PART III.

Spherically-Symmetric Motions in Stellar Atmospheres.

B. - The Propagation of a Shock-Wave in an Atmosphere of Varying Density.

Summary-Introduction.

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1. – Introduction.

There is astrophysical evidence for the existence of shocks propagating in regions of variable density. We have therefore the choice of discussing first the physics of shocks in a variable density atmosphere, or the astrophysical phenomena. Following Kaplan's preliminary report, we shall describe first the astrophysical facts, for the reader to be able to understand the connection of the physics with the astrophysics.

1·1. *Novae and Supernovae.* – There is a widespread belief that *Novae* and *Supernovae* outbursts are due to the appearance at the surface of a star of a shock front somewhere inside (LEBEDINSKY (1946), SCHATZMAN (1946*a*, *b*), ROSSELAND (1946), GUREVITCH LEBEDINSKY (1947)).

Several questions arise, concerning the production of shocks in novae:

(i) Nature of the instability initiating the shock.

(ii) Energy sources of the shock. Has the shock a nuclear origin or is it produced by some other physical process?

(iii) Propagation of the shock in layers of decreasing density. The methods used for describing that process will be given later in this paper.

It is not necessary to recall here Milne's picture of the nova phenomena

(*) A considerable help in prepating this report was the preliminary report of Dr. S. A. KAPLAN from Lvov Observatory.

Ed. Note: Last minute circumstances prohibited Dr. Kaplan's attendance at the Symposium and Dr. Schatzman kindly undertook to prepare and present this report.

as the sudden collapse of an unstable star, with liberation of gravitational energy, though it can be connected with some of the modern pictures of supernovae. Biermann's picture (1939) of a sudden release of recombination energy, with formation of a convective zone, has been objected to by Ledoux, because it cannot be a sudden phenomenon.

Schatzman (1958) has shown that vibrational instability cannot lead to any acceptable picture of the recurrence of a nova, as the time scale of the recurrence would then be of the same order of magnitude as the Helmholtz-Kelvin time scale of the contraction. He has shown that vibrational instability in the presence of a resonance-induced oscillation of finite amplitude could lead to a reasonable theory of the recurrence.

It is a great temptation to suppose that the shock is initiated by a detonation wave, the exploding fuel being some convenient nuclear species. However, for most kinds of nuclear fuels, it can be shown (Schatzman, 1951) that the thickness of a detonation wave is much larger than the radius of the star, unless the cross-section of the energy producing nuclear reaction is exceptionally large and the abundance of the nuclear fuel great enough. The conclusion is that the energy appearing in the novae phenomenon, about 10^{45} ergs, has to be liberated in a time shorter than the half period of oscillation of the star (about 10^4 s), in a non-linear phenomenon, the surface appearance of the shock being only due to the propagation of a wave in regions of decreasing density.

Spectroscopic observation of novae shows the existence of systems of lines, with different radial velocities. It seems that the envelope which has been ejected is made of several shells catching up with each other. (See, for example, the data collected by C. Payne-Gaposhkin (1957), and the well-known book of Vorontzov-Velyaminov: *Gaseous Nebulae and Novae* (1948)).

There is a large variety of novae, and it is not the place here to classify them. However, it should be mentioned that it is unlikely that one process only is producing the novae outbursts. Several nuclear reactions, depending upon the range of density, temperature and chemical abundances can lead to explosive processes.

The problem of supernovae is likely to be different, in the sense that the whole star seems to be blown apart by the explosion. The total amount of energy liberated is of the order of 10^{49} erg (the energy at rest of the whole sun is 2.10^{54} erg, and its gravitational energy is of the order of 4.10^{50} erg). Evolution of a contracting star can eventually lead to nuclear reactions which make the star dynamically unstable. A collapse, with a large temperature and density increase, can favor a large variety of nuclear reactions which have been investigated by Burbidge, Burbidge, Hoyle and Fowler (1957). However, the hydrodynamics of the collapse and the generation of the shock-wave have not been investigated except by Colgate (1959).

1'2. *Cepheids.* – The problem of shocks in a variable density atmosphere is now considered as a standard problem of *Cepheids*, and has been discussed by WHITNEY in his introductory report.

1'3. *Solar chromosphere and corona.* – According to BIERMANN (1948), SCHWARZSCHILD (1948), SCHATZMAN (1949*b*), the heating of the solar chromosphere is due to energy dissipation of compression waves, created by granulation. THOMAS (1948) has suggested that the heating is due to the dissipation of the kinetic energy of the spicules.

The production of sound waves by turbulence and their propagation out of the turbulent regions is a well-known observed fact (cf. the summary *Aspects of the Turbulence Problem* by H. LIEPMANN, 1952).

Therefore, it can be considered as certain that compression waves, produced in the hydrogen convective zone, do propagate outside, towards the chromosphere and corona, though no astrophysical fact can be considered as a direct proof of these waves.

