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Plurals and modals

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ABSTRACT
Consider one of several things. Is the one thing *necessarily* one of the several? This key question in the modal logic of plurals is clarified. Some defenses of an affirmative answer are developed and compared. Various remarks are made about the broader philosophical significance of the question.

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1. Introduction

Consider some things, and choose any one of them. Is the chosen thing *necessarily* one of the things from which it was chosen? It is frequently assumed that it is (at least on the assumption that all of the things in question still exist).[1] If some things did not include our chosen thing, then these things would simply not be the things with which we started, that is, the things from which we made a choice. If the things from which we chose exist at all, then *necessarily* (subject to the mentioned existential assumption) they include the chosen thing. Likewise, if some other thing is *not* one of things from which we chose, then this too is a matter of (conditional) necessity. These two frequently assumed thoughts can be formalized as follows:[2]

$$x \prec yy \rightarrow \Box(Ex \wedge Eyy \rightarrow x \prec yy) \qquad (\text{Rgd}^{+})$$

$$x \not\prec yy \rightarrow \Box(Ex \wedge Eyy \rightarrow x \not\prec yy) \qquad (\text{Rgd}^{-})$$

Let their conjunction be the claim that pluralities are *rigid*, which we abbreviate as (Rgd).[3]

The rigidity principles can be regarded as strong extensionality principles, which state that pluralities are tracked extensionally across possible worlds. The ordinary principle of extensionality states that if every one of *these things* is one of *those things* and vice versa, then *these things* are the very same things

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as *those things*. (We shall return to the question of how best to formalize this principle.) This is widely assumed to provide a criterion of identity for pluralities; and like criteria of identity in general, the principle is widely thought to hold of necessity. Even so, the principle provides no information about how pluralities are tracked across possible worlds. The rigidity principles fill this gap. They tell us that necessarily any given things have their members by necessity. A plurality is therefore not allowed to vary in its membership across possible worlds. Any variation in membership would result in our talking about some other things, not the things in question.

It is important not to misunderstand the rigidity claims. Assume Sophia is one of the philosophers. Does it follow that she is *necessarily* one of the philosophers? (For simplicity, I suppress the assumption that the objects in question still exist.) If so, this would imply the wildly implausible claim that necessarily Sophia is a philosopher. Fortunately, Kripke long ago taught us how to respond. Let *pp* be the things such that anything is one of them if and only it is in fact a philosopher. What is necessarily the case is that Sophia is one of *pp*. But it is not necessary that *pp* are all and only the philosophers. Sophia might have become a psychologist, not a philosopher. Then, she would not have been included in the ranks of the philosophers, although she would still have been one of *pp*.

To appreciate how the rigidity claims for pluralities have real bite, even after this pre-Kripkean confusion is cleared up, it is useful to contrast pluralities with what I shall call *groups*, such as teams, clubs, committees, and the like.[4] Consider a philosophy department. The department might have had other members than it in fact has: Sophus might have been hired instead of Sophia. Someone who *is* a member of the department might not have been so; and someone who *is not* a member, might nevertheless have been one. To ensure ourselves that there is a genuine difference between the modal profile of pluralities and groups, not merely the result of some undetected use of non-rigid designators, we can run the same argument using a proper name or a variable to pick out Sophia's department. A similar contrast obtains between plurals and predicates, which are clearly not rigid, as is illustrated by the following examples:

(1) Timothy Williamson is a philosopher but he might not have been one.
(2) Bill Clinton is not a philosopher but he might have been one.

In the light of the existence of non-rigid groups, we may want to reconsider the received view that pluralities are rigid. Indeed, some apparent counterexamples to the rigidity of pluralities are found in natural language. A nice example is the following ad which I once saw for a gym:

(3) Join, and become one of us!

The plural pronoun 'us' is naturally taken to stand for a plurality. But when so interpreted, the message presupposes that it is possible to become a member of a plurality of which one is not already a member.[5]

How are we to respond to these apparent counterexamples to the rigidity of pluralities? Interesting though they are, the examples are inconclusive. Plural pronouns ordinarily stand for pluralities. But the apparent counterexamples can be explained away if we allow that a plural pronoun can sometimes stand for a group or function as a covert description. Either way, non-rigid would be unsurprisingly. Consider, for instance, a bohemian parent who upon seeing some particularly smug business school students tells her daughter:

(4) I'm glad you're not one of them.

It is natural to understand the parent as expressing joy that her offspring is not (in some salient respect) *like* the students in question rather than pleasure with a fact about plural non-membership. Thus, (4) poses no more of a challenge to the rigidity of pluralities than the following sentence poses to the necessity of identity:

(5) I'm glad you're not him.

To make progress, we need more systematic considerations for or against the rigidity of pluralities.

The basic thought on which I shall base my defense of plural rigidity is that a plurality is nothing over and above its members. More precisely, a plurality is fully specified when we have circumscribed the things that are its members. By contrast, a group is something over and above its members, as it can have a non-trivial membership criterion, which allows a non-trivial tracking of the group across possible worlds. A group is therefore not fully specified merely by circumscribing its members; we additionally need to specify (at the very least) its membership criterion. Since a plurality is nothing over and above its members, there is no material available that might underwrite a non-trivial tracking across possible worlds. All we have to go on are the members. So the only way to track a plurality is the trivial one, which ensures plural rigidity. On this picture, pluralities manifest extensionality in its purest and strongest form.[6]

In what follows, I shall attempt to clarify and develop these ideas. I shall be particularly interested in the extent to which broadly logic considerations support plural rigidity. The result will be a disentangling and clarification of several strains of the basic thought. We shall find that plural rigidity figures at the heart of a network of ideas having to do with the extensionality of pluralities. Since the ideas in this network are true of core uses of our plural resources in ordinary language and thought, I commend them as an explication of these resources. However, because I accept the existence of non-rigid groups, it would not trouble me particularly if my view had to be regarded as partially a matter of stipulation. Should it turn out that non-rigid groups can sometimes serve as an interpretation of plurals, I would restrict myself to 'strict pluralities,' which, I would claim, are correctly described by the mentioned network of interlocking ideas.

2. Why plural rigidity matters

The question whether pluralities are rigid might seem abstruse and of limited interest. In fact, the question has vast repercussions for a number of debates in philosophical logic, metaphysics, and the philosophy of mathematics.

One example is the debate about what forms of higher order logic there are. Can plural logic be replaced by monadic second-order logic or even reduced to it? Or is some reduction in the opposite direction possible? If pluralities are rigid, then the two forms of logic have different modal profiles: as we have seen, predicates are non-rigid.[7] This makes either kind of reduction implausible and suggests instead that the two kinds of logic should be taken at face value and allowed to co-exist without one swallowing the other.

