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NOTES

A NOTE ON MONETARY POLICY, ASSET PRICES, AND MODEL UNCERTAINTY

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Using a macroeconomic model with asset prices, we analyze how optimal monetary policy and macroeconomic dynamics and performance are affected by a central bank's desire to be robust against model misspecifications. We show that a higher central bank preference for robustness implies a more aggressive reaction of the nominal interest rate to the expected future inflation rate and inflation shocks. The dynamic stability of the equilibrium is not modified for a sufficiently high preference for robustness. However, the speed of dynamic convergence is lower under robust control compared to a benchmark case without it and implies supplementary economic costs. Finally, an increase in the preference for robustness comes at the cost of higher macroeconomic and financial volatility in the presence of inflation shocks. It has no effect on the reaction of inflation, output gap, and asset price gap to shocks affecting goods and financial markets.

Keywords: Monetary Policy, Asset Prices, Model Uncertainty, Macro-financial Stability

1. INTRODUCTION

In the last decade, many central banks have succeeded in stabilizing inflation at low levels by introducing the inflation-targeting framework. However, some economists [e.g., Borio and Lowe (2002) and Svensson (2009)] are concerned that this environment might have favored large swings in asset prices. According to the Bank for International Settlements [BIS (2007)], "our understanding of economic processes may even be less today than it was in the past." The reason is an increase in fundamental uncertainties about how the economy works.

In this economic environment, recent research has developed different approaches to robust monetary policy design in order to tackle the economic and

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financial uncertainties. Without the possibility of a complete description of reality, a policy maker is likely to prefer basing policy on principles that are also valid if the assumptions on which the model is founded differ from reality. In other words, policy prescriptions should be robust to reasonable deviations from the benchmark model. The growing literature on monetary policy robustness has been mainly developed in three directions. The first leads to what has been called robustly optimal instrument rules [Giannoni and Woodford (2003a, 2003b) and Svensson and Woodford (2004)]. As these instrument rules do not depend on the specification of the generating processes of exogenous disturbances in the model, they are robust to misspecification in these processes. The second one, initiated by Hansen and Sargent (2001, 2003, 2007), corresponds to a robust control approach to the decision problem of agents who face model uncertainty. This approach to model uncertainty focuses on the worst-case outcome within a set of admissible models. In the sense of Hansen and Sargent, robust monetary policies are designed to perform well in worst-case scenarios, by minimizing the consequences of the worst-case specification of the policy maker's reference model. The third approach to robustness considers structured Knightian uncertainty. It is assumed that the uncertainty is located in one or more specific parameters of the model, but the true values of these parameters are known only to be bounded between minimum and maximum conceivable values [Giannoni (2002, 2007), Onatski and Stock (2002), and Tetlow and von zur Muehlen (2004)]. What these three approaches share is a focus on the concept of *uncertainty* in the sense of Knight instead of that of *risk*.

The literature on monetary policy robustness generally treats model uncertainties as affecting the Phillips curve and the IS equation (in closed-economy models) as well as the uncovered interest rate parity (in open economy models), neglecting the role of asset prices and the misspecification affecting asset pricing. In contrast, the work of Tetlow (2006) studies model uncertainty affecting only asset prices.

Meanwhile, ignoring model misspecifications, some recent studies consider the benefits of allowing the monetary authority to respond to asset prices in a monetary policy rule. The essential question is not whether the central bank's objective function should include asset prices, but how an inflation-targeting central bank can most effectively fulfill its objectives. A more general case can be made for central banks to react to asset prices in the normal course of policy making without trying to target asset prices [e.g., Cecchetti et al. (2000, 2003), Filardo (2000), Gilchrist and Leahy (2002), and Gilchrist and Saito (2008)]. However, Bernanke and Gertler (2001) suggest that monetary policy should not respond to changes in asset prices, except in so far as they signal changes in expected inflation.

Using the dynamic framework of Bernanke et al. (1999), Tetlow (2006) introduces parametric model uncertainty by assuming that the central bank only knows the range in which the growth rate of stock prices lies. He focuses only on misspecifications of the expected growth rate of stock price bubbles, with an application to the U.S. economy in the presence of stock market bubbles. Using numerical simulations, he has found that a direct reaction to stock prices in a policy rule reduces inflation and output volatility only marginally.

