other markets. We now briefly summarize these results; refer to the Supplementary Material for full details.

A. Other Markets

The Supplementary Material presents estimation results for 3 other markets: the Nikkei 225 index, the S&P 500 index, and the FTSE 100 index. For the latter 2 indices, we use OTC dividend swap data. In general, the results for these 3 other markets are similar to what we find for the Eurostoxx 50 index. We find a good fit of dividend derivative prices, and in all markets, the first factor exhibits fast mean reversion, whereas the second factor has slower mean reversion. We also find that stock market returns are explained well by the 2-state model, using the

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reconciliation analysis of Section V. There are, of course, some differences. For the OTC markets, we have to resort to a monthly frequency for the analysis because the daily data exhibit stale prices. Also, for the S&P 500 index and the FTSE 100 index, the reconciliation analysis shows that the first dividend derivative explains more and the price–dividend ratio explains less compared with the Eurostoxx 50 and Nikkei 225 results.

In the Supplementary Material, we also show the dividend volatility term structure, as implied by our 2-state model estimated on U.S. OTC dividend swap data. We compare this to the dividend volatility term structures implied by the habit-formation model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004), both calibrated to U.S. data. We find that our empirical estimates of the volatility term structure are broadly in line with the long-run risk model. In contrast, the habit-formation model generates a volatility term structure that strongly differs from our estimates.

B. Robustness Checks

The first robustness check concerns the modeling of the measurement errors η_t in the measurement equations of the state-space model in equation (13). We allow for serial correlation in the measurement error of the first measurement equation because we see some residual autocorrelation in the pricing errors of the benchmark model. The results show that our benchmark results are robust to allowing for such serial correlation.

In a second robustness check, we use an alternative modeling approach. Instead of modeling growth rates, interest rates, and risk premiums at once, we now model $g_{t+1}-\theta_{t+1}$ and then use observed interest rates to calculate the present values of dividends, which is possible if one assumes that interest rates and $g_{t+1}-\theta_{t+1}$ are independent and uses the expectations hypothesis for bond markets. The results in the Supplementary Material show that this alternative approach does not work well. This demonstrates the advantage and importance of modeling the 3 components at once, which does not require specific assumptions on the dependence of these variables.

VIII. Conclusion

In this article, we analyze the term structure of stock index dividend prices. We show that modeling a single variable is sufficient to describe the dynamics and level of this term structure. This variable is equal to the dividend growth minus the risk-free rate and a term capturing the risk premium. We propose a 2-factor model for this discounted risk-adjusted growth variable, capturing the dynamics of short- and medium-term dividend growth. The 2 factors shape a term structure of dividend growth that fits the data well, and they determine the dynamics of the price–dividend ratio. Applied to the Eurostoxx 50, most of the daily variation of the stock market can be explained using this model.

The model estimates show that the short-term factor reflects a horizon of less than 1 year, and the medium-term factor reflects a horizon of several years. Deploying two states next to each other allows some distinction between sudden occurrences and those at business-cycle proportions. The state-space model imposes the return to a constant mean level of growth in the long run. Given the fast mean reversion of the 2 factors, this suggests that investors do not change their opinion about growth rates beyond business-cycle horizons much. Thus, long-run growth does not seem to be a major source of stock market variation. Interest rates are a part of the discounted risk-adjusted dividend growth, and they are observable to investors. Our results then suggest that at these long horizons, interest rate variation is offset by variation in the difference between the dividend growth rate and a risk-premium correction.

Appendix A. Measurement Equations

Appendix A describes the details of the derivation of the measurement equations. We rewrite the state equations (equations (10) and (11)) in vector form and then derive the discrete-time implications of the model. Denote $Q_t = \binom{p_t}{\tilde{p}_t}$ the 2×1 vector of the factors and $\overline{Q} = \binom{\overline{p}}{\overline{p}}$ as the 2×1 vector of the constant infinite growth rate. In a 2-equation matrix format, the system becomes

$$\text{(A-1)} \qquad dQ_t = \begin{pmatrix} dp_t \\ d\tilde{p}_t \end{pmatrix} = \left[\begin{pmatrix} -\varphi & \varphi \\ 0 & -\psi \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \psi \overline{p} \end{pmatrix} \right] dt + \begin{bmatrix} \sigma_p & 0 \\ 0 & \sigma_{\tilde{p}} \end{bmatrix} \begin{pmatrix} dW_p \\ dW_{\tilde{p}} \end{pmatrix}.$$

