ASYMPTOTIC MODEL FOR LARGE - SCALE QUASIHELIOSTROPHIC FLOW

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ABSTRACT. Starting from the ideal MHD equations written in spherical coordinates, using the regular perturbation method, quasiheliostrophic equations are derived as generalisation of quasigeostrophic models for atmospheric flow. From the fourth order momentum and induction equations, two prognostic equations for the stream function and magnetic flux function are derived. The magnetohydrodynamic model based on these equations is a suitable framework for describing the interaction between large-scale dynamic and magnetic features and the presence of specific patterns like active longitudes in the solar activity cycle.

1. Introduction

As is extensively argued by De Luca and Gilman (1991),there are many reasons to consider that the dynamo seat is the boundary layer at the interface between the radiative core and the convection zone, but it is very difficult for the surface topology of large-scale dynamic and magnetic features to be determined only by the structure of the thin layer below the convection zone. The main reason is the turbulence acting at many scales in the convection zone.

Then, even if the magnetic flux concentrations and flux tubes formation take place at the base of the convection zone, and the energetic transport is driven by convective motions, the surface distribution of large-scale features characteristic of the solar activity is, if not determined, then at least influenced by the photospheric hydromagnetic flow.

The purpose of this paper is to obtain by mean of asymptotic methods, a suitable model for describing more realisticaly the large-scale hydromagnetic flow in the solar photosphere.

2. General Physical Model

A possible explanation for the existence of active longitudes is the resonant interaction between Rossby and Alfvén waves that could help the flux tubes rising in the regions of maximum vorticity in the photospheric hydromagnetic flow.

Ward (1965) suggested the presence of a Rossby regime rather than one of Hadley type for the large scale photospheric flow and Gilman (1967) developed some global circulation models supporting this idea. Martres and al (1973) related the development and the decay of an active region to the sense

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of rotation in the cyclonic vortex, while Ambrož (1986) observed that active regions and maximum velocity zones overlap for the case studies randomly chosen. Wolff and Blizard (1986) reviewed and studied some properties of low-frequency r-modes for the Sun. Using a "shallow fluid" model with magnetic field, Lou (1987) found a particular solution coresponding to Rossby-Alfvén waves.

Here, I will start from the nine MHD equations in spherical coordinates (r-radial distance, θ-latitude, φ-longitude).

$$\frac{du}{dt}+\frac{uw}{r}-\frac{uv}{r}\tan\theta-2\Omega v\sin\theta+2\Omega w\cos\theta=-\frac{1}{\rho r\cos\theta}\frac{\partial}{\partial\varphi}\left(p+\frac{B^2}{2\mu}\right)+$$
$$+\frac{B_\varphi}{\mu\rho r\cos\theta}\frac{\partial B_\varphi}{\partial\varphi}+\frac{B_\theta}{\mu\rho r}\frac{\partial B_\varphi}{\partial\theta}+\frac{B_r}{\mu\rho}\frac{\partial B_\varphi}{\partial r}-\frac{B_\varphi B_\theta}{\mu\rho r}\tan\theta+\frac{B_\varphi B_r}{\mu\rho r},$$

$$\frac{dv}{dt}+\frac{vw}{r}+\frac{u^2}{r}\tan\theta+2\Omega u\sin\theta=-\frac{1}{\rho r}\frac{\partial}{\partial\theta}\left(p+\frac{B^2}{2\mu}\right)+\frac{B_\varphi}{\mu\rho r\cos\theta}\frac{\partial B_\theta}{\partial\varphi}+$$
$$+\frac{B_\theta}{\mu\rho r}\frac{\partial B_\theta}{\partial\theta}+\frac{B_r}{\mu\rho}\frac{\partial B_\theta}{\partial r}+\frac{B_r B_\theta}{\mu\rho r}+\frac{B^2{}_\varphi}{\mu\rho r}\tan\theta,$$

$$\frac{dw}{dt}-\frac{u^2+v^2}{r}-2\Omega u\cos\theta=-\frac{1}{\rho}\frac{\partial}{\partial r}\left(p+\frac{B^2}{2\mu}\right)-g+\frac{B_\varphi}{\mu\rho r\cos\theta}\frac{\partial B_r}{\partial\varphi}+$$
$$+\frac{B_\theta}{\mu\rho r}\frac{\partial B_r}{\partial\theta}+\frac{B_r}{\mu\rho}\frac{\partial B_r}{\partial r}-\frac{B^2{}_\varphi+B^2{}_\theta}{\mu\rho r},$$