The aim of our paper is to contribute to the literature of robust monetary policy by studying how the adoption of the robust control approach of Hansen and Sargent (2007) affects the relationship between monetary policies and asset prices as well as the dynamic adjustment of the economic system. The choice of this approach is motivated by the fact that it allows us to obtain closed-form solutions for optimal robust policy and equilibrium behavior of the economy. We can therefore examine analytically the implications of model misspecification for macroeconomic performance and dynamics.

Following Leitemo and Söderström (2008a), we also allow the policy maker's preference for robustness to differ across equations, reflecting the confidence the policy maker has in each relationship. Hence, we consider several different types of misspecification within the model, affecting firms' price-setting, consumer behavior, and determination of asset prices. Thus, it is possible to examine the effect of each particular misspecification on the robust monetary policy. The importance of the ability to focus on specification errors in particular equations is justified on the ground that policy makers are more confident in some relationships than in others, and so regard some types of specification errors as more important than others. Adopting this approach, we are able to analyze the design of monetary policy under specific model uncertainty while keeping other potential sources of misspecification fixed.

One central objective of our study is to investigate whether the introduction of asset prices modifies the implications of the robust control approach for monetary policy decisions. An important practical implication of this approach is that the attenuation principle established by Brainard (1967) may not always hold. The concern about worst-case scenarios emphasized by robust control may likewise lead to amplification rather than attenuation in the response of the optimal monetary policy to shocks in a closed economy [e.g., Giannoni (2002), Onatski and Stock (2002), Giordani and Söderlind (2004), and Leitemo and Söderström (2008b)]. In effect, the precautionary central bank takes stronger action to prevent particularly costly outcomes. However, Leitemo and Söderström (2008a) show that this result does not carry over to an open economy where the optimal robust policy can be either more aggressive or more cautious than the nonrobust policy.[1]

The remainder of the paper is organized as follows. In the next section, we lay down a stylized macroeconomic model with asset prices. In the section after, we derive the optimal robust policy for the worst-case model and examine the effect of the preference for robustness on the dynamic adjustment of macroeconomic and financial variables. In the fourth section, we solve for the equilibrium solutions of endogenous variables and study their sensibilities with regard to the preference for robustness. We summarize our findings in the last section.

2. A STYLIZED MACROECONOMIC MODEL WITH ASSET PRICES

We use a linear model based on Nisticò (2011), who extends Yaari (1965) and Blanchard (1985)'s OLG-perpetual youth models to include risky equities and adapts it to a New Keynesian framework.[2] Similarly to Milani (2008), we introduce persistent shocks in the model as follows:

$$x_t = \frac{1}{1+\psi}E_t x_{t+1} + \frac{\psi}{1+\psi}s_t - \frac{1}{1+\psi}(r_t - E_t\pi_{t+1} - rr^n) + v_t^d, \quad (1)$$

$$s_t = \tilde{\beta}E_t s_{t+1} - \lambda E_t x_{t+1} - (r_t - E_t\pi_{t+1} - rr^n) + v_t^s, \quad \text{with} \quad \tilde{\beta} = \frac{\beta}{1+\psi}, \quad (2)$$

$$\pi_t = \tilde{\beta}E_t\pi_{t+1} + \kappa x_t + v_t^\pi, \quad (3)$$

$$v_t^j = \rho_j v_{t-1}^j + e_t^j, \quad \text{with} \quad j = \pi, d, s; 0 \le \rho_j \le 1 \quad \text{and} \quad E_{t-1}e_t^j = 0, \quad (4)$$

where π_t denotes the inflation rate, $E_t\pi_{t+1}$ the expected future inflation rate with E_t as the expectation operator reflecting the hypothesis of rational expectations, x_t the output gap, $E_t x_{t+1}$ the expected future output gap, s_t the stock (or asset) price gap, $E_t s_{t+1}$ the expected future stock price gap, r_t the nominal interest rate, rr^n the natural rate of interest, v_t^π an inflation or cost shock, v_t^d a demand shock, and v_t^s a disturbance that represents an equity premium shock.

