# Self-organization in a driven dissipative plasma system

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**Abstract.** We perform a fully self-consistent three-dimensional numerical simulation for a compressible, dissipative magnetoplasma driven by large-scale perturbations, that contain a fairly broad spectrum of characteristic modes, ranging from largest scales to intermediate scales and down to the smallest scales, where the energy of the system is dissipated by collisional (ohmic) and viscous dissipations. Additionally, our simulation includes nonlinear interactions amongst a wide range of fluctuations that are initialized with random spectral amplitudes, leading to the cascade of spectral energy in the inertial range spectrum, and takes into account large-scale as well as small-scale perturbations that may have been induced by the background plasma fluctuations, as well as the non-adiabatic exchange of energy leading to the migration of energy from the energy-containing modes or randomly injected energy driven by perturbations and further dissipated by the smaller scales. Besides demonstrating the comparative decays of the total energy and the dissipation rate of the energy, our results show the existence of a perpendicular component of the current, thus clearly confirming that the self-organized state is non-force free.

### 1. Introduction and motivation

The phenomenon of self-organization, in which a continuous system naturally evolves towards a state exhibiting some form of order on large scales, is deeply rooted in nature. The ordered states are remarkably robust; their detailed structure remains relatively invariant across experimental realization; these preferred states are independent of the way the system is prepared (Hasegawa 1985; Ortolani and Schnack 1993).

Several broad classes of physical processes, such as first- and second-order phase transitions, crystallization, structure formations in space and astrophysics and cosmology can be described as examples of self-organization. In chemistry, examples of self-organizations include reaction—diffusion systems such as Belousov—Zhabotinsky reactions, self-assembled monolayers, molecular self-assembly, etc. Biological systems are swarmed with examples of self-organization; besides the examples of pattern formation and morphogenesis, the origin of life itself from self-organizing chemical systems is the supreme example of self-organization. Most of

the systems exhibiting self-organization in nature are intrinsically nonlinear and often are not isolated—they are driven by some external input. Self-organization occurs in open systems which are far from thermal equilibrium. Nicolis and Prigogine (1977) had envisaged new types of self-organized states for driven dissipative systems and called these states 'dissipative structures'. Such structures provide striking examples of non-equilibrium as a source of order.

Over the past decades, nonlinear dynamics and non-equilibrium thermodynamics have developed along with plasma physics, providing qualitatively new approaches to complex problems. It is thus widely accepted nowadays, for instance, that nonequilibrium may be a source of order in dissipative systems, allowing the emergence of self-organization. Self-organizations have been observed in varieties of laboratory magnetically confined systems. Most plasma systems are dissipative, externally driven and far from equilibrium, thus sharing some essential features with other complex nonlinear systems that show spatial and temporal coherence. Such a system is described by a set of nonlinear partial differential equations. During its relaxation towards a self-organized state, it takes advantage of instabilities that lead it, for instance, to a preferred state, under certain constraints that prevent it from falling into a trivial unconfined state. Such constraints appear as quadratic or higher-order quantities, which are conserved in the absence of dissipation. The relevant feature is that, in the presence of dissipation, one of these quantities decays faster than the others. Generally, it remains to be determined which is the relevant variational principle underlying the relaxation mechanisms described above that is appropriate to characterize the self-organized state. In the space and Solar plasma flows, a number of commonly observed processes including flares, prominences, filaments and/or (magnetic) loop-like structures that are generated often during the active Solar period can be modeled by relaxation of plasma through self-organization (Bhattacharya et al. 2007). The kinematic as well as dynamics of these entities are far more complex than ever thought and we still lack an in-depth insight into their evolutionary characteristics despite the availability of enormous databases from various spacecraft missions.