This system of differential equations in matrix notation is as follows:

$$\text{(A-2)} \qquad dQ_t = C[Q_t - \overline{Q}]dt + \Sigma\, dW,$$

which has the following general solution:

$$\text{(A-3)} \qquad Q_{t+1} = \overline{Q} + \Phi(Q_t - \overline{Q}) + \varepsilon_{t+1},$$

and of which the eigenmatrix solves to

$$\text{(A-4)} \qquad \Phi = \begin{pmatrix} e^{-\varphi} & \dfrac{\varphi}{\varphi - \psi}(e^{-\varphi} - e^{-\psi}) \\ 0 & e^{-\psi} \end{pmatrix}.$$

Substituting the expression for the eigenmatrix into equation (B-3) delivers state equations:

$$\text{(A-5)} \qquad \begin{pmatrix} p_{t+1} \\ \tilde{p}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 - e^{-\varphi} & -\dfrac{\varphi}{\varphi - \psi}(e^{-\psi} - e^{-\varphi}) \\ 0 & 1 - e^{-\psi} \end{pmatrix} \begin{pmatrix} \overline{p} \\ \overline{p} \end{pmatrix} + \begin{pmatrix} e^{-\varphi} & \dfrac{\varphi}{\varphi - \psi}(e^{-\psi} - e^{-\varphi}) \\ 0 & e^{-\psi} \end{pmatrix} \begin{pmatrix} p_t \\ \tilde{p}_t \end{pmatrix} + \varepsilon_{t+1}.$$

We model the correlation between the innovation in the growth rate ν_{t+1} and the errors ε_{t+1} in these state equations as $\nu_{t+1} = \beta' \epsilon_{t+1}$, where $\beta = (\beta_p, \beta_{\tilde{p}})'$ is a 2-by-1 vector. Next, we use this process to write the n-period ahead growth rate as a function of the following factors:

$$\text{(A-6)} \qquad \pi_{t+n} = \alpha'(\overline{Q} + \Phi^{n-1}(Q_t - \overline{Q})) + \alpha' \sum_{i=1}^{n-1} \Phi^{n-i} \varepsilon_{t+i} + \beta' \varepsilon_{t+n},$$

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in which $\alpha' = (1\ 0)$. This can be substituted into the pricing equation:

$$\text{(A-7)} \qquad \ln P_{t,n} - \ln D_t = E_t\left(\sum_{i=1}^{n} \pi_{t+i}\right) + \frac{1}{2} Var_t\left(\sum_{i=1}^{n} \pi_{t+i}\right).$$

The right-hand side can be worked out as follows:

$$\text{(A-8)} \qquad \begin{aligned} \ln P_{t,n} - \ln D_t &= \alpha'(n\overline{Q} + B_n(Q_t - \overline{p})) \\ &\quad + \frac{1}{2}\text{Var}_t\left(\sum_{i=1}^{n}\left(\alpha' \sum_{j=1}^{i-1} \Phi^{n-j}\varepsilon_{t+j} + \beta'\varepsilon_{t+i}\right)\right), \end{aligned}$$

which in turn implies the following:

$$\text{(A-9)} \qquad \ln P_{t,n} - \ln D_t = \alpha'(n\overline{Q} + B_n(Q_t - \overline{p})) + \frac{1}{2}\sum_{i=1}^{n}(\beta' + \alpha' B_i)\Sigma(\beta + B_i{}'\alpha),$$

where matrix B_i is an expression constructed from the eigenmatrix:

$$\text{(A-10)} \qquad B_i = (I + \Phi + \cdots + \Phi^{i-1}) = (I - \Phi)^{-1}(I - \Phi^i).$$

