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RISK ASSESSMENT TECHNIQUES FOR SPLIT CAPITAL INVESTMENT TRUSTS

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ABSTRACT

The split capital investment trust crisis brought into focus the need for more reliable risk assessment techniques for shares in the sector. We discuss the strengths and weaknesses of traditional pricing and risk description measures for split capital investment trusts (e.g. gross redemption yield, cover, hurdle rates) and ways of making these more useful. We then examine the application of traditional option pricing techniques and discuss the problems encountered in this approach. Finally, we propose the use of stochastic modelling to deal more effectively with the complexities involved in both pricing shares and understanding their risks.

KEYWORDS

Risk Assessment; Split Capital Investment Trusts; Closed-End Funds; Option Pricing; Stochastic Modelling

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1. INTRODUCTION

Split capital closed-end funds ('splits') may be defined as investment companies or investment trust companies ('trusts') with more than one main class of share capital, offering different rights to income and capital. Their assets consist of a portfolio of shares or other securities. They are designed to meet simultaneously the needs of different types of investor. Ultimate responsibility for running the affairs of the company lies with the Board of Directors, but day-to-day administration and investment management are generally delegated to an investment management company. Splits are normally designed to be wound up at some future date, with most splits having an original term of seven to ten years. When the company is wound up, its assets are sold and the proceeds are used to pay off the various classes of share capital after meeting the entitlements of holders of debt, if any.[1]

The first modern day split, Dualvest, was launched in May 1965. The ordinary share capital of this trust was split into two distinct categories — income shares and capital shares. Holders of income shares were entitled to all the distributed income and a predetermined capital value on liquidation. Thus, they received a much higher yield than that on the underlying portfolio, since they were also entitled to the income arising from the capital shareholders' interest in the trust. Holders of the capital shares received no income, but were entitled to the remaining assets on liquidation after the income shares had been redeemed. Thus, they obtained geared capital growth. The success of Dualvest led to the creation of a number of similar vehicles in the United Kingdom, which would all be known nowadays as 'traditional splits', and in the United States of America, known as 'dual purpose funds'.[2]

In October 1987, Scottish National Trust reorganised into a split with certain novel features. In particular, a more aggressive structure was created by the inclusion of zero dividend preference shares (ZDPs). This was the first time ZDPs (often known as 'zeros') had been issued by an investment trust. ZDPs are designed to pay a pre-determined capital sum when the trust is wound up before any distribution of capital can be made to other lower-ranking shareholders. They provide gearing to the lower-ranking shareholders, but the cost to the company is not charged to the revenue account. Thus, the interest cost effectively rolls up, allowing higher dividends to be paid to other shareholders. ZDPs have no entitlement to income, so that, importantly, there is no liability to income tax for the investor.

ZDPs offered flexibility when designing capital structures, and became popular in the late 1980s, with the rise of a new type of split (sometimes known as a 'quasi-split'). In its simplest form, this type of split has, on the liabilities side of its balance sheet, two classes of share — ZDPs and 'ordinary income shares' (also known as 'income & residual capital shares'). The assets side of the balance sheet might simply consist of a broad portfolio of U.K. equities with an above average yield. When the company is wound up, ZDPs are repaid first (after any prior charges). The ordinary income shares are designed to offer high income plus all the remaining assets at the wind-up date after the ZDPs have received their capital entitlement. Quasi-splits avoided some of the conflicts of interest which could arise in traditional

[1] Shareholders always have the option to take cash, but, in practice, the directors and managers often try to retain the funds under management by encouraging roll-over into another trust or restructuring, rather than liquidation. Indeed, they are all but certain to offer a roll-over option for capital gains tax purposes.

[2] Other related products introduced in the U.S.A. include 'primes and scores' — see Jarrow & O'Hara (1989).

splits (the pursuit of high income tends to result in lower capital growth) and were sensible structures from a taxation perspective. Issues of this type of split were common right up to 1997.

With the marked fall in interest rates over the second half of the 1990s, the quest for income among private investors provided an opportunity for the issue of innovative income products. In satisfying this demand for yield, the product providers used various devices which led to the creation of complicated, unstable trust structures. These devices included:

(1) *Substantial levels of bank debt financing*. During the period of falling interest rates, the gross redemption yield[3] on a typical ZDP remained high, at say 9% p.a., whereas bank debt finance could be arranged at perhaps 6.5% p.a. So, it was considerably cheaper to finance gearing through bank debt rather than through ZDPs; but the inclusion of bank debt in a split's capital structure meant that, in the event of a breach of covenant, the bank had the right to foreclose and demand either early repayment (in full or in part, with breakage costs), restructuring of the underlying portfolio or dividend freezes/reductions.

(2) *Investment in the income-bearing shares of other splits*. Significant proportions of the income shares and ordinary income shares in the new issues of splits were typically placed with other splits (hence 'cross-holdings'), thus helping them to meet their own yield requirements. Apart from the problems of accountability and transparency, this generally created gearing upon gearing within the structures.

The above digressions from prudent practice were often combined with a thematic investment strategy in the so-called 'barbell' investment trusts (Adams & Angus, 2001). The first of these, Technology & Income Trust, was launched in July 1999, and many of the new issues of trusts in the following two years were of this novel barbell type. Barbell trusts held two distinct portfolios of investments — an income portfolio and a growth portfolio. In pictorial form, this asset structure can look like a 'barbell' such as is used in weightlifting: assets are held at either end of the income/growth spectrum, with nothing in the middle. The income portfolio typically consisted of bonds with varying degrees of risk, together with income-bearing shares of other splits. The 'growth' portfolio was typically invested in a sector or market which was popular at the time of issue (such as technology stocks).

The new wave of splits had a variety of different capital structures which could include bank debt, ZDPs, income shares, ordinary income shares, capital shares or other classes of shares. Often the shares in these structures did not have the same well-known characteristics as shares with the same title (e.g. ZDPs) issued by earlier simple splits. The shares were generally

[3] Gross redemption yield is the internal rate of return, before any tax, based on the assumption that the ZDP is held to redemption and redeemed in full.

Table 1. Capital structure of splits as at 31 December 2004

Capital structure	Number of splits
Prior charge + ZDP + Ordinary income	24
Prior charge + ZDP + Income + Capital	11
ZDP + Ordinary income	9
Prior charge + Income + Capital	7
Prior charge + ZDP + Income + Ordinary income	3
Prior charge + Convertible + ZDP + Ordinary income	3
Other splits	16
Total	73

Source: Cazenove (2005)

difficult to value and analyse, and investors often significantly underestimated the downside risk.

The severe bear market which started in 2000 brought home the dangers of bank debt combined with cross-holdings. The impact of falling markets, accompanied by equity dividend cuts, led to collapsing market prices and dividend cuts for the income-bearing shares of many splits. The substantial cross-holdings then caused dividend cuts to compound themselves across a section of the splits sector, and share prices fell yet further. Even the market prices of a number of ZDPs fell sharply, a type of share which, until then, had generally been regarded as low risk. Confidence in splits then collapsed in the early months of 2002, leading to a major Financial Services Authority investigation and a House of Commons Treasury Committee inquiry (HCTC, 2003).

It is important to emphasise that it was the new wave of aggressively structured cross-invested splits launched in the late 1990s and beyond which was flawed. The basic concept of a split capital investment trust which meets, simultaneously, the risk, income and tax preferences of different types of investor remains a sensible concept. Seventy-three splits still remain, with a variety of capital structures, as shown in Table 1. Once confidence returns, it is possible that we will see renewed expansion in the splits sector.

