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Robust control of robot manipulators with an adaptive fuzzy unmodelled parameter estimation law

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Summary

For robot manipulators, there are two types of disturbances. One is model parametric uncertainty; the other is unmodelled parameters such as joint friction forces and external disturbances. Unmodelled joint frictions and external disturbances reduce performance in terms of positioning accuracy and repeatability. In order to compensate for unmodelled parameters, the design of a new controller is considered. First, the modelled and unmodelled parameters are included in a dynamic model. Then, based on the dynamic model, a new Lyapunov function is developed. After that, new nonlinear joint friction and external disturbance estimation laws are derived as an analytic solution from the Lyapunov function; thus, the stability of the closed system is guaranteed. Better values of the adaptive dynamic compensators can be extracted by fuzzy rules according to the tracking error. Limitations and knowledge about friction and external disturbances are not required for the design of the controller. The controller compensates for all possible model parameter uncertainties, all possible unknown joint frictions and external disturbances.

1. Introduction

Robotic manipulators are highly nonlinear systems, and they are subject to two types of disturbance. One is model parametric uncertainty due to variations in grasped load characteristics such as the mass moment of inertia or the exact position of the centre of the payload. The other is unmodelled parameters such as joint friction forces and external disturbances. When the dynamic parameters of the robotic system are not precisely known, adaptive [1–4] and robust [5–7] control laws are typical control schemes to achieve good tracking performance for robot manipulators.

The Corless [8] or Corless–Leitmann [9] approach is popularly used in the design of robust controllers for robot manipulators [5, 6, 10–12]. In this approach, uniform ultimate boundedness of tracking error is shown using the Lyapunov-based theory of guaranteed stability of uncertain system. In a commonly robust control law, the maximum upper uncertainty bound, which is the worst case of the uncertainty bound, is used for stabilisation of control laws [5, 6]. However, a very high uncertainty bound may cause chattering and bad behaviour of the whole system, while a very low estimation of the uncertainty bound function may cause a higher tracking error. If the uncertainty bound of a parameter is large, large control gains are required. However, high gain is impractical in design, and low gains are desirable for robust control laws. This gain limitation deteriorates transient and steady-state performance [10]. In order to obtain better tracking performance, the uncertainty bound function should be updated properly [13]. For this purpose, the upper bounding function is made adaptive in some studies [11, 12, 14]. The upper bounding function is estimated depending on a regressed matrix [11], functions [12], or adaptive fuzzy rule [14].

In robust control laws [5, 6, 10–12], nominal control, which is used to eliminate the known dynamics of robotic manipulators, is recommended and regressor analysis is required. Based on function

approximation techniques, the use of a regressor matrix is eliminated, and regressor-free robust control of robot manipulators has been presented [15, 16]. In this approach, Fourier series expansion is used for uncertainty estimation [16].

However, unmodelled dynamics reduce performance such as positioning accuracy and repeatability. In order to achieve the desired performance, it is imperative to design some form of disturbance compensation. In the literature concerning robotic applications, some disturbance rejection techniques are proposed [17–19].

In order to compensate for both modelled and unmodelled parametric uncertainty, a robust control law [10] and a robust adaptive [18] control law are designed. In these control laws [10, 20], modelled and unmodelled parameters are included in the dynamic model. Then modelled parameter uncertainty is separated from unmodelled parameter uncertainty. After that, a robust control law is designed for each uncertainty group.

In the present paper, robot unmodelled parameters, joint frictions and external disturbances are included in a dynamic model. Based on the dynamic model, new joint friction and external disturbance estimation laws are developed. The Lyapunov theory, based on the Corless–Leitmann approach [9], was used to design the control law, and uniform boundedness error convergence is achieved. After the simulation results, it is observed that the proposed adaptive law compensates for not only all joint frictions and external disturbances but also all modelled parametric uncertainty.

