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RECENT MORTALITY TRENDS IN THE SPANISH POPULATION

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ABSTRACT

Our research deals with the way that calendar time affects mortality patterns in the Spanish population, and how this information can be used to elaborate predictions. A description of the observed mortality evolution has been worked out using data from 1975 to 1993. We have used Heligman-Pollard Law number two to model the evolution of Spanish mortality over the period and using univariate time series analysis, we have obtained a prognosis for years 1994 to 2010.

KEYWORDS

Graduation; Trends; Heligman-Pollard Laws; Life Tables

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1. INTRODUCTION

1.1 Currently, an '*appropriate* model' of the Spanish population probability of death does not exist. This paper aims to provide a standard graduation to compare and predict portfolio mortality. "The choice of the standard table is important, since any special feature in its graduation will be reproduced, even exaggerated, in the graduation of the new data." (Benjamin & Pollard, 1980, p328).

1.2 This paper focuses on the construction of Spanish population mortality tables incorporating calendar time effects. Our main goal is to project Spanish national mortality by class. We will start with the description and graduation of mortality for the Spanish population from 1975 to 1993. The evolution of the Spanish population mortality patterns will be accurately described. Afterwards, a univariate time series analysis will be used in order to model the time-varying process and to forecast the changes over time. The outcome will be the projection of the Spanish mortality to the nearest future.

1.3 The use of life tables in Spain has changed during the last few years, mainly as a consequence of European regulations. There are three main causes:

(1) There is evidence that, for the same territory and gender, mortality patterns are quickly changing.
(2) Each company intends to obtain its specific mortality table. One possibility is to model its own experience. Other companies choose to modify the population life table (or the national experience of several insurers).
(3) Applied statistical and mathematical techniques have been greatly innovated, and companies seem very motivated to use more accurate information on mortality.

1.4 Appendix A presents a general overview of the Spanish national mortality scenario. This helps to understand the motivations for the construction of a Spanish population mortality table, which will enable future mortality patterns to be forecast.

1.5 We have analysed the Spanish population data between 1975 and 1993, inclusive. Detailed information was used to estimate death rates, by cohorts of lives all born in the same calendar year τ. Using the census population and the civil registry of births, the number of lives was calculated for each cohort and every calendar year t. The census population data at any age are not typically reconciled with the number of lives in the relevant birth cohort minus the number of deaths recorded to that birth cohort prior to the census because of migration. Migration was neglected, and only a few approximations were necessary to avoid these kinds of discrepancies. The number of deaths for each cohort and every calendar year was calculated using the civil registry of deaths, where there is information on dates of birth (up to month and year). Migration was not a relevant phenomenon in Spain during the period of study, but it will certainly become crucial in the following years.

1.6 Both genders were studied separately, but geographical segmentation was not considered at this stage. From the database, we computed observed series of death rates, by gender, for each birth cohort and for each calendar year. It must be noted that for ages older than 90 the data are of little reliability.

1.7 Due to the available statistical information, there was not any need to make a central mortality approximation as in Navarro (1991) or Rue (1992). The main problem is that census data are collected on 1 March. These two authors used the INE (Instituto Nacional de Estadistica) printed database to estimate the probability of death q_x. Therefore, both of them considered the census data as the 'central exposed to risk', and, afterwards they approximated the q_x values with the actuarial estimate (see Macdonald, 1996a). In our research, a great effort has been made to avoid these approximations.

1.8 *Descriptive Analysis*

1.8.1 In Figure 1 the raw mortality rates for men and women in 1975 and in 1993 are plotted on a logarithmic scale. We see that the

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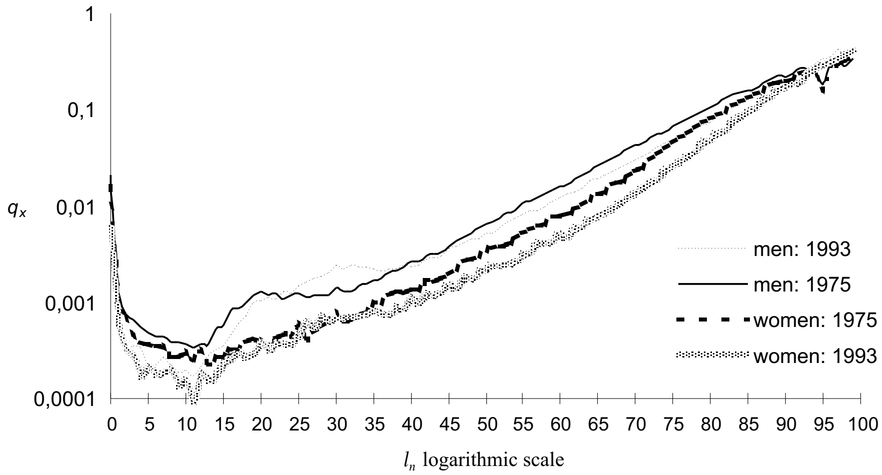

Figure 1. Observed mortality by gender

women's mortality curve shape and level is quite different to the one for men. We conclude that a separate graduation for each gender must be carried out.

1.8.2 When we compare men's probability of death in 1975 and in 1993, several changes are apparent. We notice that childhood mortality has decreased. It is also remarkable that the accident hump has suffered an increase in both the level and the spread. Moreover, its maximum is now located at older ages. For men's older adult ages we see a smoother behaviour, and, finally, mortality for the elderly (i.e., those aged over 90) has increased.

1.8.3 When considering the evolution of men's mortality, we should distinguish three age intervals. Up to age ten, we see a clear decreasing trend. From age 11 to age 40, the accident hump has become wider. From age 41 onwards, a clear decreasing trend shows up, which is very prominent around age 70.

1.8.4 We can also use Figure 1 to compare the probability of death for women in 1975 and 1993. In this case, we see that the mortality during childhood as well as in older adult ages has decreased. On the other hand, for ages older than 90, the mortality has increased. Nevertheless, we should note that for ages older than 90 our data are not reliable enough. There is also a slight accident hump becoming apparent for women. In fact, if we compare it with changes observed at other ages, the level of the accident hump seems to remain the same over the period.

1.8.5 When analysing women's evolution of mortality patterns, we should also distinguish three age intervals. Up to age 15, we see a clear

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Lines observed mortality, hyphened lines 95% confidence intervals bounds

Figure 2. Observed mortality, age 10

decreasing trend. From age 16 to age 35, no clear trend appears and the accident hump effect seems to be stable. From age 36 onwards, we notice again a clear decreasing trend, with a similar behaviour as the one observed for men.

1.8.6 As suggested by Forfar, McCutcheon & Wilkie (1988), 95% confidence intervals for the observed probability have been calculated, by age and year, using the properties of the binomial distribution. These confidence bounds will also enable us to identify possible trends.

1.8.7 For instance, in Figure 2 we see the probabilities of death and their 95% confidence intervals for ten-year-old children, from 1975 to 1993. In general, mortality in childhood and teens has decreased sharply. The average decrease over the period for the age interval between 0 and 15 is approximately 45%. Furthermore, it seems that the gender gap is steady. The excess mortality (men's against women's) for ages between 0-15 was 42% in 1975 and was 40% in 1993.

1.8.8 We have also analysed the probability of death and the 95% confidence interval for men and women aged 25. In Figure 3 the yearly estimates are plotted. We observe that the probability of death for 25-year-old males has increased over the period. This behaviour is different from the pattern observed at younger ages. At age 25, male mortality has basically remained the same until the mid 1980s and increased since then. This increasing trend seems to have stopped in 1991. Female mortality at age 25 shows no changes from 1976 to 1993. The probability of death has only decreased 5%.

Lines observed mortality, hyphened lines 95% confidence intervals bounds

Figure 3. Observed mortality, age 25

1.8.9 We know that the accident hump mainly affects ages from 20 to 35 approximately. It seems that the accident effect has increased more among men than among women. Men's probability of death has increased by an average of 65% over the period and over the age range between 20 and 35. For women, the average increase for this same age range is 1%. The men vs. women excess mortality for ages between 16-35 was 142.8% in 1975, but increased to 199.9% in 1993.

1.8.10 Could AIDS be the reason for the shift in the location of the male accident hump? Not having enough data to investigate the case, we have no clear conclusions. We should point at changes in the lifestyle of youngsters, but we suspect that AIDS could be at least one of the reasons for the shift in the accident hump effect. According to 1995 data (http://www.prous.com/ttm), 63.4% of the new infected cases are due to drug dependency and the average age at diagnosis is 33.9. In Figure 4 Spanish male mortality caused by AIDS is shown depending on age and calendar year. It seems that the peak is reached at nearly the same ages from 1990, but the death rate has increased greatly.

1.8.11 We can also perform an analysis of the evolution of mortality rates for people aged 80. The trends are shown in Figure 5. For adult ages (i.e., from age 36 onwards), the probability of death has clearly decreased for both genders. The average decrease in the interval from age 36 to age 75 is 17% for men and 36% for women. For the interval between age 76 and age 90, men's decrease is 23% on average and women's decrease is 31% on average.