Let us first consider waves of a very small amplitude. It is well known that no atmosphere is transparent to a progressive wave, unless its period is smaller than a critical period.

$$P_{\text{crit}} = \frac{4\pi H}{a},$$

where H is the scale height of the atmosphere and a the sound velocity. For an isothermal atmosphere

$$H = \frac{a^2}{\gamma g}, \qquad \text{so} \qquad P_{\text{crit}} = \frac{4\pi a}{\gamma g}.$$

For the sun

$$P_{\text{crit}} \simeq 240 \text{ s}.$$

If at some place in an atmosphere the density is ϱ_0 and the velocity of the material is v, the flux of mechanical energy F_M is

$$F_M = \tfrac{1}{2}\varrho \cdot v^2 V_{\text{group}} = \tfrac{1}{2}\varrho_0 v^2 a \sqrt{1 - \left(\frac{\sigma_{\text{crit}}}{\sigma}\right)^2},$$

where $\sigma = 2\pi/P$.

It is clear that mechanical energy can be carried in the chromosphere only by waves of a period $P < P_{\text{crit}}$, the group velocity vanishing for $P = P_{\text{crit}}$. A rough evaluation, based on a schematic theory of turbulence in the convective

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zone, shows that the main contribution to the mechanical flux is due to periods appreciably smaller than 240 s, of the order of 50 s or smaller.

SCHATZMAN (1949*b*) has even suggested that the efficient acoustic waves have a period of only 8 s. The wavelength corresponding to a period P is

$$\lambda = 4\pi H\left(\left(\frac{P_{\text{crit}}}{P}\right)^2 - 1\right)^{-\frac{1}{2}}.$$

It is readily seen that a reasonable approximation, for $P \ll P_{\text{crit}}$, is

$$\lambda \simeq aP.$$

If P is 10 s, $a = 6$ km/s, $\lambda = 60$ km $\simeq \frac{1}{2}H$.

We shall discuss later the question of the period of the acoustic waves. Let us consider first the increase of amplitude of the wave as it propagates in the atmosphere.

The amplitude increases as $\exp[\frac{1}{2}(x/H)]$. From the top of the convective zone to a height of 1000 km, we have about 10 scale heights, and the amplitude should be multiplied by 150, if the phenomenon was still linear.

However, we change from a linear phenomenon to a non-linear one, when the quadratic terms are of the order of magnitude of the linear terms in the equation. Let us consider, for example, the continuity equation

$$\varrho(1 + \operatorname{div}\xi) = \varrho_0\,,$$

and develop it to the second order

$$\varrho = \varrho_0[1 - \operatorname{div}\xi + (\operatorname{div}\xi)^2]\,.$$

The condition for the transformation of the wave into a shock-wave is

$$|\operatorname{div}\xi| \simeq 1\,.$$

As we have

$$\xi = \xi_0 \exp\left[\frac{1}{2}\,\frac{x}{H}\right]\cos\left(\sigma t - x\sqrt{\frac{\sigma^2}{a^2} - \frac{\gamma^2 g^2}{4a^4}}\right),$$

we find for $|\operatorname{div}\xi|$

$$|\operatorname{div}\xi| = \xi_0 \exp\left[\frac{1}{2}\,\frac{x}{H}\right]\sqrt{\frac{1}{4H^2} + \frac{\sigma^2}{a^2} - \frac{1}{4H^2}} = \frac{\sigma}{a}\,\xi_0 \exp\left[\frac{1}{2}\,\frac{x}{H}\right].$$

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The condition $|\operatorname{div} \xi| = 1$ gives, with $\sigma\xi_0 = v$,

$$X = 2H \ln \frac{a}{v},$$

with $v = 1$ km/s, we have

$$\frac{x}{2H} = 1.8 .$$

We can conclude that in less than 4 scale heights, the wave becomes a shock-wave.

If there were no dissipation, the amplitude of the wave would then be very large, the material velocity being of the order of the sound velocity or larger. However, as there is energy dissipation, the velocity amplitude of the wave does not exceed the sound velocity, and it remains small.

This is the main discrepancy between Schatzman and Biermann's theories. As UNSÖLD recalls it (1960), BIERMANN supposes that dissipation occurs when the Mach number is of the order 1. However, the shock front appears certainly before such a large amplitude is reached, as the velocity of propagation is larger in the regions of compression than in the regions of dilatation. As an exact theory does not exist, we satisfy ourselves by a comparison with the uniform case, where the shock front appears after a distance x':

$$x' \simeq \frac{a}{v}\,\frac{aP}{2\pi},$$

with $aP/H \simeq \frac{1}{2}$; $a/v \simeq 6$, we obtain $x'/H = \frac{1}{2}$.

Therefore, we shall consider that already in the photosphere, the compression waves are transformed into shock-waves.

After two scale heights, the velocity in the wave is about (a/e), (Mach number $M = 1/e$), but dissipation in the front is already present.

Dissipation occurs in the shock front, as a consequence of the steep change in density. It is worth considering the theory of dissipation for a viscous fluid.

The energy dissipated per second is

$$\int \frac{4}{3}\mu \left(\frac{\partial u}{\partial x}\right)^2 \mathrm{d}x = \frac{1}{\gamma - 1}\int \left(u\,\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{\gamma p}{\varrho}\,u\,\frac{\mathrm{d}\varrho}{\mathrm{d}x}\right)\mathrm{d}x\,,$$

where μ is the coefficient of viscosity. The change of specific entropy being

$$\mathrm{d}S = c_p \left(\frac{\mathrm{d}p}{P} - \gamma\,\frac{\mathrm{d}\varrho}{\varrho}\right),$$

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we see that the energy dissipated in the shock front is given by

$$\int \varrho T u \, dS ,$$

$T\,dS$ is the change of energy per gram. $\varrho T\,dS$ is the change per cubic centimeter, and $u\varrho T\,dS$ the change per square centimeter per unit of time.