A related example concerns the semantics of predication. Provided we have ordered pairs at our disposal, it is technically possible to use plurals for this purpose: we can take the semantic value of a predicate to be the plurality of tuples of which the predicate is true. However, this semantics seems contrived. In addition to the lack of homophonicity even on the intended interpretation, the need for *ad hoc* tricks to handle predicates that are true of nothing, and the artificial reliance on tuples, there is the aforementioned mismatch of modal profiles.[8]

Next, the rigidity of pluralities is a pillar of one of Williamson's main arguments for *necessitism*, the metaphysical view that necessarily everything necessarily exists.[9] The denial of this view is *contingentism*. When we go on to consider arguments for the rigidity of plurals, it will be important to keep in mind whether the argument is intended to be given in a necessitist setting (which is always easier) or a contingentist setting (which requires greater care).

Finally, the question of the rigidity of pluralities plays an essential role in an approach to mathematics and the phenomenon of indefinite extensibility that I have developed in recent work and canvass in the final section of this article.[10]

3. A Kripkean plausibility argument for the rigidity of sets

It will be useful to begin our investigation of the rigidity of pluralities by reminding ourselves of Kripke's celebrated argument for the necessity of identity and distinctness, and by observing some striking consequences that this argument has for the metaphysics of sets. Kripke's argument turns on Leibniz's law:

$$x = y \rightarrow (\phi(x) \leftrightarrow \phi(y)) \qquad \text{(Leibniz)}$$

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Given the assumption $\Box(x = x)$, the law entails $x = y \rightarrow \Box(x = y)$. Moreover, given the Brouwerian axiom

$$\phi \rightarrow \Box\Diamond\phi \tag{B}$$

we can now also derive the necessity of distinctness: $x \neq y \rightarrow \Box(x \neq y)$.[11]

However, a contingentist may object to the assumption of $\Box(x = x)$. After all, in a negative free logic, '$x = x$' can be used as an existence predicate. If so, then what is assumed is the necessary existence of x! The problem is easily circumvented. The contingentist will have no problem with the assumption that x satisfies the predicate '$\Box(x = x \rightarrow x = \ldots)$'. Applying (Leibniz), this enables us to derive formulations of the necessity of identity and distinctness that are acceptable to the contingentist:[12]

$$x = y \rightarrow \Box(x = x \rightarrow x = y) \tag{$\Box =$}$$

$$x \neq y \rightarrow \Box(x \neq y) \tag{$\Box \neq$}$$

As before, the derivation of the latter from the former relies on (B).

As Kripke realized, Leibniz's law has important metaphysical consequences. The case of sets provides a nice illustration. Consider the principle of extensionality:

$$\forall u(u \in x \leftrightarrow u \in y) \leftrightarrow x = y \tag{Ext}$$

Leibniz's law reveals a respect in which this is quite a strong principle. Let x and y be coextensional sets. By (Ext), x and y are identical. Observe now that x satisfies the predicate

$$\Box\forall u(u \in \ldots \leftrightarrow u \in x).$$

So by Leibniz's law, y too satisfies this predicate. Thus, the principle of extensionality logically entails that two coextensional sets are necessarily coextensional.[13]

This is an important consequence. Assume that sets are like groups in that they are tracked across possible worlds in some intensional way. It would then be a complete mystery why actual coextensionality should suffice for identity and therefore also for necessary coextensionality. Consider the analogous claim concerning groups: If two groups in fact have the same members, they are identical and therefore necessarily have the same members. Since membership in a group is contingent and thus is subject to 'drift' as we consider alternative possible worlds, this claim is wildly implausible. Once membership drift is permitted, there is no guarantee that groups whose members in fact coincide will not drift apart in some other possible world. Yet in the case of sets, the principle of extensionality and the necessity of identity entail that there can be

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no such drifting apart but that any drift would have to be parallel drift. The only explanation of this prohibition on drifting apart is that there can be no drift in the membership of a set at all; in other words, that sets are rigid.

Although the argument is not deductively valid, it is hard to resist. The only reason to accept the principle of extensionality – and the important consequence noted above – is that a set, unlike a group, is fully specified by its elements. Thus, when tracking a set across possible worlds, there is nothing other than the elements to go on. This ensures that the tracking is rigid.[14] By contrast, when tracking a group, there *is* more than the members to go on. But precisely because there is more to go on, we cannot accept a principle of extensionality for groups. Having the same members does nothing to ensure that two groups coincide in whatever additional factor it is that enables the non-rigid tracking. These considerations give rise to a dilemma that applies not only to sets but to any other notion of collection: either we have to give up the principle of extensionality, or else we have to accept the rigidity principles as well.[15] There is no stable middle ground. Kripke famously taught us that there can be no 'soft identity theory' in the philosophy of mind, according to which mental states are identical with physical ones but only contingently so, only the 'hard identity theory' committed to necessary identity. Our present conclusion is analogous. There can be no 'soft extensionalism' concerning sets or other kinds of collection, only 'hard extensionalism' that incorporates the rigidity claims and the idea of transworld extensionality that they embody.[16]

I shall now consider two objections. The first one is based on a mereological analogue of our plausibility argument concerning sets. Assume that x and y share all their parts; that is $\forall z(z \leq x \leftrightarrow z \leq y)$, where $\leq$ indicates parthood. Provided that parthood is reflexive and anti-symmetric, it follows that x and y are identical and thus also that they necessarily share all their parts. Yet this seems compatible with parthood being non-rigid! Does this undermine my plausibility argument concerning sets?[17] I think not. The crux of my plausibility argument is the claim that any reason to accept the principle of extensionality is also a reason to accept rigidity of elementhood. By contrast, there is reason to accept the two mereological principles which is *not* also a reason to accept rigidity of parthood. I shall have to content myself with conveying the intuitive idea. In order to make sense of contingent parthood, it is useful to think of objects as involving both matter and form.[18] For instance, a molecule that is part of me might not have been so because my form permits me to be tracked to other possible worlds on the basis of something other than just my matter. On this hylomorphic conception, it is natural to take parthood to be sensitive to both matter and form, and mutual parthood, to ensure identity not only of matter but also of form – and hence identity of the objects in question. This points to an explanation of the two mereological principle that is fully compatible with non-rigid parthood.[19]

The second objection takes its departure from the well-known fact that Leibniz's law needs to be restricted. Assume Nikita is the shortest spy. Of course, necessarily the shortest spy is the shortest spy. But it does not follow that necessarily Nikita is the shortest spy. It is often proposed that the Leibniz's law be restricted to *rigid designators* – defined as terms that refer to the same object at every world at which they refer at all – thus excluding terms like 'the shortest spy.' Ordinarily, this restriction works well. But when reasoning about sets or other kinds of collection, it threatens to undermine our Kripkean dilemma. To understand this threat, we need to distinguish between two completely different notions of rigidity. Until this paragraph, we have been concerned exclusively with a metaphysical notion of rigidity. Sets and other kinds of collection are said to be rigid if their membership is a matter of necessity, in the precise sense laid down by the kind of rigidity claims stated above. But as we have just seen, there is also the semantic notion of a rigid designator.