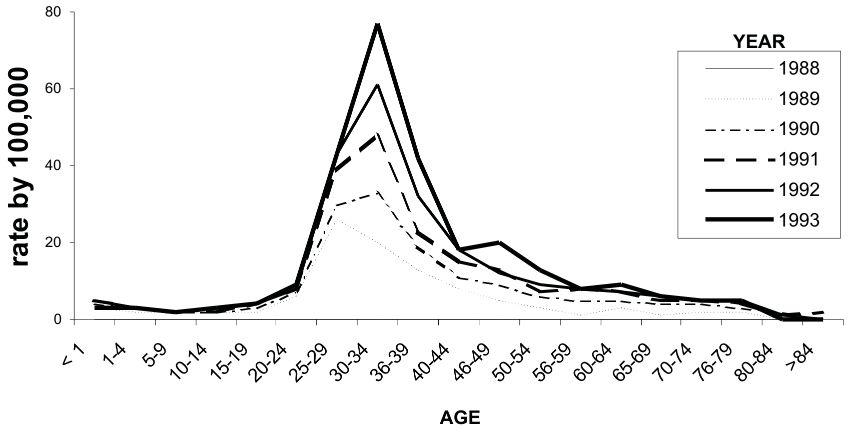
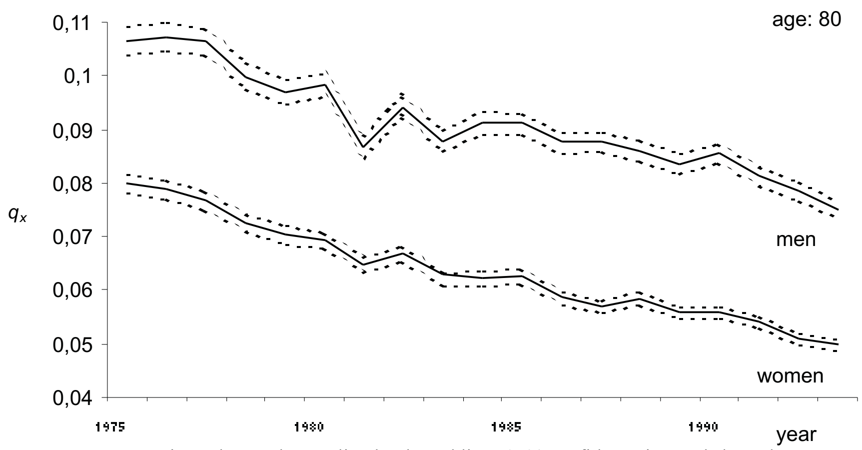


Chart from www.prous.com

Figure 4. Spanish male mortality caused by AIDS, rate by 100,000



Lines observed mortality, hyphened lines 95% confidence intervals bounds

Figure 5. Observed mortality, age 80

1.8.12 The excess mortality of men vs. women for ages between 36 and 75 was 89.2% in 1975 and 142.3% in 1993. This phenomenon was not so strong for ages between 76 and 90. The excess mortality of men to women for the elderly was 29.6% in 1975, and it increased to 46.1% in 1993. In both age ranges the gap between male and female mortality is increasing, implying that the evolution patterns are different.

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1.8.13 We conclude that, in the observed period, the mortality pattern for the Spanish population is different for men and women. Moreover, the evolution throughout the period depends on age. In the case of the male accident hump, data on AIDS seem to indicate that this is by far the most plausible reason for the change in the shape.

2. Modelling Recent Mortality Trends

2.1 In this section, we will present the methodology that we propose to model the evolution of Spanish mortality from 1975 to 1993 and to forecast future changes. We will start by defining the model. Then, we will compare the estimates with the observed data. Finally, we will forecast q_x for 2000 and 2010.

2.2 *Mortality Model for the Spanish Population*

2.2.1 We have chosen the Heligman-Pollard Laws to model q_x and to explain the differences in age and gender for each calendar year. As noted by Rogers & Gard (1991), these laws are efficient when comparing and analysing several mortality factors, such as age, gender, ethnicity, territory and time. In previous works (Rue, 1992), the Heligman-Pollard Law number two has been found to be adequate to model Spanish mortality. In addition, the nine parameters have a demographic or biological interpretation (Heligman & Pollard, 1980), which could help us to understand the whole process.

2.2.2 We assume that the probability of death in the Spanish population can be modelled using Heligman-Pollard Law number two, for each calendar year t and gender:

$$q_{x,t} = A_t^{(x+B_t)^{C_t}} + D_t e^{-E_t(\ln x - \ln F_t)^2} + \frac{G_t H_t^x}{1 + K_t G_t H_t^x}. \tag{1}$$

2.2.3 In the previous expression, the first right-hand-side term models childhood mortality, the second one captures the accident hump and the third term refers to the 'natural' mortality caused by senescence (Heligman & Pollard, 1980).

2.2.4 We have estimated the model parameters using non-linear weighted least squares for each year. The covariance and correlation matrices have been obtained, and confidence intervals for both the parameters and the adjusted q_x themselves were constructed. The studentised residuals were also analysed to check the model adequacy.

2.2.5 In the model, calendar time only affects the curve's structure, i.e., the parameters. So, we assume that some parameters must change over the period according to its biological-demographic interpretation. Once the evolution of the parameters is modelled using univariate time series analysis, we obtain forecasts of the q_x curve by substituting the parameter forecasts in (1). McNown & Rogers (1988) and Rogers & Gard (1991) applied an analogous methodology to analyse United States mortality using Heligman-Pollard Law number one.

2.3 *Graduation*

As the graduation varies greatly, depending on the range of ages used in the fitting process, we chose the greatest range with acceptable results based on the regression weighted sum of squares against the uncorrected total weighted sum of squares ratio. Consequently, the q_x extrapolation for the oldest ages (usually greater than 90) showed a strong bias that could not be solved by the model. Thus, the model was graduated from ages 0 to 90, this age range being greater than usual. As suggested by Rogers & Gard (1991), a measure of the goodness-of-fit is the MAPE (mean absolute percentage error). The sample size and MAPE results will be shown below.

2.4 *The Parameter Estimates*

2.4.1 The parameters of law (1), by year and gender, were estimated and are presented in Tables 1 and 2 for men and women respectively.

2.4.2 The method used for fitting the model was non-linear least squares, weighted by the inverse of the probability of death, as suggested by Heligman & Pollard. Nevertheless, several tests pointed out that the best

Table 1. Men's parameter estimates of Heligman-Pollard Law number two, for the Spanish population

t	A (10^{-3})	B (10^{-3})	C (10^{-3})	D (10^{-3})	E	F	G (10^{-5})	H	K
1975	1.167	1.924	88.862	0.700	6.405	21.994	3.964	1.106	1.307
1976	1.170	3.341	92.945	0.685	7.352	21.618	4.017	1.105	1.195
1977	1.149	2.422	86.153	0.726	8.711	21.860	3.880	1.105	1.291
1978	1.224	4.493	91.230	0.782	7.281	21.845	4.310	1.103	1.151
1979	1.108	5.366	93.922	0.654	8.942	21.368	5.219	1.098	−0.010
1980	1.118	6.458	91.402	0.687	8.511	22.203	4.173	1.103	0.693
1981	0.906	5.345	93.100	0.613	9.268	21.406	4.764	1.099	0.500
1982	0.944	5.930	89.251	0.641	8.620	22.058	4.087	1.102	0.513
1983	0.865	6.943	92.695	0.720	10.455	21.686	5.101	1.098	0.043
1984	0.775	4.823	86.173	0.699	8.729	21.642	4.941	1.098	−0.031
1985	0.807	11.310	95.825	0.805	10.116	22.004	4.891	1.098	−0.203
1986	0.753	13.362	104.424	0.919	8.241	22.298	5.359	1.096	−0.069
1987	0.739	11.635	100.090	1.004	7.408	22.874	4.974	1.097	0.064
1988	0.694	12.282	96.267	1.217	6.537	23.877	4.819	1.098	−0.056
1989	0.737	20.459	106.471	1.285	5.955	24.383	4.852	1.097	−0.184
1990	0.745	24.778	109.270	1.397	5.588	25.215	4.689	1.098	−0.078
1991	0.660	19.323	104.023	1.437	5.622	26.383	4.527	1.098	0
1992	0.637	16.625	101.585	1.351	4.968	27.821	4.326	1.098	0.043
1993	0.532	9.488	90.241	1.227	5.429	28.782	4.666	1.097	−0.397

