A.A.S. **4**, II, 241-259 (2009)

MODELLING INCOME PROTECTION CLAIM TERMINATION RATES BY CAUSE OF SICKNESS II: MORTALITY OF UK ASSURED LIVES

By S. Y. Ling, H. R. Waters and A. D. Wilkie

ABSTRACT

This is the second of three papers in which we present methods and results for the estimation and modelling of claim termination rates for Income Protection (IP) insurance, allowing for different causes of claim. In the first paper we discussed recoveries. In this and the third paper we develop models for the mortality of IP claimants.

We model this mortality as the sum of two components: a *base*, or *background*, mortality, which is a function of age and calendar year, but not of the specific cause of sickness or its current duration, and a cause-specific element which does depend on the current duration of the sickness. In this paper we discuss the modelling of the base mortality. In particular, we use data supplied by the Continuous Mortality Investigation relating to UK assured lives from 1975 (males) and 1983 (females) to 2003 to develop models of mortality which are functions of sex, age and calendar year. Such models are of interest in their own right, particularly at a time when expected future lifetimes are increasing.

The modelling of the cause-specific component of the mortality model is discussed in Paper III.

KEYWORDS

Mortality; Assured Lives; UK Data; Calendar Year; Generalised Linear Models; Income Protection Insurance

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1. Introduction

Our aim in this paper and its two accompanying papers, Ling *et al.* (2009a & 2009c), referred to as Paper I and Paper III, is to develop a model for claim termination rates, recovery and mortality, for Income Protection (IP) policyholders by cause of sickness. In Paper I we described a model for recovery rates by cause of sickness. It remains to develop a corresponding

https://doi.org/10.1017/S1748499500000725 Published online by Cambridge University Press

model for the mortality of IP policyholders — which is our objective in this paper and in Paper III.

As described in Paper I, Section 2, we have been supplied by the Continuous Mortality Investigation (CMI) with a very extensive data set relating to IP claim payments in the UK from 1975 to 2002. A natural starting point would be to use this data set to model directly the mortality of IP claimants. However, such a direct approach has its difficulties, namely:

(i) There are very few deaths (3,498) compared to recoveries (72,741) in our data set. See Paper I, Table 1.
(ii) About half the deaths (1,766) in our data set relate to two causes, neoplasms, malignant and benign or unspecified. See Paper I, Table 1.
(iii) The vast majority of deaths among IP policyholders occur within one year of falling sick, making it difficult to develop a model for the mortality of IP policyholders valid for longer durations of sickness.

Previous authors who have modelled the mortality of IP policyholders — CMIR12 (1991), Renshaw & Haberman (1995 & 2000) and CMIWP5 (2004) — faced the difficulties listed under points (i) and (ii) above. Each of these authors modelled the mortality intensity directly from data for IP policyholders and none attempted to incorporate cause of sickness as a factor. The general features of these earlier models can be summarised as follows:

(a) Sickness duration is the dominant explanatory variable, at least within the range of the data used by the various authors.
(b) The mortality intensity as a function of sickness duration has a bell-curved shape, peaking between 18 and 22 weeks, and then falling off before showing an upturn at sickness durations exceeding 8 years. This eventual up-turn has been attributed to a more dominant ageing process taking effect.

In CMIWP5 (2004), the age-effect for longer sickness durations was incorporated by adding a Gompertz formula as a function of age.

These considerations suggest a model of the following form for the intensity we are trying to model, denoted $\lambda(z, \mathbf{x})$:

$$\lambda(z, \mathbf{x}) = \lambda^*(\mathbf{x}) + v(z, \mathbf{x}) \tag{1}$$

where

z denotes the duration of the current sickness,

$\mathbf{x}$ is a set of covariates, namely sex, attained age and calendar year,

$\lambda(z, \mathbf{x})$ is the mortality intensity for a specific cause at sickness duration z given covariates $\mathbf{x}$,

$\lambda^*(\mathbf{x})$ is a base intensity, which is a function of $\mathbf{x}$ alone, and

$v(z, \mathbf{x})$ denotes the 'excess' mortality incurred from being sick with a specific cause for duration z given covariates $\mathbf{x}$. This excess mortality can be interpreted as the mortality in excess of that experienced by a comparable population as a result of being sick for duration z.

Note that the base mortality, $\lambda^*(\mathbf{x})$, does not depend on the specific cause of sickness, whereas the other two components of the model, $\lambda(z, \mathbf{x})$ and $v(z, \mathbf{x})$, do, although the dependence is suppressed in the notation. Note also that $\lambda(z, \mathbf{x})$ was denoted $v(i)_{x,z}$ in Paper I, Figure 2.

A cause-specific mortality model based on the mortality of a comparable population is common in the medical literature, particularly in relation to cancer research. See, for example, Dickman *et al.* (2004).

Our objective in this paper is to develop a model for $\lambda^*(\mathbf{x})$; the modelling of $v(z, \mathbf{x})$ is discussed in Paper III. To model $\lambda^*(\mathbf{x})$ we need appropriate data. We regard the UK assured lives population as a reasonably comparable group from which this base mortality can be modelled and we have used CMI data for assured lives relating to 1975 to 2003 (males) and 1983 to 2003 (females) for this purpose.

In Section 2 we describe the data available to us. In Section 3 we set out some preliminary information before giving details of the modelling of the mortality intensity for males and females in Sections 4 and 5, respectively. These mortality models are based on data from UK assured lives and are functions of sex, attained age and calendar year. At a time when increases in expected future lifetime are an important issue, both financially and socially, such models are of interest in their own right.

Full details of the research reported in this paper can be found in Ling (2009), particularly Chapter 4.

The Acknowledgements and References for this paper, and its accompanying papers, are given at the end of Paper I.

2. Data

Our data, supplied by the CMI, relate to UK assured lives from 1975 to 2003 (males) and 1983 to 2003 (females). The data are combined data for all policy durations: 0, 1 and 2+ years. The data are extensive for both males and females, as can be seen from the sample and summary statistics in Tables 1 and 2.