The problem is that it can be hard to disentangle the two kinds of rigidity. Assume that a term t refers at w_1 to a collection comprising a and b, where a and b are all and only the Fs at w_1. Assume that t refers at w_2 to the singleton collection of a, where a is the one and only F at w_2. Is t a rigid designator? Obviously, the question cannot be answered until we have been told how to track the relevant kind of collection from world to world. If the collections are tracked extensionally, then we are considering different collections, and accordingly the designator is non-rigid. But if the collections are tracked intensionally in terms of their membership criterion, then we may well be considering one and the same object, namely the collection of Fs, in which case the designator is rigid after all. The threat to our Kripkean dilemma is now apparent. To show that our use of Leibniz's law is permissible, we must first show that the terms in question are rigid designators. This involves showing that they refer to the same set across possible worlds. But this presupposes that we already know how to track sets across possible worlds! As we have seen, this is a matter of answering the question of metaphysical rigidity. Our Kripkean argument therefore appears powerless to answer the question of metaphysical rigidity as the permissibility of its appeal to Leibniz's law presupposes that an answer has already been given.

Fortunately, the threat can be avoided by reformulating the restriction on Leibniz's law. Say that a term is *purely referential* if its semantic contribution to linguistic contexts in which it occurs is exhausted by its referent; or, as Quine (1960) put, if it 'is used purely to specify its object, for the rest of the sentence to say something about' (177). Rather than restrict Leibniz's law to rigid designators, we can restrict the law to terms that are purely referential. After all, such terms serve merely to stand for, or pick out, their referents. Assume $t_1 = t_2$ is a true identity involving such terms. Then, of course, $\phi(t_1) \leftrightarrow \phi(t_2)$ is true as well, as this merely says of the referent that it is ϕ iff it is ϕ. When Leibniz's law is restricted to purely referential terms rather than to rigid designators, the problem we have discussed dissipates. The only terms involved in our argument are variables. And variables are purely referential: what a variable contributes to a linguistic context in which it occurs is nothing but its value. Thus, our Kripke-inspired argument for metaphysical rigidity goes through after all.

There is a more general lesson here as well. The problem of disentangling the semantic and metaphysical kinds of rigidity points to an unfortunate feature of the notion of a rigid designator: it runs together two kinds of consideration that are best kept apart. First, there is the semantic question of whether a term is purely referential. Then, there is the metaphysical question of how its referent is to be tracked from one possible world to another. It is true that every purely referential term is a rigid designator. But our discussion shows that we get a cleaner separation of the metaphysical and semantic questions by focusing on the notion of pure reference rather than rigid designation.[20]

4. A plausibility argument for plural rigidity

I would now like to extend the argument from the previous section to the case of pluralities. Kripke started with Leibniz's law. Is there an analogue of the law in the case of pluralities? The answer will depend on whether the relation of identity is defined between pluralities. It is far from obvious that it is. My proposal is that our extension of the Kripkean argument should use as its starting point the following indiscernibility principle for pluralities:

$$xx \equiv yy \rightarrow (\phi(xx) \leftrightarrow \phi(yy)) \qquad \text{(Indisc)}$$

where $xx \equiv yy$ abbreviates $\forall u(u \prec xx \leftrightarrow u \prec yy)$. If identity is defined on pluralities, then (Indisc) is merely the result of 'telescoping' the principle of extensionality for pluralities (which can then straightforwardly be expressed) and the plural analogue of Leibniz's law. And even if identity is not defined on pluralities, this 'telescoping' is still expressible. Either way, (Indisc) incorporates a plural analogue of Leibniz's law.

Two concerns arise. Firstly, as we have seen, the ordinary singular version of Leibniz's law needs to be restricted. Analogous considerations apply in the plural case. Fortunately, it is easy to see that (Indisc) is suitably restricted. Since plural variables are purely referential just as much as singular ones are (only in a plural way), (Indisc) is entirely legitimate. In particular, it presupposes no prior answer to the question of the rigidity of pluralities and can thus safely be employed in an argument for this rigidity thesis.

Secondly, is (Indisc) acceptable from a contingentist point of view? To assess this, we need to be more explicit about what semantics we adopt. It is natural to use a semantics on which $x \prec xx$ is true at a world w relative to an assignment σ iff all the things that σ assigns to xx exist at w, and $\sigma(x)$ is one of them.

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Furthermore, this semantics makes it natural to adopt a negative free logic.[21] The rules of universal elimination must then be formulated so as to make existential assumptions explicit; for instance, from $\forall x\, \phi(x)$, we can infer $Et \rightarrow \phi(t)$, and likewise for the universal plural quantifier. (We shall shortly have more to say about the plural existence predicate.) Given these choices, it is easy to verify that (Indisc) remains a valid principle even in a contingentist setting.[22]

We are now ready to develop our plausibility argument for the rigidity of pluralities. The next step is to derive from (Indisc) an analogue of the necessity of identity, which for obvious reasons I call *covariation*:

$$xx \equiv yy \rightarrow \Box(xx \equiv yy) \tag{Cov}$$

Given (B), we can derive the necessity of $\not\equiv$ as well.

We now come to the heart of the argument. Recall the case of sets. While ($\Box =$) is formally compatible with the non-rigidity of sets, it is far more plausible with rigidity. Precisely the same goes for (Cov) and pluralities. If plural membership was contingent – like membership in a group – why should actual coextensionality ensure necessary coextensionality? There would be nothing to prevent two actually coinciding pluralities from 'drifting apart' as we consider alternative possible worlds. The only explanation of the prohibition on such divergence is that plural membership is not subject to membership drift at all. So we establish the same dilemma as in the case of sets. There can be no 'soft extensionalism' concerning pluralities: either we need to give up the ordinary extensionality principle encapsulated in (Indisc), or else we have to accept the full transworld extensionality associated with the plural rigidity principles. Just as in the case of sets, the former horn is deeply unattractive, as it comes close to just changing the subject. So we conclude that plural rigidity is highly plausible.

