There are also the line shifts in the sunspots penumbra corresponding to a nearly horizontally outwards streaming gas with maximum velocity of about 3 km/s for a large spot—usually called the Evershed effect.

4. – Velocities in the chromosphere.

The observations of velocities in the solar chromosphere have been summarized by DE JAGER in his article in the *Handbuch der Physik* (1959). Velocities increase with height up to about 15 km/s in about 3 000 km above the solar limb.

5. – Velocities arising from convection in other stars.

The observational data are reviewed in the article of WRIGHT in the *IAU Transactions* (1955). Miss UNDERHILL has included the latest observations in her summary talk. The measured velocities increase with increasing effective temperatures of the stars and with decreasing surface gravity. You still have the table of Miss UNDERHILL.

B) Theory of the Hydrogen Convection Zone.

Convection occurs when

$$\nabla = \frac{\mathrm{d}\log T}{\mathrm{d}\log P_g} > \frac{\mathrm{d}\log T}{\mathrm{d}\log P_{g\ \mathrm{adiabatic}}} = \nabla_{\mathrm{ad}}\,.$$

In the high photospheric layers of a star this is not fulfilled; they are in radiative equilibrium, meaning that the whole energy transport is performed by radiation. In such an atmosphere the temperature distribution is given approximately by

$$T^4 = \tfrac{3}{4}T^4_{\mathrm{eff}}(\bar\tau + \tfrac{2}{3}) \quad \text{with} \quad \sigma T^4_{\mathrm{eff}} = \pi F\,, \quad \pi F = \text{net flux}\,, \tag{1}$$

while the distribution of the gas pressure P_g obeys the hydrostatic equation

$$\frac{\mathrm{d}P_g}{\mathrm{d}\bar\tau} = \frac{g}{\bar\varkappa/\mathrm{gr}}\,, \qquad g = \text{gravitational acceleration} \tag{2}$$

For the sun $T_{\mathrm{eff}} = 5\,800^{\circ}$, $g = 2.82\cdot 10^4$.

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From these equations one obtains

(3) $$\nabla_{\text{rad}} = \frac{\mathrm{d}\log T}{\mathrm{d}\log P_{\text{g}}}\,(\text{radiative equilibrium}) = \frac{3}{16}\,\frac{\bar{\varkappa} P_{\text{g}}}{g}\left(\frac{T_{\text{eff}}}{T}\right)^4 .$$

The gradient has to be proportional to the flux σT_{eff}^4. If this gradient becomes larger than the corresponding one for adiabatic stratification the layer will be convectively unstable. This may happen for two reasons: 1) ∇_{ad} becomes very small, or 2) ∇_{rad} becomes very large. $\nabla_{\text{ad}} = (\gamma - 1)/\gamma$, if γ were constant, where $\gamma = c_p/c_v$. ∇_{ad} becomes very small if γ comes close to unity. This happens if c_p is very large, which in stellar atmospheres occurs in those layers where the most abundant element hydrogen is ionized, which means in layers with $T \approx 10\,000°$. ∇_{rad} may become quite large when $\bar{\varkappa}$ becomes very large. This happens in stellar atmospheres for $T \geqslant 7\,000°$.

In those stellar atmospheres in which we are interested, the continuous absorption is mainly due to H^- absorption and hydrogen absorption in the Paschen continuum, that means absorption from the third quantum level of hydrogen. Around $T \sim 7000°$, the excitation degree of the third quantum level becomes high enough and increases rapidly, so that hydrogen absorption exceeds H^- absorption and increases rapidly with T, until T becomes so large that hardly any neutral hydrogen is left over. For such high temperatures the absorption coefficient will then decrease.

So ∇_{ad} decreases and ∇_{rad} becomes very large for about the same T. Both effects together cause quite an active convection.