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Table 2. Women's parameter estimates of Heligman-Pollard Law number two, for the Spanish population

t	A (10^{-3})	B (10^{-3})	C (10^{-3})	D (10^{-3})	E	F	G (10^{-5})	H	K
1975	1.026	2.218	84.604	0.535	2.658	38.3	0.523	1.129	1.093
1976	1.066	8.368	100.811	0.540	1.885	38	0.414	1.132	1.472
1977	1.104	7.292	94.203	0.588	2.369	39.2	0.318	1.136	1.755
1978	1.100	8.803	96.747	0.544	1.719	38.2	0.349	1.134	1.463
1979	0.898	3.127	80.564	0.587	2.547	40	0.281	1.136	1.726
1980	0.882	4.563	81.202	0.493	1.928	39	0.316	1.134	0.858
1981	0.815	7.653	91.743	0.476	1.619	40	0.327	1.132	0.821
1982	0.823	12.057	98.697	0.448	1.626	37	0.334	1.132	0.580
1983	0.907	25.479	111.919	0.512	1.373	39	0.283	1.134	0.896
1984	0.907	47.595	127.723	0.414	1.123	36	0.367	1.129	0.136
1985	0.733	12.849	91.397	0.422	1.536	36	0.334	1.130	0.001
1986	0.689	12.689	94.304	0.444	1.236	36.8	0.410	1.126	−0.330
1987	0.685	21.996	102.797	0.504	1.326	37	0.374	1.127	−0.175
1988	0.753	27.837	106.630	0.545	1.361	38	0.301	1.131	−0.090
1989	0.572	6.078	80.072	0.562	1.380	39	0.279	1.131	−0.126
1990	0.608	13.832	91.873	0.569	1.434	36	0.236	1.134	0.099
1991	0.568	10.461	86.396	0.646	1.583	39	0.192	1.137	0.395
1992	0.578	14.352	90.699	0.569	2.118	35	0.290	1.129	−0.602
1993	0.550	12.248	85.477	0.591	2.428	35.5	0.311	1.128	−1.083

results could be achieved when the weight is the inverse of the variance of the maximum likelihood estimate of the probability of death as a binomial parameter. The graduation for men and women was slightly different; as Spanish women have no accident hump, fitting parameter F became difficult. As a result, we constrained the F parameter inside a bounded region. We used a grid method to locate the best possible bound of parameter F for each year only for the females' data (i.e. we tried a variety of possible values for the bound value for F). Our final bound for F was chosen on the grounds of the best overall fitting in terms of least squares and MAPE. That was not necessary for men.

2.4.3 A Gauss-Newton method was used (i.e., we worked out first derivatives), and we got convergence in all cases. The parameter estimates follow asymptotically a normal distribution and the t ratios are suitable in that case. The t ratio statistics are presented in Table 3 for men and in Table 4 for women.

2.4.4 In Tables 3 and 4 it can be observed that parameter B is not significantly different from 0, and it has a great variance when compared to its estimated value. For both genders, the K parameter presents a development from a positive value to a negative one (being non-significant from 1983 to 1992 for men and from 1984 to 1992 for women). Basically, we have found that this parameter is closely related to the maximum age adjusted. Table 5 presents the maximum age considered for each gender and year. This age has remained approximately the same for all years (except for

Table 3. *t*-ratio results; men

t	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
1975	6.79	0.76	5.74	9.02	3.66	29.84	16.26	1070.03	14.14
1976	7.70	0.98	6.82	10.08	4.22	35.81	18.90	1241.74	14.05
1977	7.32	0.86	6.33	9.50	4.08	37.71	17.63	1159.11	13.97
1978	6.98	0.91	6.10	9.74	4.10	34.93	16.40	1075.91	10.93
1979	10.49	1.46	9.43	12.94	5.49	50.49	22.70	1412.54	−0.07
1980	7.83	1.12	7.11	10.27	4.31	39.10	19.41	1280.92	7.59
1981	7.95	1.12	7.28	10.75	4.64	43.25	22.65	1483.14	5.40
1982	7.30	1.02	6.58	10.11	4.33	38.78	19.66	1299.44	5.42
1983	5.94	0.88	5.50	9.06	4.04	38.96	17.66	1158.64	0.35
1984	7.76	1.03	6.77	12.46	5.42	49.04	23.65	1558.30	−0.35
1985	8.41	1.39	7.98	15.26	6.86	64.57	26.86	1770.85	−2.57
1986	7.43	1.33	7.27	16.54	7.41	61.61	24.03	1573.06	−0.72
1987	6.69	1.14	6.40	16.14	7.15	56.49	20.27	1331.18	0.58
1988	6.47	1.09	6.06	19.24	8.33	61.84	19.35	1280.62	−0.51
1989	6.57	1.29	6.51	21.32	9.06	63.72	19.25	1277.43	−1.64
1990	7.27	1.52	7.38	25.68	10.59	71.95	20.13	1340.78	−0.75
1991	5.78	1.13	5.78	21.98	8.93	60.32	15.87	1060.87	0
1992	7.64	1.44	7.58	28.14	10.52	67.19	18.52	1248.34	0.39
1993	6.05	0.97	5.69	20.93	7.78	52.27	15.28	1032.28	−2.87

Table 4. *t*-ratio results; women

t	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
1975	5.60	0.67	5.09	5.52	1.72	6.19	4.42	318.85	3.95
1976	5.02	0.79	4.76	5.98	1.66	5.08	4.69	346.33	7.16
1977	5.19	0.80	5.08	6.22	1.78	6.04	4.35	322.05	8.17
1978	5.11	0.77	4.58	6.07	1.62	4.86	4.50	333.84	6.63
1979	4.29	0.53	3.86	5.33	1.57	5.52	3.64	270.17	6.32
1980	4.42	0.55	3.67	4.79	1.36	4.29	3.80	280.45	2.89
1981	4.21	0.61	3.70	4.56	1.36	3.66	3.68	272.67	2.41
1982	4.43	0.71	3.98	6.02	1.58	4.66	4.43	325.78	2.05
1983	4.50	0.91	4.35	6.31	1.73	4.30	4.18	309.22	2.97
1984	4.42	1.07	4.34	7.03	1.69	4.08	5.03	370.94	0.49
1985	5.09	0.77	4.22	7.42	1.88	5.66	5.41	398.75	0.01
1986	4.75	0.69	3.73	6.67	1.73	4.47	4.94	364.62	−1.08
1987	4.73	0.85	4.12	8.54	2.17	5.64	5.42	404.72	−0.69
1988	4.31	0.84	3.94	7.75	2.02	5.16	4.35	322.43	−0.27
1989	4.77	0.57	3.55	8.03	2.17	5.70	4.71	351.12	−0.41
1990	4.12	0.63	3.36	9.81	2.29	6.78	4.79	358.27	0.36
1991	3.96	0.58	3.31	9.27	2.39	6.72	4.30	323.99	1.36
1992	4.37	0.73	3.99	11.11	2.79	10.14	4.78	351.21	−1.64
1993	4.64	0.74	4.25	12.08	3.15	12.29	5.23	383.73	−3.11

Table 5. Goodness of fit; men and women

Year	Men		Women	
	Maximum age	MAPE	Maximum age	MAPE
1975	90	4.374	88	9.555
1976	90	4.875	90	9.026
1977	90	4.862	90	10.125
1978	90	4.905	90	9.876
1979	85	3.316	90	11.052
1980	90	4.596	89	10.180
1981	90	3.680	89	10.888
1982	90	4.523	89	9.785
1983	90	3.859	89	9.901
1984	90	3.711	89	10.408
1985	90	3.921	89	8.750
1986	90	4.266	89	9.958
1987	90	4.945	90	9.444
1988	90	4.597	89	9.764
1989	90	4.564	89	8.432
1990	90	3.871	90	9.219
1991	90	5.264	90	9.717
1992	90	5.071	89	9.704
1993	90	5.552	89	9.427

* MAPE means mean absolute percentage error

1979 which can be considered as an outlier), while the curve was developing to get the maximum probability ($x_0/q_{x0} = 0.5$) at older ages. This development has been translated to the K parameter as an 'increased upward concavity', as in Figure 1.

2.4.5 For women, the E parameter has evolved from not being significant to becoming statistically significant since 1990. If the E parameter were 0, that would mean an infinite variance of the accident hump, and obviously that was the case, as no accident hump was observed at the beginning of the period. In the 1990s, probably because of the AIDS effect and other social behaviour (alcohol, smoking, ...), women's probabilities of death at young ages have increased to form a little accident hump. That development is reflected on the t-ratio statistics.

2.4.6 In the graduation of the law for each year the asymptotic correlation matrix was calculated. The values were very regular, although the estimates of the parameter themselves were changing. In Table 6 and 7 the mean of the asymptotic correlation matrix for each gender is displayed. The difference between the tables could be explained either by the distinct pattern of genders or the inequality constraints used for modelling female mortality.

2.4.7 To complete the information, graphs of the parameter estimates and their 95% confidence intervals by gender throughout the analysed period are shown in Appendix B.