It is worth noting that the real locomotive of the argument is (Cov), which is strictly weaker than (Indisc) with which the argument officially began. The covariance principle gives us precisely what its name suggests, namely that two overlapping pluralities necessarily covary in their membership. (Indisc) states that all properties of pluralities supervene on membership. To see that the latter principle goes beyond the former, consider a department whose statutes decree that all and only tenured faculty are to be members of the Hiring Committee and of the Graduate Admissions Committee.[23] Then, the two committees necessarily covary in membership. Nevertheless, the two committees have different powers, namely to hire new faculty and admit graduate students, respectively.

To be even more specific about the relation between (Indisc) and (Cov), I claim that the former 'factorizes' into the latter and the claim that the properties of a plurality supervene on what we may call its *modal membership profile*:

$$\Box(xx \equiv yy) \rightarrow (\phi(xx) \leftrightarrow \phi(yy)) \tag{Sup}$$

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We see this as follows. Clearly, (Cov) and (Sup) entail (Indisc), which in turn entails each of the former two principles. Moreover, (Cov) and (Sup) are logically independent and encapsulate different philosophical ideas, namely covariation in membership and supervenience of properties on modal membership profile, respectively. A more comprehensive 'factorization' of the ideas associated with the extensionality of pluralities will be offered in Section 6.

5. Formal arguments for plural rigidity

We now have what I regard as a fairly convincing plausibility argument for plural rigidity. But as far as formal arguments are concerned, a gap remains. Our best formal result so far is (Cov), which states that coextensive pluralities are necessarily coextensive. The desired rigidity claims state that a plurality has the same members at any world at which it exists. I would now like to investigate what it takes to formally bridge this remaining gap.

5.1. Plural existence

Since we are now aiming for formal rigor, the time has come to be entirely precise about the existential assumptions that are involved in the rigidity claims. This requires a plural existence predicate which we can use to say of some things *xx* that they exist. As we have seen, the existence of a single object *x* can be expressed simply as $x = x$. But what about the plural existence predicate?

One option is to define plural existence distributively in terms of singular existence; that is, to define *Exx* as $\forall x(x \prec xx \rightarrow x = x)$. But this is unsuccessful. For a contingentist, the initial quantifier ranges only over objects that exist at the relevant world, which renders the quantified claim trivially true for any plurality *xx* whatsoever. Another natural but unsuccessful option is to define *Exx*, by direct analogy with its singular cousin, as $xx \equiv xx$. This too is easily seen to trivialize, for exactly the same reason as our previous attempt.

One safe option is simply to adopt a primitive collective plural existence predicate *Exx*, which we stipulate to be satisfied by some things at a world just in case all these things exist at the world. Another option is available as well, provided we lay down the plausible and widely adopted axiom that every plurality is non-empty:

$$\forall xx \exists y(y \prec xx) \quad \text{(NE)}$$

We can then define *Exx* as $\exists y(y \prec xx)$. Given (NE) and our semantics, this definition is easily seen to work. We adopt the latter option as it is more economical.

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In the ensuing discussion, we also need a principle to the effect that any plurality is ontologically dependent on each of its members:

$$x \prec xx \rightarrow \Box(Exx \rightarrow Ex) \tag{Dep}$$

So we adopt this as an axiom.[24]

We shall now consider some formal arguments for plural rigidity. Each argument will first be developed from a necessitist point of view, as this is simpler. We shall then use our plural existence predicate to reformulate the argument so as to work in a contingentist setting.

5.2. The argument from uniform adjunction

The first argument relies on an operation $+$ of adjoining one object to a plurality. It is reasonable to assume that, necessarily, to be one of these things and that thing is to be one of these things or to be identical with that thing. We call this principle uniform adjunction:

$$\Box\forall x(x \prec xx + a \ \leftrightarrow \ x \prec xx \vee x = a) \tag{UniAdj}$$

We now argue as follows. Assume $a \prec xx$. Then, by (UniAdj), we have $xx \equiv xx + a$. So by (Cov), we have

$$\Box(xx \equiv xx + a). \tag{1}$$

Next, we observe that (UNIADJ) also entails

$$\Box(a \prec xx + a). \tag{2}$$

From (1) and (2), some simple modal logic ensures our desired conclusion that $\Box(a \prec xx)$.

Gabriel Uzquiano has raised a legitimate concern about the argument.[25] Is it permissible to assume that '$xx + a$' is a rigid designator? It can certainly be disputed that the term is purely referential. Fortunately, there is an answer, which forces us to make explicit an assumption that has so far only been implicit. Our argument assumes that uniform adjunctions exist. We can express this as the closure of the following plural comprehension principle, which asserts the existence of uniform adjunctions:

$$\exists yy\Box\forall u(u \prec yy \leftrightarrow u \prec xx \vee u = z) \tag{3}$$

This principle is very weak. Indeed, it is something that even an opponent of plural rigidity should assent to, as the principle retains its plausibility even when the plural variables are allowed to range over groups.[26]

As before, assume $a \prec xx$. By (3), let *yy* be the uniform adjunction of *a* to *xx*. From this point onward, the argument proceeds exactly as before, only with *yy* in the role previously played by $xx + a$. Notice that this argument makes no appeal to (Indisc) other than its single instance, (Cov). In this respect, the argument is like the plausibility argument from Section 4.

Let us now try to adapt the argument to a contingentist setting. Then, uniform adjunction requires the following, more guarded formulation:

$$\Box\big(Exx \wedge Ea \rightarrow \forall x(x \prec xx + a \leftrightarrow x \prec xx \vee x = a)\big) \qquad \text{(UniAdj-c)}$$

Our comprehension principle too must make existential assumptions explicit:

$$\exists yy \Box\big(Exx \wedge Ez \rightarrow \forall u(u \prec yy \leftrightarrow u \prec xx \vee u = z)\big) \qquad (5)$$

Thankfully, it is straightforward to verify that our first target, (Rgd^{+}), follows, and using the Brouwerian axiom B, so does (Rgd^{-}). The argument relies on our dependence axiom, (Dep).