Part of the problem lies with the lack of a fully self-consistent description (or physical model) of evolution of these structures in realistic environments. For instance, one of the mechanisms of the generation of magnetic field loops is often attributed to the Taylor relaxation (Taylor 1974) process where magnetic stresses overcome the pressure stresses which thereby balance all the magnetohydrodynamic (MHD) forces. Such a state is often characterized by a low plasma beta (where plasma beta is a ratio of pressure and magnetic energy). Under no external forces, the pressure gradients flatten out to nullify the magnetic pinch  $(\mathbf{J} \times \mathbf{B})$  force. This state is called a 'force-free state'. The force-free state is one of the analytic descriptions that describes a magnetic loop in terms of a magnetic field configuration where rotation of the magnetic field is proportional to the field itself, thus giving rise to a constant of proportionality. The constant of proportionality can be a real constant (linear force free) or dependent on space (nonlinear force free). The validity of a force-free model is however restricted to the formation of the non-evolutionary magnetic loops essentially in the vicinity of the Solar corona. Further, they are strictly invalid to describe the evolution dynamics. This further necessitates the development of a self-consistent description of coronal magnetic field loops that can respond to as well as interact with the realistic perturbations ubiquitously present in the Solar atmosphere.

Motivated by the issues described above, we have developed a more generic model based on three-dimensional (3D) simulations of fully compressible, non-adiabatic, driven dissipative magnetofluid plasma that contains a fairly broad spectrum of characteristic modes. The underlying modes range from largest scale (of the size of the system) to the intermediate scales (constituting the inertial range spectra) to the smallest scales where the energy of the system can be dissipated by virtue of collisional or viscous dissipation. One of the novel features of our 3D compressible MHD plasma model is that it deals with the entire spectrum of fluctuations, unlike those works that either describe a single mode of the flux of the coronal magnetic field and ignore the background small-scale realistic perturbations or describe single coherent structures (Amari and Luciani 2000), Additional features of our simulation model are that it includes: (i) nonlinear interactions amongst a wide range of fluctuations that are initialized with random spectral amplitudes; the nonlinearities in the underlying system drive turbulent processes and lead to the cascade of spectral energy in the inertial range spectrum; (ii) large-scale as well as small-scale perturbations that may have been induced by the background plasma fluctuations; (iii) non-adiabatic exchange of energy leading to the migration of energy from the energy-containing modes or randomly injected energy driven by perturbations and further dissipated by the smaller scales.

The plan of the paper is as follows: in Sec. 2 we briefly summarize some of the MHD relaxation models appropriate for driven dissipative plasma based on the principle of minimum dissipation rate (MDR) leading to non-force-free states. Section 3 presents a description of our model starting with the conservative forms of the MHD equations with applicable normalization procedures. Section 4 describes our simulation results. Section 5 includes a summary of our work, a conclusion and a discussion of future work.

## 2. Non-force-free self-organized states from MHD relaxation models

The seminal work on plasma relaxation proposed by Taylor predicts a force-free state for a magnetized plasma. For such states, the current  $\mathbf{J}$  is always along the direction of the magnetic field  $\mathbf{B}$  and the perpendicular component of the current  $J_{\perp} = 0$ . So, from the force-balance equation  $\mathbf{J} \times \mathbf{B} = \nabla p$ , it is seen that, for force-free states, the pressure gradient is zero, and such states are not suitable for devices confining plasma by magnetic fields.

A small amount of resistivity, ingrained in any realistic plasma, is essential to allow reconnective processes leading to relaxation. In fact, dissipation, along with nonlinearity, is universal in systems evolving towards self-organized states and it is natural to assume that dissipation plays a decisive role in the self-organization of a system. Alternative models of plasma relaxation, based on the principle of minimum dissipation rate (MDR) of energy, have been proposed by several authors (Montgomery and Phillips 1988; Farengo and Sobehart 1994, 1995; Farengo and Caputi 2002; Dasgupta et al. 1998, 2002, 2009; Bhattacharya and Janaki 2004). The principle of minimum dissipation rate is closely akin to the principle of minimum entropy production rate of irreversible thermodynamics as formulated by Prigogine (1946). Relaxation of a driven dissipative plasma has been formulated by Bhattacharya and Janaki (2004). As shown by these authors, the relaxed states obtained from MDR are non-force free, that is, for these states,  $\nabla p \neq 0$ . A critical signature of such a non-force-free state is that the perpendicular component of the

current  $J_{\perp} \neq 0$ . It may be mentioned that relaxation of a MHD plasma to a non-force-free state has been numerically demonstrated by Zhu et al. (1995). One of the main objectives of this work is to investigate the existence of non-zero  $J_{\perp}$  in a self-organized state of a driven system.