Table 6. Average asymptotic correlation matrix for the parameter estimates; men

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
A	1.000								
B	0.887	1.000							
C	0.825	0.985	1.000						
D	0.129	0.180	0.189	1.000					
E	−0.194	−0.304	−0.325	0.354	1.000				
F	−0.044	−0.121	−0.140	−0.057	−0.151	1.000			
G	0.079	0.173	0.194	−0.062	0.298	−0.332	1.000		
H	−0.071	−0.161	−0.182	0.066	−0.294	0.322	−0.993	1.000	
K	−0.041	−0.113	−0.130	0.069	−0.246	0.257	−0.860	0.908	1.000

* Averages are taken over calendar years

Table 7. Average asymptotic correlation matrix for the parameter estimates; women

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
A	1.000								
B	0.899	1.000							
C	0.845	0.986	1.000						
D	0.256	0.355	0.374	1.000					
E	−0.263	−0.438	−0.478	−0.538	1.000				
F	0.096	0.168	0.184	0.692	−0.836	1.000			
G	−0.098	−0.143	−0.179	−0.762	0.675	−0.809	1.000		
H	0.094	0.157	0.171	0.753	−0.654	0.790	−0.998	1.000	
K	0.070	0.115	0.125	0.647	−0.456	0.637	−0.910	0.931	1.000

* Averages are taken over calendar years

2.5 *The Results*

2.5.1 We compared adjusted and observed mortality rates, taking into account the confidence intervals. As an example, we have plotted the studentised residuals of the graduation results for men at year 1990 in Figure 6. The greatest variability is located around the older ages.

2.5.2 In Figure 7 we compare observed versus adjusted probability of death for men from ages 65 to 90 in 1990.

2.5.3 When looking at several goodness-of-fit measures (regression weighted sum of squares, studentised residuals and MAPE), we conclude that the model fits adequately for men. For women the lack of accident hump complicates the estimate process, and the studentised residuals turn out to be related to the age. Furthermore, the interpretation of the parameter estimates and their evolution becomes more difficult.

2.5.4 Although it was difficult to fit the Heligman-Pollard equation for women, we preferred to use the same model for both genders instead of using

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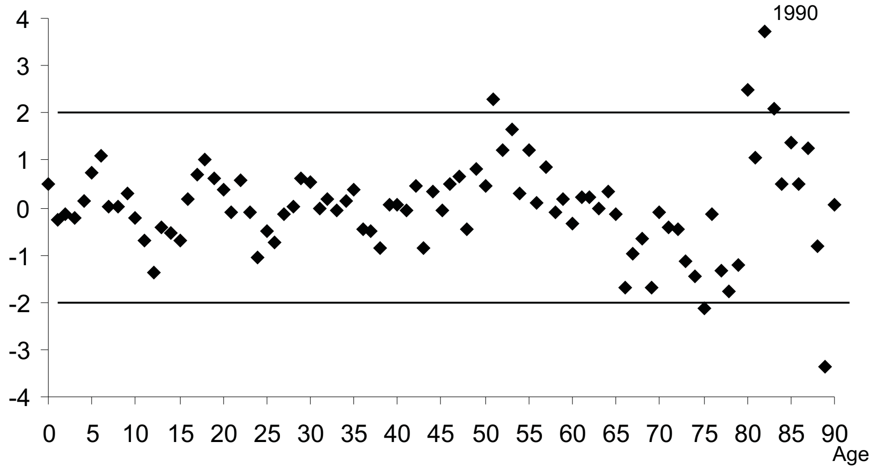

Figure 6. Studentised residuals for male probability of death graduation in 1990

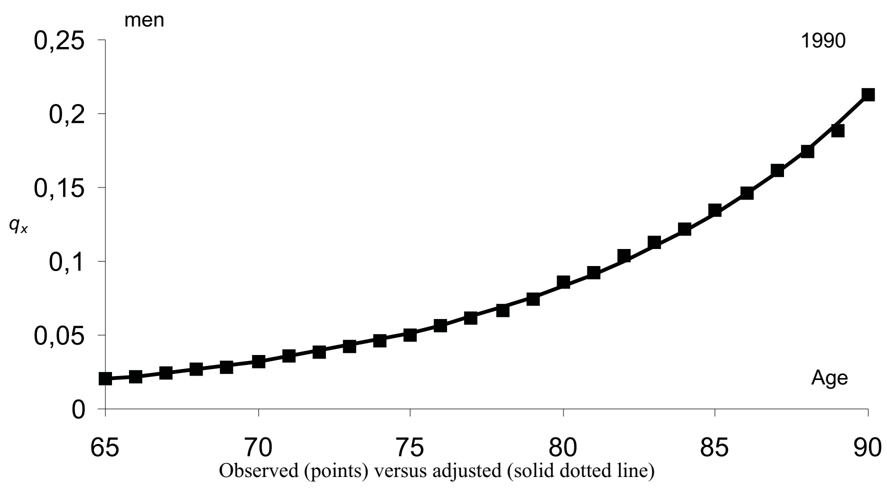

Figure 7. Observed versus adjusted probabilities of death for Spanish men in 1990; ages from 65 to 90

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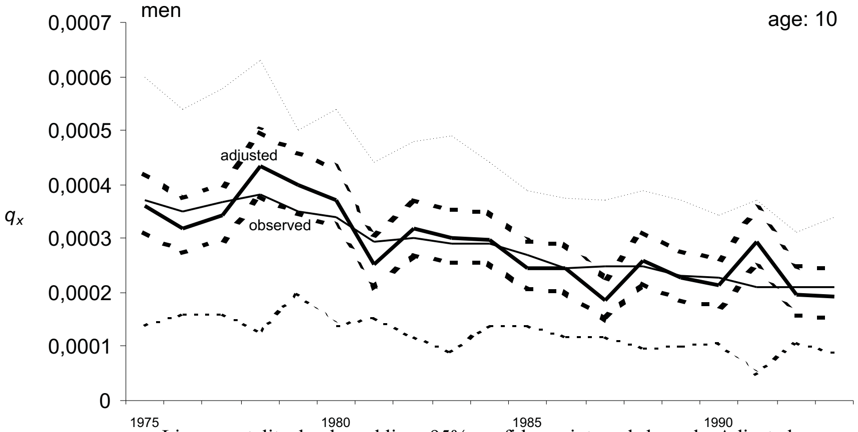
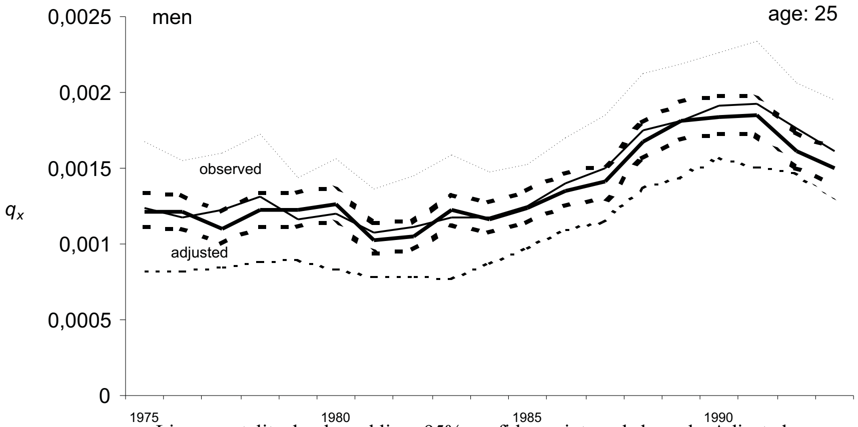


Lines mortality, hyphened lines 95% confidence intervals bounds. Adjusted (solid line) versus observed (thick solid line)

Figure 8. Adjusted versus observed confidence intervals comparison for male mortality rates, age ten



Lines mortality, hyphened lines 95% confidence intervals bounds. Adjusted (solid line) versus observed (thick solid line)

Figure 9. Adjusted versus observed confidence intervals comparison for male mortality rates, age 25

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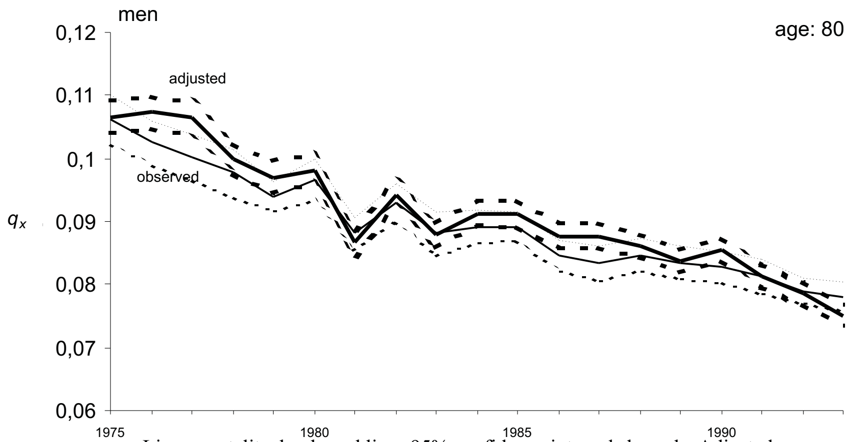

Lines mortality, hyphened lines 95% confidence intervals bounds. Adjusted (solid line) versus observed (thick solid line)

Figure 10. Adjusted versus observed confidence intervals comparison for male mortality rates, age 80

a different formula for women. For comparative purposes, it is helpful to use the same formula for both sexes.