5.3. The argument from partial rigidification

Another argument is proposed in Williamson (2010, 699–700). The argument requires that, for any plurality *xx*, there be a plurality *yy* that is a partial rigidification of *xx* in the sense that the two pluralities have the same members but it is impossible for *yy* to lose any of its members. To be precise, we assume the following plural comprehension axiom:

$$\exists yy(xx \equiv yy \wedge \forall x(x \prec yy \rightarrow \Box x \prec yy)) \qquad (6)$$

We can now argue as follows. Assume $a \prec xx$. Let *yy* be the partial rigidification of *xx*. Thus, we have $\Box a \prec yy$. By (Cov), we also have $\Box(xx \equiv yy)$. The latter two claims entail $\Box a \prec xx$, as desired.

Let us now try to develop the argument from a contingentist point of view. As usual, the comprehension axiom needs to be formulated with greater care:

$$\exists yy(xx \equiv yy \wedge \forall x(x \prec yy \rightarrow \Box(Eyy \rightarrow x \prec yy))) \qquad (7)$$

Applying the same strategy as before, we get $\Box(Eyy \rightarrow a \prec yy)$. We also have $\Box(xx \equiv yy)$. But now a problem arises. What we have proved does not entail our target claim $\Box(Exx \rightarrow a \prec xx)$ unless we can assume that the existence of *xx* ensures the existence of yy. But I submit that a theorist who doubts the rigidity of pluralities has reason to challenge this assumption. To see this, recall that such a theorist regards some pluralities as much like groups. For instance, *xx* might be the Hiring Committee, whose members happen to be *a*, *b*, and *c*. Then, the partial rigidification *yy* of *xx* is ontologically dependent on *a*, *b*, and *c*,

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whereas xx need not be subject to this ontological dependence. To clinch the argument, our theorist would have to add a third conjunct to the conjunction involved in (7), namely $Exx \rightarrow Eyy$. But this modification of (7) would be very strong and problematic from the point of view of anyone not antecedently committed to the rigidity of pluralities.

How does Williamson's argument compare with the one from uniform adjunction? Both rely on only a single instance of (Indisc), namely (Cov). But the arguments differ concerning the plural comprehension axioms that they assume: partial rigidification and uniform adjunction, respectively. From a necessitist point of view, both comprehension axioms are very plausible, and as far as I can see, plausible to roughly the same extent. From a contingentist point of view, on the other hand, the comprehension axiom that is required to complete Williamson's argument is significantly less plausible than that required by the argument from uniform adjunction. Thus, the contingentist has reason to favor the latter.

5.4. The argument from uniform traversability

The final argument for plural rigidity that we shall consider is due to Rumfitt (2005). As before, we first give a simple version of the argument that is acceptable from a necessitist point of view, and then consider how the argument can be adapted by a contingentist.

A finite plurality can be *traversed*, in the sense that its members can be exhaustively listed. Assume, for instance, that xx is the plurality whose members are a, b, and c. Then, we can also assert that this traversability is uniform, in the sense that it holds by necessity:[27]

$$\Box\forall x(x \prec xx \leftrightarrow x = a \vee x = b \vee x = c)$$

What about infinite pluralities? A straightforward generalization is available if we allow infinitary disjunctions and assume that every object a has a name $\bar{a}$:

$$\Box\forall x(x \prec xx \leftrightarrow \bigvee_{a \prec xx} x = \bar{a}) \qquad \text{(UniTrav)}$$

We now argue as follows. Assume $u \prec xx$. Then, we can find a such that $u = \bar{a}$. By the necessity of identity, we have $\Box(u = \bar{a})$. This entails the necessitation of $\bigvee_{a \prec xx} x = \bar{a}$. Some simple modal logic now ensures our target $\Box(u \prec xx)$.

Let us now consider matters from a contingentist point of view. (UniTrav) must then be reformulated so as to make all existential presuppositions explicit. We do this as follows. Given any x, we can name all of its members and use this to state that, provided that xx still exist, to be one of xx is just to be identical with one of the aforementioned members. In symbols:

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$$\Box(Exx \rightarrow \forall x(x \prec xx \leftrightarrow \bigvee_{a \prec xx} x = \bar{a})) \qquad \text{(UniTrav-c)}$$

As far as I can see, this modified principle is as plausible, given contingentism, as the original principle is, given necessitism. It is therefore satisfying to be able to verify that the original argument for rigidity goes through much as before. (The argument relies on (Dep).)

How does this formal argument for plural rigidity compare with the previous two? In my view, the argument has limited explanatory value as its premise of universal traversability is little other than an infinitary restatement of our target claim that a plurality is fixed in its membership as we shift our attention from one possible world to another. In order to substantiate this view, I shall now show just how strong the mentioned premise is and how substantial the ideological resources used in the argument are.

The clearest way to appreciate the strength of universal traversability is by observing that it entails all the premises of the previous two arguments. For example, let us verify that its necessitist version, (UniTrav), entails covariation. So assume $xx \equiv yy$. Then, we can find a bunch of names $\bar{a}$ such that (UniTrav) holds, and another bunch $\bar{b}$ such that the analogous claim holds concerning *yy* and the $\bar{b}$'s. Each of the *a*'s is identical with one of the *b*'s and vice versa. Since each of the identities holds of necessity, so too does their conjunction. Thus, the two disjunctions $\bigvee_{a \prec xx} x = \bar{a}$ and $\bigvee_{b \prec yy} x = \bar{b}$ are necessarily coextensional. Thus, via (UniTrav) and its analogue concerned with *yy* and the *b*'s, so too are *xx* and *yy*.[28] But clearly, the converse entailment does not hold. For covariation is formally compatible with membership drift, so long as two coextensional pluralities are never allowed to drift apart. It can also be verified that uniform traversability entails the comprehension principles associated with uniform adjunction and partial rigidification.

As for ideology, the argument obviously relies on infinitary resources. It is useful to separate these resources clearly from the modal claims that they are used to express. We can do so by considering what we may call *traversability*, which is (UniTrav) without the initial necessity operator. Even this plain version of traversability has some strong consequences. It makes available what Bernays (1935) calls 'quasi-combinatorial' reasoning; that is, reasoning with infinite totalities as if they were finite and subject to combinatorial operations such as formation of arbitrary subsets and effective traversal. Let me mention two important examples.