## 3. Description of the simulation model

The fluid model describing nonlinear turbulent processes in the magnetofluid plasma, in the presence of a background magnetic field, can be cast into plasma density  $(\rho_p)$ , velocity  $(\mathbf{U}_p)$ , magnetic field  $(\mathbf{B})$  and pressure  $(P_p)$  components according to the conservative form

$$\frac{\partial \mathbf{F}_{\mathbf{p}}}{\partial t} + \nabla \cdot \mathbf{Q}_{\mathbf{p}} = \mathcal{Q},\tag{3.1}$$

where

$$\mathbf{F}_{\mathrm{p}} = egin{bmatrix} 
ho_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} \ 
ho_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} \ 
ho_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} \end{bmatrix}, \quad \mathbf{Q}_{\mathrm{p}} = egin{bmatrix} 
ho_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} + rac{P_{\mathrm{p}}}{\gamma - 1} \ ar{\mathbf{I}} + rac{B^{2}}{2} \ ar{\mathbf{I}} - \mathbf{B} \mathbf{B} \ 
ho_{\mathrm{p}} \mathbf{H}_{\mathrm{p}} - \mathbf{B} \mathbf{U}_{\mathrm{p}} \ 
ho_{\mathrm{p}} \mathbf{U}_{\mathrm{p}} - \mathbf{B} (\mathbf{U}_{\mathrm{p}} \cdot \mathbf{B}) \end{bmatrix},$$

$$\mathbf{f}_{\mathbf{M}}(\mathbf{r}, t) + \mu \mathbf{\nabla}^{2} \mathbf{U}_{\mathrm{p}} + \eta \mathbf{\nabla} (\mathbf{\nabla} \cdot \mathbf{U}_{\mathrm{p}}) \end{bmatrix}$$

$$\mathscr{Q} = egin{bmatrix} \mathbf{f}_{M}\left(\mathbf{r},t
ight) + \mu 
abla^{2} \mathbf{U}_{\mathrm{p}} + \eta 
abla\left(
abla \cdot \mathbf{U}_{\mathrm{p}}
ight) \ \eta 
abla^{2} \mathbf{B} \ 0 \end{bmatrix}$$

and

$$e_{\rm p} = \frac{1}{2}\rho_{\rm p}U_{\rm p}^2 + \frac{P_{\rm p}}{\gamma - 1} + \frac{B^2}{8\pi}.$$

Equations (3.1) are normalized by typical length  $\ell_0$  and time  $t_0 = \ell_0/V_A$  scales in our simulations such that  $\bar{\nabla} = \ell_0 \nabla$ ,  $\partial/\partial \bar{t} = t_0 \partial/\partial t$ ,  $\bar{\mathbf{U}}_{\rm p} = \mathbf{U}_{\rm p}/V_A$ ,  $\bar{\mathbf{B}} = \mathbf{B}/V_A (4\pi\rho_0)^{1/2}$ ,  $\bar{P} = P/\rho_0 V_A^2$ ,  $\bar{e}_{\rm p} = e_{\rm p}/\rho_0 V_A^2$  and  $\bar{\rho} = \rho/\rho_0$ . The bars are removed from the normalized equations (3.1). Here  $V_A = B_0/(4\pi\rho_0)^{1/2}$  is the Alfvén speed and  $\bar{\mathbf{I}}$  is a unit tensor.