In the case of infinitely weak shocks, the energy dissipated is $\varrho_0 T_0 u_0 \Delta S$. It is well known that ΔS is then proportional to $(\Delta p)^3$. We are led to a formula which is similar to the formula given by BRINKLEY and KIRKWOOD (1948), and to the formula used by SCHATZMAN (1949), by DUBOV (1960) and WEYMANN (1960)

$$\Delta W = -\frac{\gamma+1}{12}\,\varrho_0\,\frac{(\Delta V)^3}{V}, \tag{1}$$

where ΔV is the velocity behind the shock front and V the sound velocity. The matter is supposed to be at rest ahead of the shock front. ΔW is the energy dissipated for 1 cm of propagation.

The question now is naturally of finding the energy W corresponding to the dissipation ΔW. If we can suppose that we have N-shaped waves, we have simply

$$W = \tfrac{1}{2}\varrho_0 (\Delta V)^2 V t_0 ,$$

where Vt_0 is the length between two successive shock fronts (DUBOV, 1960). Using a similarity argument, SCHATZMAN (1949) was led to a similar formula, but his time t_0 was not rigorously a constant.

The choice of t_0 is naturally very important, as it relates the flux of mechanical energy and the rate of dissipation. DUBOV (1960) takes $t_0 = 10$ s; SCHATZMAN (1949) as mentioned above, takes $t_0 = 8$ s.

UNNO and KAWABATA (1955) deduced from the theory of turbulence in the convective zone $t_0 = 4.6$ s.

As mentioned by DE JAGER (1961) Vt_0 is likely to be the length of the wake behind the shock front.

In the case of N-waves, the velocity behind the shock front is related to the mean square velocity $\overline{W^2}$ by the relation

$$(\Delta V)^2 = 9\overline{W^2} .$$

If E is the energy radiated away per gram per second, we have the relation

$$W = \left(\frac{E V t_0}{3}\right)^{\frac{1}{3}} .$$

For $E = 10^{10}$ erg g^{-1} s^{-1}, $V = 6\cdot 10^5$ cm s^{-1}, we find $W = 2.2$ km/s, corre-

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sponding to $\Delta V \simeq V$. For such a velocity, the shock cannot be considered any more as a weak shock. However, the approximate formula (1), is still a good approximation, as has been shown by SCHATZMAN (1949 *b*).

In fact, an exact value of W can be found only as a result of the theory of transfer in the low chromosphere. The equilibrium theory of the chromosphere supposes an exact balance between the heat generated by shocks and the energy radiated away.

An interesting remark has been made by DUBOV (1960), supposing that the energy is radiated away either by hydrogen or by helium. He shows that if the energy dissipated by acoustic waves increases, the temperature has to jump from about 6000 to 12000°. He suggests that the appearance of the spicules is due to a rapid change in the thermal balance from a « cold » to a « hot » plasma. However, his results should be revised, in order to take into account the exact solution of the non-local-thermodynamic-equilibrium conditions, as for example in POTTASCH and THOMAS (1960).

Compression waves can dissipate energy, as long as the mean free path is not too large. In the corona, where the conductivity of the gas becomes very large, there is no dissipation any more by shock-waves, and the corona becomes almost isothermal. Already mentioned by ALFVÉN (1941), the effect of conductivity has been especially taken into account by SCHATZMAN (1949) and recently studied in more detail by UNSÖLD (1960).

Dissipation in magnetohydrodynamic waves has been considered by PIDDINGTON (1955 *a* and *b*, 1956) and by COWLING (1956). The main effect, in transverse waves, is due to the fact that all particles (neutral atoms, ions and electrons) do not move exactly together. The calculation of the coefficient of damping of transverse waves by neutral friction has been done by Miss A. BAGLIN (1960), starting from the microscopic theory of a plasma with a high number of collisions.

If $\nu_{\alpha\beta}$ is the number of collisions per second of one particle of species α against all particles β, we have for the constant of damping:

$$-K_{\text{mag}} = \omega^2 \left\{ \frac{n_a}{n_i} \frac{\omega_p}{c\nu_{ae}(1+n_a/n_i)^{\frac{1}{2}}(\omega_L \Omega_L)^{\frac{1}{2}}} + \right.$$
$$\left. + \frac{(\nu_{ei}+\nu_{ie}+(\nu_{ia}\nu_{ae}+\nu_{ea}\nu_{ai})/(\nu_{ai}+\nu_{ae}))\,\omega_p(1+n_a/n_i)^{\frac{3}{2}}}{c(\omega_L\Omega_L)^{\frac{3}{2}}} \right\},$$

where ω_p is the plasma frequency

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e},$$

ω_L and Ω_L the gyrofrequency of the electrons and the ions.