Figures 1 and 2 show the crude mortality rates as a function of calendar year for selected ages for males and females. The former shows a less ragged pattern than the latter since the data for males is even more extensive than the data for females. These two figures show similar patterns: mortality rates increase with age, are higher for males than for females and have been decreasing as functions of calendar year, though not at a uniform rate for all ages.

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Table 1. Data for female assured lives (1983 to 2003)

	1983		1995		2003	
Age	Exposure (years)	Deaths	Exposure (years)	Deaths	Exposure (years)	Deaths
25	29,427.2	8	20,042.7	3	2,260.0	0
55	23,255.5	68	32,939.9	89	22,200.5	53
85	124.8	14	576.1	36	767.5	52
20 to 90	1,345,951	2,162	1,543,174	3,590	697,426.5	2,674

Table 2. Data for male assured lives (1975 to 2003)

	1975		1995		2003	
Age	Exposure (years)	Deaths	Exposure (years)	Deaths	Exposure (years)	Deaths
25	129,525.4	70	27,367.5	11	2,657.5	1
55	128,576.4	911	97,478.5	334	53,573.5	155
85	1,218.0	182	1,762.8	171	1,672.5	153
20 to 90	6,213,746	23,610	3,536,872	12,842	1,329,791	7,731

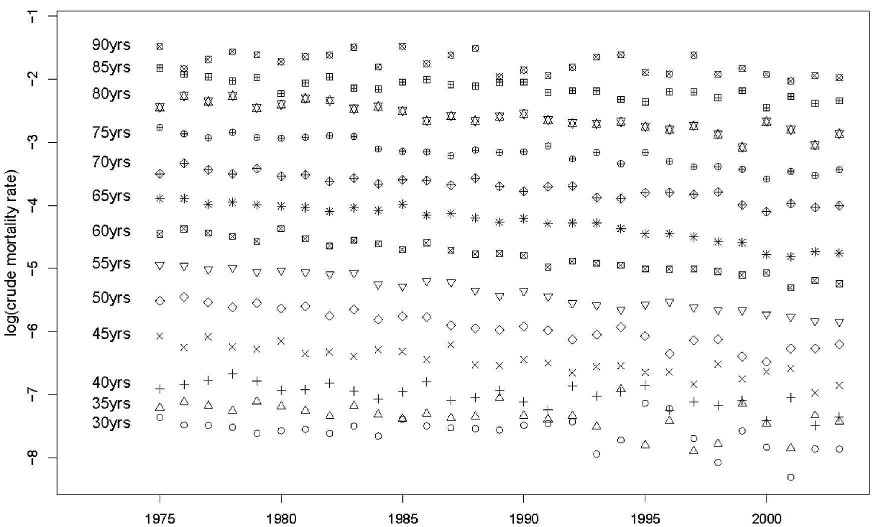

Figure 1. Crude mortality intensities for selected ages for males

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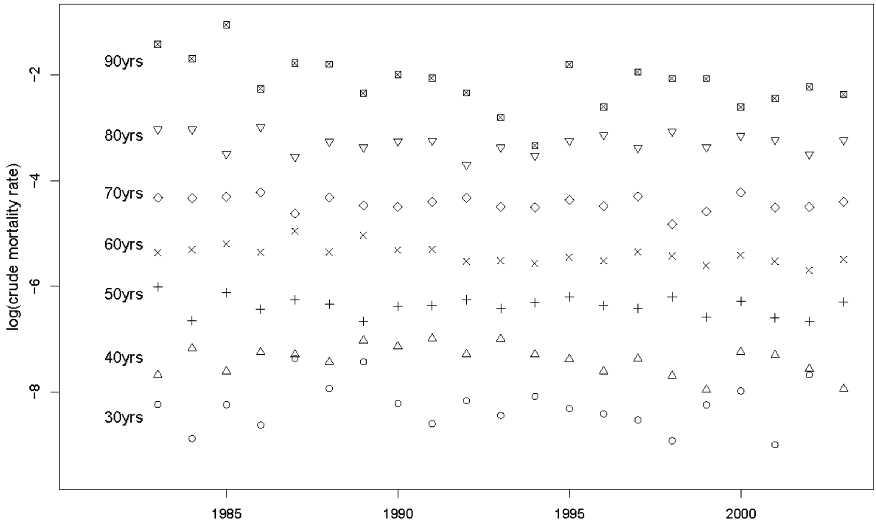

Figure 2. Crude mortality intensities for selected ages for females

The IP data used in Papers I and III to model recovery rates and cause-specific mortality, respectively, relates to years 1975 to 2002. It is convenient to have a model for the background mortality based on data from a corresponding period, particularly since mortality rates have been changing over this period. The assured lives data for males meets this condition but the data for females goes back only to 1983. However, CMIR6 (1983) gives the ultimate mortality of UK female assured lives, expressed as a formula for q_x, based on data from the period 1975 to 1978. We can obtain corresponding values for μ_x for $x = 20, 21, \ldots, 90$ from the formula:

$$\mu_x = -\log(1 - q_{x-1/2}).$$

We treat these rates as appropriate to 1977, the mid point of 1975 to 1978, and refer to them as FA77 rates. In Section 4, when we discuss the modelling of the mortality data for females, we will ensure that when we extrapolate our rates back to 1977 we obtain values consistent with the FA77 rates.

Since the data for females is so extensive, and is even more extensive for males, and because the pattern of mortality is sufficiently different for the two sexes, we modelled the mortality of males and females separately. For each sex the intensity $\lambda^*(\mathbf{x})$ is a function only of age, x, and calendar year, y. We denote this $\lambda_{x,y}$. Throughout this paper, and Paper III, 'age' means exact attained age.

