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Order and chaos in ETG-driven drift–dissipative waves with sheared flows

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Abstract. We derive a system of nonlinear equations that govern the dynamics of low-frequency short-wavelength electromagnetic waves in the presence of equilibrium density, temperature, magnetic field and velocity gradients. In the linear limit, a local dispersion relation is obtained and analyzed. New η_e-driven electromagnetic drift modes and instabilities are shown to exist. In the nonlinear case, the temporal behaviour of a nonlinear dissipative system can be written in the form of Lorenz- and Stenflo-type equations that admit chaotic trajectories. On the other hand, the stationary solutions of the nonlinear system can be represented in the form of dipolar and vortex-chain solutions.

1. Introduction

In recent years, there has been increasing interest in numerous types of drift-wave instabilities in order to explain the enhanced fluctuations causing anomalous particle and heat transport (Kadomtsev 1965; Mikhailovskii 1974; Hasegawa and Mima 1978). It is well known that electron-temperature-gradient, η_e, modes may be responsible for the anomalous electron energy transport for various toroidal devices. Several authors have investigated $\eta_e(\eta_i)$ electrostatic modes in slab geometry for a uniform magnetized plasma (Coppi et al. 1967; Liu 1971; Rozhanskii 1981; Shukla 1987). However, when the plasma β (particle kinetic pressure/magnetic pressure) exceeds the electron-to-ion mass ratio, the electromagnetic effects on η_e modes must be taken into account.

In this paper, we investigate the linear and nonlinear properties of low-frequency ($\omega \ll \omega_{ce}$) and short-wavelength electromagnetic drift–dissipative waves in an electron plasma with equilibrium density, temperature, magnetic field and velocity gradients. For this purpose, we employ the hydrodynamic equations of Braginskii (1965) and derive a system of nonlinear equations. In the linear limit, a new dispersion relation under the local approximation is derived, and a number of interesting limiting cases are discussed. On the other hand, in the nonlinear case, we discuss possible stationary and non-stationary solutions of the newly derived nonlinear equations.

Let us consider the nonlinear propagation of low-frequency ($\omega \ll \omega_{ce}$, where ω_{ce} is the electron cyclotron frequency) electromagnetic waves in the presence

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of equilibrium density, temperature and magnetic field gradients along with equilibrium velocity gradients $d_x v_{e0}(x) = v'_{e0}$. Here $v_{e0}(x)$ is the magnetic-field-aligned plasma sheared flow. We assume that the difference in the equilibrium flow velocities of ions and electrons leads to a small shear component of the magnetic field.

The electron and ion velocity under the drift approximation are

$$\mathbf{v}_e \approx \mathbf{v}_{EB} + \mathbf{v}_{De} + (v_{e0} + v_{ez}) \frac{\mathbf{B}_\perp}{B_0} + \hat{\mathbf{z}} v_{ez},$$

$$\mathbf{v}_i \approx \mathbf{v}_{EB} - \frac{c}{B_0 \omega_{ci}} (\partial_t + \nu_e + \mathbf{v}_{EB} \cdot \boldsymbol{\nabla} - \mu_i \nabla_\perp^2) \nabla_\perp \phi,$$

where

$$\mathbf{v}_{EB} = c\hat{\mathbf{z}} \times \frac{\boldsymbol{\nabla}\phi}{B_0},$$

$$\mathbf{v}_{De} = -\frac{c}{eB_0 n_{e0}} \hat{\mathbf{z}} \times \boldsymbol{\nabla}(n_e T_e)$$

are the usual $\mathbf{E} \times \mathbf{B}_0$ and diamagnetic drifts respectively. Here

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{1}{c} \partial_t A_z \hat{\mathbf{z}}$$

is the electric field vector, $\phi(A_z)$ is the electrostatic (parallel component of the vector) potential, $\mathbf{B}_\perp = \boldsymbol{\nabla} A_z \times \hat{\mathbf{z}}$ is the perpendicular component of the wave magnetic field, c is the speed of light and ν_e is the electron collision frequency. The parallel component of the electron velocity perturbation is determined from the parallel component of Ampère's law, giving

$$v_{ez} \approx \frac{c}{4\pi e n_e} \nabla_\perp^2 A_z,$$

where $\nabla_\perp^2 = \partial^2 x + \partial^2 y$.