2. Dynamic Model

The dynamic model of an n-link manipulator can be written as [10]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + B\dot{q} + f_l(q,\dot{q}) + d = \tau, \tag{1}$$

where q denotes joint position, τ is the n-dimensional vector of the actuating input, $C(q,\dot{q})\dot{q}$ is the n-dimensional vector of Coriolis and centripetal forces, $G(q)$ is the n-dimensional gravitational force vector, $f_l(q,\dot{q})$ represents the static and low speed friction, $B\dot{q}$ is the viscous friction forces at the joints and d is the external disturbance at the joints. The joint friction can be modelled as [10]

$$f = f_l(q,\dot{q}) + B\dot{q}, \tag{2}$$

where $f_l(q,\dot{q})$ represents the static and low speed friction; it is a function of joint angle and velocity and usually difficult to model and to compensate for accurately. Then the dynamic model (1) can be written as

$$Y(q,\dot{q},\ddot{q})\pi + f + d = \tau \tag{3}$$

where π is the p-dimensional vector of robot parameters and $Y(q,\dot{q},\ddot{q})$ is the $n \times p$ dimensional matrix containing position, velocity and acceleration of joints. For any specific trajectory, q_d, $\dot{q}_d$ and $\ddot{q}_d$ show the desired position, velocity and acceleration vectors, respectively. The parameters $\tilde{q} = q - q_d$ and $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ are the measured actual position and velocity errors. Using the above information, the corrected desired velocity and acceleration vectors for nonlinearities and decoupling effects are given.

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}; \ \ddot{q}_r = \ddot{q}_d - \Lambda\dot{\tilde{q}} \tag{4}$$

The error σ is given as

$$\sigma = \dot{q} - \dot{q}_r = \dot{\tilde{q}} + \Lambda\tilde{q}, \tag{5}$$

where Λ is an $n \times n$ dimensional positive definite diagonal matrix. During motion, the inertia parameter vector π is constant, not known exactly, but bounded. The upper uncertainty bound is defined as follows:

$$\pi_0 - \pi = \tilde{\pi} \leq \rho \tag{6}$$

It is assumed that $\pi_0 \in R^p$ and $\rho \in R^p$ are known. Then the following robust control law is given [5]:

$$\tau_r = M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)u_p - K\sigma$$

$$= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\pi_0 + u_p) - K\sigma \tag{7}$$

Here M_0, C_0, G_0 and π_0 are the known nominal control parameters, u_p is the robust control input and $K\sigma$ is the PD action.

3. Proposed Controller

Considering the dynamic model and the nominal control law (7), the overall control law is proposed as

$$\tau = \tau_r - \hat{f} + \hat{d} + u_d = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\pi_0 + u_p) - \hat{f} + \hat{d} + u_d - K\sigma, \tag{8}$$

where $\hat{f}$ is the estimation law for joint frictions f, $\hat{d}$ is the estimation law for external disturbance d, u_p is the control input designed robust to modelled parametric uncertainty and u_d is the control input designed for joint frictions and disturbances. Substituting (8) into (1) and considering Equations 1–6, Equation 9 is obtained.

$$M(q)\dot{\sigma} + C(q,\dot{q})\sigma = \tilde{M}(q)\ddot{q}_r + \tilde{C}(q,\dot{q})\dot{q}_r - \hat{f} + \hat{d} + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)u_p + u_d$$

$$+ f + d - K\sigma = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\tilde{\pi} + u_p) + u_d - \hat{f} + \hat{d} + f + d - K\sigma \tag{9}$$

Here $\tilde{M} = M_0 - M$, $\tilde{C} = C_0 - C$, $\tilde{G} = G_0 - G$ and $\pi_0 - \pi = \tilde{\pi}$. The unknown friction and external disturbance parameter estimation error vector $\tilde{\eta}$ are defined as

$$\tilde{\eta} = \hat{f} - \hat{d} \tag{10}$$

It is assumed that friction f and external disturbance d are not constant but bounded as

$$\|f\| + \|d\| \leq \omega \tag{11}$$

Since $\omega \in R$ is assumed unknown, ω should be estimated with an estimation law to control the system properly. The estimate of ω is $\hat{\omega}$ and the estimation error $\tilde{\omega}$ is defined as

$$\tilde{\omega} = \omega - \hat{\omega} \tag{12}$$

4. Theorem and Proof of Stability

In order to derive a new controller, the following theorem is given.