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3. Preliminaries

To improve stability in parameter estimation, we use Chebycheff polynomials, $C_i(.)$, $i = 0, 1, 2 \ldots$ (see Paper I, Section 4.3), and transform x and y to x' and y' by centring each at the midpoint of its range and scaling so that its range is approximately from -1 to $+1$:

$$\begin{aligned} x' &= \frac{x - 55}{35} && \text{for } x = 20, 21, \ldots, 90 \\ y' &= \frac{y - 1989}{14} && \text{for } y = 1975, 1976, \ldots, 2003. \end{aligned} \tag{2}$$

We will also make use of natural cubic splines, $\{w_j(x)\}_{j=1}^{k-1}$, defined in terms of k fixed knots $\tau_1 < \tau_2 < \ldots < \tau_k$ as follows:

$$\begin{aligned} w_1(x) &= x \\ w_j(x) &= (x - \tau_{j-1})_+^3 - (x - \tau_{k-1})_+^3 \frac{(\tau_k - \tau_{j-1})}{(\tau_k - \tau_{k-1})} + (x - \tau_k)_+^3 \frac{(\tau_{k-1} - \tau_{j-1})}{(\tau_k - \tau_{k-1})} \\ & \qquad j = 2, 3, \ldots, k-1 \end{aligned}$$

where for any t

$$(x - t)_+ = \begin{cases} (x - t) & \text{if } \ x \geq t \\ 0 & \text{otherwise} \end{cases} .$$

See Devlin & Weeks (1986).

Note that for $j \geq 2$ the function $w_j(x)$ is zero for $x < \tau_{j-1}$, piecewise cubic for $\tau_{j-1} \leq x \leq \tau_k$ and linear for $x > \tau_k$. Hence, any linear combination of $\{w_j(x)\}_{j=1}^{k-1}$ is linear for $x < \tau_1$, piecewise cubic for $\tau_1 \leq x \leq \tau_k$ and linear for $x > \tau_k$.

4. Mortality Model for Male Assured Lives

4.1 *Initial Modelling*

Our starting point for a model for λ_{xy} for males is:

$$\log \lambda_{xy} = h_x + \sum_{i=1}^{n} (\beta_i + \beta_{ix}) C_i(y') \qquad \text{for } x = 20, 21, \ldots, 90$$

where

β_i is a set of constants,

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β_{ix} is a set of constants subject to the constraints: $\beta_{i20} = 0, i = 1, 2, \ldots, n$,
β_x is a set of constants.

In the spirit of the Lee–Carter model (Lee & Carter (1992)), we can interpret h_x as the average age-specific pattern of mortality, $C_i(y')$ as the main year trend at all ages and β_{ix} as measuring the age-specific deviation from the main year trend.

In the spirit of the Cox (1972) Proportional Hazards model, we can regard the model for λ_{xy} as the product of a baseline hazard, $\exp(h_x)$, and a relative risk, $\exp(\sum_i (\beta_i + \beta_{ix}) C_i(y'))$, where the former is a function only of age and the latter is a function of year, with age-dependent coefficients.

The first task is to determine the value of n. Including parameters in the model sequentially, $h_x, +\beta_1, +\beta_{1x}, +\beta_2, \ldots,$ fitting by maximum likelihood and calculating the Akaike Information Criterion (AIC) index (see Paper I, Section 4.3), showed that parameters up to β_{4x} decreased the AIC, but including β_5 and β_{5x} increased the AIC. It was decided that n should be set at 4. The resulting model

$$\log \lambda_{xy} = h_x + \sum_{i=1}^{4} (\beta_i + \beta_{ix}) C_i(y') \tag{3}$$

has a total of 355 parameters, which is too many. We reduce the number of parameters by modelling $\{\beta_{ix}\}$ and $\{h_x\}$ as functions of x. In principle, we can do this by estimating the parameters using maximum likelihood for a variety of different model structures. In practice, this would be computationally prohibitive and so we adopt a pragramatic approach, much as we did in Paper I — determining a model structure on the basis of a partial likelihood and then estimating all the parameters by maximising the full likelihood. This process is described in Sections 4.2 and 4.3.

4.2 *Modelling the Relative Risk*

To determine the structure of a model for $\{\beta_{ix}\}$, we will treat $\{h_x\}$ as a set of constants, at least initially. By doing this we can investigate model structures by maximising the (partial) likelihood without having to specify the values for $\{h_x\}$, just as would be the case in a Cox (1972) Proportional Hazards model.

Figure 3 shows the estimates of β_{ix} as functions of x for $i = 1, \ldots, 4$, as calculated in Section 4.1. Also shown in Figure 3 as solid lines are the final models for $\{\beta_{ix}\}$ (see Section 4.3).

We model $\{\beta_{ix}\}$ using natural cubic splines with k knots, so that our model for the age-dependent coefficient of $C_i(y')$, $\beta_i + \beta_{ix}$, is

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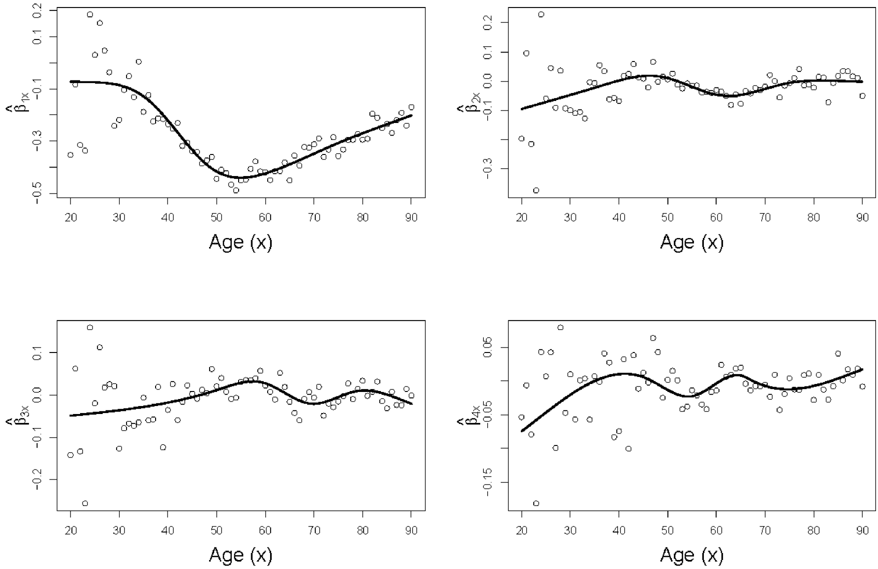

Figure 3. Age-dependent year coefficients for males

$$\beta_i + \sum_{j=1}^{k-1} \gamma_{ij} w_j(x'). \tag{4}$$

The structure of this part of the model is determined by the number and placement of the knots. We explore the effect of different structures by maximising the partial likelihood and calculating the AIC index. For a given number of knots, it is standard practice to place the knots at fixed quantiles in the data. This is our starting point, but occasionally we adjust the placements in an *ad hoc* fashion to see if we can improve the overall fit.