The dynamics of electromagnetic waves is governed by the equations of continuity, momentum and energy balance, which are supplemented by the charge-neutrality condition and Ampère's law. From these equations, after letting

$$n = n_{e0} + \delta n_e, \qquad T = T_{e0} + \delta T_e,$$

where $\delta n_e \ll n_{e0}$ and $\delta T_e \ll T_{e0}$, we readily obtain

$$(\mathscr{D}_t^e + \mathbf{v}_{Be} \cdot \boldsymbol{\nabla} - \nu_e \rho_e^2 \nabla_\perp^2) \delta n_e + \frac{e n_{e0}}{T_{e0}} (\mathbf{v}_n - \mathbf{v}_{Be}) \cdot \boldsymbol{\nabla}\phi$$
$$- \boldsymbol{\nabla} A_z \times \hat{\mathbf{z}} \cdot \boldsymbol{\nabla} \left(\frac{J_{e0}}{eB_0} \right) + \frac{n_{e0}}{T_{e0}} \mathbf{v}_{Be} \cdot \boldsymbol{\nabla} T_{e1} + \frac{c}{4\pi e} \mathscr{D}_z (\nabla_\perp^2 A_z) = 0, \quad (1)$$

$$(\mathscr{D}_t^e + \mathbf{v}_{D0}^e \cdot \boldsymbol{\nabla}) A_z - \lambda_e^2 (\mathscr{D}_t^e + \mathbf{v}_{D0}^e \cdot \boldsymbol{\nabla} + \boldsymbol{\nu}_e) \nabla_\perp^2 A_z$$
$$+ c(\partial_z + \mathbf{S}_{v0} \cdot \boldsymbol{\nabla}) \phi - \frac{c T_{e0}}{e n_{e0}} \mathscr{D}_z \delta n_e - \frac{c}{e} \mathscr{D}_z \delta T_e = 0, \quad (2)$$

$$\left(\mathscr{D}_t^e+\frac{5}{3}\mathbf{v}_{Be}\cdot\boldsymbol{\nabla}-\frac{2\chi_e}{3n_0}\nabla_\perp^2\right)\delta T_e+e\left(\eta_e-\frac{2}{3}\right)\mathbf{v}_n\cdot\boldsymbol{\nabla}\phi-\frac{2T_{e0}}{3n_{e0}}\mathscr{D}_t^e\,\delta n_e$$
$$+0.48\lambda_{De}^2\,ce\partial_z\nabla_\perp^2 A_z=0, \quad (3)$$

$$\mathscr{D}_t^e\nabla_\perp^2\phi+\frac{B_0\,\omega_{ci}}{cn_{e0}}\left[\mathbf{v}_{Be}\cdot\boldsymbol{\nabla}\,\delta n_e+\frac{n_{e0}}{T_{e0}}\mathbf{v}_{Be}\cdot\boldsymbol{\nabla}\,\delta T_e\right.$$
$$\left.-\boldsymbol{\nabla}A_z\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla}\left(\frac{J_{e0}}{eB_0}\right)\right]+\frac{v_A^2}{c}\mathscr{D}_z(\boldsymbol{\nabla}_\perp^2 A_z)=0, \quad (4)$$

where

$$\mathscr{D}_t^e\equiv\partial_t+\mathbf{v}_{EB}\cdot\boldsymbol{\nabla}+(v_{e0}+v_{ez})\,\partial_z,$$

$$\mathscr{D}_z\equiv\partial_z+\frac{1}{B_0}\boldsymbol{\nabla}A_z\times\hat{\mathbf{z}}\cdot\boldsymbol{\nabla},$$

$$S_{v0}\equiv\frac{\hat{\mathbf{z}}\times\boldsymbol{\nabla}v_{e0}}{\omega_{ce}},$$