The right-hand side in the momentum equation denotes a forcing function  $(\mathbf{f}_{\mathrm{M}}(\mathbf{r},t))$  that essentially influences the plasma momentum at the larger length scale in our simulation model. With the help of this function, we drive energy in the large-scale eddies to sustain the magnetized turbulent interactions. In the absence of forcing, the turbulence continues to decay freely. While the driving term modifies the momentum of the plasma, we conserve density (since we neglect photoionization and recombination). The large-scale random driving of turbulence can correspond to external forces or instabilities, for example fast and slow streams, a merged interaction region, etc., in the Solar wind, supernova explosions, stellar winds in the interstellar medium (ISM), etc. The magnetic field evolution is governed by the usual induction equation and obeys the frozen-in-field theorem unless a nonlinear dissipative mechanism introduces small-scale damping. Note carefully that the MHD plasma momentum equation contains nonlinear terms on the right-hand side. This means that mode-coupling processes can potentially be mediated by nonlinear interactions, in addition to the damping associated with the small-scale

turbulent motion. Thus, nonlinear turbulent cascades are not only responsible for the spectral transfer of energy in the inertial range, but are also likely to damp the plasma motion in a complex manner. Nonetheless, the spatiotemporal scale in the nonlinear damping can be distinct from that of the linear dissipation. We restrict forcing of plasma momentum fluctuations in a region specified by the wavenumbers  $k < k_{\rm f}$  such that energy is injected in the large-scale plasma fluctuations. The driving is random in time and space. The amplitude of the driving force is also chosen randomly between zero and one. We further make sure that the random number generator used for the driving force is isotropic and uniform in the spectral space and does not lead to spectral anisotropy.

Turbulent-relaxation evolution studies in three dimensions are performed to investigate the nonlinear mode coupling interaction of a decaying compressible MHD turbulence described by the closed set of equations (3.1). For this purpose, we have developed a full three-dimensional (3D) compressible MHD code (Shaikh et al. 2008). All the fluctuations are initialized isotropically (no mean fields are assumed) with random phases and amplitudes in Fourier space and evolved further by integration of (3.1) using a fully de-aliased pseudospectral numerical scheme. Fourier spectral methods are remarkably successful in describing turbulent flows in a variety of plasma and hydrodynamic (i.e. non-magnetized) fluids. Not only do they provide an accurate representation of the fluid fluctuations in the Fourier space, but they are also non-dissipative. Because of the latter, nonlinear mode coupling interactions preserve ideal rugged invariants of fluid flows, unlike finite difference or finite volume methods. The conservation of the ideal invariants (energy, entropy, magnetic potential, helicity, etc.) in turbulence is an extremely important feature in general, and particularly in our simulations, because these quantities describe the cascade of energy in the inertial regime, where turbulence is, in principle, free from large-scale forcing as well as small-scale dissipation. The precise measurement of the decay rates associated with the MHD invariants is therefore one of the major concerns in the study of MDR. Dissipation is nonetheless added physically in our simulations to push the spectral cascades further down to the smallest scales and also to allow minimal dissipation. The evolution variables are discretized in Fourier space and we use periodic boundary conditions. The initial isotropic turbulent spectrum was chosen to be close to  $k^{-2}$  with random phases in all three directions. The choice of such (or even a flatter than -2) spectrum does not influence the dynamical evolution of the turbulent fluctuations as the final state in all our simulations leads to the identical results that are consistent with the proposed analytic theory. The equations are advanced in time using a secondorder predictor-corrector scheme. The code is made stable by a proper de-aliasing of spurious Fourier modes and choosing a relatively small time step in the simulations. Additionally, the code preserves the  $\nabla \cdot \mathbf{B} = 0$  condition at each time step. Our code is massively parallelized using message passing interface (MPI) libraries to facilitate higher resolution in a 3D volume. Kinetic and magnetic energies are also equipartitioned between the initial velocity and the magnetic fields. The latter helps treat the transverse or shear Alfvén and the fast/slow magnetosonic waves on an equal footing, at least during the early phase of the simulations.