Equation (1) is the aggregate demand (or IS) equation. The term $\frac{\psi}{1+\psi}s_t$ reflects the wealth effect on the household consumption. This effect is explained by the coexistence of older households having accumulated wealth and younger households without assets.[3] The magnitude of the wealth effect depends on the composite coefficient ψ, which is a combination of structural parameters, i.e., $\psi \equiv \gamma\frac{1-\beta(1-\gamma)}{1-\gamma}\frac{\omega}{PC}$, where γ denotes the span of the agents' planning horizon and $\frac{\omega}{PC}$ denotes the steady-state real financial wealth–to–consumption ratio. Thus, the wealth effect depends positively on the structural parameter γ and the ratio ω/PC.

Equation (2) characterizes the stock price gap dynamics. Stock prices are forward-looking; i.e., the stock price gap depends on its own one period–ahead expectations as well as the expected future output gap. The parameter β represents the private discount factor, which is positive but less than unity. The composite coefficient $\lambda \equiv \tilde{\beta}\frac{1+\varphi}{\mu}\frac{Y}{Q} - (1-\tilde{\beta}) > 0$ (which depends on the steady-state markup μ, on the inverse of the steady-state elasticity of labor supply φ, and on the steady-state real output–to–stock price ratio Y/Q) enters with a negative sign in (2); i.e., an increase in future output gap will have a negative effect on the stock price gap.[4]

Equation (3) is a forward-looking Phillips curve based on optimizing private sector behavior and nominal rigidities, which has been extensively used in the recent literature on monetary policy [Clarida et al. (1999)]. The composite parameter $\kappa \equiv \frac{(1-\vartheta)(1-\vartheta\tilde{\beta})}{\vartheta}(1+\varphi)$, where ϑ represents the fraction of firms keeping their prices constant in period t, is the output-gap elasticity of inflation and captures the effects of the output gap on real marginal costs and hence on inflation.

Finally, equation (4) defines the shocks in the system as first-order autoregressive processes where ρ_j represents the degree of persistence and $e_t^j \sim N(0, \sigma_j^2)$ is the stochastic component.

3. MONETARY POLICY UNDER MODEL UNCERTAINTY

Although the central bank perceives the benchmark model described by equations (1)–(3) as the most likely specification, it realizes that the true model may deviate from the benchmark, without, however, being able to specify a probability distribution for deviations. To take account of that, we introduce into (1)–(3) a second type of disturbances, denoted by w_t, u_t, and h_t (i.e., output, asset pricing, and inflation misspecifications, respectively). The disturbances are controlled, in the sense of Hansen and Sargent (2007), by a fictitious "evil agent" representing the policy maker's worst fears concerning specification errors. Thus, the worst-case model is given by

$$x_t = \frac{1}{1+\psi}E_t x_{t+1} + \frac{\psi}{1+\psi}s_t - \frac{1}{1+\psi}(r_t - E_t\pi_{t+1} - rr^n) + v_t^d + w_t, \quad (5)$$

$$s_t = \tilde{\beta}E_t s_{t+1} - \lambda E_t x_{t+1} - (r_t - E_t\pi_{t+1} - rr^n) + v_t^s + u_t, \quad (6)$$

$$\pi_t = \tilde{\beta}E_t\pi_{t+1} + \kappa x_t + v_t^\pi + h_t. \quad (7)$$

As it is common in the robust control literature, we assume that the central bank allocates a budget χ^2 to the evil agent, who creates misspecifications under the following budget constraints:

$$E_t\sum_{t=0}^{\infty}\beta^t z_{t+n}^2 \le \chi^2, \quad \text{with} \quad z = w, u, h. \quad (8)$$

To design the robust monetary policy, the central bank takes into account of model misspecifications by minimizing its objective function in the worst possible model within a given set of plausible models. We assume that the monetary policy maker takes expected future variables as given in following a monetary strategy without commitment (discretion). Monetary policy is implemented to minimize the conditional expectation of the loss function. The robust monetary policy is obtained by solving the min–max problem

$$\min_{\{r_t\}}\max_{\{h_t,w_t,u_t\}}\sum_{0}^{\infty}\frac{1}{2}\beta^t E_t\left[\alpha(\pi_t - \pi^T)^2 + (x_t - x^*)^2 - \theta_h h_t^2 - \theta_w w_t^2 - \theta_u u_t^2\right], \quad (9)$$

subject to the misspecified model (5)–(7) and the evil agent's budget constraints (8). The parameter α denotes the relative weight that the central bank assigns to the inflation target, π^T, and the output gap target, x^*. The central bank sets the nominal interest rate to minimize the value of its intertemporal loss function, whereas the evil agent creates misspecifications to maximize the central bank's loss given its budget constraints. Parameters $\theta_z \in]0, +\infty[$, $z = w, u, h$, determine the set of models available to the evil agent that the policy maker wants to be robust against. They represent the central bank's preferences for robustness: the higher the value of θ_z, the lower is the preference for robustness. They are related to the evil agent's budget: as $\chi^2 \to 0$, $\theta_z \to \infty$, and the model specification errors approach zero.