After a lengthy process — details are in Ling (2009) — we settled on 6 knots placed at ages 23.5, 45.5, 55, 65, 67.5 and 88.25. Given this structure, preliminary estimates of the parameters β_i and γ_{ij}, $i = 1, \ldots, 4$, $j = 1, \ldots, 5$ are determined by maximising the partial likelihood. We denote this set of parameters $\hat{\beta}$.

4.3 *Modelling the Baseline Hazard*

We model $\{h_x\}$ using a GM(r, s) formula, expressed as Chebycheff polynomials in x', which gives considerable flexibility. For various combinations of r and s, we fit the parameters of the GM(r, s) formula by maximum likelihood, in each case keeping the β and γ parameters for the relative risk at

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Table 3. Parameter estimates for formula (5) with their standard errors

Symbol	Estimate	Standard error	Symbol	Estimate	Standard error
b_0	−5.008173	0.007158	γ_{21}	0.167464	0.123156
b_1	3.112883	0.013179	γ_{22}	0.028993	0.227137
b_2	0.449331	0.010706	γ_{23}	−0.800877	0.573081
b_3	−0.338143	0.008173	γ_{24}	2.362281	0.580483
b_4	0.092917	0.006055	γ_{25}	−2.815544	0.462222
b_5	−0.037030	0.004121	γ_{31}	0.043105	0.055076
b_6	−0.019695	0.003306	γ_{32}	0.044698	0.034306
β_1	−0.088583	0.097105	γ_{33}	−1.420710	0.327058
β_2	0.071923	0.080383	γ_{34}	4.172928	0.732956
β_3	−0.005599	0.029076	γ_{35}	−5.386457	0.953359
β_4	0.120374	0.034071	γ_{41}	0.194873	0.057476
γ_{11}	−0.016607	0.148566	γ_{42}	−0.254007	0.059278
γ_{12}	−1.390934	0.274117	γ_{43}	2.108121	0.413333
γ_{13}	4.361826	0.689810	γ_{44}	−4.670528	0.873335
γ_{14}	−4.299791	0.693569	γ_{45}	11.037816	2.300782
γ_{15}	0.963943	0.544455			

the values determined in Section 4.2, $\hat{\beta}$. We then examine the likelihoods to determine the optimal values for r and s. This process resulted in a GM(0,7) formula being adopted for the baseline hazard, so that our final model is

$$\log \lambda_{xy} = \sum_{i=0}^{6} b_i C_i(x') + \sum_{i=1}^{4}\left(\beta_i + \sum_{j=1}^{5} \gamma_{ij} w_j(x')\right) C_i(y'). \qquad (5)$$

Note that a computational advantage of choosing $r = 0$ is that we have a log-linear structure for our model and all the parameters can be estimated in the GLM framework. All the parameters in formula (5), including the β and γ parameters, are (re-)estimated using maximum likelihood. The final parameter estimates and their standard errors are set out in Table 3. None of the re-estimated parameter values in Table 3 is significantly different from its initial estimate, $\hat{\beta}$.

The solid lines in Figure 3 show the modelled values for the age-dependent coefficients $\{\beta_{ix}\}$, calculated using the final parameter values in Table 3.

4.4 *Results — Males*

Figure 4 shows the crude and modelled mortality intensities (on the log scale) plotted against age for calendar years 1975, 1995 and 2003. Also shown are 95% confidence intervals for the crude estimates.

Table 4 gives a summary of some of the formal statistical tests of a graduation, applied separately to all 29 calendar years. These tests are

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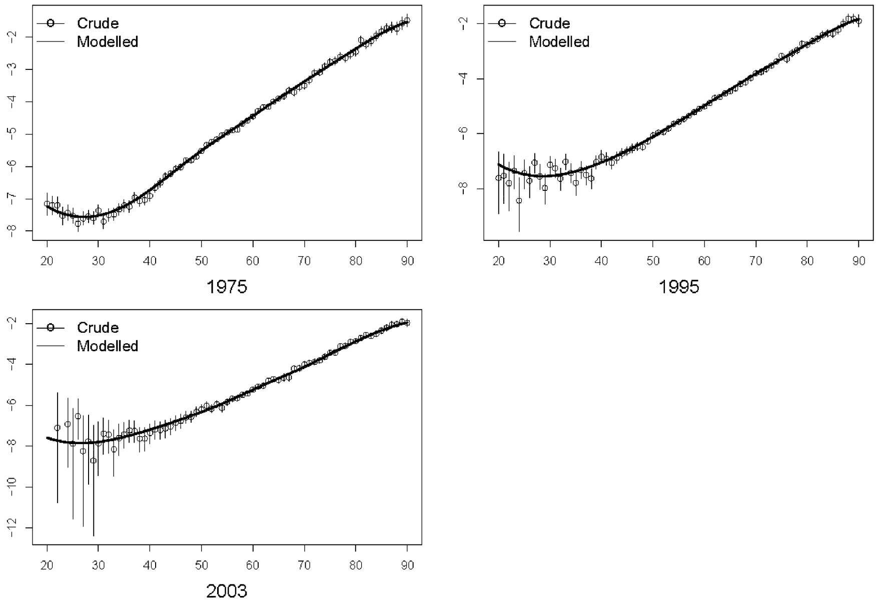

Figure 4. Mortality intensities (log scale) for males

carried out on the relative deviation after fitting the model in Equation (5) to the data. The *p*-values for the sign test, runs test, Kolmogorov–Smirnov test, serial correlation test (with lags 1, 2 and 3) are all shown in Table 4. These statistical tests are discussed in detail in Forfar *et al.* (1988). Any *p*-value less than 5%, indicating failure of the test concerned, is highlighted by an asterisk. The results in Table 4 show that our model for the mortality of male assured lives fits reasonably well. Of the 174 *p*-values shown, only seven (less than 5%) are significant in the sense of being less than 5%. Also, there are no years with more than a single significant *p*-value, so that the model does not fit badly in any particular year.