#### 4. Simulation results

Our major focus in the paper is to study turbulent relaxation of magnetofluid plasma through nonlinear interactions. For this purpose, we let magnetized fully

compressible MHD turbulence evolve under the action of nonlinear interactions in which random initial fluctuations, containing sources of free energy, lead to the excitation of unstable modes. These modes often deviate the initial evolutionary system substantially away from its equilibrium state. When the instability tends to saturate, unstable modes lead to fully developed turbulence in which larger eddies transfer their energy to smaller ones through a forward cascade until the process is terminated by the small-scale dissipation. During this process, MHD turbulent fluctuations are dissipated gradually due to the finite Reynolds number, thereby damping small-scale motion as well. The energy in the smaller Fourier modes migrates towards the higher Fourier modes following essentially the vector triad interactions  $\mathbf{k} + \mathbf{p} = \mathbf{q}$ . These interactions involve the neighboring Fourier components  $(\mathbf{k}, \mathbf{p}, \mathbf{q})$  that are excited in the local inertial range turbulence. We conjectured in our earlier work (Shaikh et al. 2008) that MDR plasma relaxes towards a nonlinear non-force-free state through nonlinear evolution. In our simulations, we consider this point in the presence of driven turbulence. Our simulations are carried out in the presence of a mean magnetic field (taken along the z direction). One of the ways we ensure the nonlinear non-force-free evolution is by determining whether or not there develops any perpendicular component of the plasma current. In the following, we explain why it is essential to self-consistently produce a perpendicular component of the plasma current  $(J_{\perp})$  that facilitates the nonlinear non-force-free interactions.

The nonlinear interactions in magnetoplasma turbulence are determined predominantly by  $\bar{J} \times \bar{B}$  forces in the plasma momentum equation. Similarly, since  $\bar{V} \propto \bar{B}$  (through  $\bar{J}$ ), the same nonlinear interactions also govern the magnetic induction equation. In the presence of a mean magnetic field along the z direction, the parallel and perpendicular components can be denoted respectively by  $\bar{J}_{\parallel}$  and  $\bar{J}_{\perp}$ . For obvious reasons,  $\bar{J}_{\parallel} \times \bar{B} = 0$  for magnetic field fluctuations that lie strictly along the  $\bar{J}_{\parallel}$  component of the plasma current. Hence, this term contributes negligibly in the nonlinear interactions. By contrast, it is only the  $\bar{J}_{\perp}$  component, emerging from the  $\bar{J}_{\perp} \times \bar{B}$  term, that contributes largely to the nonlinear interactions. Thus, current fluctuations orthogonal to the mean or fluctuating magnetic field component predominantly govern the nonlinear interactions.

Figure 1 shows isosurfaces of the x component of the magnetic field by the evolution of random initial turbulent fluctuations leading to the formation of relatively small-scale isotropic structures. Figure 1 is a typical snapshot of nonlinear turbulent fluctuations during an early phase of evolution. The initial condition in combination with the random forcing leads to the state depicted in Fig. 1. In Fig. 2, we follow the evolution of both  $\bar{J}_{\parallel}$  and  $\bar{J}_{\perp}$  components of the currents to quantitatively measure their progressive development. Clearly, the two components gradually develop and evolve self-consistently in our simulations. This further leads to  $\bar{J}_{\perp} = \bar{J} - (\mathbf{J} \cdot \bar{\mathbf{B}}/|\bar{\mathbf{B}}|)\hat{b}$ —where  $\hat{b} = \bar{\mathbf{B}}/|\bar{\mathbf{B}}|$  is the unit vector along  $\bar{\mathbf{B}}$ —which is nonzero, thus proving conclusively that the resultant state is non-force free. For a force-free state,  $J_{\perp}$  is zero.