The Lagrangian for this problem is given by

$$L = \sum_0^\infty \left\{ \begin{array}{l} \frac{1}{2} E_t \left[\alpha(\pi_t - \pi^T)^2 + (x_t - x^*)^2 - \theta_h h_t^2 - \theta_w w_t^2 - \theta_u u_t^2 \right] \\ -\mu_t^x \big[x_t - \frac{1}{1+\psi} E_t x_{t+1} - \frac{\psi}{1+\psi} s_t + \frac{1}{1+\psi}(r_t - E_t \pi_{t+1} - rr^n) \\ \quad -v_t^d - w_t \big] \\ -\mu_t^s \left[s_t - \tilde{\beta} E_t s_{t+1} + \lambda E_t x_{t+1} + (r_t - E_t \pi_{t+1} - rr^n) - v_t^s - u_t \right] \\ \quad - \mu_t^\pi \left[\pi_t - \tilde{\beta} E_t \pi_{t+1} - \kappa x_t - v_t^\pi - h_t \right] \end{array} \right\},$$

where μ_t^x, μ_t^s, and μ_t^π are Lagrange multipliers on the constraints (5)–(7), respectively. The first-order conditions for the min–max problem are

$$\frac{\partial L}{\partial \pi_t} = \alpha(\pi_t - \pi^T) - \mu_t^\pi = 0, \tag{10}$$

$$\frac{\partial L}{\partial x_t} = (x_t - x^*) + \kappa \mu_t^\pi - \mu_t^x = 0, \tag{11}$$

$$\frac{\partial L}{\partial r_t} = -\frac{1}{1+\psi} \mu_t^x - \mu_t^s = 0, \tag{12}$$

$$\frac{\partial L}{\partial s_t} = \frac{\psi}{1+\psi} \mu_t^x - \mu_t^s = 0, \tag{13}$$

$$\frac{\partial L}{\partial w_t} = -\theta_w w_t + \mu_t^x = 0, \tag{14}$$

$$\frac{\partial L}{\partial u_t} = -\theta_u u_t + \mu_t^s = 0. \tag{15}$$

$$\frac{\partial L}{\partial h_t} = -\theta_h h_t + \mu_t^\pi = 0.. \tag{16}$$

From (12) and (13), it follows that $\mu_t^x = \mu_t^s = 0$. The first order conditions (10) and (11) imply that the preference for robustness does not influence the optimal trade-off between inflation and the output gap. Using $\mu_t^x = \mu_t^s = 0$ in (14) and (15), we obtain $w_t = u_t = 0$. Thus, the optimal misspecifications in the output and asset pricing equations are always zero, as the central bank is able to neutralize misspecifications in these equations by an appropriate adjustment of the interest rate. In effect, as discussed by Leitemo and Soderstrom (2008a, 2008b), the central bank does not fear such misspecifications because its loss function is not affected by the interest rate movements. We remark that, in Leitemo and

Soderstrom (2008a), the exchange rate equation is prone to misspecification. Even if asset prices and the exchange rate have similar impacts on the output equation, the asset pricing equation in our model is not affected by misspecification. The explanation is that the asset price gap does not directly affect the Phillips curve, whereas the latter is influenced by the exchange rate in their model.