The next section looks at traditional risk assessment techniques, why they became misleading, and how they could be made more useful. Section 3 discusses the application of option pricing techniques to splits, and Section 4 outlines the problems with this approach. Section 5 proposes the use of Monte Carlo simulation in the pricing and risk assessment of splits, and Section 6 is the conclusion.

2. Traditional Risk Assessment Statistics and their Problems

2.1 *Traditional Risk Assessment Statistics*

We now discuss the traditional risk assessment and valuation statistics for four classes of shares which are commonly found in splits: ZDPs, ordinary

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income shares, income shares and capital shares. These statistics are produced routinely by stockbrokers and other financial practitioners.

Zero dividend preference shares

The main statistics used by financial analysts in assessing the risk of ZDPs, ever since they were first issued in 1987, have been asset cover and hurdle rate.

Asset cover (also known as *final asset cover* or just *cover*) is the ratio of gross assets (less any prior ranking capital) to the assets required to pay the predetermined redemption amount of the ZDPs at the redemption date. In a simple split trust, with no prior ranking capital and invested in a broad portfolio of U.K. equities, it gives a rough indication of the risk of the ZDP shareholders not receiving their full entitlement at redemption, but does not take the term to redemption into account. It can be defined in different ways. Some analysts, for example, deduct future annual costs (management fees, interest costs) charged to the capital account in arriving at the final total assets figure.

Hurdle rate (also known as *fulcrum point*, *growth to cover* or *final value hurdle rate*) is the required annual growth rate of gross assets to pay the full redemption amount of ZDPs. A negative hurdle rate indicates that gross assets could fall each year by the rate indicated and still be sufficient to repay the ZDPs in full. Hurdle rate is a crude measure of the risk, even for a simple quasi-split structure, because it does not take into account the yield on the underlying portfolio of assets. Clearly, the higher the yield on the underlying portfolio, the more difficult it is, in general, to achieve the required annual capital growth rate, as higher yielding securities do not necessarily have higher total returns than lower yielding securities. Hurdle rates, generally, take future annual costs charged to capital into account. As with asset cover, however, care needs to be taken to establish exactly which definition has been used in the calculation. A variation on hurdle rate is *wipe-out rate*, which measures the annualised rate of decrease in gross assets which would just lead to no capital payment (or 'wipe-out') on wind-up. This provides a more useful guide to downside risk, but was rarely given in the prospectuses of splits before the splits crisis broke. *Current price hurdle rate* and *initial price hurdle rate* are other such variations.

Gross Redemption Yield (GRY) of a ZDP can be compared to that of a British Government bond of similar duration. Thus, *Points over Gilt GRY* gives a measure of the risk premium priced into a ZDP. *GRY no growth* measures the GRY of a ZDP, on the assumption that the capital value of the underlying gross assets remains unchanged between the present time and wind-up. Plots of GRY or GRY no growth against cover or hurdle rate attempt to compare possible returns for ZDPs against some measure of risk.

Income shares

Key statistics for income shares are asset cover, hurdle rate and net redemption yield. The asset cover calculation is based on an income share's full redemption value. Wipe-out rate is also commonly used for income shares. *Net redemption yield* (NRY), often used in conjunction with various assumed annual growth rates for gross assets and dividends paid to income shares, measures the annualised return to income shareholders net of income tax. The range of NRYs for different assumed growth rates can be useful in risk assessment.

Ordinary income shares (income & residual capital shares)

Hurdle rate (to current share price), wipe-out rate and net redemption yield under different growth assumptions for the underlying portfolio are also used for ordinary income shares. Gearing is used as a risk measure, and may be defined as the ratio of gross assets to the assets attributable to ordinary income shareholders.

Capital shares

If there are ZDPs in issue, the normal practice of 'stepping up' the ZDP by a pre-determined periodic amount, instead of taking the full redemption value, complicates measurement of the net asset value of capital shares. As a result, Newlands (2000) describes quoted discount to NAV figures for capital shares, a popular valuation method, as virtually meaningless. As with ordinary income shares, hurdle rate, wipe-out rate and gearing are used as risk measures for capital shares.

2.2 *Why Traditional Risk Assessment Statistics became Misleading*

Following the collapse of many of the aggressively structured splits issued in the late 1990s and beyond, it should now be clear to all investment practitioners that statistics such as asset cover and hurdle rate should not have been used in isolation. Other factors, such as the quality of the underlying portfolio of assets, capital structure, expense structure, sources of income, portfolio volatility and, critically, the amount of bank debt and details of bank covenants should have been considered, if such information was available or could have been obtained.

Hurdle rates for shares in the aggressively structured trusts appeared deceptively easy to achieve, for the following reasons:

(1) Hurdle rates are based on the growth of the whole underlying portfolio, ignoring the fact that a substantial part of the portfolio may not be held in growth assets, and thus could not possibly be expected to grow at anything like the projected rate of growth of the growth assets.
(2) Investors' expectations of returns from equities had been derived from the bull market of the previous quarter of a century.

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(3) Many investors were unaware that the expected nominal return on equities would be low in an environment of low inflation and low interest rates. If there were holdings in the high-yielding shares of other splits, with dividend yield exceeding expected total return, it was, in fact, reasonable to expect erosion of their capital value.
(4) Any holdings in high-yielding bonds could suffer defaults and capital loss.

The last two reasons also made the asset cover statistic misleading. This statistic was even more misleading if the trust's costs up to maturity were not deducted from gross assets.

It is possible to find two ZDPs with similar hurdle rates and cover, but with very different risk profiles. Appendix A presents a hypothetical case of a geared ZDP with equal hurdle rate and superior cover to an ungeared ZDP, but with greater sensitivity to large falls in the underlying asset portfolio.

2.3 *Making Traditional Statistics more Useful*

We have shown that traditional risk measures can be dangerously misleading in certain circumstances. However, there are ways in which they can be made more useful.

Total return to cover could be used instead of hurdle rate. This represents the total return (i.e. capital gain plus dividend yield) required on the underlying portfolio of assets for the ZDP to reach a cover of one. The total return to cover can then be compared with the expected return on the total underlying portfolio, to gauge whether or not full payment of the ZDP or income share redemption amount is realistic. Another variation on hurdle rate (known as *equity hurdle rate*) would be to measure the hurdle rate (or total return to cover) based only on that part of the underlying portfolio invested in growth assets. This would present a more realistic picture of the growth required on the growth assets within a 'barbell' trust.

Greater use of sensitivity analysis, showing GRY or NRY for various *negative* and positive total return scenarios (e.g. −10% p.a., −5% p.a., ..., +15% p.a., +20% p.a.) would aid understanding of the risks involved for a particular class of share. It is important that a number of negative market scenarios are included in any such analysis. A plot of GRY (% p.a.) against percentage change (per annum) in the underlying portfolio, such as shown in Appendix A, would also aid in the assessment of risk for a class of share.

A more developed approach (Merrill Lynch, 1999) to describing risk for a ZDP is to assign 'credit ratings' to ZDPs, as with non-government bonds. This involves estimating the probability of default and the returns expected on default. By comparing these numbers with the historical experience of a range of bonds with different credit ratings, a credit rating can be assigned to the ZDP. Finally, an appropriate credit spread can be determined for each rating by reference to the bond market. The anticipated spread is then compared to the 'Points over Gilt GRY' to see if risk is appropriately priced. This approach takes account of the probability of default, so is generally more useful than traditional risk assessment statistics. It can also be used to explain the ZDP market's behaviour at times when bond market credit spreads widen. An example of this was the widening of ZDP credit spreads during the 1998 LTCM/ Russian bond crisis.