Figure 5 shows a comparison between our model for the mortality of male assured lives and information relating to the AM92 graduation (CMIR 17 (1999)). This is of interest since AM92 was graduated using a subset of our data and our model provides a 'graduation' for 1992 by setting the Year covariate to 1992. In detail, Figure 5 shows:

(i) The log of the mortality intensity from our model with Year set to 1992 (equivalently $y' = 3/14$).
(ii) The point estimates of the log of the mortality intensity, plus and minus two standard deviations, for the durations 2+ data relating to

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Table 4. Test statistics (*p*-values %) for the model for male assured lives

Year	Sign test	Runs test	KS test	Serial correlation test 1	2	3
1975	50.0	23.7	78.5	4.9*	23.6	69.8
1976	95.2	17.3	86.9	3.6*	88.4	85.3
1977	11.7	38.8	66.0	81.8	85.1	21.4
1978	23.8	18.9	81.5	21.0	45.8	40.4
1979	11.7	38.8	29.0	27.9	95.1	54.1
1980	50.0	11.6	96.2	61.3	95.1	13.5
1981	31.7	42.1	48.9	24.6	59.6	90.7
1982	50.0	83.1	39.8	77.0	14.1	39.7
1983	82.9	85.3	66.0	73.7	12.7	92.7
1984	4.8*	55.8	64.3	24.4	80.3	28.5
1985	59.4	83.1	26.4	71.2	63.0	34.4
1986	76.2	61.0	7.2	33.0	10.4	74.9
1987	7.7	41.9	6.5	50.8	2.9*	57.3
1988	40.6	32.2	48.8	12.4	12.9	97.2
1989	31.8	33.1	95.2	37.8	39.6	40.4
1990	99.2	55.0	75.3	28.6	19.5	77.3
1991	82.9	71.4	65.0	70.8	79.5	14.1
1992	50.0	76.3	98.0	36.8	88.9	91.4
1993	23.8	8.6	35.9	1.0*	60.4	41.4
1994	11.8	22.0	76.9	8.1	76.5	86.6
1995	0.8*	51.2	94.1	27.6	89.9	23.9
1996	7.7	99.9	30.9	48.2	96.4	63.6
1997	59.4	59.4	81.3	22.1	21.0	85.9
1998	97.2	87.6	44.4	69.8	27.3	62.1
1999	68.2	60.0	99.7	56.0	48.1	71.3
2000	31.8	1.7*	9.5	6.0	48.8	13.1
2001	7.7	61.4	98.4	83.5	68.7	83.0
2002	40.6	24.1	30.8	36.1	80.9	87.7
2003	76.2	12.3	87.1	17.0	9.7	94.1

permanent assurances for males, 1991-4. These values have been calculated from the adjusted R_x and adjusted A_x values in CMIR 17 (1999, Table 1.15).

(iii) The log of the graduated mortality intensity for AM92, durations 2+. These values have been calculated from the GM(2,3) formula with the parameters given in CMIR 17 (1999, Table 1.6).

It can be seen that our model and AM92 are very close at all ages, except the youngest ages. At the youngest ages our model fits the data rather better than the AM92 graduation. Factors to bear in mind are that our model was fitted to data for all durations, not just durations 2+, and that our model has considerably more parameters than a GM(2,3) model.

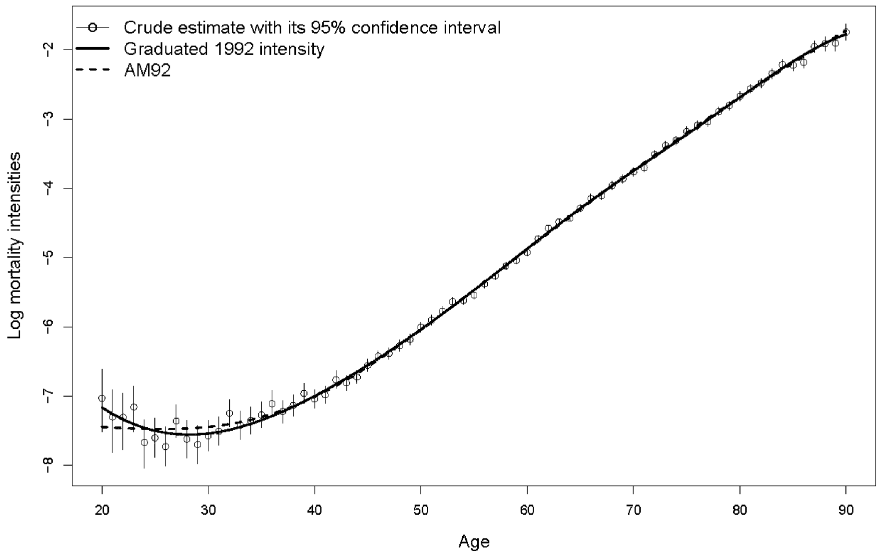

Figure 5. Mortality intensities (log scale) for males: a comparison with AM92

5. Mortality Model for Female Assured Lives

5.1 *Modelling the Relative Risk*

The modelling of the mortality for female assured lives follows broadly the same process used in Section 4, though the details differ. Our starting point is a proportional hazards-type model:

$$\log \lambda_{xy} = h_x + \phi_y \tag{6}$$

where the baseline hazard is a function of age only, and the relative risk is a function of calendar year, but, as we shall see, with age-dependent coefficients.