The equations are written without vector notation. By the definition of Φ, B_n is worked out as follows:

$$\text{(A-11)} \qquad \begin{aligned} B_n &= \begin{pmatrix} \dfrac{(1-e^{-n\varphi})}{(1-e^{-\varphi})} & \dfrac{\varphi}{\varphi - \psi}\left(\dfrac{(1-e^{-n\psi})}{(1-e^{-\psi})} - \dfrac{(1-e^{-n\phi})}{(1-e^{-\phi})}\right) \\ 0 & \dfrac{(1-e^{-n\psi})}{(1-e^{-\psi})} \end{pmatrix} \\ &= \begin{pmatrix} \varphi_n & \dfrac{\varphi}{\varphi-\psi}(\psi_n - \varphi_n) \\ 0 & \psi_n \end{pmatrix}, \end{aligned}$$

with shorthand notation:

$$\text{(A-12)} \qquad \phi_n = \frac{(1-e^{-n\phi})}{(1-e^{-\phi})}$$

$$\text{(A-13)} \qquad \psi_n = \frac{(1-e^{-n\psi})}{(1-e^{-\psi})}.$$

An expression that consists of scalars only is obtained by substituting all elements of the previous equation in the measurement equation:

$$\text{(A-14)} \qquad \begin{aligned} &\ln P_{t,n} - \ln D_t \\ &= (1\ \ 0)\left(n\begin{pmatrix}\overline{p}\\ \overline{p}\end{pmatrix} + \begin{pmatrix} \varphi_n & \dfrac{\varphi}{\varphi-\psi}(\psi_n - \varphi_n) \\ 0 & \psi_n \end{pmatrix}\begin{pmatrix} p_t - \overline{p} \\ \tilde{p}_t - \overline{p}\end{pmatrix}\right) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\left((\beta_p \beta_{\tilde{p}}) + (1\ \ 0)\begin{pmatrix} \varphi_i & \dfrac{\varphi}{\varphi-\psi}(\psi_i - \varphi_i) \\ 0 & \psi_i \end{pmatrix}\right)\begin{pmatrix} \sigma^2 & 0 \\ 0 & \tilde{\sigma}^2 \end{pmatrix} \\ &\quad \times \left(\begin{pmatrix}\beta_p \\ \beta_{\tilde{p}}\end{pmatrix} + \begin{pmatrix} \varphi_i & 0 \\ \dfrac{\varphi}{\varphi-\psi}(\psi_i - \varphi_i) & \psi_i \end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right) + \eta_{t,n}. \end{aligned}$$

From this, an expression that consists of scalars only is obtained:

$$\text{(A-15)} \quad \begin{aligned} &\ln P_{t,n} - \ln D_t \\ &= n\overline{p} + \varphi_n(p_t - \overline{p}) + \frac{\varphi}{\varphi - \psi}(\psi_n - \varphi_n)(\tilde{p}_t - \overline{p}) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\left(\sigma_p^2(\beta_p + \varphi_i)^2 + \sigma_{\tilde{p}}^2\left(\beta_{\tilde{p}} + \frac{\varphi}{\varphi - \psi}(\psi_i - \varphi_i)\right)^2\right) + \eta_{t,n}, \end{aligned}$$

which is the same as equation (13) in the main text. The right-hand term on the right-hand side is referred to in the article as the "convexity term." Dividend return variance follows from equation (B-9). Conditional variance is reduced to the following:

$$\text{(A-16)} \quad \sigma_t^2(\ln P_{t+1,n} - \ln P_{t,n}) = \sigma_t^2(\ln D_{t+1}) + \tfrac{1}{2}\alpha' B_{n-1} \Sigma B_{n-1}' \alpha.$$

Substituting for the variables in the 2-state model yields equation (16), from which the volatilities in Figure 3 are shown by taking square roots.

Appendix B. Dividend Derivatives Data

1. Constant-Maturity Construction

Dividend derivatives usually expire at a fixed date near the end of the calendar year,[17] and therefore their time to maturity shortens by 1 day for each day that passes. For application in the state-space model, growth rates of a constant horizon are required. The horizons of the measurement equations regard annual increments, and the state equations regard 1-day increments. To obtain growth rates from prices with constant maturities, we interpolate derivatives with adjacent expiry dates. The interpolation is weighted by a scheme that reflects the uneven distribution of dividends through the year. For example, in the spring season, 60% of the Eurostoxx 50 dividends of a full index year are paid in a matter of a few weeks (Figure B1).