Theorem 1: *Let* $\alpha_1, \alpha_2, \ldots, \alpha_p$, $\beta_1, \beta_2, \cdots, \beta_p, \lambda_1, \lambda_2 \ldots, \lambda_p, b \in R$. *The estimation law for the joint friction force is* $\hat{f}$, *the estimation for external disturbances is* $\hat{d}$ *and the estimation law for the uncertainty bound of the unknown parameters of f and d is* $\hat{\omega}$. *The estimation laws* $\hat{f}$, $\hat{d}$ *and* $\hat{\omega}$ *are defined as*

$$\hat{f} = diag\left((\beta_i^2 \alpha_i)\frac{arctan(\alpha\int\sigma\,dt)_i}{(\alpha\int\sigma\,dt)_i^2 + 1} \right); \hat{d} = diag\left(\frac{\lambda_i}{(\alpha\int\sigma\,dt)_i^2 + 1} \right) \; i = 1, 2, \cdots n; \dot{\hat{\omega}} = b_1\|\sigma\| \tag{13}$$

The additional control input u_p *and* u_d *are defined as*

$$u_p = \begin{cases} -\dfrac{Y^T\sigma}{\|Y^T\sigma\|}\|\rho\| & if \; \|Y^T\sigma\| > \varepsilon_p \\ -\dfrac{Y^T\sigma}{\varepsilon_p}\|\rho\| & if \; \|Y^T\sigma\| \leq \varepsilon_p \end{cases} = \; ; \; u_d = \begin{cases} -\dfrac{\sigma}{\|\sigma\|}\hat{\omega} & if \; \|\sigma\| > \varepsilon_d \\ -\dfrac{\sigma}{\varepsilon_d}\hat{\omega} & if \; \|\sigma\| \leq \varepsilon_d \end{cases} \tag{14}$$

If the control laws (13) and (14) are substituted into the control input (8) for control of robot manipulators, then the control law (8) will be continuous and the closed-loop system will be uniformly ultimately bounded.

Proof. In order to prove the theorem, the following Lyapunov function candidate is proposed:

$$V(\sigma, \tilde{q}, \tilde{\pi}, \tilde{\omega}, \tilde{\eta}, \Phi) = \frac{1}{2}\sigma^{T}M(q)\sigma + \frac{1}{2}\tilde{q}^{T}D\tilde{q} + \frac{1}{2b_{1}}\tilde{\omega}^{2} + \frac{1}{2}\tilde{\eta}^{T}(K_{T} + \Phi^{2})^{2}\tilde{\eta} \geq 0, \qquad (15)$$

where $D \in R^{n \times n}$ is a positive diagonal matrix and Φ is chosen as an $n \times n$ dimensional diagonal matrix changing in time. The time derivative of V throughout the system (9) is

$$\dot{V} = \sigma^{T}M(q)\dot{\sigma} + \frac{1}{2}\sigma^{T}\dot{M}(q)\sigma + \tilde{q}^{T}D\dot{\tilde{q}} + \frac{1}{b_{1}}\tilde{\omega}\dot{\tilde{\omega}} + 2\tilde{\eta}^{T}(K_{T} + \Phi^{2})\Phi\dot{\Phi}\tilde{\eta}$$
$$+ \tilde{\eta}^{T}(K_{T} + \Phi^{2})^{2}\dot{\tilde{\eta}} \qquad (16)$$