Preliminary investigations assumed that the relative risk, $\exp(\phi_y)$ in formula (6), had the following functional form:

$$\phi_y = \sum_{j=1}^{k-1} a_j w_j(y')$$

where the functions $w_j(.)$ are natural cubic splines defined relative to a set of k knots. See Section 3. The Akaike Information Criterion indicated that six

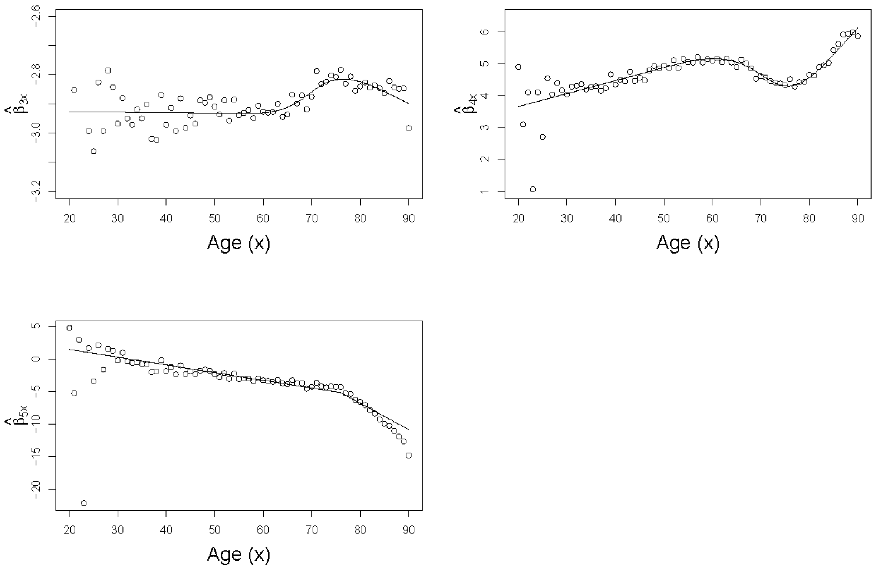

Figure 6. Age-dependent year coefficients for females

knots, placed at the years 1975, 1983.7, 1988.5, 1993, 1997.5 and 2002, is optimal. This structure was adopted.

Since $w_1(y')$ and $w_2(y')$ determine the spline between the first two knots, 1975 and 1983.7, and since we have no real data for years 1975 to 1982, it was decided that the coefficients of $w_1(y')$ and $w_2(y')$ should not depend on age, whereas the coefficients of the other splines could. The coefficients of $w_1(y')$ and $w_2(y')$ are denoted β_1 and β_2 and the coefficients of the other spline functions are denoted $\beta_i + \beta_{ix}$, $i = 3, 4, 5$. Plots of the crude estimates of β_{ix} as functions of age are shown in Figure 6. These estimates have been calculated using the same procedure as for males — see Section 4.1.

A lengthy trial and error process, guided by the plots in Figure 6, concluded with: β_{3x} being modelled by a natural cubic spline with four knots, at ages 55, 68, 72 and 86. β_{4x} being modelled by a natural cubic spline with four knots, at ages 50, 68, 75 and 86. β_{5x} being modelled by a linear spline with one knot, at age 76.

The natural cubic splines modelling β_{ix} for $i = 3, 4$ can be written:

$$\gamma_{i1} x' + \sum_{j=2}^{3} \gamma_{ij} w_j(x')$$

and the natural cubic spline modelling β_{5x} can be written:

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$$\gamma_{51}x' + \gamma_{52}(x' - (76-55)/35)_+.$$

Hence, our model for the mortality of female assured lives is now:

$$\log \lambda_{xy} = h_x + \beta_1 w_1(y') + \beta_2 w_2(y') + \sum_{i=3}^{4}\left(\beta_i + \gamma_{i1}x' + \sum_{j=2}^{3}\gamma_{ij} w_j(x')\right) w_i(y') + (\beta_5 + \gamma_{51}x' + \gamma_{52}(x' - (76-55)/35)_+) w_5(y').$$

By treating $\{h_x\}$ as a set of constants, the parameters in this formula can be estimated by maximising the partial likelihood. We denote this initial set of parameters $\hat{\beta}$.

5.2 *Modelling the Baseline Hazard*

As with the model for males, we model $\{h_x\}$ using a GM(r, s) formula, expressed as Chebycheff polynomials in x', choosing r and s to maximise the likelihood with the parameters in the relative risk fixed at $\hat{\beta}$. A GM(0,8) formula was chosen, so that our final model is

$$\log \lambda_{xy} = \sum_{i=0}^{7} b_i C_i(x') + \beta_1 w_1(y') + \beta_2 w_2(y') + \sum_{i=3}^{4}(\beta_i + \gamma_{i1}x' + \sum_{j=2}^{3}\gamma_{ij} w_j(x')) w_i(y') + (\beta_5 + \gamma_{51}x' + \gamma_{52}(x' - (76-55)/35)_+) w_5(y'). \quad (7)$$

All the parameters in this model are then (re-)estimated by maximum likelihood; the values and their standard errors are shown in Table 5.

5.3 *Results — Females*

Figure 7 shows the crude and modelled mortality intensities (on the log scale) plotted against age for calendar years 1983, 1995 and 2003. Also shown are 95% confidence intervals for the crude estimates.

Table 6 gives a summary of some of the formal statistical tests of a graduation, applied separately to all 21 calendar years. These results are calculated and displayed in the same way as those in Table 4. The results are even better than those for males — only two p-values, out of a total of 126, are less than 5%, and both of those are close to 5%. These results indicate that the model for the mortality of female assured lives fits the data very well.

Figure 8 shows the crude and modelled mortality intensities, on a log scale, for selected ages for females as functions of calendar year. The crude rates for 1977 are the FA77 rates.

Table 5. Parameter estimates for formula (7) with their standard errors

Parameter	Estimate	Standard error
b_0	−5.939330	0.614450
b_1	3.367757	0.032079
b_2	0.309219	0.025177
b_3	−0.061689	0.021643
b_4	0.111592	0.018500
b_5	−0.078545	0.015911
b_6	−0.005641	0.011097
b_7	−0.028507	0.010097
β_1	−0.748567	1.166394
β_2	0.496324	0.756464
β_3	−2.416903	2.550120
β_4	3.758088	2.540982
β_5	−2.651587	1.384525
γ_{31}	−0.693227	0.259576
γ_{32}	4.177638	1.009590
γ_{33}	−50.898874	9.424659
γ_{41}	2.755805	1.069499
γ_{42}	−3.670702	1.209876
γ_{43}	68.604318	15.027415
γ_{51}	−3.534043	1.879462
γ_{52}	−15.510799	5.744604