Firstly, traversability ensures the permissibility of impredicative plural comprehension axioms of the following form:

$$\exists x(\phi(x) \wedge x \prec xx) \rightarrow \exists yy \forall u\big(u \prec yy \leftrightarrow \phi(u) \wedge u \prec xx\big) \qquad (8)$$

To see this, we first find a bunch of names $\bar{a}$ which provide a traversal of *xx*. We would like another bunch of names $\bar{b}$ which provide a traversal of just those

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members of *xx* that satisfy ϕ. This is easy: just go through the former bunch and delete every item that names a non-ϕ. The resulting sub-traversal yields a quantifier free – and thus fully predicative – definition of the desired plurality. The upshot is that traversability functions like a principle of reducibility, in the sense of Russell and Whitehead's famous axiom of reducibility. This principle of reducibility becomes particularly far-researching if there is an all-encompassing or universal plurality, as is standardly assumed. We then get a justification of the full impredicative comprehension scheme.

Secondly, when we work in an intuitionistic theory, traversability ensures the availability of classical quantification restricted to any plurality. Assume that a formula $\psi(x)$ (which may have further free variables) has the following decidability property:

$$\forall x(\psi(x) \vee \neg\psi(x))$$

In effect, this means that the property $\lambda x.\psi(x)$ behaves classically. Then, tranversability ensures that quantification restricted to *xx* behaves classically as well, in the precise sense that we have the following decidability property:

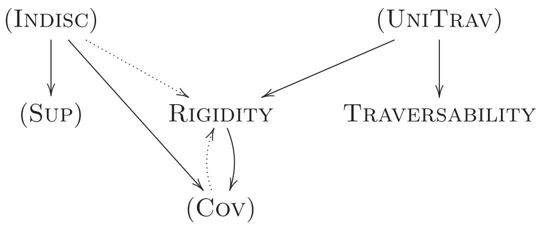
$$(\forall x \prec xx)\psi(x) \vee \neg(\forall x \prec xx)\psi(x)$$

To see this, observe that by traversability, this restricted quantification reduces to a conjunction of its instances, each of which has been assumed to behave classically.

All these considerations support my view that the argument from universal traversability has little suasive force. Its official premise is very strong and close to the desired target. In addition, the argument relies on very strong ideological resources. Despite these complaints, however, the argument serves to highlight a very important aspect of the extensionality of pluralities, namely their traversability

6. Three 'factors' of the extensionality of pluralities

Let us return to the basic thought from the introduction, namely that a plurality is 'nothing over and above' the circumscribed lot of objects that it comprises. The intervening discussion has disentangled several strains of this thought, whose implications and relations of non-deductive support have been explored and can, for a necessitist, be summarized as follows:



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Solid arrows represent one-way implications. Dotted arrows represent non-deductive support, but which can be transformed into implications by adding suitable comprehension axioms, as discussed in Sections 5.2 and 5.3. Formal theses are in parentheses, as usual. Rigidity abbreviates the conjunction of the rigidity claims (Rgd^+) and (Rgd^-), and, in a contingentist setting, also the dependence claim (Dep). A contingentist can use the same diagram except that (UniTrav) must be replaced with (UniTrav-C), whose left diagonal implication then only yields the two rigidity claims, not (Dep).[29]

My next claim is that the items on the diagram's second floor yield a 'factorization' of all the strains of the extensionality of pluralities that are represented in the above diagram.[30] We begin by observing that each item represents a simple and natural idea.

(a) The properties of any plurality supervene on its modal membership profile, as expressed by (Sup).

(b) A plurality has a rigid membership profile: it has the very same members at any possible world at which it exists.

(c) A plurality is traversable, thus ensuring the permissibility of quasi-combinatorial reasoning applied to the plurality.

Next, we observe that the three 'factors' entail each of the strains of the extensionality of pluralities. It suffices to verify that the items on the top floor of the diagram are entailed by those on the second. For (Indisc), this is implicit in work already done (see the previous footnote). And it can be verified that (UniTrav) (or its contingentist cousin) is entailed by Rigidity and Traversability.

What remains is to verify that the three 'factors' are logically independent of one another. To see that property supervenience, (Sup), does not follow from the other two aspects of extensionality, consider again the case of committees. Imagine an oligarchic department where three senior academics *a*, *b*, and *c* have written into the department statutes that they, and they alone, are to be on the Hiring Committee and the Graduate Admissions Committee. Both committees have a rigid membership profile and are clearly traversable. Yet the two committees are not subject to property supervenience as different powers of decision are vested in them.

Next, to show that a rigid membership profile does not follow from the other two aspects, consider the case of properties, understood as objects that are individuated by the necessary coextensionality of their defining concepts or conditions, and tracked across possible worlds in terms of this concept or condition. Thus understood, properties exemplify the second aspect of extensionality: all the characteristics of any given property are shared by any necessarily coextensive property. However, properties can be subject to contingent membership (or, perhaps better, contingent application), including

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on domains that are traversable. And as we have seen, the traversability of a domain ensures the traversability of any property on this domain.[31]

Finally, we observe that traversability is not a formal consequence of the other two aspects of extensionality. The principles that explicate these other two aspects do not ensure the availability of the infinitary resources needed for traversability. As Bernays observed, traversability is based on an extrapolation from the finite into the infinite. How far are we willing to extrapolate? The first two aspects of the extensionality of pluralities do not, by themselves, provide any answer to this question.[32]

7. The status of plural comprehension

I wish to end with some remarks about the status of the plural comprehension axioms. Many philosophers appear to regard such axioms as utterly trivial and insubstantial.[33] Provided that a condition is well defined and has at least one instance, *of course* the condition can be used to define a plurality of all and only its instances. This view seems to me misguided and a result of an excessive concern with ontology at the expense of all other questions. Because plural logic is thought to incur no additional ontological commitments over and above those already incurred by the singular quantifiers, the plural comprehension axioms are assumed to be free of the only kind of commitment that one cares about. One of the main upshots of this article is that, irrespective of the question of ontological commitment, pluralities are governed by strong extensionality principles whose satisfaction is a non-trivial matter. Since the plural comprehension axioms make claims about how the plural variables can be interpreted, and since each of these interpretations is governed by non-trivial extensionality principles, these axioms too should be regarded as non-trivial.

To elaborate, let us consider the three 'factors' of the extensionality of pluralities. We begin with the two easier cases. It is not hard to see that traversability is a non-trivial assumption. To say that plural comprehension is permissible on a condition ϕ is to say that we may reason quasi-combinatorially about all the ϕ's. A number of disputes in the foundations of mathematics testify to the non-triviality of this assumption.[34] Nor is property supervenience a trivial matter. Consider the following:

(6) The Hiring Committee met yesterday. They decided to make an offer to Sophia.