Since the upper boundary of this unstable layer occurs in $\tau = 0.8$, the convection zone contributes appreciably to the observed radiation. Therefore astrophysicists are interested especially in the temperature stratification of these layers. The temperature stratification depends on the amount of energy which is transported by radiation. As I said, the gradient ∇ is proportional to the radiative flux πF_{rad}. If $\pi F_{\text{rad}} < \sigma T_{\text{eff}}^4$, we have to put πF_{rad} into eq. (3) instead of σT_{eff}^4 and obtain

(4) $$\nabla = \frac{3}{16}\,\frac{\bar{\varkappa} P_{\text{g}}}{g}\,\frac{\pi F_{\text{rad}}}{\sigma T^4} .$$

If we know πF_{rad} we can calculate ∇ and the temperature-pressure stratification

(5) $$\Delta \log T = \int_{P_{\text{g}_0}}^{P_{\text{g}}} \nabla \,\mathrm{d}\log P_{\text{g}} .$$

In equilibrium the amount of energy transported through the atmosphere must

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be independent of depths: $dF/dt = 0$ which means

$$\pi F_{rad} + \pi F_{conv} = \pi F = \sigma T_{eff}^4 . \tag{6}$$

(Energy transport by conduction may be neglected.) All we have to know is πF_{conv}. So the primary interest of astrophysicists is the amount of convective energy transport, which can be expressed as

$$\pi F_k = c_p \varrho T \frac{\overline{\Delta T}}{T} \cdot \bar{v} , \tag{7}$$

where the mean should be taken over the horizontal plane in question. The problem then is to calculate $\overline{\Delta T}$ and $\bar{v}$.

To my knowledge this has only been done in the approximation of a so-called mixing length theory, which in this connection means something different from the mixing length theory applied to turbulent shear flow and should perhaps be better called « characteristic-scale » approximation. The Rayleigh numbers in stellar atmospheres are very large due to the vast dimensions, so we may expect the convection to be turbulent. In the mixing length approximation it is assumed that only turbulence elements of size l exist and that they will travel this same length l and then disappear as turbulence elements.

This kind of theory was first applied to stellar atmospheres by SIEDENTOPF (1935) and BIERMANN (1942).

In the convection zone at a given point P we have the following situation: The mean logarithmic temperature gradient is ∇. The adiabatic gradient for the given T and P_g is ∇_a, which is much smaller than ∇. A bubble that would start rising in P will rise with a somewhat steeper gradient ∇' than ∇_a, due to energy exchange with the surrounding matter.

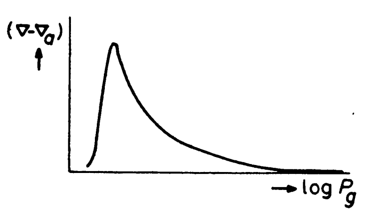

Fig. 2.

The temperature difference ΔT is then proportional to $\nabla' - \nabla$.

With the above assumptions of mixing length theory (VITENSE, 1953) we obtain

$$\frac{\overline{\Delta T}}{T} \sim (\nabla - \nabla') \frac{l}{2H} \quad \text{giving} \quad \pi F_k = c_p \varrho T \bar{v} (\nabla - \nabla') \frac{l}{2H} , \tag{8}$$

where $H = RT/\mu g$.

In deriving this equation we have assumed that $\nabla - \nabla'$ is constant over a distance l and that $\overline{\Delta T} = \Delta T(x = l/2)$.

1178

The velocity is derived by the integral

$$\frac{m}{2}v^2 = \int_0^x K(x)\,\mathrm{d}x\,, \tag{9}$$

with $K = -g\cdot\Delta\varrho\cdot$volume of the bubble. $\Delta\varrho$ is connected with ΔT by $\Delta\varrho/\varrho = (\Delta T/T)\cdot Q$, where $Q = 1 - \partial\log\mu/\partial\log T$ takes care of the change in mean atomic weight due to changes in the degree of ionization. x is the co-ordinate, corresponding to geometrical depth.

The difference $\nabla_{ad} - \nabla'$, is determined by the energy loss of the bubble on its way, which means by radiative energy exchange.

With these equations we can calculate $\overline{\Delta T}$ and $\bar{v}$ and the stratification in the convective layer, if we find a proper size for the characteristic scale l, which we did not yet determine.