2.5.5 From Figures 8 to 10 we compare adjusted and observed mortality rates for men aged ten, 25 and 80. Graphically, for a given age, it is possible to compare the observed q_x and their 95% binomial confidence intervals with the graduated q_x and their 95% confidence intervals throughout the period. Generally, the graduated q_x should be located between the binomial bounds.

3. Time Series Modelling of the Parameters

3.1 *Time Series Modelling*

3.1.1 After the graduation, a univariate analysis has been performed to forecast the changes of the parameters over the period. For instance, an AR(1) model with constant was adjusted to parameter A estimates. Thus, parameter A forecasts for years 2000 and 2010 are 0.47×10^{-3} and 0.39×10^{-3} respectively. Once the forecasting model is estimated, we can construct what-if scenarios with different shocks on different parameters. Afterwards, the corresponding forecast q_x-curve can be retrieved.

3.1.2 In general, an autoregressive model was estimated. This means that a shock only affects a parameter until a new value (structure) is reached. An autoregressive model also implies that projections in excess of ten years contain only long-term trends, thus its reliability is unknown, as the effects of current

Table 8. Univariate time series analysis models for the parameter estimates

	Men	Cons	Trans	Women	Cons	Trans
A	ARIMA(1,0,0)	X		ARIMA(1,0,0)		
B	ARIMA(1,0,0)	X	X	ARIMA(1,0,0)	X	X
C	ARIMA(1,0,0)	X		ARIMA(0,0,0)	X	X
D	ARIMA(2,0,0)	X		ARIMA(1,0,0)	X	
E	ARIMA(1,1,1)			ARIMA(2,0,0)	X	X
F	ARIMA(1,1,0)		X	ARIMA(1,0,0)		
G	ARIMA(0,0,0)	X	X	ARIMA(1,0,0)	X	X
H	ARIMA(1,1,0)			ARIMA(1,0,0)	X	X
K	ARIMA(0,1,1)	X		ARIMA(1,0,0)		

Cons indicates that the model incorporates a constant in its specification
Trans indicates that the variable in the model is the natural logarithm of the parameter

shocks will have ceased and no new shocks will have been incorporated. The only way of incorporating new shocks and their corresponding effects is through the construction of what-if scenarios.

3.1.3 The univariate models finally applied are shown in Table 8. ARIMA(s_1, s_2, s_3) means that the model is autorregresive of order s_1, moving average of order s_3, and that s_2 differences of the series have been used (see, for example, Greene, 1998). In order to choose the final model, six tests of goodness of fit and five tests on the residuals were applied, but, in several cases, the election of a model was difficult.

3.2 *Model Forecasts*

3.2.2 Applying parameter forecasts to (1), we obtained the corresponding q_x forecasts for years 1994 to 2010. Projections for years 2000 and 2010 are presented in Appendix C. For instance, in Figure 11 we plot Spanish male q_x-curve for year 1993 and the forecasted curves for years 2000 and 2010. The corresponding female forecasts on q_x-curves are shown in Figure 12.

3.2.3 Some trends can be identified in Figures 11 and 12. The decreasing trend in children's mortality continues. The 'accident hump' ages will not show an increasing mortality trend for men and it will be maintained for women. For older ages, mortality will eventually change its decreasing trend and probabilities will become greater in the near future. Concluding that mortality may be forecast to deteriorate in the future, rather than to continue its very strong improvement of the past few decades, seems rather surprising. We may advocate that the favourable trends shown in the past cannot be maintained in the future. Two reasons for this finding may be, on the one hand, that the short memory of time series models implies that only

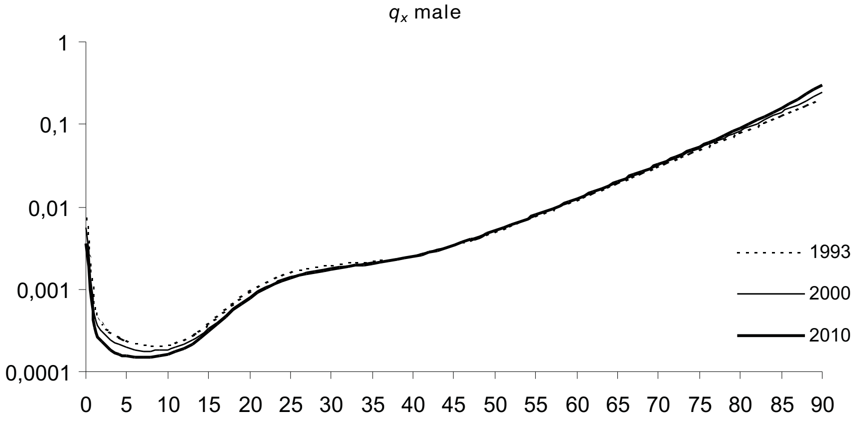

Figure 11. Male forecast probability of death for years 2000 and 2010

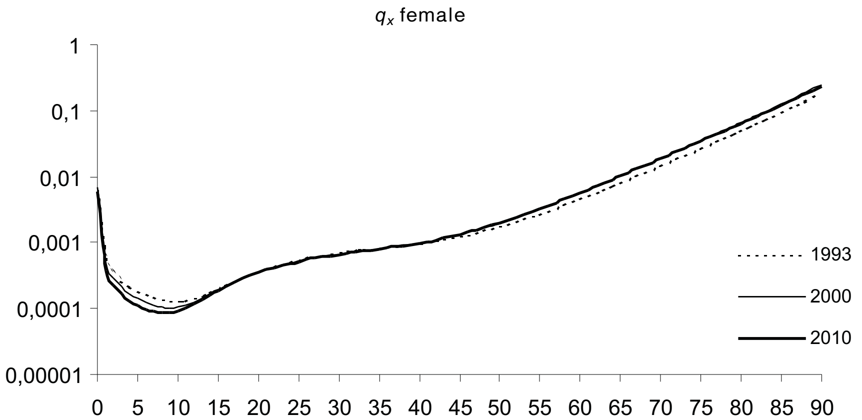

Figure 12. Female forecast probability of death for years 2000 and 2010

the changes in the latest years count or, on the other hand, that the Spanish society is more similar to other European societies, so that the improvements do decelerate.

3.2.4 The increase in older ages was also observed in Kastenholz (1995) when comparing some European countries.

3.2.5 When looking at Figures 11 and 12, the forecast of mortality rates for ages older than 65 is surprising, but two main reasons can be pointed out.

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On the one hand, we have to bear in mind that we are studying different birth cohorts. Today, elderly people are the survivors of a quite selected cohort in its youth (Spanish influenza, civil war, ...). Citizens who are just reaching the retirement age have not suffered this strong selection, in fact they have benefited from progress (nutrition, medical conditions, welfare, increasing income, ...). As more people, less selected, are surviving, the life expectancy at retirement age will slightly decrease, as the span of life is not increasing rapidly enough to compensate this process. In Kastenholz (1995, p11), seven European countries' mortality rates are compared. We think that male Spanish mortality has an evolution pattern similar to the one in the United Kingdom or the Netherlands. On the other hand, as shown in Figure 10, our graduation law tends to overestimate the probability of death at older ages; although in most cases it is inside the binomial confidence interval, it is always near the upper bound. Finally, we expect that the span of life will continue to increase.

4. Conclusions and Further Research

4.1 There is a clear evolution of the Spanish mortality patterns from 1975 to 1993, which are different by gender.

4.2 We have used Heligman-Pollard Law number two to model the evolution of Spanish mortality over the period, and, using univariate time series analysis, we have obtained a prognosis for years 1994 to 2010.

4.3 The number of parameters in the model is small, and each parameter has its own interpretation and evolution. Different shocks were detected over the analysed period. We also constructed confidence intervals for the parameters and the q_x-curves themselves.

4.4 The implementation of the whole model, including time series analysis, was relatively quick, although results depend on the truncation at older ages, and fitting the model was clearly more difficult for the female group. In addition, there is always a possibility that the time series model is unclear. It must also be pointed out that forecasting is of little reliability for ages beyond the truncation age.

4.5 We note that it is also necessary to combine the two errors together (estimated adjusted error and estimated time-series error), in order to generate a confidence interval for the forecasts of the q_x curve and to obtain more precise what-if scenarios.

4.6 The univariate time-series analysis on the parameter estimates indicates the widespread existence of autoregressive structures; therefore, with limited memory. This implies that: information concerning the later years has a large influence on forecasts; and that a 20 to 30-year experience is enough to identify long-term trends. In fact, if a longer period is used the forecasts could deteriorate. The ARIMA models easily predict the long-term trends, but the future shocks are not predictable. As a consequence, a mortality table must be checked every five years or so, to ensure that new shock effects are incorporated.