Substituting Eq. (9) into (16), the following equation is obtained:

$$\dot{V} = \sigma^{T}\left[\frac{1}{2}\dot{M}(q) - C(q, \dot{q})\right]\sigma - \sigma^{T}K\sigma + \sigma^{T}Y(\tilde{\pi} + u_{p}) + \sigma^{T}(f + d - \hat{f} + \hat{d} + u_{d})$$
$$+ \tilde{q}^{T}D\dot{\tilde{q}} - \frac{1}{b_{1}}(\omega - \hat{\omega})\dot{\hat{\omega}} + 2\tilde{\eta}^{T}(K_{T} + \Phi^{2})\Phi\dot{\Phi}\tilde{\eta} + \tilde{\eta}^{T}(K_{T} + \Phi^{2})^{2}\dot{\tilde{\eta}} \qquad (17)$$

Note that $\dot{\tilde{\omega}} = -\dot{\hat{\omega}}$ since ω is constant. Using the property $\sigma^{T}[\dot{M}(q) - 2C(q, \dot{q})]\sigma = 0 \ \forall \sigma \in R^{n}$ and taking $D = 2\Lambda K$ [5], Eq. (17) becomes

$$\dot{V} = -\dot{\tilde{q}}^{T}K\dot{\tilde{q}} - \tilde{q}^{T}\Lambda K\Lambda\tilde{q} + \sigma^{T}Y(\tilde{\pi} + u_{p}) - \|\sigma\|\,\omega + \|\sigma\|\,\hat{\omega} + \sigma^{T}u_{d} + \sigma^{T}f + \sigma^{T}d$$
$$+ \sigma^{T}(-\hat{f} + \hat{d}) + 2\tilde{\eta}^{T}(K_{T} + \Phi^{2})\Phi\dot{\Phi}\tilde{\eta} + \tilde{\eta}^{T}(K_{T} + \Phi^{2})^{2}\dot{\tilde{\eta}} \qquad (18)$$

K_{T} and Φ are defined as a $n \times n$ dimensional diagonal matrix such that

$$\Phi = diag\left(\frac{(\alpha \int \sigma\, dt)_{i}}{\sqrt{\alpha_{i}\beta_{i}}}\right); K_{T} = diag\left(\frac{1}{\alpha_{i}\beta_{i}}\right); K_{T} + \Phi^{2} = diag\left(\frac{(\alpha \int \sigma\, dt)_{i}^{2} + 1}{\alpha_{i}\beta_{i}}\right); i = 1, 2, \cdots n \quad (19)$$

$$\dot{\Phi} = diag\left(\frac{(\alpha\sigma)_{i}}{\sqrt{\alpha_{i}\beta_{i}}}\right); i = 1, 2, \cdots, n. \qquad (20)$$

From Eq. (13), the friction and external disturbance parameter estimation error vector $\tilde{\eta} = \hat{f} - \hat{d}$ is defined as

$$\tilde{\eta}_{i} = \frac{\beta_{i}^{2}\alpha_{i}^{2}arctan(\alpha \int \sigma\, dt)_{i} - \lambda_{i}}{1 + (\alpha \int \sigma\, dt)_{i}^{2}} \qquad (21)$$

The time derivative of $\tilde{\eta}$ is

$$\dot{\tilde{\eta}}_{i} = \frac{2(\alpha\sigma)_{i}(\alpha \int \sigma\, dt)_{i} - \lambda_{i}}{1 + (\alpha \int \sigma\, dt)_{i}^{2}}\tilde{\eta}_{i} + \frac{(\beta_{i}^{2}\alpha_{i}^{2})\sigma_{i}}{\left(1 + (\alpha \int \sigma\, dt)_{i}^{2}\right)^{2}} \qquad (22)$$