$J_{e0}=-en_{e0}\,v_{e0}$ is the electron equilibrium current, $\lambda_{De}^2=T_e/4\pi n_0\,e^2$ is the Debye length, $\lambda_e=c/\omega_{pe}$ is the electron skin depth and

$$\eta_e=\frac{d_x\ln T_{e0}}{d_x\ln n_{e0}},$$

with

$$\mathbf{v}_n=-\frac{cT_{e0}}{eB_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\ln n_{e0}(x),$$

$$\mathbf{v}_{Be}=-\frac{cT_{e0}}{eB_0}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\ln B_0(x),$$

$$\mathbf{v}_{D0}=-\frac{c}{eB_0\,n_{e0}}\hat{\mathbf{z}}\times\boldsymbol{\nabla}(n_{e0}\,T_{e0})=\mathbf{v}_n(1+\eta_e).$$

Equations (1)–(4) are the desired nonlinear coupled equations describing electromagnetic fluctuations in a non-uniform magnetoplasma with equilibrium density, temperature and magnetic field gradients and with plasma sheared flows.

In the linear limit, assuming that all the perturbed quantities are proportional to $\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$, where $\mathbf{k}$ and ω are the wavevector and the frequency respectively, Fourier transforming (1)–(4) gives the following dispersion relation:

$$(W_e+\omega_{Be})\left[\Omega k_\perp^2\rho_s^2+\omega_{Be}\left(1+\frac{\mathbf{k}\cdot\mathbf{S}_{v0}}{k_z}\right)\right](\Omega_0\,F_e-0.48c^2k_z^2\,k_\perp^2\,\lambda_{De}^2)$$
$$-\frac{ck_z\,T_{e0}}{en_{e0}}\left\{(W_e+\omega_{Be})\left[\Omega_0\left(1+\frac{\mathbf{k}\cdot\mathbf{S}_{v0}}{k_z}\right)+\left(\eta_e-\frac{2}{3}\right)\omega_n\right]\right.$$
$$\left.-\left[\left(\frac{2}{3}\Omega+\Omega_0\right)(\omega_n-\omega_{Be}-k_\perp^2\,\rho_s^2\,\Omega)\right]\right\}\left(\beta_0+\frac{en_{e0}\,\omega_{Be}\,F_e}{ck_z\,T_{e0}}\right)=0, \quad (5)$$

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where

$$\Omega^* = \Omega + iD_e k_\perp^2, \qquad \Omega = \omega - k_z v_{e0},$$
$$W_e = \Omega^* - \omega_{Be},$$
$$\Omega_0 = \Omega - \frac{5}{3}\omega_{Be} + \frac{i2\chi_e k_\perp^2}{3n_0},$$
$$b_e = k_\perp^2 \lambda_e^2, \qquad F_e = (1+b_e)(\Omega - \omega_D) + i\nu_e b_e,$$
$$\beta_0 = k_y \partial_x \left(\frac{J_{e0}}{eB_0}\right) + \frac{ck_z k_\perp^2}{4\pi e},$$

and

$$\rho_s = \frac{(T_e/m_i)^{1/2}}{\omega_{ci}}$$

is the ion Larmor radius with the electron temperature. For a uniform $\mathbf{B}_0$ and for non-dissipative case, the dispersion relation takes the following simple form:

$$\Omega\left[(1+b_e)(\Omega^2 - \omega_D \Omega) - 0.48c^2 k_z^2 k_\perp^2 \lambda_{De}^2 - \frac{5T_{e0} ck_z \beta_0}{3en_{e0}}\right] k_\perp^2 \rho_s^2 - \frac{ck_z T_{e0}\beta_0}{en_{e0}}\left[\Omega\left(1 + \frac{\mathbf{k}\cdot\mathbf{S}_{v0}}{k_z}\right) - (\eta_e + 1)\omega_n\right] = 0, \quad (6)$$