We did introduce the characteristic length l in order to find an equilibrium value for $\bar{v}$ and $\overline{\Delta T}$. The normal instability calculations yield a circulating motion with increasing velocity as long as we regard only the linear terms in the hydrostatic equations. We should find an equilibrium value for v if we take into account all the energy dissipating terms. A first step in this direction was made by Malkus and Veronis (1958) who considered a case with a relatively small Rayleigh number. I heard that Schwarzschild, Ledoux and Spiegel have tried to include turbulent viscosity. We shall probably hear about these attempts later.

We started from another viewpoint, assuming that the circulating motion does not really exist as a full circle, but that the velocity increase of rising bubbles is terminated because they are disturbed so much on their way that they do not exist any more as a unique feature. The question then is how far can they travel without losing their identity. This length we shall take as the characteristic length l. According to the assumptions of a mixing length theory this same length will then also determine the linear extension of the bubbles. For the numerical calculations $l = H$ was assumed for the following reasons: Primarily we want to calculate the convective energy transport. Small bubbles will lose their surplus energy rather quickly due to radiative energy exchange. The largest bubbles will lose the least amount of energy and therefore transport most of it. On the other hand, with the assumption of rising bubbles we cannot make the bubbles very large, for otherwise they could not exist as a unique feature. Also the bubbles will have changed their internal structure appreciably after having traveled one scale height and will therefore essentially lose their identity. These considerations give an upper limit $l \leqslant H\cdot a$, where a is of the order of unity. But the bubble cannot, of course, be assumed to be larger than the whole unstable layer. If the unstable layer is less thick than one scale height we have to assume the characteristic length to be of the order of the height of the unstable layer.

The results that have been derived with these assumptions are given in a paper by BÖHM-VITENSE (1958).

The convective energy transport can be neglected down to optical depths $\bar{\tau} = 2$. The extension of the convection zone in the sun is 60 000 km; it is larger for lower effective temperatures (90 000 km for $T_{eff} = 5\,000°$) and smaller for higher effective temperatures (4 000 km for 7 000°). For even higher temperatures the height of the convective layer comes out to be smaller than the scale height. For these stars convection must stop rather abruptly because we have to assume the most unstable bubbles to be smaller than the scale height. Smaller bubbles will have a much greater energy exchange, so the energy transport is reduced. This makes the convection zone still narrower. (The gradient ∇ becomes steeper, so higher temperatures are already reached for relatively low pressures, and the hydrogen is already ionized in higher layers.) The size of the bubbles has to be reduced again, and so on. We obtain a very narrow unstable zone in radiative equilibrium. The velocities that can be expected are of the order of 1 cm/s. For main sequence stars this occurs for $T_{eff} \geqslant 8\,000°$. For giants and supergiants it occurs for much lower temperatures (4 400° for very bright supergiants).

The calculated velocities are of the same order of magnitude as the observed ones (perhaps, somewhat lower). For the sun one calculates, for example, close to the upper boundary of the convection zone $\bar{v} = 1.7$ km/s. (A factor $\frac{1}{2}$ was introduced in $\overline{v^2}$ in order to take into account the turbulent friction. Probably this should not be done in a mixing length theory.)

The calculated velocities show the same trend as the observed ones, becoming larger with higher T_{eff} and lower surface gravity, but suddenly decreasing when convective energy transport becomes negligible. This last result does not agree with observations. For hot stars we must therefore look for another mechanism which can give high turbulent velocities. Perhaps we should come back to this point in the discussion.

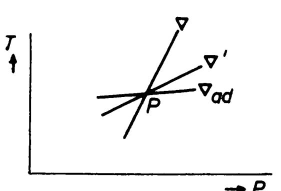

Fig. 3.

The assumption $l = H$ was first introduced into theory of stellar convection zones by Biermann. During the last several years it has been subject to much criticism. What other length could be introduced as the characteristic length? In any case we should take the size of the most unstable bubbles which transport the main amount of energy. There do exist a number of instability investigations.

The distribution of instability $(\nabla - \nabla_{ad})$ is generally given by the above graph.

The layer, which is really very unstable, extends only a few hundred km, usually the same order of magnitude as the scale height. (This result does not depend sensitively on the assumption about mixing length.)