Derivatives prices that have a constant horizon from any observation date are constructed from observed derivatives prices. Such constant-maturity (CM) derivative prices $F_{t,n}^{\text{CM}}$ take the following shape, attaching the seasonal pattern of the dividend index as weights to the observed derivatives prices w_i, with i standing for the day in the dividend index year, $i = 1$ being the first day of the count of the dividend index (which is the first trading day following the expiry date of a dividend derivatives contract):

$$\text{(B-1)} \quad F_{t,n}^{\text{CM}} = (1 - w_i)F_{t,n} + w_i F_{t,n+1}.$$

The weight w_i of the dividend index reflects the cash dividend amount paid as a proportion of the total amount during a dividend index year. The average of the years 2005–2013 is taken. $F_{t,n}$ is the observed price of the derivative that expires nth in line into the future from the observation date onward, with $F_{t,n+1}$ expiring the following year. This weighting scheme reduces the impact of the nth derivative to expire on the CM derivative as time passes by the proportion w_i of dividends that have actually been declared. Its complement $(1 - w_i)$ is the proportion that remains to be declared until the expiry date and is therefore an expectation of undeclared dividends for year n at the observation date. In order to produce a derivative price with constant maturities, this undeclared amount is balanced by the proportion of the price of the derivative expiring the year after. In so doing, the CM price reflects no seasonal pattern while still accounting for the seasonal shift in impact from the nth derivative to the

[17]The Nikkei 225 dividend index runs until the last trading day in March.

next. For example, during the dividend season in the spring, the weight is shifted more quickly from the first to the second derivative[18] than in other parts of the year.[19]

2. The First-to-Expire CM Derivative

The weighting scheme in equation (B-18) is applied to obtain all CM derivatives prices, except for the first CM derivative, because the proposed approach carries measurement problems. At time t the expected dividend to be delivered at the expiration of the first derivative $E_t(D_1)$ is the sum of the dividend index DI_t as it accretes throughout the year and its unknown complement $E_t(UD_1)$:

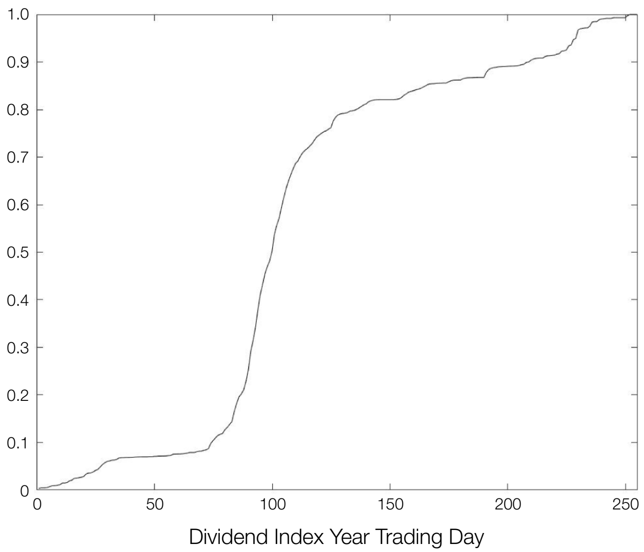
$$E_t(D_1) = \mathrm{DI}_t + E_t(UD_1). \tag{B-2}$$

For CM derivatives with horizons longer than the first, the weight w_i in equation (B-17) is the average seasonal pattern in the preceding decade, which may not necessarily resemble that of a particular dividend index year $\mathrm{DI}_t/E_t(D_1)$. The difference between the two is shown in Figure B2; for example, in Apr. 2013 the payments of Eurostoxx 50 dividends had already reached 33% of the annual total, whereas on average in the years 2005–2013 it stood at 20%. This advance did not drop below 10% until a month later. In general, dividend payments in 2012 and 2013 seem to have taken place earlier in the calendar year than usual in the preceding years. Weighting the first derivative by the average of the preced-

FIGURE B1

Proportion of Dividend Payments throughout the Eurostoxx 50 Dividend Index Year

Figure A1 shows the proportion of dividend payments throughout the Eurostoxx 50 dividend index year. The first trading day of a dividend index year is the Monday following the third Friday of December. The chart depicts the average of the years 2005–2013.