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Numerically, we find

$$- K_{\text{mag}} = \omega^2 \left\{ 10^{-1.93} \frac{1-x}{x} \frac{1}{B T^{\frac{1}{2}} N^{\frac{1}{2}}} + 10^{-30.35} \frac{1-x}{x} \frac{N^{\frac{3}{2}} T^{\frac{1}{2}}}{B^3} + 10^{21.17} \frac{N^{\frac{3}{2}} T^{-\frac{3}{2}}}{B^3} \right\} .$$

x being the degree of ionization.

If we compare K_{mag} to the path of a transverse wave in a time t_0, we obtain

$$\frac{1}{\omega^2} |K_{\text{mag}} V_A t_0| = 10^{10.51} \frac{1-x}{x} \frac{1}{N T^{\frac{1}{2}}} + 10^{-18.01} \frac{1-x}{x} \frac{N T^{\frac{1}{2}}}{B^2} + 10^{-8.83} \frac{N T^{-\frac{3}{2}}}{B^2} .$$

The velocity of the transverse wave is equal to the velocity of the Alfvén wave, $B/\sqrt{4\pi\varrho}$, where ϱ is the total density of the gas.

The consequences of the above expression have not been worked out yet. However, it can be seen that no transverse wave can propagate in the lower chromosphere, unless the magnetic field is large enough to prevent complete damping. For example, for $N = 10^{16}$, $(1-x)/x = 10^{3.5}$, $T = 6\,000°$, $\omega = 0.6$, the second term gives

$$|K_{\text{mag}} V_A t_0| \simeq \frac{800}{B^2} .$$

Roughly speaking, the magnetic field has to be larger than 30 G for magneto-hydrodynamic waves to propagate in the photosphere. The appearance of MHD waves in the upper chromosphere can explain the transfer of mechanical energy in the corona.

In the regions of low density, the damping of MHD waves becomes very small, unless we have to deal with shock waves. The production of a shock results from the fact that the plasma is compressible.

Let us consider, with K. O. Friedrichs (1959) a surface $S(t)$ with a characteristic velocity of propagation, c_{ch}, in the normal direction at each point of the surface $S(t)$.

If we consider the normal component of the flow velocity

$$u_n = (\boldsymbol{n}_0 \cdot \boldsymbol{p}) ,$$

we can write for the characteristic velocity

$$c_{\text{ch}} = u_n \pm c .$$

Thus, $\pm c$ is the normal component of the characteristic velocity, relative to the flow velocity.

As is well known, there are, at any point, three values of c. The flow velocity u can be considered as the composition of three velocities; one, u_t, is a transverse velocity; the two others being along the two vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$:

$$\begin{cases} \boldsymbol{\alpha} = \dfrac{H_n \boldsymbol{H}}{4\pi} - \varrho c_{\text{fast}}^2 \boldsymbol{n} \\ \boldsymbol{\beta} = \dfrac{H_n \boldsymbol{H}}{4\pi} - \varrho c_{\text{slow}}^2 \boldsymbol{n} \, . \end{cases}$$

where c_{fast} and c_{slow} are the two velocities of propagation of the non-transverse waves. The third velocity is

$$b_n = \left(\frac{H_n^2}{4\pi\varrho}\right)^{\frac{1}{2}},$$

and is the Alfvén velocity.

Except for the pure transverse wave, the characteristic velocity differs from the velocity c. Therefore, exactly as for sound waves, these waves will have the tendency to get steeper and steeper, until they become shock-waves.

The transverse wave, on the other hand, being a shear wave, is not associated with a change of density, and has no reason for becoming a shock-wave. Moreover, in case of a transverse shock-wave, there is no change of density and no change of entropy, and therefore, no dissipation in the shock front (except when taking into account the diffusion of each kind of particles with respect to the others). Therefore, only MHD compression waves can lead to a shock and to large dissipation.

OSTERBROCK (1961) has studied in detail the dissipation by MHD shocks.

1·4. *Stellar chromospheres and corona.* – It seems very likely that for stars of late spectral types, which have a convective zone, a source of energy exists which can produce around these stars a chromosphere and a corona.

Several problems arise in that connection, which can be mentioned only briefly:

(i) In giant stars, it seems probable that a large amplitude of the acoustic waves (shock-waves), is reached already in the photosphere. Assuming that it is the case, SCHATZMAN (1949 *a*) has shown that a flux of mechanical energy F_m of the order of 1/25th of the total energy flux can provide sufficient energy for the production of large chaotic motion. It is then possible to explain the width of the lines in several stars (δ *C Ma*, ε *Aur*, η *Aql*, α *C Mi*).

(ii) However, it is quite likely that the emission features in the lines of Ca II, found by O. C. WILSON and M. K. VAINU BAPPU (1957) are a consequence of the temperature gradient in the outer layers of the star, and are similar to the emission feature in the case of these lines on the sun. JEFFERIES

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and THOMAS (1959) have shown, in the case of the sun, that these features can reasonably be explained by the temperature gradient.

The existence of such a gradient shows that most stars are surrounded by a chromosphere.

(iii) The study of the transfer problem with an energy source leads to new solutions with a temperature minimum in the outer layers of the star. BAROIN and SCHATZMAN (1950) have obtained a model with a temperature minimum at

$$\bar{\tau} \simeq 0.02 .$$

New computations of such models, with solution of the problem of line formation, should be made.