3. Option Pricing

3.1 *Traditional Splits*

Ingersoll (1976) studies U.S. dual purpose funds,[4] which are similar to U.K. traditional splits (see Section 1), but which are now extinct following a change in the U.S. tax code in 1989. The framework which he creates can be used for pricing the components of U.K. split capital investment trusts.

To derive an appropriate option pricing model for the capital shares and income shares of a traditional split, we assume a lognormal distribution for asset returns and constant interest rates. Dividends D are assumed to be paid continuously to the income shares, and are proportional to the asset value V, so that $D = \delta V$, where δ is a constant.

To simplify the discussion, we assume initially that there are no management fees. The value of a capital share is then isomorphic to a call option on a dividend paying stock and is given by:

$$f(V, T) = Ve^{-\delta T} N(x_1) - Ee^{-rT} N(x_2)$$

where:

$$x_1 = \frac{\ln\left(\dfrac{Ve^{-\delta T}}{Ee^{-rT}}\right) + \dfrac{\sigma^2}{2}\,T}{\sigma\sqrt{T}} \qquad x_2 = x_1 - \frac{\sigma\sqrt{T}}{2}$$

r is the continuously compounded risk-free rate of interest;
T is the time to maturity of the split in years;
E is the final redemption value of the income shares in the traditional split;
σ is the standard deviation of return on the underlying assets; and
$N(x)$ is the cumulative normal distribution.

The income share consists of two parts. The first component is the value

[4] Litzenberger & Sosin (1977) discuss the structure and management of U.S. dual purpose funds.

of the stream of dividends, that is:

$$V(1 - e^{-\delta T}).$$

The second component is the value of the redemption amount. At maturity time T this is equal to:

$$\begin{array}{lll} E & \text{if} & V(T) \geq E \\ V(T) & \text{if} & V(T) < E \end{array}$$

but this is just the value of a risky zero coupon bond with maturity value E. Its current price can be written as a risk-free bond minus a put option:

$$Ee^{-rT} - (Ee^{-rT}N(-x_2) - Ve^{-\delta T}N(-x_1)).$$

Using put-call parity, the current value of the maturity proceeds to the income shares can also be written as:

$$Ve^{-\delta T} - f(V, T).$$

Hence, the value of the total fund divides as follows:
(1) capital shares, with value $f(V, T)$; and
(2) income shares, with value:

$$V(1 - e^{-\delta T}) + Ve^{-\delta T} - f(V, T) = V - f(V, T).$$

Note that the total of the capital shares and the income shares is V.

We now introduce a management fee M, which is charged continuously to the revenue account and is proportional to V, so that $M = \varphi V$, where φ is a constant. This means that some of the value is siphoned off to management. The total outflow rate (dividends and fees) is now effectively $\gamma = \varphi + \delta$. The value of the payment stream of total outflows is:

$$V(1 - e^{-\gamma T}).$$

Of this amount the value of the management fees is:

$$V\frac{\varphi}{\gamma}(1 - e^{-\gamma T})$$

and the value of the dividend stream is:

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$$V\frac{\delta}{\gamma}(1-e^{-\gamma T}).$$

Note that the combined value of these two is $V(1-e^{-\gamma T})$.

The value of the capital shares is now:

$$f(V,T)=Ve^{-\gamma T}N(h_1)-Ee^{-rT}N(h_2)$$

where:

$$h_1=\frac{\ln\left(\frac{Ve^{-\gamma T}}{Ee^{-rT}}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\qquad h_2=h_1-\frac{\sigma\sqrt{T}}{2}.$$

The value of the redemption rights to the income shares is:

$$Ve^{-\gamma T}-f(V,T).$$

Hence, the total value of the income shares is:

$$\left[V\frac{\delta}{\gamma}(1-e^{-\gamma T})\right]+\left[Ve^{-\gamma T}-f(V,T)\right]=\left[V\left(\frac{\delta}{\gamma}(1-e^{-\gamma T})+e^{-\gamma T}\right)-f(V,T)\right].$$

Once again the sum of the three parts is equal to V:

$$\left[V\frac{\varphi}{\gamma}(1-e^{-\gamma T})\right]+\left[V\left(\frac{\delta}{\gamma}(1-e^{-\gamma T})+e^{-\gamma T}\right)-f(V,T)\right]+[f(V,T)]=V.$$

3.2 *Contemporary U.K. Splits*

A ZDP may be considered equal in value to the present value of a risk-free bond less the value of a put option on the trust's gross assets (Gemmill, 2002). The exercise price of the put equals the sum of the payments due at maturity on prior charges and the bond. Prior charges are assumed to be risk free. Given that lenders are able to place covenants on their loans (e.g. assets to exceed debt by 1.8 times), and demand that borrowers take action to reduce risk if the covenants are breached, the risk of making such loans is certainly low.

Denoting the theoretical price of a ZDP by Z, we have:

$$Z=F_z e^{-rT}-\text{Put}(P+F_z)$$

where:

F_z is the final redemption value of the ZDP;
P is the sum of the payments due at maturity on prior charges; and

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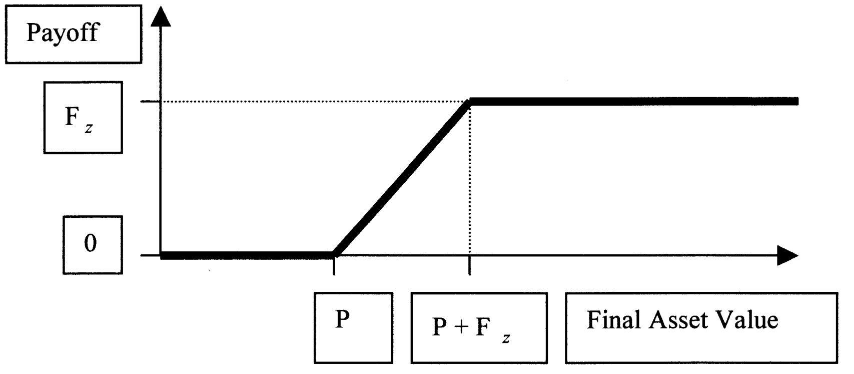

Figure 1. Payoff chart for zero dividend preference share

Put(X) denotes the theoretical value of a put option with strike price X and with expiry date equal to the wind-up date of the trust.

Figure 1 shows the payoff chart for the ZDP.

In applying the Black-Scholes (1973) model to determine the price of the put option, certain inputs must be calculated, observed or estimated:

(1) The exercise price is known (sum of any prior charges and the final ZDP payment).
(2) The interest rate r is defined above.
(3) The yield may be estimated as the gross revenues from the underlying portfolio less costs attributable to the revenue account, divided by the gross assets.
(4) The value of the underlying gross assets could be obtained from the trust managers, or estimated from NAV of the whole trust, adjusted for prior charges, capitalised charges and management fees.
(5) The volatility of the underlying gross assets could be estimated from historical norms, forecasted, or derived from a history of the level of gross assets, if available. The net asset value of a split is published periodically, but it is the volatility of the gross assets (i.e. net assets plus debt, including bank loans) which is the required input for the model. Gross asset volatility may vary over time, due to either changing market conditions or adjustments to portfolio holdings by the investment manager. Stochastic modelling techniques for gross asset volatility have been developed, and these can help with this problem.