The last two terms in Eq. (17) are written depending on the functions K_{T}, Φ, $\dot{\Phi}$, $\tilde{\eta}$ and $\dot{\tilde{\eta}}$ as

$$2(\tilde{\eta})_{i}((K_{T})_{i} + \Phi_{i}^{2})\Phi_{i}\dot{\Phi}_{i}\tilde{\eta}_{i} + (\tilde{\eta})_{i}((K_{T})_{i} + \Phi_{i}^{2})^{2}(\dot{\tilde{\eta}})_{i} = (\tilde{\eta})_{i}\frac{2(\alpha \int \sigma\, dt)_{i}^{2} + 1)(\alpha \int \sigma\, dt)_{i}(\alpha\sigma)_{i}}{\beta_{i}^{2}\alpha_{i}^{2}}(\tilde{\eta})_{i}$$

$$-(\tilde{\eta})_{i}\frac{2(\alpha \int \sigma\, dt)_{i}^{2} + 1)(\alpha \int \sigma\, dt)_{i}(\beta\sigma)_{i}}{\beta_{1}^{2}\alpha_{1}^{2}}(\tilde{\eta})_{i} + (\tilde{\eta})_{i}\frac{(\alpha \int \sigma\, dt)_{i}^{2} + 1)^{2}}{\beta_{i}^{2}\alpha_{i}^{2}}\frac{\beta_{i}^{2}\alpha_{i}^{2}(\sigma)_{i}}{(1 + (\alpha \int \sigma\, dt)_{i}^{2})^{2}} \quad (23)$$

The first and second terms cancel each other and Eq. (23) can be written as

$$2\tilde{\eta}^{T}(K_{T} + \Phi^{2})\Phi\dot{\Phi}\tilde{\eta} + \tilde{\eta}^{T}(K_{T} + \Phi^{2})^{2}\dot{\tilde{\eta}} = \tilde{\eta}^{T}\sigma \qquad (24)$$

Then the derivative of the Lyapunov function is obtained as

$$\dot{V} = -\dot{\tilde{q}}^{\mathrm{T}} K \dot{\tilde{q}} - \tilde{q}^{\mathrm{T}} \Lambda K \Lambda \tilde{q} + \sigma^{\mathrm{T}} Y(\tilde{\pi} + u_{\mathrm{p}}) - \|\sigma\| \,\omega + \|\sigma\| \,\hat{\omega} + \sigma^{\mathrm{T}} u_{\mathrm{d}} + \sigma^{\mathrm{T}}(f + d)$$

$$\leq -\dot{\tilde{q}}^{\mathrm{T}} K \dot{\tilde{q}} - \tilde{q}^{\mathrm{T}} \Lambda K \Lambda \tilde{q} + \sigma^{\mathrm{T}} Y(\tilde{\pi} + u_{\mathrm{p}}) + \|\sigma\| \,(-\omega + \|f\| + \|d\|) + \|\sigma\| \,\hat{\omega} + \sigma^{\mathrm{T}} u_{\mathrm{d}} \qquad (25)$$

For stability analysis, four different cases are considered similar to the robust control law [10].

Case 1: If $\left\| Y^T \sigma \right\| \geq \varepsilon_p$ and $\|\sigma\| \geq \varepsilon_d$. For the first case, control inputs are defined as $u_d = -\frac{\sigma}{\|\sigma\|} \hat{\omega}$ and $(u_p) = -\frac{Y^T \sigma}{\|Y^T \sigma\|} \|\rho\|$. Then Eq. (25) is obtained as

$$\dot{V} = -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y \tilde{\pi} - (Y^T \sigma)^T \frac{Y^T \sigma}{\|Y^T \sigma\|} \|\rho\| + \|\sigma\| \,\hat{\omega}$$

$$- \frac{\sigma^T \sigma \hat{\omega}}{\|\sigma\|} + \|\sigma\| \,(-\omega + \|f\| + \|d\|)$$

$$\leq -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \left\| Y^T \sigma \right\| (\|\tilde{\pi}\| - \|\rho\|) + \|\sigma\| \,(\hat{\omega} - \hat{\omega}) \leq 0 \qquad (26)$$