In the absence of an equilibrium density gradient, assuming that $k_\perp^2 \lambda_{De}^2 \ll 1$ and $\omega \gg k_z v_{e0}$, (6) shows the growth rate of instability to be

$$\gamma = \frac{k_y}{k_\perp}\left[\frac{m_e}{m_i(1 + k_\perp^2 \lambda_e^2)\, en_{e0}}\right]^{1/2} |\partial_x v_{e0}\, \partial_x J_{e0}|^{1/2}. \quad (7)$$

Equation (7) tells us that the equilibrium sheared flow is responsible for the instability. In the absence of v_{e0} and for $k_\perp^2 \lambda_{De}^2 \ll 1$ and $\eta_e = -1$, we get the threshold of instability as

$$\omega_D^2 < 4(1+b_e)^{-1}\left(1 + \frac{5}{3}k_\perp^2 \rho_s^2\right)\left(\frac{\omega_{ci} k_y k_z \partial_x J_{e0}}{k_\perp^2 en_{e0}} + k_z^2 v_A^2\right). \quad (8)$$

On the other hand, for $k_\perp^2 \lambda_{De}^2 \ll 1$, $\omega \gg k_z v_{e0}$ and $\eta_e \gg 1$ and for

$$\omega\omega_D k_\perp^2 \rho_s^2 = -k_z c\beta_0 (1 + \tfrac{5}{3}k_\perp^2 \rho_s^2),$$

we obtain another interesting limiting case, namely

$$\omega = \frac{1 + i\sqrt{3}}{2}\left|\frac{\omega_n T_{e0} \beta_0 ck_z(\eta_e + 1)}{en_{e0} k_\perp^2 \rho_s^2 (1+b_e)}\right|^{1/3}, \quad (9)$$

which shows that plasma will become unstable whenever the equilibrium electron density and temperature gradients are in the opposite direction. This also shows an oscillatory instability with a real frequency

$$\omega_r = \frac{1}{2}\left|\omega_n \frac{T_{e0}}{en_{e0}} \frac{\beta_0 ck_z(\eta_e+1)}{k_\perp^2 \rho_s^2 (1+b_e)}\right|^{1/3} \quad (10)$$

and imaginary part $\gamma = \sqrt{3}\,\omega_r$.

Next, we present the nonlinear coherent vortex solutions of the coupled equations (1)–(4), by introducing a new frame $\xi = y + \alpha z - ut$, where α and u are constants, and assuming that ϕ, A_z, δn and δT are functions of x and ξ only.

Assuming that at equilibrium the density and temperature gradients are zero and that the plasma is embedded in a constant external magnetic field such that $\lambda_{De}^2 \nabla_\perp^2 \ll 1$ and $u \gg v_{e0}$, (1)–(4) respectively can be rewritten as

$$\mathcal{O}_\phi \delta n_e = \mathcal{O}_A \left(\frac{n_{e0} \partial_x v_{e0}}{uB_0} A_z + \frac{c\alpha}{4\pi eu} \nabla_\perp^2 A_z \right), \tag{11}$$

$$\mathcal{O}_\phi \left[(1 - \lambda_e^2 \nabla_\perp^2) A_z - \frac{c\alpha_0}{u} \phi \right] + \mathcal{O}_A \left(\frac{c\alpha T_{e0}}{en_{e0} u} \delta n_e + \frac{c\alpha}{eu} \delta T_e \right) = 0, \tag{12}$$

$$\mathcal{O}_\phi \left(\delta T_e - \frac{2T_{e0}}{3n_{e0}} \delta n_e \right) \approx 0, \tag{13}$$