In developing an option pricing model for other classes of shares, we confine our attention to splits with a bank loan, ZDPs, income shares and capital shares. Traditional splits and quasi-splits effectively represent special cases of this type of structure. In the case of a traditional split, there are no ZDPs in issue; in the case of a quasi-split, the income shares and capital shares are combined as a single class of share.

Income shareholders are entitled to all the dividends paid out by the trust, plus a final redemption value on wind-up if sufficient assets are available. Dividends are generally set in advance, but managers have increased, cut or cancelled dividends in the past.

The theoretical price of an income share is:

$$\sum_{t=1}^{T} \mathrm{E}(D_t)e^{-rt} + F_i e^{-rt} - \mathrm{Put}(P + F_z + F_i) + \mathrm{Put}(P + F_z)$$

where F_i denotes the final redemption value of the income shares ranking after the ZDPs, and $\mathrm{E}(D_t)$ denotes the expected value of the dividend to be paid to income shareholders at time t.

The payoff chart for the income share is given in Figure 2.

Capital shareholders are entitled to the residual assets once all prior capital has been paid. The payoff for the capital shares can be represented as:

$$\mathrm{Max}(A - [P + F_z + F_i], 0)$$

where A is the final gross asset value of the trust.

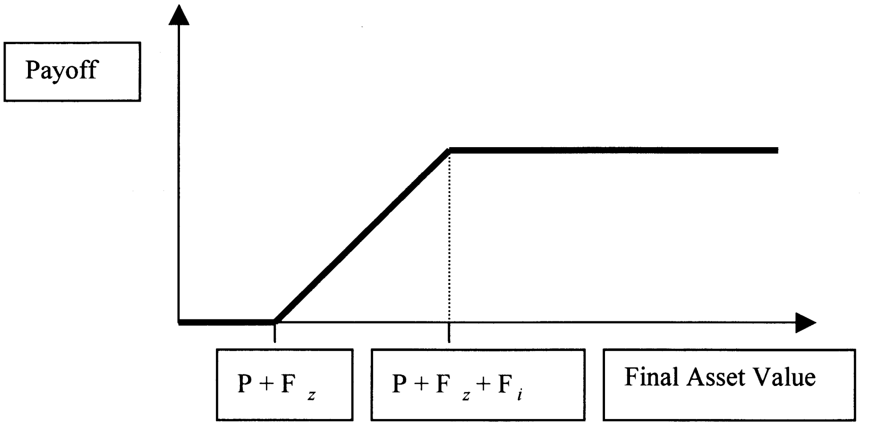

Figure 2. Payoff chart for income share

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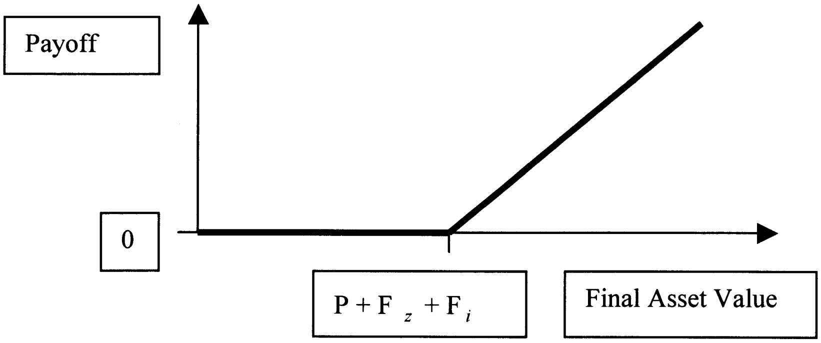

Figure 3. Payoff chart for capital share

Thus, the theoretical price of a capital share is:

$$\text{Call}(P + F_z + F_i)$$

where Call(X) denotes the theoretical value of a put option with strike price X and with expiry date equal to the wind-up date of the trust.

The payoff chart for the capital share is given in Figure 3.

Analyses by Merrill Lynch (1999), Gemmill (2001) and Cazenove & Co (2002) give a varied picture for the differences between market prices and option model prices for both ZDPs and ordinary income shares (which are effectively income shares and capital shares combined as a single class of share). An alternative approach is to calculate the implied volatility of each class of share using observed prices and a closed-form option pricing model. Implied volatility can then be compared to peer group implied volatilities or forecast volatilities. 'Unreasonable' implied volatility would suggest mis-pricing.

Sensitivity measures (known as 'Greeks' — delta, kappa, etc.) can be derived from the Black-Scholes model (see Hull, 2003). One use of these Black-Scholes sensitivities from the perspective of a manager of a split would be in marginal contribution analysis. Prior to the disposal or purchase of a security, the change in volatility of the gross assets could be estimated, and the consequent effect on the price of shares in each class deduced by calculating kappa. Note that some of the fundamental variables, such as force of inflation, considered by Adams (1999)[5] in the risk assessment of

[5] This paper was first submitted in 1997, well before the build up of aggressive structures in the splits sector. It concerned traditional splits and simple quasi-splits without bank debt or cross-holding complications.

splits are not explicitly present in the Black-Scholes model. The Black-Scholes model sensitivities thus have limited use in the risk assessment of splits, although, if a more sophisticated option pricing model was used which took account of all the dependencies, such sensitivity measures would become more useful.

4. Problems with Current Option Pricing Models for Splits

There are two sets of difficulties in applying the above option pricing models. First, some of the difficulties are not due to the option framework in general, but to the specific (and restrictive) assumptions employed. The models in Section 3 are developed from Ingersoll (1976), which was written just three years after the Black-Scholes model was first published. Since then the option model has been extended in many directions. For example, Scott (1997) presents an option pricing model with stochastic volatility, stochastic interest rates and a jump-diffusion process driving stock returns. Secondly, some of the difficulties lie outside the option model. Such difficulties which reduce the effectiveness of option models applied to split capital investment trusts include:

(1) Possible variation in the management and administration fees (for example, some trusts have negotiated reductions in fee levels in recent years). Also, expense allocation policy could be changed, thereby changing the value of different classes of share. The Statement of Recommended Practice for the Financial Statements of Investment Trust Companies (AITC, 2003) recommends that: "Investment management fees should be allocated between capital and revenue in accordance with the board's expected long-term split of returns, in the form of capital gains and income respectively, from the entire investment portfolio of the investment trust company."

(2) The possibility of changes in dividend distributions and the impact of this on other share classes. Such changes may be due to the Board's discretion over distributions. In addition, Section 265 of the Companies Act 1985 states that dividends must not be distributed if the gross assets of a trust are less than 1.5 times the trust's liabilities. To forecast future dividends, it is helpful to know historical dividend distributions and to understand how distribution changes are decided by the Board. Understanding can be assisted through knowledge of the trust's articles of association, the Board's method of achieving fairness between the different share classes, the revenue reserves and the yield on the underlying asset portfolio. An algorithm could then be created to model the Board's behaviour, and this could form the basis of a modelling approach to the problem.

(3) Cross-holdings. Several splits own the income-bearing shares of other

splits or high-yielding investment companies as a source of higher dividend income. However, the dividends from these holdings may be raised, cut or cancelled, and capital losses are possible as a result. A fall in the value of income-bearing shares in one split may have a resonant impact on other splits, due to cross-holdings. This effect was very significant during the splits crisis in 2001 and 2002, but will be less significant going forwards, due to restrictions on cross-holdings imposed by the new Listing Rules (FSA, 2003). Under the new rules, listed investment companies may not invest more than 10% of the gross assets in fellow U.K. listed investment companies, unless those companies have a stated policy which allows them to invest no more than 15% of their assets in other U.K. listed investment companies.