Since K and Λ are positive definite matrices and $\left\| Y^T \sigma \right\| (\|\tilde{\pi}\| - \|\rho\|) + \|\sigma\| \,(\hat{\omega} - \hat{\omega}) \leq 0$, the derivative of the Lyapunov function will be $\dot{V} \leq 0$. As a result, the system will be stable. Eq. (15) shows that V is a positive continuous function and V tends to a constant as $t \to \infty$ and therefore V remains bounded. Thus, $\dot{\tilde{q}}$ and $\tilde{q}$ are bounded, that is, $\dot{\tilde{q}}$ and $\tilde{q}$ converge to zero and this implies that σ is bounded and converges to zero. As a result, $\int \sigma dt$ is bounded and converges to a constant. These imply that $\hat{f}$, $\hat{d}$ and $\hat{\omega}$ are bounded.

Case 2: If $\left\| Y^T \sigma \right\| \geq \varepsilon_p$ and $\|\sigma\| \leq \varepsilon_d$. For the second case, the control inputs are defined as $(u_p) = -\frac{Y^T \sigma}{\|Y^T \sigma\|} \|\rho\|$ and $u_d = -\frac{\sigma}{\varepsilon_d} \hat{\omega}$. Then Eq. (25) is obtained as

$$\dot{V} \leq -\dot{\tilde{q}}^{\mathrm{T}} K \dot{\tilde{q}} - \tilde{q}^{\mathrm{T}} \Lambda K \Lambda \tilde{q} + \sigma^{\mathrm{T}} Y \tilde{\pi} - (Y^T \sigma)^{\mathrm{T}} \frac{Y^{\mathrm{T}} \sigma}{\|Y^{\mathrm{T}} \sigma\|} \|\rho\| + \|\sigma\| \,\hat{\omega} - \sigma^{\mathrm{T}} \frac{\sigma}{\varepsilon_{\mathrm{d}}} \hat{\omega} \leq 0$$

$$\leq -\dot{\tilde{q}}^{\mathrm{T}} K \dot{\tilde{q}} - \tilde{q}^{\mathrm{T}} \Lambda K \Lambda \tilde{q} + \left\| Y^{\mathrm{T}} \sigma \right\| (\|\tilde{\pi}\| - \|\rho\|) + \|\sigma\| \left(\hat{\omega} - \frac{\|\sigma\|}{\varepsilon_{\mathrm{d}}} \hat{\omega} \right) \leq 0$$

$$\leq -\dot{\tilde{q}}^{\mathrm{T}} K \dot{\tilde{q}} - \tilde{q}^{\mathrm{T}} \Lambda K \Lambda \tilde{q} + \|\sigma\| \left(\hat{\omega} - \frac{\|\sigma\|}{\varepsilon_{\mathrm{d}}} \hat{\omega} \right) \leq 0 \qquad (27)$$

The proof is similar to that of a robust controller [5], where $x^T = [\dot{\tilde{q}}^T, \tilde{q}^T]$ and $Q = diag[\Lambda K \Lambda K]$, provided

$$x^{\mathrm{T}} Q x \geq \varepsilon_d \frac{\hat{\omega}}{4}. \qquad (28)$$

Using the relationship

$$\delta_{\min} Q \leq x^{\mathrm{T}} Q x \leq \delta_{\max} Q \leq 0 \qquad (29)$$

where $\delta_{\min}(Q)$ and $\delta_{\max}(Q)$ denote the minimum and maximum eigenvalues of Q, respectively, it can be obtained that $\dot{V} \leq 0$ if

$$\delta_{min} Q \|x\|^2 \geq \varepsilon_d \frac{\hat{\omega}}{4} \qquad (30)$$

It is shown that $\dot{V} \leq 0$ for $\|x\|$, where

$$\|x\| \geq \sqrt{\frac{\varepsilon_d \hat{\omega}}{4 \delta_{min}(Q)}} \qquad (31)$$