$$\mathcal{O}_\phi \nabla_\perp^2 \phi = \frac{eB_0^2}{m_i n_0 c^2} \mathcal{O}_A \left(\frac{n_{e0} \partial_x v_{e0}}{uB_0} A_z + \frac{c\alpha}{4\pi eu} \nabla_\perp^2 A_z \right), \tag{14}$$

where

$$\mathcal{O}_\phi \equiv \partial_\xi - \frac{c}{uB_0} (\partial_x \phi \partial_\xi - \partial_\xi \phi \partial_x),$$

$$\mathcal{O}_A \equiv \partial_\xi - \frac{1}{\alpha B_0} (\partial_x A_z \partial_\xi - \partial_\xi A_z \partial_x), \qquad \alpha_0 = \alpha - \frac{\omega_v}{\omega_{ce}}.$$

Obtaining the stationary solutions of (11)–(14) is a rather involved process, and can only be done numerically. However, we discuss here some approximate solutions by considering first an inertial-wave case in which one may ignore the density fluctuations. We further assume that $\lambda_e^2 \nabla_\perp^2 A_z \ll A_z$ by assuming that the scale size of the vortex is much smaller than the electron skin depth. Then (12) gives

$$A_z = \frac{c\alpha_0}{u} \phi, \tag{15}$$

Substituting (15) into (14), we get

$$\left(1 - \frac{v_A^2 \alpha\alpha_0}{u^2}\right) \partial_\xi \nabla_\perp^2 \phi - \frac{c}{uB_0} \left(1 - \frac{\alpha_0^2 v_A^2}{u^2}\right) \mathcal{T}(\phi, \nabla_\perp^2 \phi) - \frac{\omega_{ci} \alpha_0 \partial_x v_{e0}}{u^2} \partial_\xi \phi = 0, \tag{16}$$

where the Jacobian

$$\mathcal{T}(f, g) \equiv \partial_x f \partial_\xi g - \partial_\xi f \partial_x g.$$

In the absence of velocity shear, (16) takes the following form of the Navier–Stokes equation:

$$\partial_\xi \nabla_\perp^2 \phi_1 - \frac{c\mu_0}{uB_0} \mathcal{T}(\phi, \nabla_\perp^2 \phi) = 0, \tag{17}$$

where

$$\mu_0 \equiv \frac{1 - \alpha_0^2 v_A^2 / u^2}{1 - v_A^2 \alpha\alpha_0 / u^2}.$$

Equation (17) is satisfied by

$$\nabla_{\perp}^{2}\phi = \frac{4\phi_0 K^2}{a_0^2}\exp\left[-\frac{2}{\phi_0}\left(\phi - \frac{uB_0}{\mu_0 c}x\right)\right], \tag{18}$$

which is satisfied by

$$\phi = \frac{uB_0}{\mu_0 c}x + \phi_0 \ln\left[2\cosh(Kx) + 2\left(1 - \frac{1}{a_0^2}\right)\cos(K\xi)\right], \tag{19}$$

where ϕ_0 and K are arbitrary constants and a_0 represents the size of a vortex. For $a_0^2 > 1$, the solution (19) represents a typical vortex-chain or Kelvin–Stuart 'cat's eyes' solution (Mikhailovskii 1974; Shukla et al. 1998).

Next, for non-inertial waves in which $\delta n_e \neq 0$ and assuming that $\lambda_e^2 \nabla_{\perp}^2 A_z \ll A_z$, (12) gives

$$\mathcal{O}_{\phi}\left(A_z - \frac{c\alpha_0}{u}\phi\right) + \mathcal{O}_A\left[\frac{c\alpha}{eu}\left(\frac{T_{e0}}{n_{e0}}\delta n_e + \delta T_e\right)\right] = 0, \tag{20}$$