Work in that direction lies in the recent paper of WEYMANN (1960). However, he did not consider flows with a discontinuity, as studied by PARKER. Therefore, his conclusions concerning the optical effects of the outgoing flow of matter cannot be considered as definitive.

(iiii) It should be mentioned that mass-loss occurs as soon as the thermal velocities of the particles is of the same order of magnitude as the velocity of escape, as mentioned by RUBRA and COWLING (1960). In supergiants, the temperature corresponding to escape can be reached before the temperature of a corona, as we have

$$T_{\text{escape}} = 10^{7.2} \frac{m}{m_\odot} \frac{R_\odot}{R} .$$

For a radius of a few times $10^2 R_\odot$, we may well have a temperature of escape of 10^5 °C, which is well below the temperature of the corona.

2. – Theory of shocks.

Before giving the analysis of the published work on propagation of shocks in a variable density atmosphere, we shall briefly recall some important references concerning shock waves:

(A) *Shock fronts*

(*a*) Theory of dissipation in a shock front:
LANDAU and LIFSCHITZ (1953);

(*b*) Propagation of shocks in a uniform gas, with dissipation:
BRINKLEY and KIRKWOOD (1948);

(*c*) Magnetohydrodynamic shocks:

F. DE HOFFMANN and E. TELLER (1950),
K. O. FRIEDRICHS (1955),
J. BAZER and W. B. ERICSON (1959),
P. GERMAIN (1959);

(*d*) Influence of radiation. Work of SACHS (1946) and ROSSELAND (1949) gives the relations between densities, pressures, temperatures, and velocities, before and after passage of the shock-wave. SCHATZMAN (1951) has calculated the velocity of propagation of a shock-wave, taking into account the relativistic effects.

Let us call U the velocity of the shock front, $U-u$ the material velocity behind the shock front, ϱ_1 and ϱ_0, P_1 and P_0, T_1 and T_0 the density, pressure, and temperatures after and before passage of the shock front. If we call x and y the ratios of the densities and temperatures

$$T_1 = yT_0\,, \qquad \varrho = x\varrho_0\,,$$

and

$$1-\beta = P_R/(P_R+P_G)\,,$$

we have the relation between x and y

$$x^2y\beta_0 + x\left[(7+y^4)(1-\beta_0)+4(1-y)\beta_0\right] - (7y^4+1)(1-\beta_0) - \beta_0 = 0\,,$$

the velocity U is given by

$$U^2 = \frac{P_0}{\varrho_0}\cdot\frac{\beta_0 xy - 1 + (1-\beta_0)y^4}{x-1}\,,$$

and $U-u$ is given by

$$x(U-u) = U\,.$$

Numerical study of these relations is under way.

H. K. SEN and A. W. GUESS (1957) have studied the problem of radiative transfer in a shock front. Their work is based entirely on the assumption of local thermodynamic equilibrium and great optical thickness. The result is expressed in terms of thickness of the shock front as a function of the particle mean free path, λ_0 ahead of the shock:

$$\text{thickness} = t_0\lambda_0\,.$$

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The following table is taken from the Sen and Guess paper, where $M_0 = u_0/c_0$ is the Mach number for the velocity u_0 of the matter with respect to the front and ahead of it (Table I)

TABLE I.

M_0	t_0	t_{0R}
1.5	9.5	27.3
2	8.5	31.4
2.5	9.7	40.8
4	14.7	87.8

t_0 is the thickness without radiation, t_{0R}, with radiation.

The exactness of these results can be contested, as MARSHALL (1956) has shown. However, SEN and GUESS, consider the possibility that electrons and ions are not at the same temperature in the front, in which case the Prandtl number

$$P_r = \frac{\mu \gamma c_V}{k},$$

(where μ is the coefficient of viscosity) is $\frac{3}{4}$. If electrons and ions were at the same temperature, P_r would be much smaller than $\frac{3}{4}$. But, if ions and electrons are not at the same temperature, what is the meaning of using the Rosseland mean calculated for L.T.E.?

KAPLAN and KLIMISHIN (1959) have also calculated some of the properties of shock-waves, including radiation, with special regard to the detonation-recombination wave.

KUBIKOWSKI (1959) has studied the cooling of matter behind the shock front when the optical thickness of the matter ahead of the shock front is small, for the purpose of application to cepheids. He obtains an expression for the distribution of temperature behind the shock front, a characteristic length being

$$l = \frac{1}{\varkappa \varrho} \left(\frac{c_p}{R} \frac{\bar{\mu} P_G}{6 P_R} \frac{u}{c} \right)^{\frac{1}{2}},$$

where u is the velocity of the shock front with respect to the matter behind. $\varkappa \varrho l = \tau_S$ is the optical thickness of the region of decay of the temperature.

For example, for $\log \varrho_2 = -8.89$, $\theta_0 = 0.16$, $\log P_G = 2.57$, we have $\log \varkappa_2 = -0.41$ behind the shock front, $c_p \mu / R = 16$. With $u = 5$ km/s, we obtain

$$\tau_S = 0.02 .$$

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A similar problem has been studied by KAPLAN and KLIMISHIN (1960), with accent on the heating of the gas ahead of the shock front.