Taking account of $\mu_t^x = u_t = w_t = 0$, (10), (11), and (16) yield

$$x_t - x^* = -\alpha\kappa(\pi_t - \pi^T), \tag{17}$$

$$x_t - x^* = -\kappa\theta_h h_t. \tag{18}$$

The second-order conditions, with respect to w_t, u_t, and h_t, for the central bank's min–max problem are obtained in deriving (14)–(16), taking account of (7), (10), (11), and $\mu_t^x = \mu_t^s = 0$:

$$\frac{\partial^2 L}{\partial w_t^2} = -\theta_w < 0; \; \frac{\partial^2 L}{\partial u_t^2} = -\theta_u < 0; \tag{19}$$

$$\frac{\partial^2 L}{\partial h_t^2} = -\theta_h + \frac{1}{\kappa^2} < 0 \Rightarrow \frac{1}{\kappa^2} < \theta_h. \tag{20}$$

The conditions (19) and (20) ensure that the objective function of the central bank is concave in misspecifications. As in Hansen and Sargent (2007), we can justify $\theta_h > 1/\kappa^2$ by showing that, for $\theta_h < 1/\kappa^2$, the fictitious evil agent will choose $h_t \to \infty$.

Using (7), (17), and (18), we get

$$\pi_t = \frac{\theta_h}{(1+\alpha\kappa^2)\theta_h - \alpha}\left(\tilde{\beta}E_t\pi_{t+1} + \kappa x^* - \pi^T + v_t^\pi\right) + \pi^T, \tag{21}$$

$$x_t = x^* - \frac{\alpha\kappa\theta_h}{(1+\alpha\kappa^2)\theta_h - \alpha}\left(\tilde{\beta}E_t\pi_{t+1} + \kappa x^* - \pi^T + v_t^\pi\right), \tag{22}$$

$$h_t = \frac{\alpha}{(1+\alpha\kappa^2)\theta_h - \alpha}\left(\tilde{\beta}E_t\pi_{t+1} + \kappa x^* - \pi^T + v_t^\pi\right), \tag{23}$$

where $(1+\alpha\kappa^2)\theta_h - \alpha > 0$ for $\theta_h > 1/\kappa^2$. Inflation shocks affect π_t, x_t, and h_t either directly or indirectly through the expected future inflation rate. These direct and indirect effects are positively impacted by an increase in the preference for robustness (i.e., a decrease in θ_h).

Substituting $w_t = 0$, x_t, given by (22), into (5), we obtain the nominal interest rate rule for the worst-case model as follows:

$$r_t = E_t\pi_{t+1} + rr^n + \frac{(1+\psi)\alpha\kappa\theta_h}{(1+\alpha\kappa^2)\theta_h - \alpha}\left(\tilde{\beta}E_t\pi_{t+1} + \kappa x^* - \pi^T + v_t^\pi\right)$$
$$+ E_t x_{t+1} - (1+\psi)x^* + \psi s_t + (1+\psi)v_t^d. \tag{24}$$

The optimal nominal interest rate reacts positively to the expected future inflation rate and output gap, current asset price gap, and inflation and demand shocks.

PROPOSITION 1. *The reaction of the optimal nominal interest rate to the expected future inflation rate and inflation shocks is increasing with the preference for robustness against inflation misspecification.*

Proof. Deriving the nominal interest rate determined by (24) with respect to the expected future inflation rate and inflation shocks respectively, and then deriving the resulting partial derivatives with regard to the preference for robustness, leads to

$$\frac{\partial^2 r_t}{\partial E_t \pi_{t+1} \partial \theta_h} = \frac{\tilde{\beta}\partial^2 r_t}{\partial v_t^{\pi} \partial \theta_h} = -\frac{\tilde{\beta}(1+\psi)\alpha^2\kappa}{[(1+\alpha\kappa^2)\theta_h - \alpha]^2} < 0. \qquad \blacksquare$$

In effect, a stronger preference for robustness, which corresponds to a lower θ_h, implies a stronger reaction of the optimal nominal interest rate relative to the expected future inflation rate. In other words, an increase in inflation misspecification is somewhat equivalent to a positive inflation shock. Furthermore, it has some implications in terms of dynamic adjustments.

The difference equation for the inflation rate is given by (21) and that for the asset price gap can be obtained using $w_t = u_t = 0$, (5)–(7), (17), and (23) to eliminate r_t, x_t, $E_t\pi_{t+1}$, and $E_t x_{t+1}$. The system of dynamic equations is presented in matrix form as