Eliminating δn_e and δT_e from (20), using (11) and (13), we readily obtain

$$\mathcal{O}_{\phi}(\nabla_{\perp}^2\phi - \chi_1\phi_1 + \chi_2 A_z) = 0, \tag{21}$$

A possible solution of (21) is

$$\nabla_{\perp}^2\phi - \chi_1\phi_1 + \chi_2 A_z = f_1\left(\phi - \frac{uB_0}{c}x\right), \tag{22}$$

where

$$\chi_1 = \frac{3\alpha_0 e^2 B_0}{5\alpha m_i T_{e0}}, \qquad \chi_2 = \frac{3eu\omega_{ci}}{5\alpha T_{e0}}$$

and f_1 is an arbitrary constant.

Similarly (11)–(14) can be combined as

$$\mathcal{O}_A^2\left\{\left[1 + \frac{5c\alpha T_{e0}}{3eun_{e0}}\left(\frac{n_{e0}\,\partial_x v_{e0}}{uB_0} + \frac{c\alpha}{4\pi eu}\nabla_{\perp}^2\right)\right]A_z - \frac{c\alpha_0}{u}\phi\right\} = 0,$$

or

$$\left[1 + \frac{5c\alpha T_{e0}}{3eun_{e0}}\left(\frac{n_{e0}\,\partial_x v_{e0}}{uB_0} + \frac{c\alpha}{4\pi eu}\nabla_{\perp}^2\right)\right]A_z - \frac{c\alpha_0}{u}\phi = f_2(A_z - \alpha B_0 x), \tag{23}$$

Eliminating A_z from (22) and (23), we get

$$\nabla_{\perp}^4\phi_1 + c_1\nabla_{\perp}^2\phi_1 + c_2\phi_1 - c_3 x = 0, \tag{24}$$

where

$$\nabla_{\perp}^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \xi^2},$$

$$c_1 = -\left(f_1 + \frac{3\alpha_0 ec\omega_{ci}}{5\alpha T_{e0}}\right), \qquad c_2 = \frac{3u^2\chi_2(c\alpha_0/u + f_2)}{5c^3\alpha^2\lambda_{De}^2}, \qquad c_3 = \frac{3u^3\chi_2 B_0 f_2}{5c^4\alpha^2\lambda_{De}^2}.$$

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Here we also assume that

$$\partial_x v_{e0} = -\frac{3eB_0 u^2}{5c\alpha T_{e0}}$$

and f_2 is a constant of integration. Equation (24) is a fourth-order differential equation that admits a spatially bounded dipolar-vortex solution (Mikhailovskii 1974; Shukla et al. 1998).

2. Chaotic behaviour of electromagnetic fluctuations

In order to study the temporal behaviour of nonlinearly interacting finite-amplitude two-dimensional electromagnetic waves (i.e. $\partial_z = 0$) in a collisional magnetoplasma without density gradient, (1)–(4) respectively can be rewritten as

$$(d_t - D_e \nabla_\perp^2)\,\delta n_e = \alpha_1 \partial_y A_z - \alpha_2(\partial_y A_z \partial_x - \partial_x A_z \partial_y)\nabla_\perp^2 A_z, \qquad (25)$$

$$[(1-\lambda_e^2\nabla_\perp^2)\,d_t - \eta_e \nabla_\perp^2]A_z = -\beta_1 \partial_y \phi + \beta_2(\partial_y A_z \partial_x - \partial_x A_z \partial_y)\,\delta n_e, \qquad (26)$$

$$(d_t + \mu_i \nabla_\perp^2)\nabla_\perp^2 \phi - \gamma_1 \partial_y A_z + \gamma_2(\partial_y A_z \partial_x - \partial_x A_z \partial_y)\nabla_\perp^2 A_z = 0, \qquad (27)$$

where

$$d_t \equiv \partial_t + \mathbf{v}_{EB}\cdot\boldsymbol{\nabla},$$

$$\eta_e = \nu_{ei}\lambda_e^2, \qquad \alpha_1 = \frac{\partial_x J_{e0}}{eB_0}, \qquad \alpha_2 = \frac{c}{4\pi e B_0},$$

$$\beta_1 = \frac{c d_x v_{e0}}{\omega_{ce}}, \qquad \beta_2 = \frac{5cT_{e0}}{3eB_0 n_{e0}},$$

$$\gamma_1 = \frac{\omega_{ci}\partial_x J_{e0}}{cen_{e0}}, \qquad \gamma_2 = \frac{v_A^2}{cB_0}.$$