Is it permissible to apply the rule of plural existential generalization to 'they'? The answer must be 'no'. Generalizing in this way would ascribe the property of making a job offer to the members of the committee *considered as a mere plurality*, where in reality, the property can only be ascribed to *the committee as such*. For it is only the committee, not the plurality of its members, that has the power to make job offers. Indeed, the property ascribed in the second sentence of (6) fails to supervene on modal membership profile. For as our earlier examples show, two committees can share the same modal membership profile while differing in the powers that are vested in them.[35]

The remaining 'factor' of extensionality is a rigid membership profile. I have argued elsewhere that this is a highly non-trivial matter, and in fact that many concepts and conditions lack an extension with a rigid membership profile.[36] Let me attempt to summarize the central idea of the view. It is useful to begin with a non-mathematical example. Assume you detest web pages that link to themselves and wish to create an inventory of all web pages that are innocent of this bad habit. That is, you wish to create a web page that links to all and only the web pages that do not link to themselves. Can your wish be fulfilled? The answer depends on how the above characterization of your wish is analyzed. Should the scope of the crucial plural description – 'the web pages that do not link to themselves' – be narrow or wide? Depending on its scope, your wish can be analyzed in either of the following two ways:

(N) You wish to design a web page *y* such that, for every web page *x*, *y* links to *x* iff *x* does not link to itself.
(W) There are some web pages *xx* such that, for every web page *x*, *x* is one of *xx* just in case *x* does not link to itself, and you wish to design a web page *y* that links to all and only *xx*.

On the narrow scope reading (N), your wish is flatly incoherent and on a par with the wish to ensure the existence of a Russellian barber:

(B) You wish there to be a barber *y* such that, for all *x*, *y* shaves *x* iff *x* does not shave himself.

But on the wide scope reading (W), I claim, there is no theoretical obstacle to the fulfillment of your wish.

The relevant difference has to do with how the target web pages are specified. On (N), the target is specified intensionally by means of the condition '*x* does not link to itself.' If you attempt to fulfill your wish by creating a new web page *y*, then *y* will be in the scope of the quantifier 'for every web page *x*' and thus be subjected to the mentioned condition, with fatal result. By contrast, on the wide scope reading (W), the target web pages are specified extensionally by means of the plurality *xx*. This makes it unproblematic to envisage – and indeed bring about – an alternative situation in which the associated wish is fulfilled: you simply create a web page *y* that links to all and only *xx*. Of course, *y* has to be a *new* web page, in the sense that it did not exist in the original situation; otherwise it would with fatal consequence fall under the quantifier 'for every web page *x*' that figures in the description of *xx* employed in (W).

It is important to notice the essential role played by the rigidity of pluralities in the argument. Although the plurality *xx* is described, in the original situation,

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by means of a potentially dangerous condition, there is no requirement that the plurality should remain so described in alternative situations. Like any other plurality, *xx* are tracked to an alternative situation in terms of the members, not in terms of any description that these members happen to satisfy.

Assume now instead that you care about pure sets, not web pages. You have no wishes concerning the latter. But you wish to define a pure set that has as elements all and only those pure sets that are not elements of themselves. It should by now be obvious that this sentence is subject to a scope ambiguity that parallels that of our example concerning web pages.

(N′) You wish to define a pure set *y* such that, for every pure set *x*, *x* is an element of *y* iff *x* is not an element of itself.

(W′) There are some pure sets *xx* such that, for every pure set *x*, *x* is one of *xx* just in case *x* is not an element of itself, and you wish to define a pure set *y* whose elements are all and only *xx*.

On the narrow scope reading, your wish is flatly incoherent. The wide scope reading is more interesting. Assume, as is customary, that standard plural comprehension holds, such that there are indeed some pure sets *xx* that include all and only the non-self-membered pure sets. On this assumption, there is no logical or mathematical obstacle to the fulfillment of your wish. We can make good mathematical sense of the envisaged pure set; for we know exactly what its elements are. Given this, it would run counter to the spirit of modern mathematics to deny that this is a definition in good mathematical standing.

Now we have a problem, however. Unlike web pages, pure sets exist of metaphysical necessity, if at all. The pure set *y* was not created through its definition but existed all along. This means that *y* is in the range of the quantifier 'for every pure set *x*' that figures in the description of *xx* employed in (W′). Once again, the result is fatal. What to do? Our only option, I argue, is to reject the assumption that there are some pure sets *xx* that include all and only the non-self-membered pure sets. That is, we must curtail plural comprehension. There are certain conditions which – despite having a sharp intension – lack an extension with a rigid membership profile. Such conditions cannot figure in plural comprehension axioms.

It would be wrong to think that the view is all negative, however. It is true that plural comprehension must be restricted. But this restriction opens the door to a version of naive set comprehension – every plurality can be used to define a set – which in turn is used in (what I find) an elegant and illuminating approach to set theory.[37]

Summing up, this paper has argued that pluralities are rigid and in fact that this is just one member of a small cluster of strong extensionality principles that govern pluralities. Although these principles can be split into three independent 'factors' (of which plural rigidity is one), they go naturally together as

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a package. Since the principles are far from trivial, nor are the plural comprehension axioms, which assert the existence of pluralities that are governed by the principles. This realization has an important consequence concerning the applications of the rigidity of pluralities that were outlined in Section 2. It takes us beyond the first three applications, which Williamson accepts, to the final one, canvassed just now, which Williamson is likely to reject.[38]

Notes

1. See Rumfitt (2005, Section VII), Uzquiano (2011), Williamson (2003, 456–457), Williamson (2010, 699–700), and Williamson (2013). The view is challenged in Hewitt (2012).
2. Here and in what follows, a displayed open formula will be short for the necessitation of its universal closure. It is important to realize that this convention differs from another one in discussions of modal logic, which lets a formula be short for its *modal closure*, where this is defined as the result of prefixing the formula with any string of universal quantifiers and necessity operators.
3. Here and in what follows, I use the word 'plurality' as a convenient shorthand to convey claims whose proper expression eschews this word in favor of some plural construction.
4. See e.g. Landman (1989) and Uzquiano (2004).
5. A similar example is attributed to Dorothy Edgington in Rumfitt (2005). Further examples are found in Hewitt (2012).
6. Sets – understood as on the iterative conception – come close but have the additional and complicating factor that their members are 'bound together' into a single object.
7. What about reducing plural logic to monadic second-order logic by translating a plural quantifier by means of a monadic second-order quantifier restricted to concepts that are non-empty and rigid? Technically, this should work. But philosophically, the proposal seems strained and lacking in motivation.
8. See Williamson (2003, Section IX).
9. See Williamson (2010).
10. See Linnebo (2010) and Linnebo (2013). While this approach draws inspiration from Parsons (1983) and to some extent also Putnam (1967) and Hellman (1989), these earlier views do not rely in the same way on the rigidity of pluralities.
11. In fact, as Williamson (1996) has pointed out, ($\Box \neq$) can also be derived without use of the Brouwerian axiom by invoking suitable principles of actuality.
12. Since the necessitation of (Leibniz) ensures $\Box(x = y \rightarrow x = x)$, the existential presupposition '$x = x$', present in ($\Box =$), would be redundant in ($\Box \neq$). For were $x \neq y$ to fail, the mentioned presupposition would anyway be satised. (Thanks to Williamson for this observation.)
13. Here, and in the paragraphs that follow, I leave implicit the proviso that the sets still exist.
14. This is 'the basic thought' from the end of Section 1.
15. For sets, the former option is unattractive. As Boolos (1971, 229–230) reminds us, if ever there was an example of an analytic truth, then the extensionality of sets is one.
16. This is significant for Fregean and neo-Fregean approaches to collections (or extensions, or *Wertverläufe*). These approaches regard extensionality as a