4.7 Now we have an appropriate model of the Spanish population mortality that is able to explain past behaviour and forecast future trends. When projecting insured lives mortality, it seems reasonable to assume that the same behaviour used for the population mortality projection will hold. We regard our present work as the first step in a larger project, in which we also intend to consider and analyse the trends in Spanish mortality for term life insurance, annuities and pensions.

Acknowledgements

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APPENDIX A

A GENERAL OVERVIEW OF THE SPANISH NATIONAL MORTALITY

A.1 Surprisingly enough, in a recent Spanish market survey carried out by INESE (Instituto de Estudios Superiores Financieros y de Seguros), 90% of the life insurance companies used GKM80 (Grundzahlen Kapital Männer 1980) tables, 5% used GKM95 (Grundzahlen Kapital Männer 1995) and the other 5% used PEM80 (Spanish 1980 population life table for males, Martinez, 1997).

A.2 Spanish mortality tables (PEF stands for Spanish females' population life table, and PEM stands for Spanish males' population life table) have traditionally been graduated using the Makeham Law of mortality. Recently, other models, such as Logit Gompertz-Makeham (3,3) (Navarro, 1991) and Heligman-Pollard Laws (Rue, 1992) have been tested; but Spanish insurance companies, mainly due to the influence of Swiss reinsurance, make an extensive use of the Swiss tables. All GRM80 (Grundzahlen Renten Männer 1980), GRF80 (Grundzahlen Renten Frauen 1980), GKM80, GKF80 (Grundzahlen Kapital Frauen 1980) and the new 95 series are based on Swiss insurance companies national portfolios, and, therefore, it is unlikely that they reflect the true Spanish population mortality. In addition, past experience shows that when Spanish insurance companies used Swiss tables they earned large profits due to over-rating in term insurance portfolios.

A.3 Although term insurance has been the main life insurance line of business in Spain, pension and annuities portfolios have been rapidly growing since 1995 (Banco de España, http://www.bde.es). The yearly average increase of mathematical and pension provisions is currently 10 billion dollars approximately, as opposed to 0.3 billion dollars increase on average between 1981 and 1984. At the same time, some institutions in this market have noticed a major problem; the mortality tables used in practice seem to over-estimate the probability of death at older ages. This implies a great social and financial risk for the insurers.

A.4 We have compared the Swiss and Spanish mortality tables with the Spanish population mortality rates and their corresponding 95% binomial confidence intervals. We have used our raw mortality rates and confidence intervals using expression 2.6.3 and 2.6.4 in Forfar, McCutcheon & Wilkie (1988, 11-12), following the secant method and the useful starting values that they suggested. Our results are based on population data, the comparison for men is shown in Tables A.1 and A.2 for ages 20, 30, 40, 50, 60 and 70.

A.5 The existing Spanish life tables (UNESPA, Unión Española de Entidades Aseguradoras y Reaseguradoras) use a pre-smoothed life table produced by INE (Instituto Nacional de Estadistica). King and Hardy's method for the Makeham law is applied, so that deaths and exposures are

Table A.1. Male mortality comparison for 1975-1979 and 1980-1984

Age	GKM-70	PEM-70	L75-79	U75-79	GKM-80	GRM-80	PEM-80	L80-84	U80-84
20	1.270	1.139	1.083	1.294	1.140	0.799	1.034	0.943	1.129
30	1.420	1.502	1.182	1.418	1.262	1.020	1.170	1.067	1.293
40	2.619	2.824	2.309	2.674	2.221	2.011	2.206	2.122	2.456
50	6.799	6.858	5.994	6.552	6.094	4.946	5.762	5.413	5.928
60	17.479	17.209	15.231	16.317	16.093	10.757	14.397	13.778	14.686
70	44.770	43.471	38.290	40.217	42.554	25.647	35.780	34.018	35.787

Rates in thousands. Average 95% confidence intervals throughout the period.
L75-79 and L80-84 indicate lower bound averages and U75-79 and U80-84 indicate upper bound averages.

Table A.2. Male mortality comparison for 1985-1989 and 1990-1993

Age	GKM-80	GRM-80	PEM-80	L85-89	U85-89	GKM-95	GRM-95	PEM-90	L90-93	U90-93
20	1.140	0.799	1.034	1.192	1.396	1.550	1.294	1.206	1.166	1.367
30	1.262	1.020	1.170	1.391	1.630	1.229	1.306	1.619	2.162	2.448
40	2.221	2.011	2.206	2.076	2.390	1.869	1.862	2.697	2.289	2.619
50	6.094	4.946	5.762	5.258	5.816	4.308	4.218	5.503	4.976	5.500
60	16.093	10.757	14.397	13.012	13.858	11.552	9.374	12.777	12.094	12.885
70	42.554	25.647	35.780	32.245	33.977	31.371	19.886	31.495	30.347	31.826

Rates in thousands. Average 95% confidence intervals throughout the period.
L85-89 and L90-93 indicate lower bound averages and U85-89 and U90-93 indicate upper bound averages.

smoothed by moving average methods before actually calculating death rates. The Spanish PEM tables do not fall within the 95% population confidence intervals in Tables A.1 and A.2, because either they refer to a previous period or they have been pre-smoothed.

A.6 Tables A.1 and A.2 show evidence that the mortality tables used in the Spanish market do not properly reflect the current mortality or its evolution.

A.7 An appropriate model of the population mortality, meaning that it is able to fit and to forecast, is required to analyse the Spanish mortality experience and to understand the behaviour and perspectives of a life insurance portfolio in this market.

APPENDIX B

PARAMETER ESTIMATES TRENDS

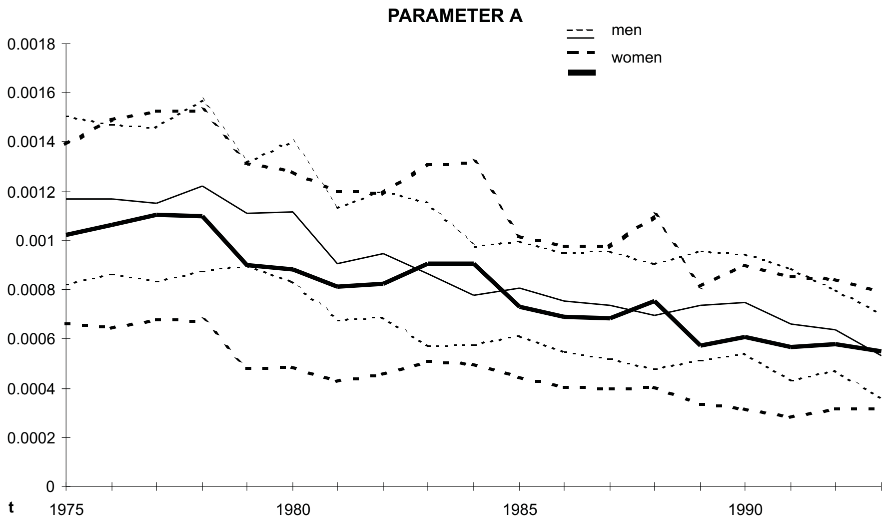

Figure A.1. Parameter *A*, trend comparison by gender and 95% confidence bands

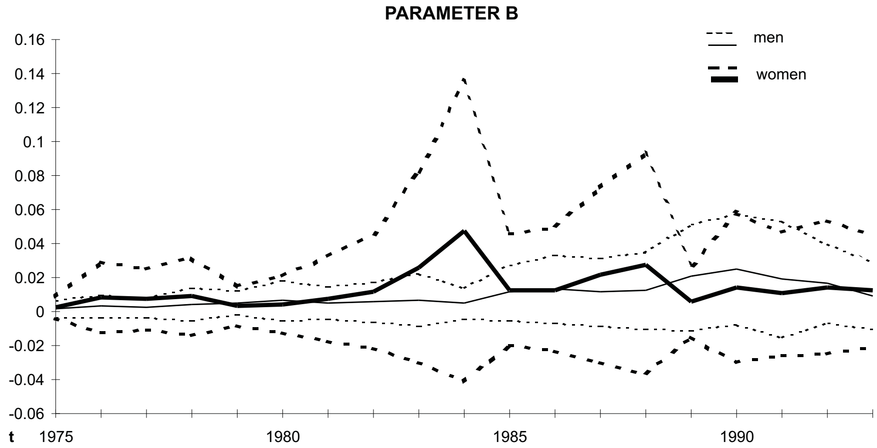

Figure A.2. Parameter *B*, trend comparison by gender and 95% confidence bands

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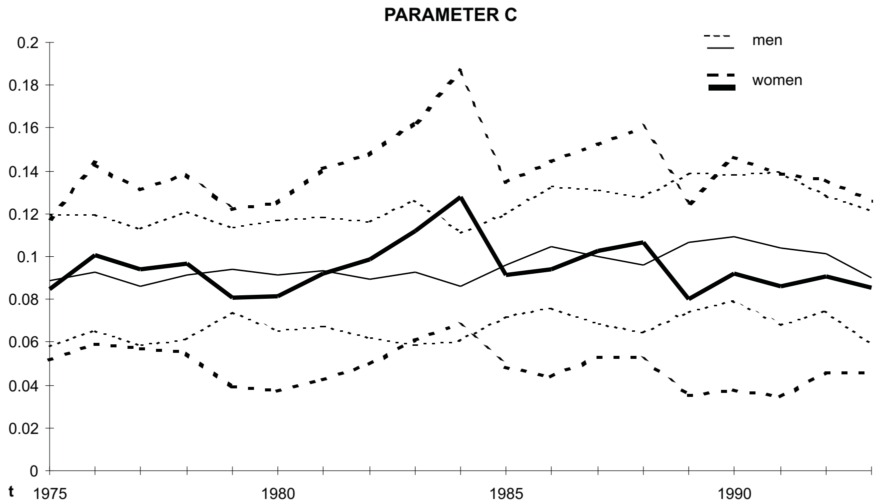