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t s_{t+1} \end{bmatrix} = \begin{bmatrix} \dfrac{(1+\alpha\kappa^2)\theta_h - \alpha}{\tilde{\beta}\theta_h} & 0 \\ \dfrac{\alpha\tilde{\beta}\kappa\theta_h(1+\psi) - \alpha\kappa(1+\lambda)[(1+\alpha\kappa^2)\theta_h - \alpha]}{\tilde{\beta}^2\theta_h} & \dfrac{1+\psi}{\tilde{\beta}} \end{bmatrix} \begin{bmatrix} \pi_t \\ s_t \end{bmatrix}$$
$$+ \begin{bmatrix} -\dfrac{\kappa}{\tilde{\beta}}x^* - \dfrac{\alpha\kappa^2\theta_h - \alpha}{\tilde{\beta}\theta_h}\pi^T - \dfrac{1}{\tilde{\beta}}v_t^{\pi} \\ \dfrac{\tilde{\beta}[\lambda - \psi] + \alpha\kappa^2(1+\varepsilon+\lambda)}{\tilde{\beta}^2}x^* \\ + \dfrac{\alpha^2\kappa(1+\lambda)(\kappa^2\theta_h - 1) + \alpha\tilde{\beta}\kappa\theta_h(\lambda - \psi)}{\tilde{\beta}^2\theta_h}\pi^T \\ + \dfrac{\alpha\kappa(1+\lambda)}{\tilde{\beta}^2}v_t^{\pi} + \dfrac{(1+\psi)}{\tilde{\beta}}v_t^d - \dfrac{1}{\tilde{\beta}}v_t^s \end{bmatrix}. \qquad \textbf{(25)}$$

In (25), we notice that the parameter θ_h is associated with π_t but not with s_t. If $\theta_h \to \infty$, system (25) describes the dynamics in the benchmark case where the central bank knows the true structure of the economy exactly.

PROPOSITION 2. *In the benchmark model without misspecifications, the dynamic system has a stable equilibrium.*

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Proof. Under the inflation-targeting regime without robust control, i.e., $\theta_h \to \infty$, the stability matrix of (25) has two eigenvalues $E_1 = \frac{1+\alpha\kappa^2}{\tilde{\beta}}$ and $E_2 = \frac{(1+\psi)}{\tilde{\beta}} = 1/\beta$. Both eigenvalues are greater than unity. That ensures convergence to equilibrium when the system is perturbed. ■

Because there are two stable eigenvalues, the system has a continuum of trajectories converging to the equilibrium. Even though this might cause some coordination difficulties between private agents when they form expectations about the future, one such system is more resilient to the effects of exogenous shocks than a system with a unique converging path (saddlepoint stable equilibrium).[5] The latter is exposed to the risk of developing speculative bubbles.

In the following, we examine how the robust monetary policy, by amplifying the reaction of the nominal interest rate to inflation shocks, affects the dynamic adjustment of the system.

PROPOSITION 3. *The equilibrium is stable if the preference for robustness is sufficiently low, i.e., $\theta_h > 1/\kappa^2$, with the speed of convergence to equilibrium being lower under robust control than in the benchmark case.*

Proof. Under the inflation-targeting regime with robust control, the stability matrix of (25) has two eigenvalues: $E_1^r = \frac{(1+\alpha\kappa^2)\theta_h-\alpha}{\tilde{\beta}\theta_h} = \frac{\theta_h+\alpha(\kappa^2\theta_h-1)}{\tilde{\beta}\theta_h} > 1$ if $\theta_h > 1/\kappa^2$, and $E_2^r = 1/\beta > 1$. As both eigenvalues are superior to unity, the system is stable. Comparing E_1 and E_1^r, we find that $E_1^r = \frac{(1+\alpha\kappa^2)\theta_h-\alpha}{\tilde{\beta}\theta_h} < \frac{1+\alpha\kappa^2}{\tilde{\beta}} = E_1$. As the second eigenvalue is the same under the two monetary policy regimes, the speed of convergence to equilibrium is reduced if the central bank has a stronger preference for robustness (i.e., a smaller θ_h). ■

When the preference for model robustness varies in the interval $\theta_h \in]1/\kappa^2, +\infty[$, it has no impact on the stability properties of the system but affects the speed of convergence to equilibrium. Departing from the benchmark model, where $\theta_h \to \infty$, a reduction in θ_h will incite the central bank to amplify the reaction of the optimal nominal interest rate to the expected future inflation rate and inflation shocks. Stronger response of optimal monetary policy makes the system converge less rapidly to equilibrium. Slower dynamic adjustment implies that the economy will stay longer in each state of disequilibrium and hence induces additional costs in terms of welfare during the adjustment process, which are not captured by the measurement of macroeconomic volatility.