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criterion of identity and are thus committed to soft extensionalism. But they also view a collection as in some way 'obtained from' its defining (Fregean) concept and are thus potentially on collision course with hard extensionalism. See Parsons (2012) for a discussion of Frege's concept of extension.

17. Thanks to Jeremy Goodman for articulating this objection.
18. Abstract objects would be a limiting case where the material contribution is nil.
19. The proper analogue of the set theoretic principle of extensionality would be the principle that sameness of *material* parts ensures identity. Now the analogy that seemed to cause trouble for my plausibility argument does work: any reason to accept this principle is also a reason to accept rigidity of material parthood. Of course, anyone attracted to non-rigid parts should respond to this observation by denying that sameness of material parts ensures identity.
20. See Stalnaker (1997) and essays 1–3 of Fine (2005) for some closely related considerations.
21. Notice that this enables us to drop the existential assumptions *Ex* and *Eyy* from (Rgd$^-$) on p. 1.
22. Here it is essential to observe that (Indisc) is short for the necessitation of its universal closure; cf. footnote 2. Had (Indisc) instead been short for its modal closure, we could have derived $\Box(xx \equiv yy \rightarrow \Box(xx \equiv yy))$, which has contingentist S5 countermodels. (Let *xx* and *yy* be distinct pluralities at w_1, neither of which exists at w_2. Then $xx \equiv yy \rightarrow \Box(xx \equiv yy)$ holds at w_2. Since the antecedent is vacuously true at w_2, so is the consequent. But this contradicts the distinctness of *xx* and *yy* at w_1.) Thanks to Tim Williamson for questions that prompted this clarification.
23. I am assuming that the statutes are partially constitutive of the committees, in the sense that, were one to change the statutes, the original committees would cease to exist and be replaced by new ones. If necessary, this persistence condition for the committees can be written into the statutes.
24. Notice that this enables us to drop the existential assumptions *Ex* from (Rgd$^+$) on p. 1.
25. A closely related concern is expressed in Uzquiano (2014).
26. Again, it is instructive to consider the mereological analogue, which in the case of (3) is the principle that for any objects *x* and *y* there is a sum. A natural formalization of this principle would be:

$$\exists z \Box(x \leq z \wedge y \leq z \wedge \forall u(x \leq u \wedge y \leq u) \rightarrow z \leq u) \qquad (4)$$

Assume a molecule *m* is part of a cat *c*. For convenience, write $m + c$ for their sum. We easily derive $c \leq m + c$ and $m + c \leq c$, and hence by anti-symmetry, $c = m + c$. Since $m + c$ necessarily has *m* as part, by Leibniz's law, so does *c*. However, unlike its plural analogue, this argument involves controversial steps. If $x \leq y$ requires merely that the matter of *x* be included in that of *y*, then anti-symmetry can plausibly be denied. On the other hand, if $x \leq y$ is sensitive also to the formal aspects of *x* and *y*, then the relevant instance of (4) can plausibly be denied. True, there is a sum $m + c$ which necessarily has both *m* and *c* as parts. But this sum has a formal aspect which falsifies the third conjunct of the relevant instance of (4). Although $m \leq c \wedge c \leq c$, we do not have $m + c \leq c$: for this sum has a formal aspect, manifested in its modal profile, that goes beyond anything found in *c*.

27. In fact, as Jeremy Goodman observed, if a singleton plurality is uniformly traversed by its sole member, then Uniform Adjunction allows us to *prove* that any finite plurality is uniformly traversed by its members.
28. In fact, the entailment of covariance goes through on the contingentist version as well.
29. If desired, one can tweak (UniTrav-C) so as to ensure that (Dep) too follows, namely by adding the following as a third (and perfectly sensible) conjunct: $(Exx \leftrightarrow \bigwedge_{a \prec xx} E\bar{a})$.
30. Recall the claim from p. 662 that (Indisc) can be 'factorized' into (Sup) and (Cov).
31. Of course, *uniform* traversability is another matter, as demonstrated by the argument from Section 5.4.
32. There is a more indirect connection, however. In mathematics, the prevailing view has come to be that quasi-combinatorial reasoning should be extrapolated 'as far as possible.' How far is that? Given (Sup), Rigidity opens for the possibility of extrapolating such reasoning very far, namely to any plurality. The reason is that such reasoning ensures traversability, and thus by (Sup) also uniform traversability, whence by the argument developed in Section 5.4, also Rigidity. Thus, without Rigidity, it would not be permissible to extend quasi-combinatorial reasoning to any plurality.
33. Consider e.g. the breezy arguments for the status of the axioms of plural as 'genuine logical truths' found in Boolos (1985, 342) (corresponding to Boolos (1998, 167)) and Hossack (2000, 422).
34. See Feferman (2005) for a survey of debates concerning the legitimacy of impredicative reasoning in mathematics.
35. Our example from Section 6 of the Hiring Committee and the Graduate Admissions Committee will do.
36. See Linnebo (2010).
37. See Linnebo (2013). This approach uses a non-metaphysical modality to identify the legitimate forms of plural comprehension. This alternative modality allows us to represent the stages of 'the process of set formation'. Plural comprehension is permissible on any condition whose instances can be exhausted by one of these stages.
38. I am grateful for comments from Salvatore Florio, Jeremy Goodman, Simon Hewitt, Timothy Williamson, Gabriel Uzquiano, as well as participants at a London research seminar and a Montreal workshop where this work was presented. Much of the research was undertaken while benefiting from an ERC Starting Grant.

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