Figure A.3. Parameter *C*, trend comparison by gender and 95% confidence bands

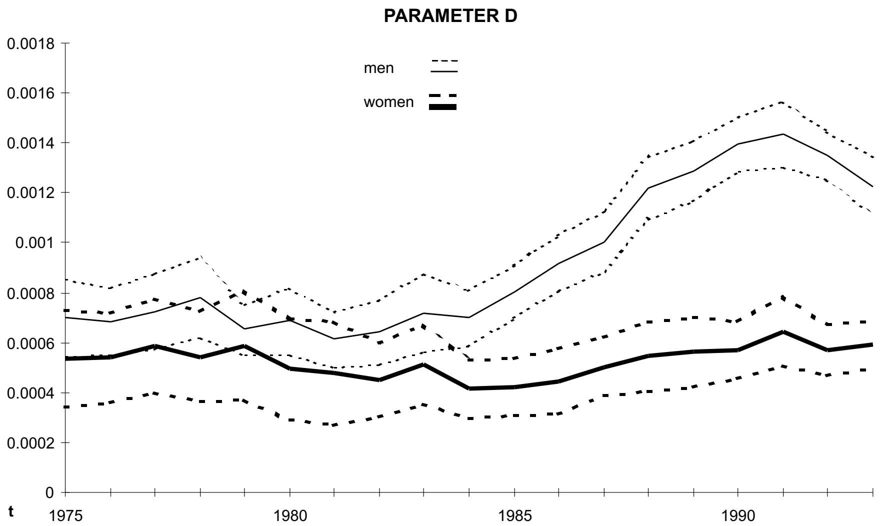

Figure A.4. Parameter *D*, trend comparison by gender and 95% confidence bands

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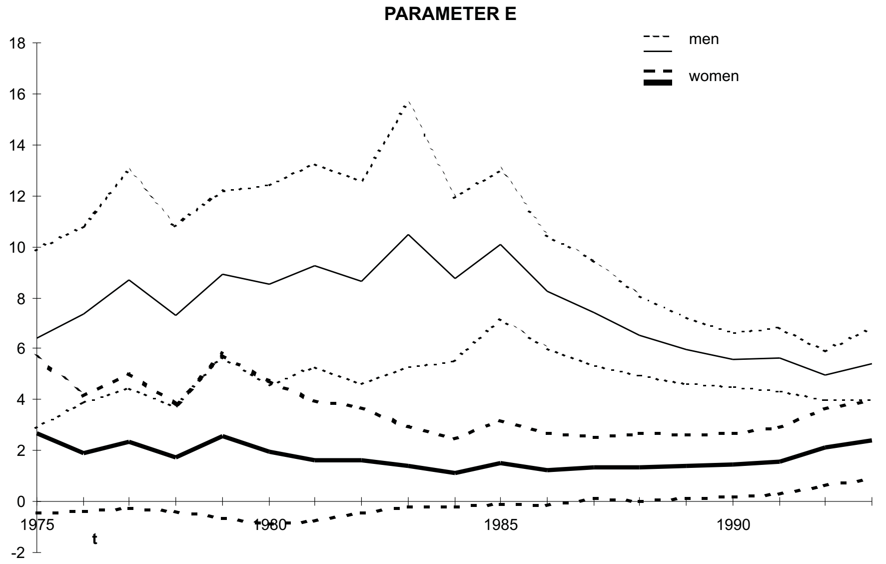

Figure A.5. Parameter E, trend comparison by gender and 95% confidence bands

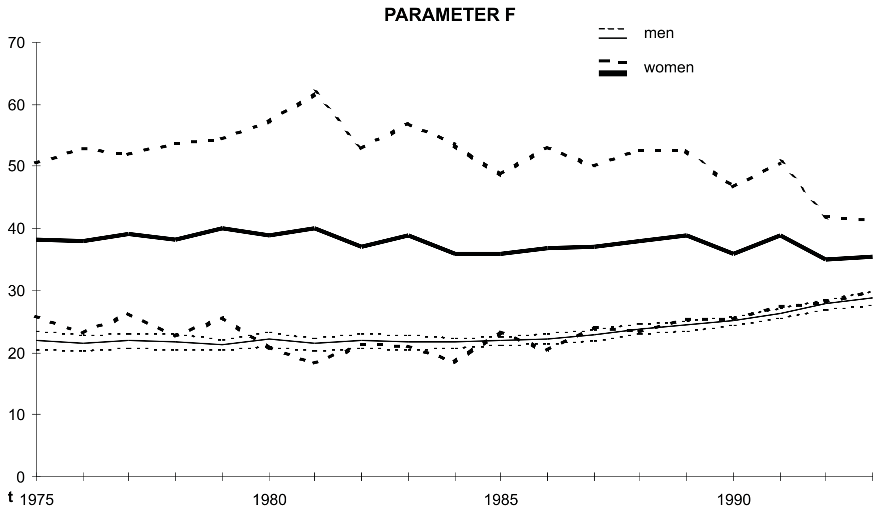

Figure A.6. Parameter F, trend comparison by gender and 95% confidence bands

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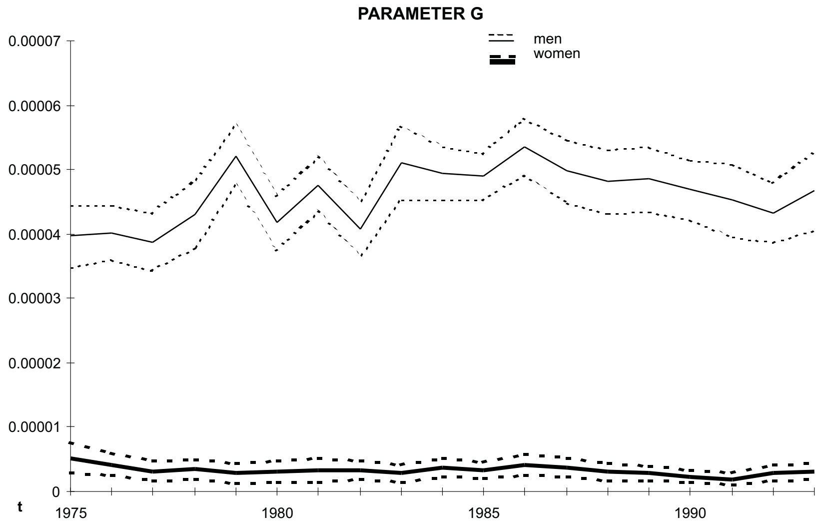

Figure A.7. Parameter G, trend comparison by gender and 95% confidence bands

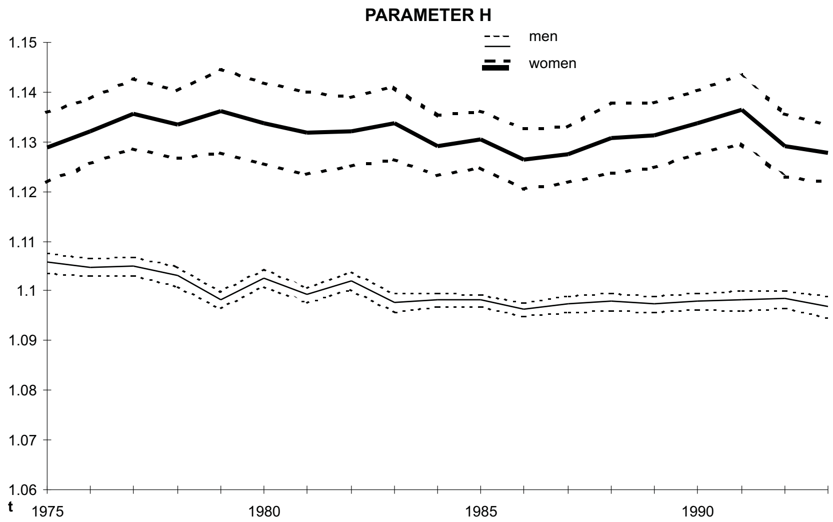

Figure A.8. Parameter H, trend comparison by gender and 95% confidence bands

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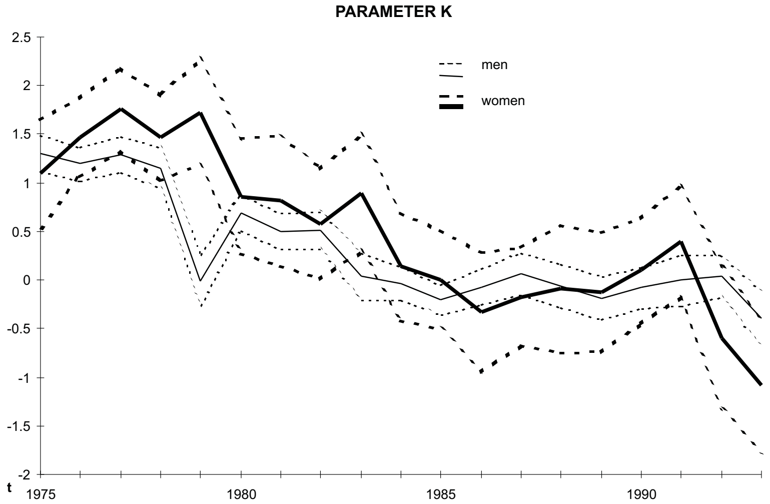