(4) Banking covenants and possible breaches of those covenants. If a covenant is breached, the lender may require the trust to sell assets to raise cash, thereby restoring the asset to debt ratio to an acceptable level and reducing the risk to the lender. This can create a path dependency problem which makes the use of closed-form or lattice methods of implementing option pricing ineffective. Selling assets to raise cash will also lower the volatility of the underlying asset portfolio, which will have a negative impact on the value of any capital shares, particularly if they are out-of-the-money.[6] Note that banking covenants may be renegotiated during the life of the trust, further complicating the analysis. One example of a renegotiated covenant can be seen in the Chairman's Statement of the Annual Report for Danae Investment Trust PLC (February 14th, 2003), which states: "We are required to maintain cover of 165% in acceptable assets over the outstanding amount of the loan; however the Bank [of Scotland] has granted us the right to offset cash held within the portfolio against the loan's value for the purpose of that calculation. This has proved an invaluable option."

(5) Share buy-back programmes. The manager can enhance NAV per share and possibly smooth movements in the discount to NAV by buying ZDPs at a discount and cancelling them. A buy-back may also change two of the inputs to the option model. Firstly, gross assets per share will rise; secondly, the volatility of gross assets may increase (gearing will increase if there is bank debt or prior ranking capital, but no increase in the allocation to cash or bonds following a buy-back). As regards ZDP pricing, the increase in gross assets per share is likely to overwhelm the effect of an increase in volatility, although the manager's tactics need to be considered for a full understanding of the impact of buy-backs. Many managers will sell the 'marginally least attractive' asset in the portfolio

[6] A call option is out-of-the-money if the strike price is above the current price of the underlying asset.

at the time of a buy-back to finance that buy-back; others may use cash holdings within the portfolio at the time of the buy-back. Capital shares will benefit from any increase in volatility (see Moles, 2004). However, Section 264 of the Companies Act, which provides a net asset test, may restrict share buy-backs.

5. Pricing and Risk Assessment using Monte Carlo Simulation

5.1 *Building the Model*

With the complexities of many split trust structures undermining the effectiveness of current closed-form option pricing approaches, the use of Monte Carlo simulation represents a suitable alternative implementation method (Boyle, 1977; Barrie & Hibbert, 2003). Simulations can deal with many of the problems described above. Potential distributions of dividends and capital payments to each class of shareholder over the lifetime of the trust can then be estimated.

In building a model for the risk assessment of splits, the starting point is to ascertain the structure of the investment portfolio, including percentages held in various asset classes (e.g. equity, possibly broken down by region or sector; bonds, described by coupon, maturity, rating and seniority; income-bearing shares in other splits, etc.). The trust's liabilities must then be ascertained, including proportional and fixed fees and their allocation to revenue and capital accounts; loan interest and allocation; corporation tax; loan covenants and rules to be followed if these covenants are breached. The trust's share capital structure is also a required input to the model. Some of the key features of the model are as follows:

(1) The model incorporates a set of rules for generating asset returns and paying liabilities over each time period.
(2) A run of 1,000 simulations is undertaken. The model's time step is at the discretion of the user. For this study, a quarter-year period was used.
(3) Using stochastic models for interest rates, equity returns and credit transitions, the returns on each of the trust's asset classes are derived. Details of these stochastic models are given in Section 5.2. Both capital returns and income are calculated, with income added to the trust's revenue account.
(4) Fixed expenses and expenses proportional to the underlying gross (or net) assets are charged to the capital and revenue accounts, using the specified allocation rule.
(5) Interest is charged to the capital and revenue accounts, using the specified allocation rule.
(6) Dividends paid by the trust are charged to the revenue account.
(7) Corporation tax rates are applied to unfranked income, after offsetting

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the trust's expenses and interest payments.

(8) Banking covenants are checked, and, if breached, portfolio re-allocation is carried out according to a specified covenant breach rule.

(9) If the test in Section 265 of the Companies Act 1985 rule is not met, dividends which would otherwise have been paid are held in a revenue reserve account, for payment if compliance is achieved later. If compliance is not achieved later, this reserve would be distributed as capital if all prior charges have been paid.

(10) On wind-up of the trust, capital distributions are made to each class of share, according to pre-determined rules.

5.2 *Stochastic Models Employed*

Term structure model

To model the term structure, we assume that the short-term interest rate $r(t)$ follows a two-factor version of the *Vasicek* model. This model can be described as follows:

(1) The short-term rate of interest is assumed to follow a mean-reverting stochastic process; that is that the short rate is subject to (apparently) random disturbances over time, but is pulled towards some 'natural' (mean reversion) level over the long term. This is sometimes called an *elastic random walk*.

(2) The 'natural' (mean reversion) level for the short-term interest rate also follows a mean-reverting stochastic process.

The stochastic differential equations (SDEs) which govern the behaviour of the short rate $r(t)$ and reversion level $u(t)$ are:

$$dr = \alpha_1(u(t) - r(t))dt + \sigma_1(dZ_1 + \kappa dt)$$
$$du = \alpha_2(\mu - u(t))dt + \sigma_2(dZ_2 + \kappa dt)$$

where:

$r(t)$ is the short rate at time t;
$u(t)$ is the mean reversion level for the short rate at time t;
α_1 is the strength of mean-reversion of r;
σ_1 is the annualised volatility (standard deviation) of the short rate;
α_2 is the strength of mean-reversion of u;
μ is the mean reversion level for $u(t)$;
σ_2 is the annualised volatility (standard deviation) of the short rate mean reversion level; and
Z_1 and Z_2 are *independent* Brownian Motions.

The 'market price of risk' parameter κ controls the risk premium on bonds relative to cash. Specifically, we can show that the return on an

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irredeemable bond exceeds the return on cash by an amount

$$\left(-\frac{\sigma_1}{\alpha_1}-\frac{\sigma_2}{\alpha_2}\right)\times\kappa.$$

When the model is used for risk-neutral valuation purposes, the market price of risk κ is set to zero.

Equity portfolio

The trust's equity portfolio is assumed to be composed of two different types of equities:

(1) 'plain vanilla' equities; and
(2) cross-holdings in other splits.

We model equity total returns *in excess* of the short-term risk-free interest rate. This means that the total returns on the trusts' equity portfolios are constructed by modelling an excess return, and then adding this to the risk-free rate. Specifically, we assume that the total-return index on the plain vanilla equity portfolio $S(t)$ follows the following SDE:

$$\frac{dS}{S}=(r(t)+erp)dt+\sigma_M dZ_M$$

where:

erp is the equity arithmetic risk premium (set to zero for valuation purposes);
σ_M is the equity volatility; and
Z_M is a Brownian motion.

The total return consists of both capital return and dividends. To model the dividend, we assume that the *log* of the dividend yield follows a mean-reverting process:

$$d\ln(y)=\alpha_y\big(\mu_y-y(t)\big)dt+\sigma_M dZ_y(t)$$

where:

α_y is the strength of mean-reversion of y;
σ_M is the equity volatility;
μ_y is the mean-reversion level for $\ln(y)$; and
Z_y is a Brownian motion.

So long as a high negative correlation is imposed on the shocks to the yield model and the equity returns model, this specification produces equity yields which move — in the short term — with price changes. Over longer periods *strong* equity returns tend to be *followed by* above-average dividend growth (as the dividend yield reverts to mean) and equity declines will be *followed by* below-average dividend growth or falling dividends. The model is simple and has a natural economic interpretation.