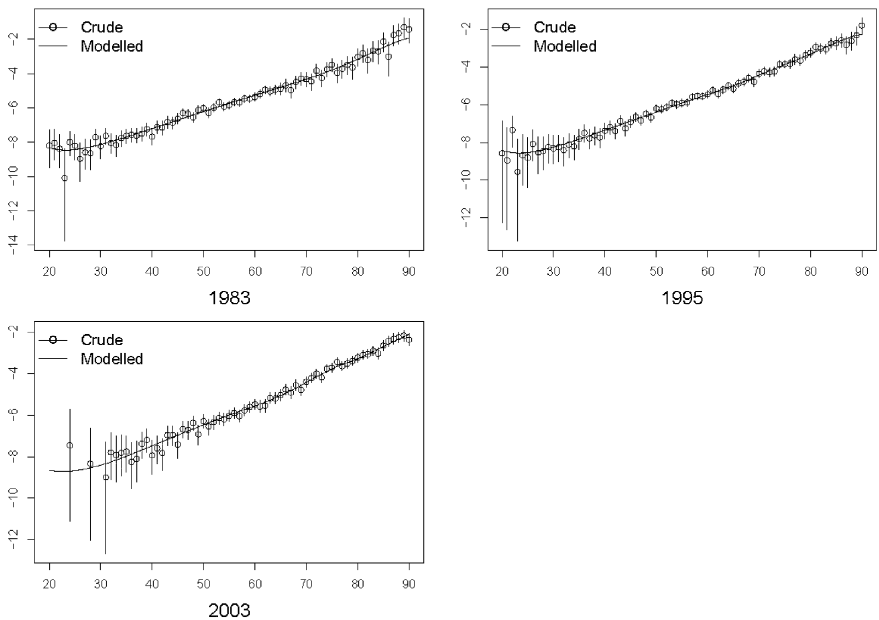

Figure 7. Mortality intensities (log scale) for females

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Table 6. Test statistics (p-values %) for the model for female assured lives

Year	Sign test	Runs test	KS test	Serial correlation test 1	2	3
1983	40.6	60.0	93.0	57.5	63.8	4.8*
1984	7.7	11.7	78.9	16.2	33.9	41.1
1985	68.2	17.3	90.9	58.8	80.4	51.6
1986	11.8	48.5	78.5	68.9	74.7	73.7
1987	17.1	20.2	34.6	9.4	35.8	67.6
1988	59.4	88.4	99.4	92.1	47.9	71.8
1989	50.0	68.5	51.0	56.3	69.1	79.8
1990	40.6	95.5	77.2	50.3	20.3	29.1
1991	50.0	92.5	99.2	96.0	67.5	57.7
1992	76.2	95.8	82.7	66.8	32.7	95.0
1993	88.2	55.4	59.4	89.0	6.5	78.4
1994	7.7	85.1	87.3	70.0	75.6	35.9
1995	50.0	83.1	98.3	95.8	58.0	37.7
1996	68.2	7.8	65.6	6.4	95.8	90.2
1997	59.4	83.1	89.0	49.6	86.9	90.7
1998	4.8*	65.1	63.9	55.3	31.4	15.5
1999	11.8	29.9	93.2	50.4	66.4	90.6
2000	31.8	8.1	97.2	37.1	22.3	86.6
2001	7.7	70.6	99.9	84.6	46.8	65.4
2002	50.0	40.6	64.9	85.0	11.0	7.4
2003	59.4	59.4	96.9	96.0	44.3	98.8

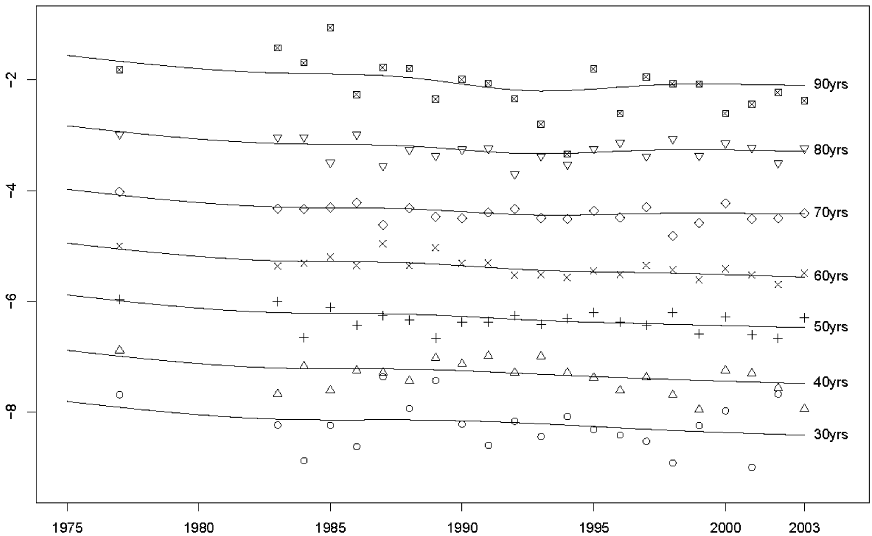

Figure 8. Crude and modelled mortality intensities (log scale) for selected ages for females

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Figure 9 shows a comparison between our model for the mortality of female assured lives and information relating to the AF92 graduation (CMIR 17 (1999)). In detail, Figure 9 shows:

(i) The log of the mortality intensity from our model with Year set to 1992 (equivalently $y' = 3/14$).
(ii) The point estimates of the log of the mortality intensity, plus and minus two standard deviations, for the durations 2+ data relating to permanent assurances for females, 1991-4. These values have been calculated from the adjusted R_x and adjusted A_x values in CMIR 17 (1999, Table 1.17).
(iii) The log of the graduated mortality intensity for AF92, durations 2+. These values have been calculated from the GM(1,2) formula with the parameters given in CMIR 17 (1999, Table 1.11).

It can be seen that our model and AF92 are very close at all ages, as was the case for the comparison with AM92 for males in Figure 5.