We follow the approach of Stenflo (1996) and introduce the ansatz

$$\phi = \phi_1(t)\sin(K_x x)\sin(K_y y), \qquad (28)$$

$$\delta n_e = n_1(t)\sin(K_x x)\sin(K_y y), \qquad (29)$$

$$A_z = A_1(t)\sin(K_x x)\cos(K_y y) - A_2(t)\sin(2K_x x), \qquad (30)$$

where K_x and K_y are constant parameters, and ϕ_1, n_1, A_1 and A_2 are some time-dependent amplitudes. Substituting (28)–(30) into (25)–(27), and after appropriate normalization, we obtain the following 4×4 matrix that describes the nonlinear coupling between various amplitudes:

$$\begin{pmatrix} d_\tau X \\ d_\tau Y \\ d_\tau Z \\ d_\tau U \end{pmatrix} = \begin{pmatrix} -\sigma_0 & \sigma_0 & s_0 Y & 0 \\ r & -1 & -X & s_1 Z \\ Y & 0 & -b & s_2 Y \\ 0 & \sigma_1 & s_3 Y & -\sigma_1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ U \end{pmatrix}. \qquad (31)$$

Here

$$\sigma_0 = \frac{\mu_i(1+k^2\lambda_e^2)}{\eta_e}, \qquad \sigma_1 = \frac{K_\perp^2 D_e(1+k^2\lambda_e^2)}{\eta_e K^2},$$

$$r = -\frac{\beta_1 k_y a_1}{\eta_e K^2 a_2}, \qquad b = \frac{4K_x^2(1+K^2\lambda_e^2)}{K^2(1+4K_x^2\lambda_e^2)},$$

$$s_0 = -\frac{\gamma_2(k^2-4k_x^2)(1+k^2\lambda_e^2)k_x k_y a_2 a_3}{a_1 \eta_e K^4},$$

$$s_1 = \frac{\beta_2 K_x K_y a_3 a_4}{a_2 \eta_e K^2},$$

$$s_2 = \frac{\beta_2 K_x K_y(1+k^2\lambda_e^2)a_2 a_4}{2\eta_e K^2(1+4K_x^2\lambda_e^2)a_3},$$

$$s_3 = \frac{\alpha_2 K_x K_y(k^2-4k_x^2)(1+K^2\lambda_e^2)a_2 a_3}{\eta_e K^2 a_4},$$

with $K^2 = K_x^2 + K_y^2$. It is worth mentioning here that we have dropped the terms proportional to $\sin(3K_x x)$ in the derivation of (31).

If we take $K_y^2 = 4K_x^2$ then (31) reduces to a Lorenz–Stenflo (Lorenz 1964; Stenflo 1996) type equation. However, the normalizations used here are

$$\phi_1 = a_1 X = \pm\frac{\sqrt{2}\eta_e K_\perp^2 B_0}{cK_x K_y\sqrt{(1+K^2\lambda_e^2)(1+K^2\lambda_e^2-12K_x^2\lambda_e^2)}},$$

$$A_1 = a_2 Y = \pm\frac{\sqrt{2}\mu_i\eta_e K_\perp^6 B_0}{cK_x K_y^2\gamma_1\sqrt{(1+K_\perp^2\lambda_e^2)(1+K_\perp^2\lambda_e^2-12K_x^2\lambda_e^2)}},$$