Modelling cross-holdings in a split was a considerable challenge before the splits crisis unfolded. However, changes to the Listing Rules (FSA, 2003) impose a severe limit on cross-holdings, thereby eliminating the possibility of a downward spiral in prices as experienced by some splits during the recent prolonged bear market. In the model in this paper, cross-holdings in other splits are treated as high-beta shares. Specifically, the total return index on the portfolio of cross-holdings $C(t)$ is assumed to satisfy:

$$\frac{dC}{C} = r(t)dt + \beta\left(\frac{dS}{S} - r(t)dt\right)dt + \sigma_C dZ_C$$

where:

β is the beta of the portfolio of cross-holdings;
σ_C is the cross-holdings' specific risk; and
Z_C is an independent Brownian Motion.

Corporate bond portfolio

To model the corporate bond portfolio we use the Jarrow-Lando-Turnbull (JLT) model (Jarrow *et al.*, 1997). With this model, the probability of a bond issuer of a given credit rating making a transition to any other credit rating in one year (including default) is described by a transition matrix. This matrix is transformed into a risk-neutral matrix for the purposes of corporate bond pricing.

5.3 *Using Simulation Outcomes to Illustrate Risk*

Given a series of simulations, a cumulative probability distribution curve for gross assets at the wind-up date can be constructed (see Appendix B). A chart can also be constructed to show the mean path and, say, the 5th and 95th percentiles of paths for gross assets at each stage in the simulation process (Appendix C).

A fair value for each share can be estimated from knowledge of the mean redemption values and all dividend payments across the simulations (calculated under the risk-neutral measure). Possible reasons for differences between estimated fair values and current market prices include poor liquidity in the splits market, difficulties in undertaking arbitrage (due to poor splits transparency and difficulties in short-selling), mis-specification of the model, or an investor sentiment effect (e.g. investors shun a segment of the stock market which has been under regulatory scrutiny).

A distribution of fair values for each class of share can be determined at any point in the future as part of a risk analysis. Strictly speaking, shifts in the difference between estimated fair values and market prices should also be considered in the analysis. However, such shifts are difficult to predict, and will be relatively small in the context of the spread of outcomes shown by the stochastic projections. In the case of conventional U.K. investment trusts, empirical evidence suggests that discount volatility represents only a small proportion of the total variance of returns, particularly for longer (e.g. greater than six-monthly) return intervals (Adams, 2003).

To understand better the risks associated with each class of share, we propose the use of separate 'payoff cumulative probability charts' for each class of share. For each of these charts, the horizontal axis shows the payoff (in £s) for the share at wind-up. The vertical axis shows the cumulative probability of such a payoff. Based on the outcomes from a series of simulations, the chart thus shows the probability that the payoff is equal to or less than any given value on the horizontal axis. Figures 4, 5 and 6 illustrate this method, using an asset generating model and the capital structure of a theoretical split with bank loan, ZDPs, income shares and capital shares, as described in Appendix D.

To illustrate risk/return profiles, we advocate the use of 'return

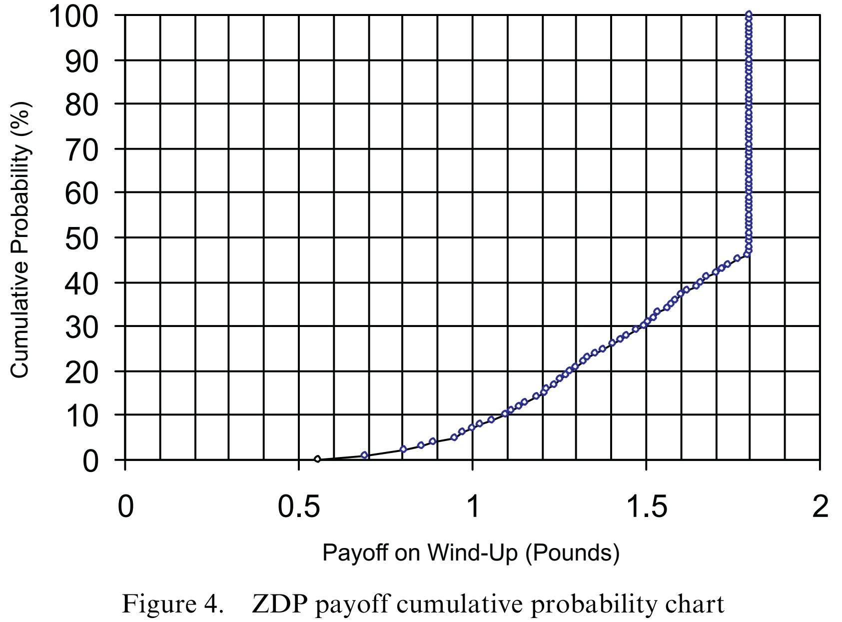

Figure 4. ZDP payoff cumulative probability chart

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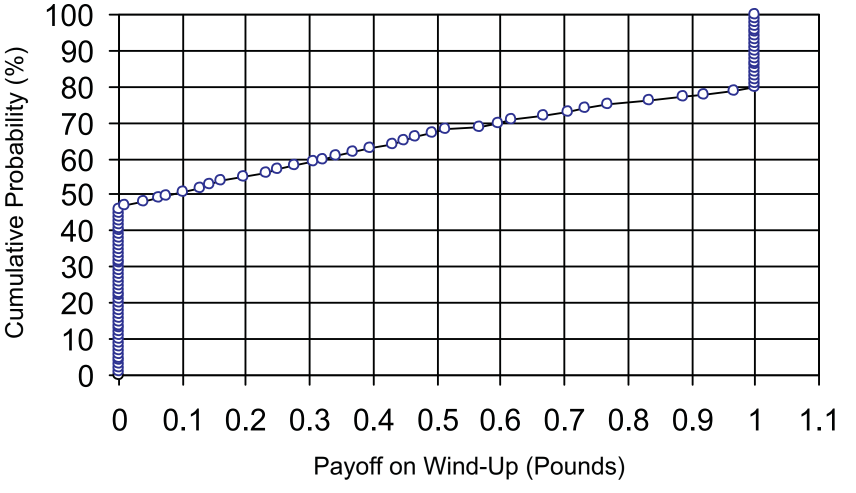

Figure 5. Income share payoff cumulative probability chart

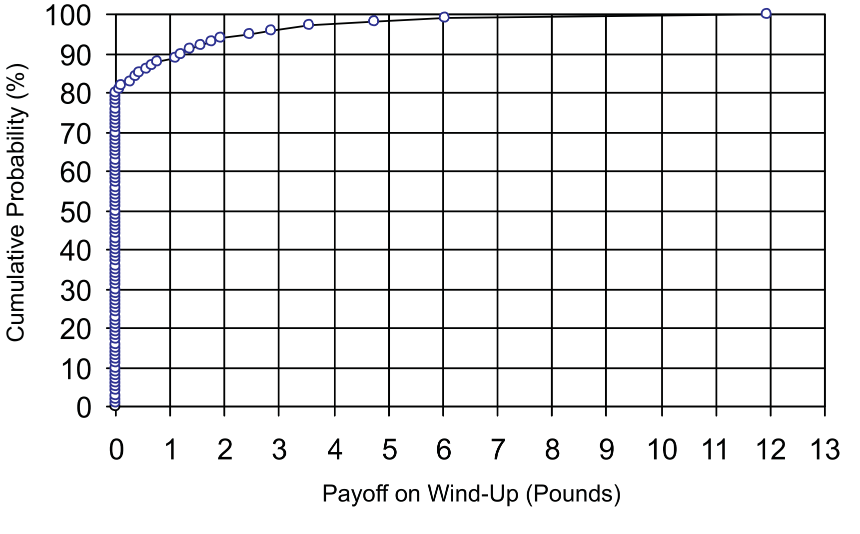

Figure 6. Capital share payoff cumulative probability chart

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probability histograms' for each class of share. The annualised returns up to the wind-up date for a class of share are based on the outcome of a series of simulations. The probability associated with each range of returns is calculated as the proportion of simulation returns which fell within that range. For the theoretical split detailed in Appendix D, the return probability histograms for each class of share are shown in Figures 7, 8 and 9.