Revision of the theory is needed.

(B) *Shocks in variable density atmosphere.*

(*a*) Method of similarity. Already, at the first meeting, BURGERS (1949) discussed the problem of propagation of a shock-wave in a variable density atmosphere. Since that time, the method of similarity has been developed by SEDOV (1957). It has been used several times, for example by SEDOV (1955), by KOPAL (1954), and CARRUS, FOX, HAAS, KOPAL (1951 *a*, *b*), and by M. H. ROGERS (1957) for an infinitely strong shock (gravity being negligible).

For spherical shocks, the similarity method can be applied only for a distribution of density and other parameters given by a power law, *e.g.*, $\varrho = Ar^{-\alpha}$ where A and α are constants. The solution is obtained as a function of time and radius through a function $\xi = (t\psi/r)$. Moreover, there must be only two characteristic constant parameters, dimensionally independent (A is one of these parameters).

This second assumption is very restrictive as we usually have more than one characteristic parameter (except A) with different dimensions; for example, the constant of gravitation G, the energy of the explosion E, the temperature in the center of the star, and so on.

Therefore, similarity solutions can be obtained only by neglecting some parameters. KOPAL (1954) claims that his 1954 solution is very close to actual shocks; though KAPLAN in his preliminary report doubts that similarity solutions can represent astrophysical phenomena.

The case $\alpha = \frac{5}{2}$ is singular and allows one to choose three parameters, A, G, and E, of which only two have independent dimensions: $E \sim GA^2$. The equation of the movement of the shock is $r \sim (GA)^{\frac{2}{5}} t^{\frac{4}{5}}$ (SEDOV 1957, CARRUS, FOX, HAAS, KOPAL 1951 *b*). All parameters (density, velocity, pressures ...) behind shock front depend only on the dimensionless parameter,

$$\eta = r/(GA)^{\frac{2}{5}} t^{\frac{4}{5}},$$

and therefore are similar.

If we have two characteristic parameters A and G, then E depends on the time, but α is arbitrary (KOPAL, 1956).

If $\xi = t^{2/\alpha}/r$, ξ_1 represents the position of the shock front:

$$r_1 = t^{2/\alpha}/\xi_1 .$$

The Mach number of the shock is given by

$$M^2 = \frac{4(3-\alpha)(\alpha-1)}{\alpha^2\gamma\xi_1\alpha} .$$

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There is, inside ξ_1 a sphere ξ_2 which is a contact discontinuity, corresponding to the presence of vacuum inside (ejection of a shell).

We should notice here the constancy of the Mach number during shock propagation. In real stellar conditions, it is certainly not true.

A series of papers are devoted to the applications of the method of similarity solutions to the movement of shocks in stars. To the above mentioned papers, we must add SEDOV (1956), JAVORSKAYA (1956), LIDOV (1957), ROGERS (1956). The book of BAUM, KAPLAN and STANYKOVICH (1958) collects a number of important results.

Special applications of the theory of similarity flow has been made to the motion of the shock near the surface of a star (GANDELMANN, FRANK-KAMENETZKY, 1956). It was shown that the equation of motion of the shock near the surface is $(R - r) \sim t^{0.59}$. The numerical value 0.590 of the power of t was found for the stellar envelope with the Kramers law of opacity. In that solution, the velocity and temperature behind the shock increases to infinity when the shock approaches the surface. As a result from the above-mentioned work, radiation would change considerably this result.

(*b*) Discontinuous medium. Another method, and still an exact method, consists in replacing the variable density medium by a series of layers with different densities. The problem is then to study the effect of passage and reflection across the discontinuities, and this method was suggested at the first meeting (1949). It has been used by CHISNELL (1955) and applied by a group of Japanese scientists (ONO, SAKASHITA and YAMAKAZI, 1960) to the propagation of plane shocks in a plane atmosphere. They show that the intensity of the shock is approximately proportional to the power -0.6 of the pressure ahead of the shock front, and therefore increases considerably when approaching the surface.

This method is more elaborate than the similarity method an can be applied to a larger variety of cases. It could be improved by introducing radiative loss.

An important work has been done by HAZLEHURST (1961) in order to explain the novae ejection.

(*c*) Weak shocks. Motions of weak shocks, as shown by WHITHAN (1953), can be investigated with the linearized equations.

SCHATZMAN (1954) has used a Fourier analysis to study the propagation of a given perturbation in an atmosphere. It is worth giving the result, as it has some implications for the heating of the solar chromosphere. The amplitude of the wave can be written

$$S = S(\omega) \exp \left[\frac{\gamma g z}{2a^2} + i \left(\sigma t - i \sqrt{\frac{\sigma^2}{a^2} - \frac{\gamma^2 g^2}{4a^4}}\, z \right) \right].$$

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If we suppose a displacement at $z = 0$:

$S = 0$ for $t < 0$, $S = 1$ for $0 < t < \theta$, $S = 0$ for $t > \theta$, we have

$$S(\omega) = \frac{\exp\,[i\sigma a] - 1}{4\pi i\sigma},$$

and by integration over σ, we find the amplitude

$$S = \exp\left[\frac{\gamma g z}{a^2}\right] - 1,$$

for $at < z < a(t + \theta)$.