The condition $\theta_h > 1/\kappa^2$ is important not only for the central bank's min–max problem to have a solution but also for ensuring the dynamic stability of the system. If we relax the constraint $\theta_h > 1/\kappa^2$ so that the preference for robustness is sufficiently high, i.e., $\theta_h < \frac{\alpha}{1+\alpha\kappa^2-\tilde{\beta}}$, the equilibrium could be saddlepoint stable and is less favorable in terms of macroeconomic stabilization. In this case, because the inflation rate and asset price gap are forward-looking, they can jump instantaneously onto the unique stable path converging to equilibrium. However, the system may be prone to the formation of rational speculative bubbles and hence generates greater dynamic volatility.

4. MODEL UNCERTAINTY AND MACROECONOMIC PERFORMANCE

At equilibrium, the inflation rate and output gap are only affected by inflation shocks, as the optimal monetary policy has neutralized the effects of other shocks. Meanwhile, the asset price gap is influenced by shocks affecting goods and financial markets. Even if the preference for robustness affects the reactions of endogenous variables to inflation shocks, it does not modify their sensitivity with regard to other shocks. The effects of a variation in the preference for robustness are summarized in the following proposition.

PROPOSITION 4. *An increase in the preference for robustness strengthens the reaction of* π_t, $E_t\pi_{t+1}$, x_t, $E_t x_{t+1}$, A_t *and* $E_t A_{t+1}$ *to inflation shocks. The preference for robustness has no effect on the reaction of these variables to shocks affecting goods and financial markets.*

Proof. See Appendix. ■

As we have previously discussed, the central bank does not fear misspecification in the output and asset pricing equations. Therefore, the effects of shocks affecting these two equations are not associated with the preference for robustness.

However, the central bank could fear misspecification in the Phillips curve and would respond more aggressively to inflation shocks in the worst-case model. This misspecification becomes larger when the expected future inflation rate is higher following a positive inflation shock. It induces an increase in the inflation rate, either directly through the misspecification or indirectly through a larger output gap. Thus, in order to achieve the desired trade-off between inflation and the output gap, the central bank increases the output gap in response to the misspecification. As the preference for robustness is positively (θ_h is negatively) related to the misspecification in the Phillips curve, an increase in this preference reinforces the reactions of inflation and the output gap to inflation shocks and hence positively affects their volatility. Affected by inflation shocks through the channels of the expected future inflation rate and real output gap as well as the nominal interest rate, the volatility of asset price gap will be amplified following an increase in the preference for model robustness. Our results suggest that the attenuation principle of Brainard does not hold when monetary policy decisions are based on the robust control approach, even though the asset prices are taken into account.

5. CONCLUSIONS

Considering the central bank's worst-case model with asset prices, we have solved for the optimal robust monetary policy and examined its dynamic implications as well as its effects on macroeconomic performance. It is shown that the reaction of the optimal nominal interest rate becomes more sensitive to the expected future inflation rate and inflation shocks, but remains unchanged in the presence of shocks affecting goods and financial markets. This implies that the central bank can focus on the misspecification in the Phillips curve and ignore misspecifications concerning goods and asset markets.

We find that the dynamic stability nature of the equilibrium is not modified by the robust monetary policy if the central bank limits its preference for robustness to a certain level. However, the speed of convergence to the equilibrium is smaller under robust control compared to the benchmark case without it. Furthermore, the boundary condition for the preference for robustness is also justified to prevent a saddlepoint stable equilibrium, which is prone to speculative bubbles and hence generates more dynamic volatility.

In terms of macroeconomic performance, an increase in the preference for robustness reinforces the reaction of the inflation rate, asset price gap, and output gap to inflation shocks. Nevertheless, the preference for robustness has no effect on the reaction of inflation, output gap and asset price gap to shocks affecting goods and financial markets.

In practice, the concern about worst-case scenarios emphasized by the robust-control approach leads to amplification in the response of the optimal monetary policy to inflation shocks. Stronger action by the precautionary central bank to prevent particularly costly outcomes may be justified. However, the central bank will face the trade-off between the benefit of avoiding very bad outcomes in worst-case scenarios and the economic costs due to slower convergence to equilibrium as well as the costs due to higher volatility in the inflation rate, output gap, and asset price gap.