$$\frac{d\rho}{dt}+\rho\nabla\boldsymbol{v}=0, \qquad (1)$$

$$c_v\frac{dT}{dt}+RT\nabla\boldsymbol{v}=0,$$

$$p=\rho RT,$$

$$\frac{dB_\varphi}{dt}=\frac{B_\theta}{r}\frac{\partial u}{\partial\theta}-\frac{B_\varphi}{r}\frac{\partial v}{\partial\theta}+B_r\frac{\partial u}{\partial r}-B_\varphi\frac{\partial w}{\partial r}-\frac{u}{r}B_\theta\tan\theta+$$
$$+B_r\frac{\partial u}{\partial r}-2B_\varphi\frac{w}{r}+\frac{v}{r}B_\varphi\tan\theta,$$

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$$\frac{dB_\theta}{dt} = \frac{1}{r\cos\theta}\left(B_\varphi\frac{\partial v}{\partial\varphi} - B_\theta\frac{\partial u}{\partial\varphi}\right) + B_r\frac{\partial v}{\partial r} - B_\theta\frac{\partial w}{\partial r} + \frac{u}{r}B_\varphi\tan\theta + \\ + B_r\frac{v}{r} - 2B_\theta\frac{w}{r} + \frac{v}{r}B_\theta\tan\theta,$$

$$\frac{dB_r}{dt} = \frac{1}{r\cos\theta}\left(B_\varphi\frac{\partial w}{\partial\varphi} - B_r\frac{\partial u}{\partial\varphi}\right) + \frac{1}{r}\left(B_\varphi\frac{\partial w}{\partial\theta} - B_r\frac{\partial v}{\partial\theta}\right) - \frac{B_\varphi u - B_\theta v}{r} - \\ - 2B_r\frac{w}{r} + \frac{v}{r}B_r\tan\theta.$$

The notations are the same as in Ghizaru (1992) as well as the scaling assumptions and the dimensionless system obtained in terms of the ten non-dimensional parameters.

3. Quasiheliostrophic Model

The regular perturbation method will be applied in the sequel to the dimensionless system.

Taking account of the quasihorizontal character of the photospheric flow, the vertical velocity is expanded as

$$w \sim \varepsilon^2 w_0 + \varepsilon^3 w_1 + \ldots$$

The other unknown functions are expanded as

$$X \sim X_0 + \varepsilon X_1 + \varepsilon^2 X_2 + \varepsilon^3 X_3 + \ldots$$

and

$$\sin(\theta_0 + \lambda y) \sim \sin\theta_0 + \lambda y\cos\theta_0 - \frac{1}{2}\lambda^2 y^2\cos\theta_0 \ldots$$

$$\cos(\theta_0 + \lambda y) \sim \cos\theta_0 - \lambda y\sin\theta_0 - \frac{1}{2}\lambda^2 y^2\cos\theta_0 \ldots$$

for $\varepsilon \to 0$.

Following Vamoş and Georgescu (1990) it is assumed that $T_0=1$ and p_0, ρ_0, T_1, p_1 and ρ_1 are time-independent.

Using the same characteristic scales as in Ghizaru (1992), the small parameter will be the Rossby number: $R_0=O(\varepsilon)$ for $\varepsilon \to 0$.

Another important hypothesis is that the Alfvén velocity is of the same order of magnitude as the flow velocity. Then, the Alfvén-Mach number squared is: $M_A=O(1)$ for $\varepsilon \to 0$.

Following Priest (1982), the plasma beta is chosen to be $\beta_p=10^3$, so that $\beta_p=O(\varepsilon^{-3})$ for $\varepsilon \to 0$.

The order of magnitude of the other parameters is given in Ghizaru (1992). After matching, the quasiheliostrophic equations are obtained in the fourth order. The first three order equations are obtained in the same form as in Vamoş and Georgescu (1990).