For the income shares, dividends received over the life of the fund should also be considered. Due to the possibly different tax treatment of income and capital, taxable investors may wish to keep income and capital return charts separate. However, the two may be combined to produce a total return histogram, using a particular investor's tax rates for income and for capital gains in the total return calculation, if required.

5.4 *Sensitivity and Scenario Analysis using Stochastic Modelling*

It is possible to study the sensitivity of the output to different assumptions in the asset-return generating model or to changes in the trust's structure. This helps in understanding the risks involved, and may be useful in the design of new splits, as well as in assessing existing splits. It can also be used to complement the limited sensitivity analysis which is possible using

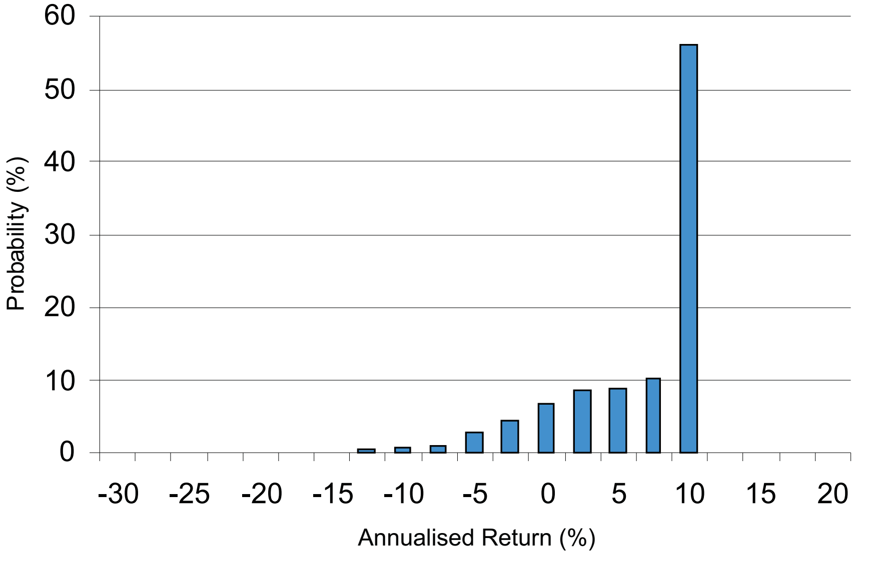

Figure 7. ZDP return histogram

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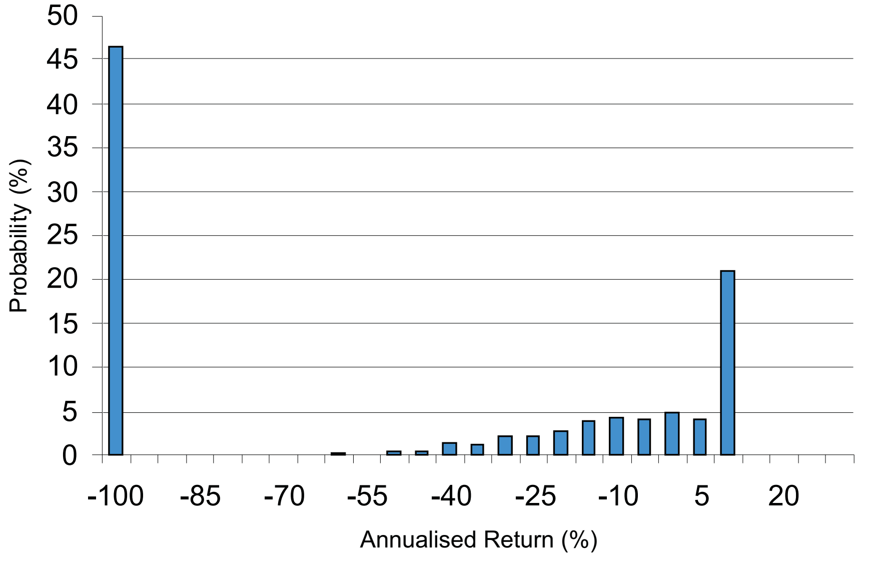

Figure 8. Income share capital return histogram

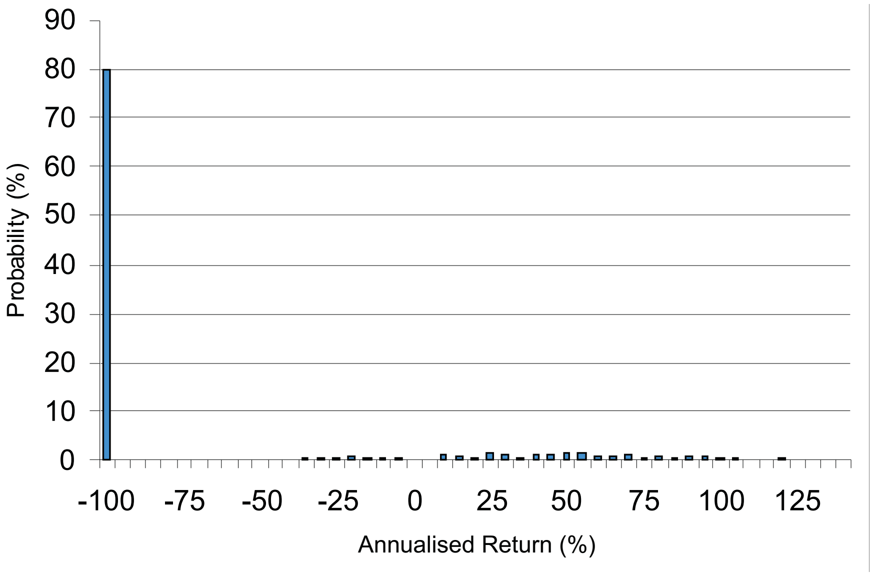

Figure 9. Capital share return histogram

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Table 2. Scenario analysis example

	Initial state (as per Appendix D)	Replace high yield portfolio with bonds; set equity portfolio dividend yield to 4% from 3.5%	Alter capital structure: replace 5 million of ZDPs with bank loan (£5.5 million)	Replace £10 million of equities with bonds; set number of bonds to 20 from ten
Capital share wipe-out probability (%)	80	83	77	85
Probability of capital gain on capital shares (%)	17	15	20	13
Income share wipe-out probability (%)	47	44	41	44
Probability of capital gain on income shares (%)	30	28	33	25
Probability that ZDP pays full redemption value (%)	53	56	58	55

closed-form option pricing models. Table 2 illustrates some of the ways in which scenario analysis can be applied using simulation techniques, for the theoretical split outlined in Appendix D.

Scenario analysis could be used to create a capital structure at launch, or an asset allocation at any stage in the fund's life, such that the probability of the ZDP paying out its full redemption value meets a certain value (e.g. 95%). Various share buy-back proposals could also be studied using this technique, to assess the impact on each class of share, and thus to ensure fairness across all classes.