$$A_2 = a_3 Z = -\frac{\mu_i\eta_e K_\perp^6 B_0}{cK_x K_y^2\gamma_1(1+K_\perp^2\lambda_e^2-12K_x^2\lambda_e^2)}Z,$$

$$n_1 = a_4 U = \mp\frac{\sqrt{2}\mu_i\eta_e\alpha_1 K_\perp^4 B_0}{cK_x K_y D_e\gamma_1\sqrt{(1+K_\perp^2\lambda_e^2)(1+K_\perp^2\lambda_e^2-12K_x^2\lambda_e^2)}}.$$

Equations (31) are the generalized Lorenz–Stenflo equations, whose properties can be studied both analytically and numerically by means of standard techniques. The equilibrium points of (31) can be obtained by setting time-derivative terms equal to zero and solving this nonlinear set of coupled equations. The 3×3 matrix case has been studied in some detail by Mirza and Shukla (1997). It is worth mentioning that the detailed behaviour of the chaotic motion can be studied by solving (31) numerically. However, this investigation is beyond the scope of this paper.

The stability of the stationary states can be studied by a simple linear analysis. Letting

$$X = X_s + X_1, \qquad Y = Y_s + Y_1,$$

$$Z = Z_s + Z_1, \qquad U = U_s + U_1,$$

the linearized system is

$$\begin{pmatrix} d_\tau X_1 \\ d_\tau Y_1 \\ d_\tau Z_1 \\ d_\tau U_1 \end{pmatrix} = \begin{pmatrix} -\sigma_0 & \sigma_0 + s_0 Z_s & s_0 Y_s & 0 \\ r - Z_s & -1 & -X_s + s_1 U_s & s_1 Z_s \\ Y_s & X_s + s_2 U_s & -b & s_2 Y_s \\ 0 & \sigma_1 + s_3 Z_s & s_3 Y_s & -\sigma_1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \\ U_1 \end{pmatrix}, \tag{32}$$

where $X_1 \ll X_s$, $Y_1 \ll Y_s$, $Z_1 \ll Z_s$, $U_1 \ll U_s$, and (X_s, Y_s, Z_s, U_s) represents a stationary state. The corresponding characteristic equation is thus

$$(\sigma_1 + \lambda)(\lambda + b)[\lambda^2 + (1 + \sigma_0)\lambda + \sigma_0(1 - r)] = 0, \tag{33}$$

which governs the linear stability of the stationary state. If we set $\sigma_1 = s_1 = s_2 = s_3 = 0$ then we recover the results of our earlier investigation (Mirza and Shukla 1997).

3. Discussion and conclusions

In summary, we have derived a set of nonlinear equations that govern the coupling of low-frequency electromagnetic waves in a non-uniform collisional magnetoplasma that has an equilibrium density gradient as well as a sheared plasma flow. The physical mechanism of the present instability is the coupling of free energy stored in the sheared equilibrium plasma flow to Alfvén-like modes. For a collision-dominated magnetoplasma without a density gradient, there is the possibility of a resistive instability of Alfvén-like waves in the presence of equilibrium sheared ion flows. We have also shown that the stationary solutions of the nonlinear system without dissipation and density gradients can be represented in the form of dipolar and vortex chains. Furthermore, linearly excited finite-amplitude electromagnetic waves interact among themselves and lead to a chaotic state due to mode couplings. This has been demonstrated by seeking a time-dependent solution of the nonlinear equations that govern the dynamics of finite-amplitude electromagnetic waves in a resistive medium. We have found that the nonlinear dynamics of electromagnetic turbulence in the presence of sheared plasma flows without the density gradient can be expressed as a set of four coupled-mode equations, or simply the generalized Lorenz–Stenflo equations, which admit chaotic trajectories. The results of our investigation should be helpful in understanding the properties of electromagnetic turbulence in low-temperature laboratory and space plasmas.

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