Böhm (1958) calculated the size of the most unstable wave numbers in the Rayleigh way for convection zones consisting of two layers with very different degrees of instability. In the very unstable zone the most unstable wave numbers are those which correspond to the height of the zone with high instability, regardless of the extension of the less unstable zone. (Density variations were neglected.) If this length ought to be taken as the characteristic length it would again be equal to the scale height within a factor of 2 or 3.

Skumanich (1955) made investigations of the instability for an atmosphere of decreasing density but unique degree of instability. His result was an increasing instability for smaller wave numbers, but Böhm and Richter (1959) repeated the same calculations taking into account the radiative energy exchange which will of course reduce the instability of small wave numbers. They found that for conditions in the sun one should expect the largest instability for wave lengths > 300 km/s (perhaps larger by a factor of 2).

So all the various heights that might be suggested by these investigations lead to the same order of magnitude for l as was assumed at least for the high layers (*).

The observations show a size of granules of 700 km, probably corresponding to the most unstable wave number in the upper part of the convection zone. On the other hand, 700 km $= \frac{3}{2} H$ for the depth from which those bubbles should rise, which we see on the surface.

So for the high layers our numerical results, concerning the stratification in the convective zone, may be expected to be right within a factor of 2 or 3 (with regard to πF_{conv}) if we assume $l = H$. For the deeper layers this assumption may not always be right because H increases with increasing T and possibly l should be fixed and connected with the extension of the unstable layer. But in deep convective layers we shall always find $\nabla \approx \nabla_{\mathrm{ad}}$ regardless of the assumptions about l.

There has been criticism against using this kind of mixing length theory at all. Of course it can only be regarded as a first order approximation. What

(*) The order of magnitude agreement between the size of the most unstable zone and the size of the most unstable wave number is due to the fact that ∇ becomes about equal to ∇_{ad}, meaning that instability becomes small when radiative energy exchange over a distance l is negligible. The most unstable wave length also corresponds to the smallest extension for which radiative energy exchange is negligible. Since $H = RT/\mu g$ is always of the same order of magnitude as the geometrical depths to the point in question, all the possible characteristic scale heights appear to be necessarily of the same order of magnitude.

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ought to be done is to solve the exact hydrodynamic equations for a stationary state, *e.g.*, with a first approximation convection zone (as, for instance, the ones described above); then one has to calculate the energy transport and πF_{rad}. Having this, one could calculate a better stratification for the convection zone, solve again the hydrostatic equations, and so on. To my knowledge, nobody has as yet succeeded in doing so, but we shall probably hear from MALKUS about investigations that may be used for a step in this direction. The basic difference of the stellar case in comparison with laboratory experiments seem to be that we do not know the lower boundary conditions for the convection zone. These themselves depend on the solution for πF_{conv}.

There has also been criticism against regarding the observed granulation as rising and falling gas. In 1953 and 1954, SCHATZMAN and THOMAS proposed the granules to be the appearance of acoustic waves (see also WHITNEY, 1958). According to the investigations of LIGHTHILL (1955), there will be generated acoustic waves in a turbulent velocity field. Part of these will certainly travel upwards and will be amplified due to the rapidly decreasing density in high photospheric and chromospheric layers (SCHIRMER, 1950). Probably they will finally become shock-waves, which as far as we know are the main agency for the rising temperature in the chromosphere and corona (SCHWARZSCHILD, 1948; BIERMANN, 1948). However I do not think that these acoustic waves have any important influence on the appearance of the granulation. I do not see any reason against assuming the granules to be rising and falling matter even in the convectively stable radiative zone, because they can easily overshoot. This means the rising gas will reach the upper boundary of the convective layer with a surplus temperature, and therefore will still be accelerated into the radiative zone. Theoretical investigations (UNNO, 1957; BÖHM and RICHTER, 1960) show that we have to expect a circulation in the radiative zone with nearly the same absolute velocities as in the upper part of the convection zone, exactly in the way that is observed by ALLEN, WADDELL, and SUEMOTO.

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