The corresponding decay rates associated with turbulent relaxation of the rugged ideal invariants of MHD, namely magnetic helicity  $(K = \int \mathbf{A} \cdot \mathbf{B} \, d\mathbf{v})$ , magnetic energy  $(E = 1/2 \int B^2 \, d\mathbf{v})$  and energy dissipation rate  $(R = \eta \int J^2 \, d\mathbf{v})$ , are shown simultaneously in Fig. 3. Clearly, the magnetic energy E decays faster than the magnetic helicity K, and the energy dissipation (R) decays even faster than the two invariants. This state corresponds to a minimum dissipation in which selective decay

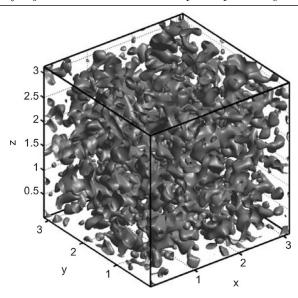


Figure 1. Evolution of random initial turbulent fluctuations leads to the formation of relatively small-scale isotropic structures in 3D compressible MHD simulations. The numerical resolution is  $128^3$  in a cubic box of volume  $\pi^3$ . The dissipation parameter  $\eta = \nu = 10^{-4}$ . Shown are the isosurfaces of the x component of the magnetic field.

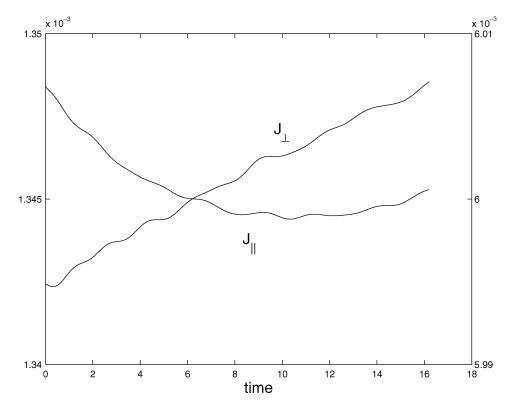


Figure 2. Evolution of currents associated with the parallel and perpendicular components of the magnetic field fluctuations in driven MHD plasma system. The evolution of a finite  $J_{\perp}$  establishes the fact that the relaxed state is a non-force-free state, i.e.  $\mathbf{J} \times \mathbf{B} \neq 0$ .

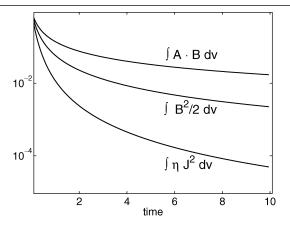
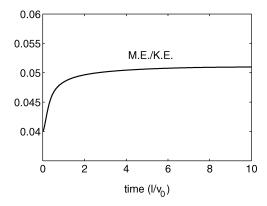


Figure 3. Evolution of decay rates associated with turbulent relaxation of the rugged ideal invariants of MHD, namely magnetic helicity  $(K = \int \mathbf{A} \cdot \mathbf{B} \, d\mathbf{v})$ , magnetic energy  $(E = 1/2 \int B^2 \, d\mathbf{v})$  and dissipative current  $(R = \eta \int J^2 \, d\mathbf{v})$  are shown simultaneously.



**Figure 4.** The decay rates of kinetic energy of turbulent fluctuations are initially higher than the magnetic energy. Hence, the ratio of magnetic to kinetic energies shows a sharp rise in the initial evolution. The two decay rates eventually become identical, thereby leading to a constant value of the ratio.