The fourth order induction and horizontal momentum equations read

$$\rho_0\left(\frac{D_0 u_0}{Dt} - 2yv_0\tan^{-1}2\theta_0 - v_1\right) - \rho_1 v_0 = -\frac{\partial}{\partial x}(p_3 + \boldsymbol{B}_0^2) + (\boldsymbol{B}_0\nabla)B_{\varphi_0},$$

$$\rho_0\left(\frac{D_0 v_0}{Dt} + 2yu_0\tan^{-1}2\theta_0 + u_1\right) + \rho_1 u_0 = -\frac{\partial}{\partial y}(p_3 + \boldsymbol{B}_0^2) + y\frac{\partial p_2}{\partial y}\tan\theta_0 + (\boldsymbol{B}_0\nabla)B_{\theta_0},$$

$$\frac{D_0 B_{\varphi_0}}{Dt} = B_{\theta_0}\frac{\partial u_0}{\partial y} - B_{\varphi_0}\frac{\partial v_0}{\partial y} + B_{r_0}\frac{\partial u_0}{\partial z}, \tag{2}$$

$$\frac{D_0 B_{\theta_0}}{Dt} = B_{\varphi_0}\frac{\partial v_0}{\partial x} - B_{\theta_0}\frac{\partial u_0}{\partial x} + B_{r_0}\frac{\partial v_0}{\partial z},$$

$$B_{r_0} v_0 \tan\theta_0 - (B_{\varphi_0}u_0 + B_{\theta_0}v_0) = 0.$$

Here,

$$\frac{D_0}{Dt} = \frac{\partial}{\partial t} + u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}.$$

A simplified form of the equations is obtained if the radial component of the induction equation is expanded as

$$B_r \sim \varepsilon B_{r0} + \varepsilon^2 B_{r1} + \varepsilon^3 B_{r2} + \dots \qquad for\ \varepsilon \to 0.$$

After cross differentiation of momentum equations with respect to x and y, the following equations, obtained in first, second and third order are used:

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \quad \frac{\partial v_1}{\partial y} - y\frac{\partial v_0}{\partial y}\tan\theta_0 - v_0\tan\theta_0\frac{\partial u_1}{\partial x} = 0,$$

$$u_0 = -\frac{1}{\rho_0}\frac{\partial p_2}{\partial y}, \quad v_0 = \frac{1}{\rho_0}\frac{\partial p_2}{\partial x}.$$

If in addition we assume

$$u_0 = -\frac{\partial\psi}{\partial y}\ ,\ v_0 = \frac{\partial\psi}{\partial x}\ ,\ B_{\varphi_0} = -\frac{\partial A}{\partial y}\ ,\ B_{\theta_0} = \frac{\partial A}{\partial x}\ , \qquad (3)$$

where A is the magnetic flux function and $\psi=p_2/\rho_0$ the stream function, and we note the Jacobian with:

$$J(\alpha,\beta) = \frac{\partial\alpha}{\partial x}\frac{\partial\beta}{\partial y} - \frac{\partial\alpha}{\partial y}\frac{\partial\beta}{\partial x}\ ,$$

we obtain:

$$\frac{\partial}{\partial t}(\nabla_h^2\psi) + J(\psi,\nabla_h^2\psi) + \frac{\partial\psi}{\partial x}\cot\theta_0 = J(A,\nabla_h^2 A)\ , \qquad (4)$$

$$\frac{\partial}{\partial t}(\nabla_h^2 A) + J(\psi,\nabla_h^2 A) + 2J\left(\frac{\partial\psi}{\partial x},\frac{\partial A}{\partial x}\right) + 2J\left(\frac{\partial\psi}{\partial y},\frac{\partial A}{\partial y}\right) = J(A,\nabla_h^2\psi)\ . \qquad (5)$$

Here ∇_h is the horizontal divergence.

Adding necessary initial and boundary conditions for ψ and A, (4) and (5) form a suitable model for the large-scale photospheric flow characterisation. The numerical model will be presented elsewhere.

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References

Ambrož,P. 1986, in *Proceedings of the 12th Regional Consultation in Solar Physics*, Smolenice, 501.

De Luca, E.E. and Gilman, P.A., 1991, in A.N.Cox, W.C.Livingston and M.S.Matthews (eds) *Solar Interior and Atmospheres*, 275.

Ghizaru, M. ,1992, submitted to *Romanian Astronomical Journal.*

Gilman, P.A., 1967, *J.Atm.Sci.*, 24,101.

Lou, Y. Q., 1987, *Asprophys. J.*, 322, 862.

Martres, M-J., I. Soru-Escaut and J. Rayrole, 1973, *Solar Phys.*, 32, 365.

Priest, E. R., 1982, *Solar Magnetohydrodynamics*, Reidel Publishing Company, Dordrecht.

Vamoş, C. and Georgescu, A., 1990, *Z. Meteorol.*, 40, 14.

Ward, F., 1965, *Astrophys. J.*, 141,534.

Wolff, C.L. and Blizard, J.B., 1986, *Solar Phys.*, 105,1.