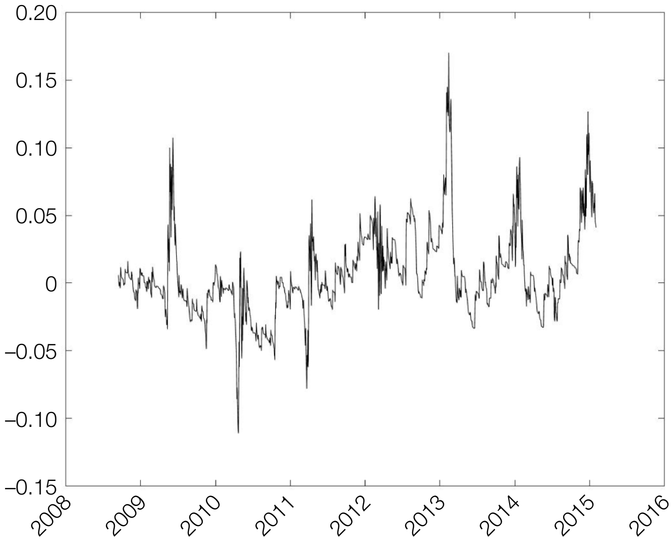
[18] *First and second derivatives* is shorthand for the derivatives that are first and second to expire.

[19] A linear weighting scheme would reflect the adjacent derivative prices unevenly. For example, halfway through the dividend index year, 80% of annual dividends are already declared and paid. Linear weighting would then overemphasize the information contained in the price of the derivative in equation (B-17) that is the soonest to expire.

FIGURE B2

Eurostoxx 50 Dividends Paid Out up to a Given Date Relative to Their Average

Figure A2 shows the degree to which dividends vary year by year as to the exact dates on which they are paid out. Dividends paid from the start of a dividend index year up to a given day in that year deviate from the average paid up to that particular calendar day (2009–2015).



ing decade when dividends realize sooner in the year than the average, as was the case in Apr. 2013, overemphasizes the importance of that first derivative to the 1-year CM derivative. This first CM derivative will then contain backward-looking information as well as underemphasize the unrealized proportion of the contemporaneous dividend index, both to the tune of the difference between the historical average and the realized dividend index. To avoid this issue, the first CM derivative is construed by defining the weight as the proportion of the dividend index that has been realized of the total expected dividend for that year only:

$$F_{t,1}^{\mathrm{CM}} \quad = \quad F_{t,1} - \mathrm{DI}_t + \frac{\mathrm{DI}_t}{F_{t,1}} F_{t,2}. \tag{B-3}$$

For building a first CM derivative with a constant 1-year horizon as a stochastic variable, we include unknown $E_t(UD_1)$ and exclude known DI_t. The expectation of full-year dividends is proxied by the equivalent observation. Later CM derivatives do not weight variables that have already been partly realized; hence, the weighting issue of the first CM derivative does not reoccur. For $n \geq 2$, the prices of CM derivatives remain constructed as in the weighting equation (equation (B-18)).

3. Calculating Seasonal Weights for Different Dividend Index Years

Expiry years do not have the same number of trading days every year or across markets. Not only do trading holidays differ, but also the expiry date is set to the third Friday in December in every expiry year. This day falls anywhere between Dec. 15 and Dec. 21,[20] and the number of trading days fluctuates accordingly.

[20]With the exception of the Nikkei 225.

To establish a seasonal pattern for w_i that is correct for the actual number of trading days in each expiry year, realized dividends are normalized and averaged. First, the amount of dividends paid on a given day is expressed as a percentage of the total dividends paid in the matching dividend index year. Next, for each expiry year, these percentages are normalized to a set number of trading days. Finally, they are averaged. For calculating the values in the weighting equation, they are rescaled to the actual number of trading days in the dividend index year in question. This approach guarantees that in every expiry year, weight w_i starts at 0 and ends the year at 100%, regardless of the number of trading days.

Supplementary Material

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