The increase of pressure is

$$\Delta P = Cte\;\gamma a^2 \varrho_0 \left(1 - \exp\left[-\frac{\gamma g(z - z_0)}{a^2}\right]\right).$$

As a function of time, the relative decrease of the pressure behind the shock front is characterized by a time

$$\delta = \frac{a}{\gamma g}.$$

For the sun, $\delta \simeq 20$ s. This characteristic value is an essential result of the structure of the atmosphere, and is much smaller than the critical period of the atmosphere (indeed, 4π times smaller). It corresponds very closely to the period which had to be introduced in the decay theory in order to express in a simple way the kinetic energy of the shock.

In his preliminary report, Kaplan mentions, in connection with the problem of weak shocks, a work of PICKELNER (1959) in which he studied the gravitational damping of acoustic waves.

(*d*) Solitary waves. The theory of simple waves (Riemann solution) is well-known. BAUM, KAPLAN, and STANYKOVICH (1958) have studied the movement of these waves in a gravitational field. They can show how long it takes for a non-linear flow to turn into a shock. If the initial pressure is P_1, the velocity of sound C_i, and the gravity g, the disturbance of the pressure turns to a shock at the point where the pressure is P_S.

If τ is a characteristic time of the disturbance of the pressure, we have for $\gamma = \frac{5}{3}$

$$\frac{gP_S}{4P_i} - 1 = \frac{C_i}{5g\tau}\left[1 - \left(\frac{P_S}{P_i}\right)^{\frac{1}{5}}\right].$$

(*e*) Approximate methods. LEBEDINSKY (1946) and SCHATZMAN (1951) have applied the law of conservation of energy, with the result that the velocity of propagation is given by $v \sim (\varrho r^2)^{-\frac{1}{3}}$.

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However, the assumption of conservation of energy is questionable in case of strong shocks.

ODGERS and KUSHWAHA (1957) assumed the constancy of the pressure-time curve for every element of the gas. They found a fast damping of the isothermal shocks.

SAKURAI (1956) found a solution for the equation of the movement of the shock created (with energy E) in the center of a polytropic sphere. The solution is given by a series in (r/r_0), where $r_0 = (E/3\pi p_c)^{\frac{1}{3}}$, p_c being the pressure at the center of the star. The series converges for $(r/r_0) < 1$, and therefore is not applicable to the movement of the shock in the outer layers of a star.

WEYMANN (1960 *b*) takes an average of the equation of energy for N-waves, assuming a profile for these waves. The result is an equation of energy which allows one to calculate the heat transfer in the chromosphere:

$$\frac{d}{d\xi}\left[\frac{\sigma^2}{12}\left(\frac{F_0^3\tau_0}{\gamma}\right)^{\frac{1}{2}}\right] + \frac{F_0}{\tau_0} = 0 ,$$

where ξ is the Lagrangian co-ordinate, F_0 the average radiation loss, τ_0 a reference specific volume, and $\sigma = (p_2 - p_1)/p_0$ is the shock strength parameter.

However, he has not taken into account the refraction of the waves, which was considered by SCHATZMAN (1949 *b*). Due account of the refraction can be found both in DE JAGER (1961) and OSTERBROCK (1961) papers.

3. – Conclusion.

Much progress has still to be done in the theory of propagation of shock waves in variable density atmospheres. Much attention should be given to the numerical work of WHITNEY, using the theory of characteristics.

The astrophysicists wish certainly to receive some help from the aerodynamicists to succeed in solving one of the major problems of astrophysics.

BIBLIOGRAPHY

ALFVÉN H., 1941, *Ark. Mat. Astr. Och. Fys.*, **27**, A no. 25.

BAGLIN A., 1960, *Jour. de Phys.* (in proofs).

BAROIN M. and SCHATZMAN E., 1950, *C. R.*, **231**, 757.

BAUM PH. A., KAPLAN S. A. and STANYKOVICH K. P., 1958, *Introduction to Cosmical Gas Dynamics*, Moscow.

BAZER J. and ERICSON W. B., 1959, *Ap. J.*, **129**, 758.

1062

https://doi.org/10.1017/S0074180900104450 Published online by Cambridge University Press