Figure A.9. Parameter K, trend comparison by gender and 95% confidence bands

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APPENDIX C

FORECAST PROBABILITY OF DEATH

Table C.1. Men and women forecast probability of death for years 2000 and 2010

	Forecast probability of death			
	Women		Men	
Age	2000	2010	2000	2010
0	0.0069462301	0.0060451317	0.0056082964	0.0035345987
1	0.0004731248	0.0003815298	0.0004802062	0.0003503683
2	0.0002855140	0.0002273634	0.0003109803	0.0002298502
3	0.0002097956	0.0001659658	0.0002461019	0.0001854509
4	0.0001679766	0.0001324145	0.0002130391	0.0001640719
5	0.0001414580	0.0001114475	0.0001944660	0.0001531738
6	0.0001236091	0.0000977816	0.0001840107	0.0001481945
7	0.0001117598	0.0000894333	0.0001787871	0.0001470959
8	0.0001049188	0.0000857828	0.0001773899	0.0001489195
9	0.0001028556	0.0000867749	0.0001793054	0.0001533941
10	0.0001056311	0.0000925152	0.0001848330	0.0001609367
11	0.0001133343	0.0001030552	0.0001951516	0.0001727608
12	0.0001259429	0.0001182949	0.0002122903	0.0001908817
13	0.0001432649	0.0001379559	0.0002388688	0.0002178920
14	0.0001649355	0.0001615990	0.0002776175	0.0002565103
15	0.0001904442	0.0001886635	0.0003307983	0.0003090220
16	0.0002191777	0.0002185144	0.0003996918	0.0003767708
17	0.0002504670	0.0002504880	0.0004842784	0.0004598343
18	0.0002836311	0.0002839312	0.0005831728	0.0005569454
19	0.0003180139	0.0003182335	0.0006937947	0.0006656477
20	0.0003530129	0.0003528492	0.0008127081	0.0007826225
21	0.0003880992	0.0003873123	0.0009360434	0.0009041012
22	0.0004228310	0.0004212449	0.0010599207	0.0010262841
23	0.0004568595	0.0004543601	0.0011808131	0.0011457027
24	0.0004899315	0.0004864618	0.0012958165	0.0012594908
25	0.0005218871	0.0005174409	0.0014028133	0.0013655511
26	0.0005526563	0.0005472704	0.0015005375	0.0014626244
27	0.0005822532	0.0005760002	0.0015885586	0.0015502766
28	0.0006107705	0.0006037509	0.0016672062	0.0016288262
29	0.0006383732	0.0006307085	0.0017374583	0.0016992351
30	0.0006652931	0.0006571197	0.0018008113	0.0017629812
31	0.0006918245	0.0006832877	0.0018591506	0.0018219295
32	0.0007183206	0.0007095699	0.0019146289	0.0018782120
33	0.0007451904	0.0007363759	0.0019695623	0.0019341251
34	0.0007728984	0.0007641673	0.0020263461	0.0019920449
35	0.0008019643	0.0007934587	0.0020873919	0.0020543659
36	0.0008329645	0.0008248197	0.0021550837	0.0021234572
37	0.0008665358	0.0008588791	0.0022317537	0.0022016385
38	0.0009033791	0.0008963294	0.0023196718	0.0022911708
39	0.0009442664	0.0009379336	0.0024210488	0.0023942593
40	0.0009900478	0.0009845335	0.0025380488	0.0025130669

Table C.1. (continued) Men and women forecast probability of death for years 2000 and 2010 (continued)

	Forecast probability of death			
	Women		Men	
Age	2000	2010	2000	2010
41	0.0010416612	0.0010370588	0.0026728100	0.0026497353
42	0.0011001436	0.0010965392	0.0028274701	0.0028064110
43	0.0011666443	0.0011641173	0.0030041959	0.0029852762
44	0.0012424396	0.0012410640	0.0032052163	0.0031885818
45	0.0013289512	0.0013287965	0.0034328561	0.0034186830
46	0.0014277658	0.0014288979	0.0036895714	0.0036780758
47	0.0015406579	0.0015431406	0.0039779868	0.0039694356
48	0.0016696166	0.0016735118	0.0043009322	0.0042956559
49	0.0018168745	0.0018222437	0.0046614817	0.0046598900
50	0.0019849422	0.0019918463	0.0050629931	0.0050655928
51	0.0021766457	0.0021851459	0.0055091499	0.0055165664
52	0.0023951705	0.0024053277	0.0060040042	0.0060170077
53	0.0026441108	0.0026559842	0.0065520236	0.0065715601
54	0.0029275248	0.0029411711	0.0071581408	0.0071853696
55	0.0032499992	0.0032654689	0.0078278070	0.0078641465
56	0.0036167204	0.0036340540	0.0085670507	0.0086142333
57	0.0040335574	0.0040527790	0.0093825421	0.0094426809
58	0.0045071548	0.0045282635	0.0102816636	0.0103573342
59	0.0050450395	0.0050679972	0.0112725893	0.0113669282
60	0.0056557421	0.0056804576	0.0123643719	0.0124811975
61	0.0063489356	0.0063752425	0.0135670417	0.0137110016
62	0.0071355943	0.0071632219	0.0148917168	0.0150684676
63	0.0080281750	0.0080567098	0.0163507272	0.0165671552
64	0.0090408257	0.0090696608	0.0179577557	0.0182222471
65	0.0101896251	0.0102178927	0.0197279969	0.0200507702
66	0.0114928579	0.0115193420	0.0216783387	0.0220718529
67	0.0129713331	0.0129943547	0.0238275687	0.0243070276
68	0.0146487517	0.0146660190	0.0261966106	0.0267805858
69	0.0165521336	0.0165605459	0.0288087956	0.0295199980
70	0.0187123133	0.0187077065	0.0316901747	0.0325564138
71	0.0211645194	0.0211413327	0.0348698802	0.0359252587
72	0.0239490522	0.0238998953	0.0383805447	0.0396669526
73	0.0271120811	0.0270271686	0.0422587895	0.0438277784
74	0.0307065855	0.0305729996	0.0465457965	0.0484609404
75	0.0347934713	0.0345941994	0.0512879798	0.0536278626
76	0.0394429018	0.0391555776	0.0565377794	0.0593997931
77	0.0447358935	0.0443311496	0.0623546043	0.0658598021
78	0.0507662402	0.0502055472	0.0688059570	0.0731052942
79	0.0576428503	0.0568756752	0.0759687852	0.0812511934
80	0.0654926023	0.0644526640	0.0839311142	0.0904340242
81	0.0744638642	0.0730641817	0.0927940322	0.1008171976
82	0.0847308639	0.0828571853	0.1026741216	0.1125979384
83	0.0964991666	0.0940012124	0.1137064584	0.1260164833
84	0.1100126035	0.1066923419	0.1260483423	0.1413684683
85	0.1255621305	0.1211579907	0.1398839749	0.1590218761
86	0.1434972793	0.1376627621	0.1554303808	0.1794406317

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Table C.1. (continued) Men and women forecast probability of death for years 2000 and 2010 (continued)

	Forecast probability of death			
	Women		Men	
Age	2000	2010	2000	2010
87	0.1642411428	0.1565156299	0.1729449767	0.2032181004
88	0.1883102462	0.1780788333	0.1927353510	0.2311257062
89	0.2163412841	0.2027789857	0.2151720472	0.2641852777
90	0.2491276750	0.2311210787	0.2407054878	0.3037798293
91	0.2876704317	0.2637063127	0.2698886953	0.3518289176
92	0.3332503648	0.3012550497	0.3034082672	0.4110772080
93	0.3875328656	0.3446367085	0.3421273336	0.4855918052
94	0.4527238376	0.3949092084	0.3871462785	0.5816687026
95	0.5318085111	0.4533717443	0.4398904318	0.7096033793

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