6. Conclusion

As the complexity of split capital investment trusts has increased, with greater use of bank loans tied to covenants, more complicated share capital structures and the problem of cross-investment within the sector, so too has the need for more sophisticated risk assessment techniques. The use of traditional measures in such an environment can give a dangerously false sense of understanding of the risks. The applicability of option pricing models has also been severely limited by the complexity of many trust structures. We advocate a stochastic modelling approach to deal with these complexities, and suggest the use of simple descriptive diagrams derived from this approach to illustrate the risks inherent in each class of share. In particular, we propose the use of 'return probability histograms' to illustrate the risk/return profiles for each class of share.

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APPENDIX A

COMPARISON OF THE ZDPS OF A HYPOTHETICAL GEARED TRUST TO A HYPOTHETICAL UNGEARED TRUST

Assume that both ZDPs have two years until redemption, and that interest and management fees are charged to the revenue account. The underlying portfolios of investments are identical.

	Ungeared trust	Geared trust
Total assets	100	100
Debt	0	50
Final ZDP repayment	80	30
Cover	1.25	1.67
Hurdle rate (%)	−10.56	−10.56
Wipe-out rate (%)	−100.00	−29.29

Both ZDPs have the same hurdle rate, and the geared trust has a higher cover. However, this does not mean that the geared ZDP is less risky than the ungeared ZDP. The geared ZDP has greater sensitivity to large falls in the underlying portfolio. Assuming both ZDPs trade on a gross redemption yield of 7% p.a., Figure A.1 shows how sensitive each ZDP is to changes in the underlying portfolio:

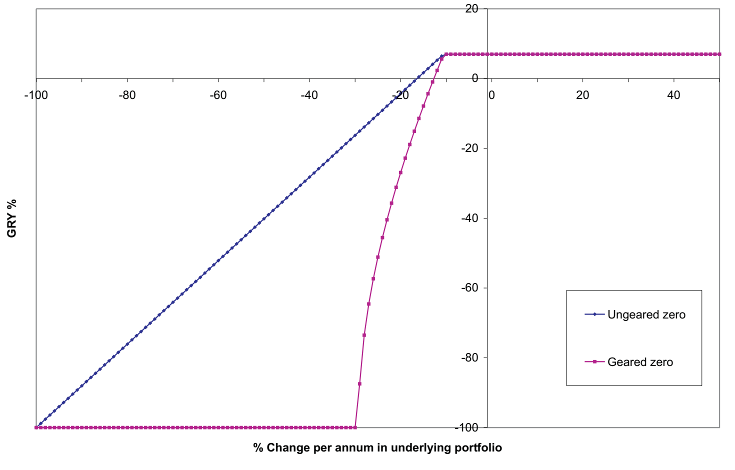

Figure A.1. Sensitivity of a geared and an ungeared zero to changes in the underlying portfolio

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APPENDIX B

CUMULATIVE PROBABILITY DISTRIBUTION CURVE FOR GROSS ASSETS

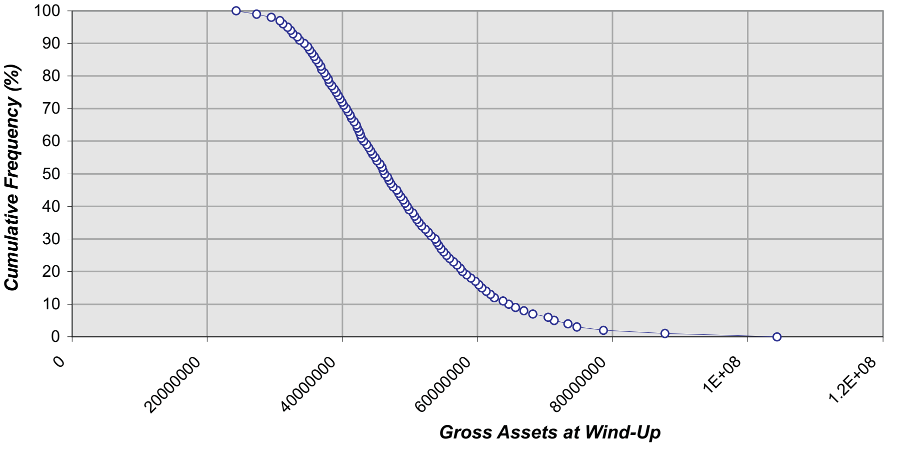

Figure B.1

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APPENDIX C

PATH OF GROSS ASSETS AT EACH STAGE OF THE SIMULATION PROCESS (1st, 5th, 25th, 50th, 75th, 95th AND 99th PERCENTILES)

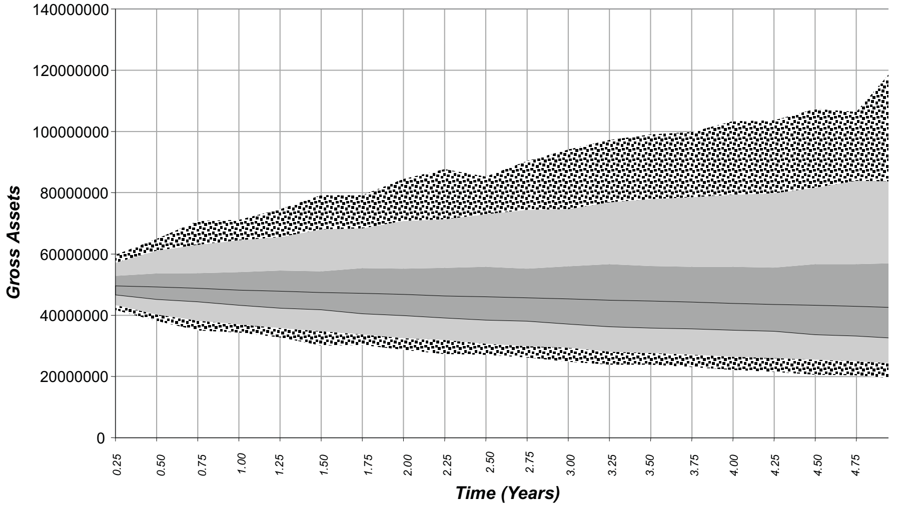

Figure C.1

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APPENDIX D

CAPITAL STRUCTURE OF A THEORETICAL SPLIT

Break-down of asset portfolio

Portfolio breakdown	£
Initial value of equity portfolio	35,000,000
Initial value of high yield portfolio	4,000,000
Initial value of bond portfolio	10,000,000
Cash	
Cash at bank	1,000,000
Debtors less creditors	0
Total	50,000,000

Corporate bond portfolio details

Number of holdings 10

Coupon	Maturity (years)	Coupon frequency	Credit rating	Seniority
7%	10	4	3	3

Equity model		
	Equity risk premium	3.50%
	Volatility	20.00%
Dividend-yield model		
	Dividend yield reversion rate	0.25
	Dividend yield reversion level	−3.02
	Dividend yield volatility	20%
	Initial dividend yield	3.50%
High-yield equity portfolio		
	Beta	2
	Stock-specific risk	10%
	Dividend yield multiple	2

	Classes of capital			
Name of capital class	Bank loans	ZDPs	Income shares	Capital shares
Income entitlement	TRUE	FALSE	TRUE	FALSE
Capital entitlement	TRUE	TRUE	TRUE	TRUE
Capital priority	1	2	3	4
Nominal or loan amount/number of shares	5,000,000	20,000,000	20,000,000	10,000,000
Fixed income payment per nominal/ share (£ p.a.)	0.07		0.08	
Fixed capital payment per nominal/share (£)	1	1.8	1	

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