BIERMANN L., 1939, *Z. f. Ap.*, **18**, 344.
BIERMANN L., 1948, *Z. f. Ap.*, **25**, 161.
BRINKLEY S. R. and KIRKWOOD J. G., 1947, *Phys. Rev.*, **71**, 606.
BURBIDGE G. R., BURBIDGE E. M., HOYLE F. and FOWLER W. A., 1957, *Rev. Mod. Phys.*, **29**, 547.
CARRUS P. A., FOX P. A., HAAS F. and KOPAL Z., 1951*a*, *Ap. J.*, **113**, 193.
CARRUS P. A., FOX P. A., HAAS F. and KOPAL Z., 1951*b*, *Ap. J.*, **113**, 496.
CHISNELL R. E., 1955, *Proc. Roy. Soc. A.*, **232**, 350.
COLGATE S. E., 1959, *Phys. Fluids*, **2**, 485.
COWLING T. G., 1956, *M. N.*, **116**, 114.
DE HOFFMANN F. and TELLER E., 1950, *Phys. Rev.*, **80**, 692.
DE JAGER C., 1961, preprint.
DUBOV E. E., 1960, *News of Crimean Astrophysical Observatory*, **22**, 101.
FRIEDRICHS K. O., 1959, *Non-linear wave motion in magnetohydrodynamics*, Center Report No. 1845, Los Alamos.
GANDELMANN G. M. and FRANK-KAMENETZKY D. A., 1956, *C.R.U.R.S.S.*, **107**, 811.
GERMAIN P., 1959, *N.E.R.A.*, Publ. No. 97.
HAZLEHURST J., 1961, *Progress in Astronomy and Astrophysics* (in the press).
JAVORSKAYA I. M., 1956, *C.R.U.R.S.S.*, **111**, 783.
JEFFERIES J. T. and THOMAS R. N., 1959, *Ap. J.*, **129**, 401.
KAPLAN S. A. and KLIMISHIN I. A., 1959, *A.J.U.R.S.S.*, **36**, 410.
KAPLAN S. A. and KLIMISHIN I. A., 1960, *A.J.U.R.S.S.*, **37**, 281.
KOPAL Z., 1954, *Ap. J.*, **120**, 159.
KUBIKOWSKI J., 1959, *Ann. d'Ap.*, **22**, 741.
LANDAU L. D. and LIFSCHITZ E. M., 1953, *Mechanics of Continuous Media.*
LEBEDINSKY A. I., 1946, *A.J.U.R.S.S.*, **23**, 15.
LIDOV M. L., 1957, *A.J.U.R.S.S.*, **34**, 603.
LIEPMANN H., 1952, *ZAMP*, **3**, 321, 407.
MARSHALL W., 1956, *Phys. Rev.*, **103**, 1900.
MILNE E. A., 1930, *M. N.*, **91**, 4.
ODGERS G. J. and KUSHWAHA R. S., 1957, *Ap. J.*, **62**, 95.
ONO Y., SAKASHITA S. and YAMAZAKI H., 1960, *Propagation of shock waves in inhomogenous gases*, preprint referred to by S. A. KAPLAN in his preliminary report.
OSTERBROCK O., 1961, preprint.
PAYNE-GAPOSHKIN C., 1957, *The Galactic Novae*, Amsterdam - New York
PIDDINGTON J. H., 1955*a*, *M. N.*, **114**, 638.
PIDDINGTON J. H., 1955*b*, *M. N.*, **114**, 651.
PIDDINGTON J. H., 1956, *M. N.*, **116**, 314.
POTTASCH S. R. and THOMAS R. N., 1960, *Ap. J.*, **132**, 195.
ROSSELAND S., 1946, *Ap. J.*, **104**, 329.
ROSSELAND S., 1949, *Pulsation Theory of Variable Stars*, London.
ROGERS M. H., 1956, *Proc. Roy. Soc. A.*, **235**, 120.
ROGERS M. H., 1957, *Ap. J.*, **125**, 478.
RUBRA F. T. and COWLING T. G., 1960, *Mem. Soc. Roy. Sci. Liege*, **16**, 274.
SACHS R., 1946, *Phys. Rev.*, **69**, 514.
SAKURAI A., 1956, *J. Fluid Mechanics*, **1**, 436.
SCHATZMAN E., 1946*a*, *C. R.*, **222**, 722.
SCHATZMAN E., 1946*b*, *Ann. d'Ap.*, **9**, 199.
SCHATZMAN E., 1949*a*, *C. R.*, **228**, 814.
SCHATZMAN E., 1949*b*, *Ann. d'Ap.*, **12**, 203.

SCHATZMAN E., 1951, *Ann. d'Ap.*, **14**, 294.
SCHATZMAN E., 1954, *Bull. Acad. Roy. Belgique*, **40**, 139.
SCHATZMAN E., 1958, *Ann. d'Ap.*, **21**, 1.
SCHWARZSCHILD M., 1948, *Ap. J.*, **107**, 1.
SEDOV L., 1955, *4-th Meeting on Questions of Cosmogony*, 133.
SEDOV L., 1956, *C.R.U.R.S.S.*, **111**, 770.
SEDOV L., 1957, *Similarity and Dimensional Methods in Mechanics*.
SEN H. K. and GUESS A. W., 1957, *Phys. Rev.*, **108**, 560.
THOMAS R. N., 1948, *Ap. J.*, **108**, 130.
UNNO W. and KAWABATA K., 1955, *Publ. Ast. Soc. Japan*, **7**, 21.
UNSÖLD A., 1960, *Z. f. Ap.*, **50**, 57.
VORONTZOV-VELYAMINOV, 1948, *Gaseous Nebulae and Novae*.
WEYMANN R., 1960*a*, *Ap. J.*, **132**, 370.
WEYMANN R., 1960*b*, *Ap. J.*, **132**, 452.
WILSON O. C. and VAINU BAPPU M. K., 1957, *Ap. J.*, **125**, 661.
WHITHAN G. B., 1953, *Com. pure and applied math.*, **6**, 397.

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