NOTES

1. For a more extensive discussion of policy implications of the robust control approach, see Walsh (2003).

2. See Nisticò (2011) for detailed micro-foundations of the New Keynesian model with asset prices. For an open economy version of this model, see Di Giorgio and Nisticò (2007).

3. Other effects of asset prices on aggregate demand, such as Tobin's q effect, are not included in the model.

4. There is traditionally a positive relation between stock returns and subsequent growth rates of real activity [Fama (1990)]. This relation disappeared in the early 1980s [Binswanger (2000, 2004)] for some countries, whereas it is confirmed by Tsouma (2009), who uses more recent data. Milani (2008) suggests that the negative relation between stock price gap and future real output gap in Nisticò's model (2011) is explained by the assumption of a flexible labor market, which would generate countercyclical profits and dividends, and the sign before λ could be positive if labor rigidities were allowed for.

5. The equilibrium is saddlepoint stable if the number of stable eigenvalues is greater than or equal to the number of predetermined variables but less than the total number of dynamic variables [see Buiter (1984)].

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APPENDIX: PROOF OF PROPOSITION 4

Using the method of indeterminate coefficients [McCallum (1983)] to solve the decomposable dynamic system (25) and (17), we can obtain the equilibrium solutions for the worst-case model as follows:

$$\pi_t = \frac{\alpha(\kappa^2\theta_h - 1)\pi^T + \theta_h\kappa x^*}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}} + \frac{\theta_h}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi} v_t^\pi,$$

$$E_t\pi_{t+1} = \frac{\alpha(\kappa^2\theta_h - 1)\pi^T + \theta_h\kappa x^*}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}} + \frac{\theta_h\rho_\pi}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi} v_t^\pi,$$

$$x_t = \frac{\alpha\kappa\theta_h(1-\tilde{\beta})\pi^T + [\theta_h(1-\tilde{\beta}) - \alpha]x^*}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}} - \frac{\alpha\kappa\theta_h}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi} v_t^\pi,$$

$$E_t x_{t+1} = \frac{\alpha\kappa\theta_h(1-\tilde{\beta})\pi^T + [\theta_h(1-\tilde{\beta}) - \alpha]x^*}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}} - \frac{\alpha\kappa\theta_h\rho_\pi}{(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi} v_t^\pi,$$

$$s_t = -\frac{(\lambda-\psi)\{\alpha\kappa\theta_h(1-\tilde{\beta})\pi^T + [\theta_h(1-\tilde{\beta})-\alpha]x^*\}}{(1+\psi-\tilde{\beta})[(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}]} - \frac{1+\psi}{1+\psi-\tilde{\beta}\rho_d}v_t^d$$
$$+ \frac{1}{1+\psi-\tilde{\beta}\rho_s}v_t^s - \frac{\alpha\kappa\theta_h[1+\psi-(1+\lambda)\rho_\pi]}{(1+\psi-\tilde{\beta}\rho_\pi)[(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi]}v_t^\pi,$$

$$E_t s_{t+1} = -\frac{(\lambda-\psi)\{\alpha\kappa\theta_h(1-\tilde{\beta})\pi^T + [\theta_h(1-\tilde{\beta})-\alpha]x^*\}}{(1+\psi-\tilde{\beta})[(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}]} - \frac{1+\psi}{1+\psi-\tilde{\beta}\rho_d}\rho_d v_t^d$$
$$+ \frac{1}{1+\psi-\tilde{\beta}\rho_s}\rho_s v_t^s - \frac{\alpha\kappa\theta_h[1+\psi-(1+\lambda)\rho_\pi]}{(1+\psi)-\tilde{\beta}\rho_\pi)[(1+\alpha\kappa^2)\theta_h - \alpha - \theta_h\tilde{\beta}\rho_\pi]\rho_\pi v_t^\pi}.$$

To investigate the impact of an increase in the preference for robustness (i.e., a decrease in θ_h), we first derive these solutions with respect to different shocks and then with respect to the preference for robustness. In the presence of inflation shocks, the first and second derivatives of each variable are of opposite signs, meaning that an increase in the preference for robustness reinforces the reaction of all variables. In the presence of demand and financial shocks, only the first derivatives of asset price gap are different from zero, and they are independent of θ_h. ■