processes lead to the faster decay rates of the magnetic energy (when compared with the magnetic helicity decay rates). Furthermore, the time evolutions of the volume averages of global helicity, magnetic energy and the dissipation rates are plotted in Fig. 4. Thus, these quantities are averaged over the fluctuations, and hence they show a regular behavior. So, the decays of the global helicity, magnetic energy and the dissipation rates are not due to any linear decay; they are the results of nonlinear turbulence in the system. The selective decay processes (in addition to dissipations) depend critically on the cascade properties associated with the rugged MHD invariants that eventually govern the spectral transfer in the inertial range. This can be elucidated as follows (Biskamp 2003). The magnetic vector potential in 3D MHD dominates, over the magnetic field fluctuations, at the smaller Fourier modes, which in turn leads to a domination of the magnetic helicity invariant over the magnetic energy. On the other hand, dissipation occurs predominantly at the higher Fourier modes, which give rise to a rapid damping

of the energy-dissipative quantity R. A heuristic argument for this process can be formulated in the following way. The decay rates of helicity K and dissipation rate  $R = \int_V \eta j^2 dV$  in the dimensionless form, with the magnetic field Fourier decomposed as  $\mathbf{B}(\mathbf{k},t) = \Sigma_k \mathbf{b_k} \exp(i\mathbf{k} \cdot \mathbf{r})$ , are

$$\frac{dK}{dt} = -\frac{2\eta}{S} \Sigma_k k \mathbf{b_k}^2, \quad \frac{dR}{dt} = -\frac{2\eta^2}{S^2} \Sigma_k k^4 \mathbf{b_k}^2, \tag{4.1}$$

where  $S = \tau_{\rm R}/\tau_{\rm A}$  is the Landquist number and  $\tau_{\rm R}$  and  $\tau_{\rm A}$  are the resistive and Alfvén time scales, respectively. The Landquist number in our simulations varies between  $10^6$  and  $10^7$ . We find that at scale lengths for which  $k \approx S^{1/2}$ , the decay rate of energy dissipation is  $\sim O(1)$ . However, at these scale lengths, helicity dissipation is only  $\sim O(S^{-1/2}) \leq 1$ . This physical scenario is further consistent with our 3D simulations. Interestingly, the decay rates of kinetic energy of turbulent fluctuations are initially higher than the magnetic energy. Hence, the ratio of magnetic to kinetic energies shows a sharp rise in the initial evolution, as shown in Fig. 4. However, as the evolution progresses, the two decay rates become identical and the ratio eventually approaches a constant value. Another important outcome to emerge from our investigations is that a state corresponding to the minimum dissipation rates is more plausible in a driven dissipative plasma. So, we may conclude, notwithstanding the Taylor hypothesis of a force-free state, that our simulations clearly demonstrate that the nonlinear selective decay processes lead the plasma fluctuations to relax towards a non-force-free state in a rather natural and selfconsistent manner.

## 5. Summary, conclusion and future work

We have demonstrated through a fully self-consistent 3D numerical simulation that a compressible, dissipative magnetoplasma driven by large-scale perturbations can give rise to a self-organized state, which is not force free. This is corroborated by the presence of a perpendicular component of the current  $J_{\perp}$  in the self-organized state. In addition, the comparative decay rates of global helicity, magnetic energy and the (ohmic) dissipation rate amply indicate that the dissipation rate can also serve as an effective minimizer for a driven dissipative plasma.

It is to be noted that the underlying system has a finite dissipation. When the rate of dissipation exceeds that of the forcing, dissipative processes dominate the evolution. In such a case, both the energy and helicity decrease rapidly. Note that dissipation is necessary in our simulation to validate the hypothesis of minimum dissipation rates that lead eventually to a self-organization state in a dissipative MHD plasma by selectively operating on magnetic energy and helicity. In the event of initially zero fluctuations, the forcing leads to the population of the turbulent spectra during the early phase of evolution, which then decay owing to the finite dissipation in the system. The final results in such cases give rise to the same conclusions that are described in the context of the MDR state elsewhere in our paper.

Further, we would like to indicate that our results are in line with Prigogine's idea of the emergence of a new type of self-organized states called 'dissipative structures' for systems far from equilibrium, where the nonlinear interactions of fluctuations create an ordered state in the system. A detailed investigation of the dissipative structures in plasma will be undertaken as a future work.

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