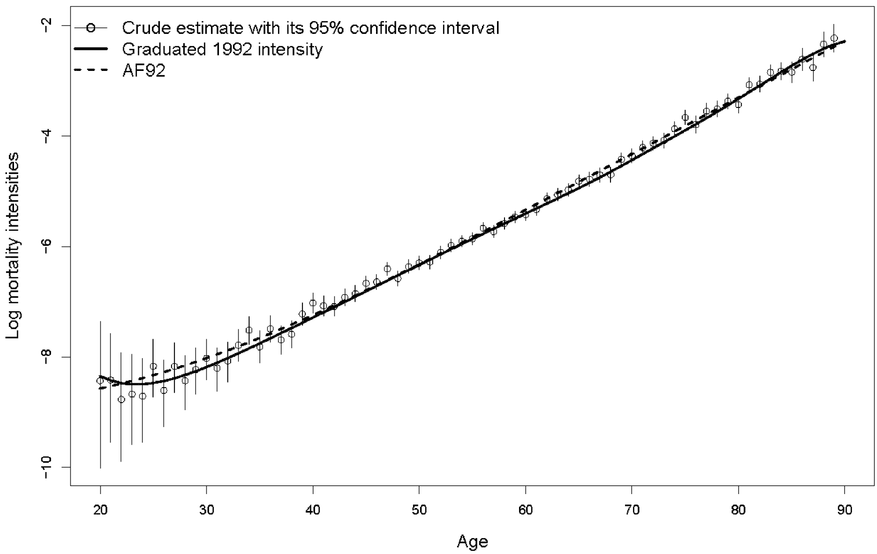

Figure 9. Mortality intensities (log scale) for females: a comparison with AF92

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6. Further Comments

Over the period in which we are interested, 1975 to 2003, mortality rates for UK assured lives have been improving, as can be seen from Figures 1 and 2, and so our models include calendar year as a covariate. The improvement in mortality among UK assured lives, other UK populations and in many other countries is of considerable importance both socially and financially. See, for example, Willetts *et al.* (2004). Given this, it is natural to check whether our models produce plausible mortality projections for years beyond 2003.

Figures 10 and 11 show projections of the mortality for selected ages for males and females, respectively, up to 2020. It can be seen that, while the projected rates from 2003 to 2020 for females appear plausible, those for males do not. The explanation for this difference can be found in the different ways in which we have modelled the effect of calendar year. For males this was through the use of Chebycheff polynomials, $C_i(y')$. See formula (5). These polynomials are stable over the range -1 to $+1$ but not stable outside this range. Our transformation of y to y', formula (5), ensures that y' is between -1 and $+1$ for the range of our data, but not beyond 2003. The calendar year effect for females is modelled using splines.

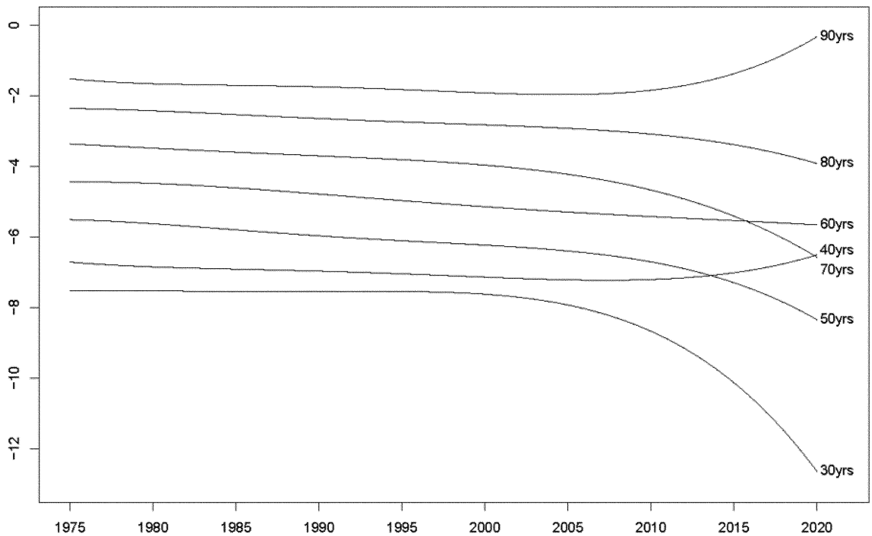

Figure 10. Projected mortality intensities (log scale) for selected ages for males

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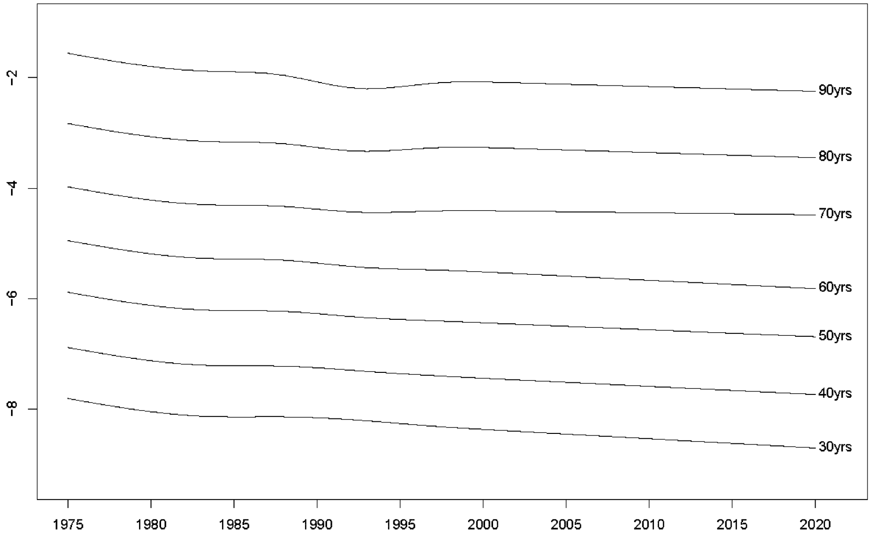

Figure 11. Projected mortality intensities (log scale) for selected ages for females

See formula (7). As noted in Section 3, these functions are linear beyond the final knot.

However, just because the projected mortality rates for females *may appear* plausible, there is no reason to believe that *they are* plausible. It was not our intention to produce rates which could be projected beyond the limits of our data.

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