

CORRECTION TO 'ADDITIVE AND SUBSTRUCTIVE BASES OF \mathbb{Z}_m IN AVERAGE'

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For $A \subseteq \mathbb{Z}_m$ and $n \in \mathbb{Z}_m$, let $\sigma_A(n)$ be the number of solutions to the equation $n = x + y$ with $x, y \in A$. Let \mathcal{H}_m be the set of subsets $A \subseteq \mathbb{Z}_m$ such that $\sigma_A(n) \geq 1$ for all $n \in \mathbb{Z}_m$. Let

$$\ell_m = \min_{A \in \mathcal{H}_m} \left\{ m^{-1} \sum_{n \in \mathbb{Z}_m} \sigma_A(n) \right\}.$$

A result of Ding and Zhao [1, Lemma 2.4] on Ruzsa's numbers implies that $\limsup_{m \rightarrow \infty} \ell_m \leq 192$. In [2], the authors improved this to $\limsup_{m \rightarrow \infty} \ell_m \leq 144$.

In the proof of [2],

$$A_1 = \{u + 2pv : (u, v) \in B\}. \tag{1}$$

As pointed by Mr. Honghu Liu, (1) implies $|A_1| \leq |B|$ rather than $|A_1| \leq 2|B|$ as stated in our article. Since $\ell_m \leq |B|^2/m$ from [2], adjusting the numbers accordingly leads to the following much stronger bound.

THEOREM 1. *We have $\limsup_{m \rightarrow \infty} \ell_m \leq 36$.*

References

- [1] Y. Ding and L. Zhao, 'A new upper bound on Ruzsa's numbers on the Erdős–Turán conjecture', *Int. J. Number Theory* **20** (2024), 1515–1523.
- [2] G. Liang, Y. Zhang and H. Zuo, 'Additive and subtractive bases of \mathbb{Z}_m in average', *Bull. Aust. Math. Soc.*, to appear. Published online (25 November 2024).

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