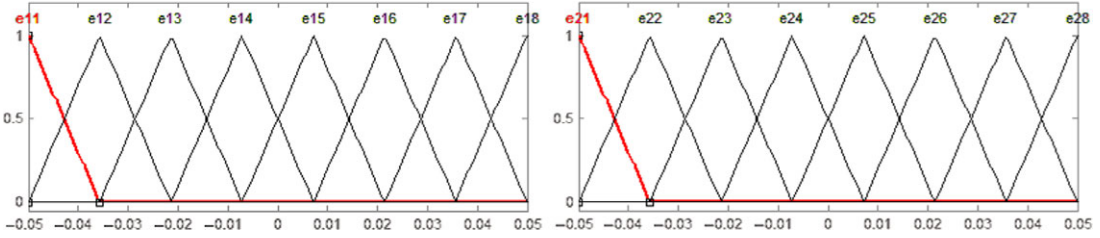**Figure 1.** *Membership functions of error values (e_1, e_2).*

Case 3: If $\left\| Y^T\sigma \right\| \le \varepsilon_p$ and $\|\sigma\| \ge \varepsilon_d$. The time derivative of the Lyapunov function is

$$\dot{V} \le -\dot{\tilde{q}}^{\mathrm{T}} K\dot{\tilde{q}} - \tilde{q}^{\mathrm{T}}\Lambda K\Lambda\tilde{q} + \sigma^{\mathrm{T}}Y\tilde{\pi} - (Y^{\mathrm{T}}\sigma)^{\mathrm{T}}\frac{Y^{\mathrm{T}}\sigma}{\varepsilon_p}\|\rho\| + \|\sigma\|\,\hat{\omega} - \frac{\sigma^{\mathrm{T}}\sigma\hat{\omega}}{\|\sigma\|} \le 0$$

$$\le -\dot{\tilde{q}}^{\mathrm{T}} K\dot{\tilde{q}} - \tilde{q}^{\mathrm{T}}\Lambda K\Lambda\tilde{q} + \left\| Y^{\mathrm{T}}\sigma \right\| \left(\|\tilde{\pi}\| - \frac{Y^{\mathrm{T}}\sigma}{\varepsilon_p}\|\rho\| \right) \le 0 \tag{32}$$

The maximum term of Eq. (32) will be $\varepsilon_p\frac{\|\rho\|}{4}$. The rest of the proof is given elsewhere [5].

Case 4: If $\left\| Y^T\sigma \right\| \le \varepsilon_p$ and $\|\sigma\| \le \varepsilon_d$. For the fourth case, Eq. (25) will be

$$\dot{V} = -\dot{\tilde{q}}^{\mathrm{T}} K\dot{\tilde{q}} - \tilde{q}^{\mathrm{T}}\Lambda K\Lambda\tilde{q} + Y^{\mathrm{T}}\sigma\tilde{\pi} - (Y^{\mathrm{T}}\sigma)^{\mathrm{T}}\frac{\left\| Y^{\mathrm{T}}\sigma \right\|}{\varepsilon_p}\|\rho\| + \|\sigma\|\,\hat{\omega} - \frac{\sigma^{\mathrm{T}}\sigma}{\varepsilon_d}\hat{\omega} \le 0$$

$$\le -\dot{\tilde{q}}^{\mathrm{T}} K\dot{\tilde{q}} - \tilde{q}^{\mathrm{T}}\Lambda K\Lambda\tilde{q} + \left\| Y^{\mathrm{T}}\sigma \right\| \left(\|\tilde{\pi}\| - \frac{\left\| Y^{\mathrm{T}}\sigma \right\|}{\varepsilon_p}\|\rho\| \right) + \|\sigma\| \left(\hat{\omega} - \frac{\|\sigma\|}{\varepsilon_d}\hat{\omega} \right) \le 0 \tag{33}$$

The values $\left\| Y^T\sigma \right\| \, (\|\tilde{\pi}\| - \frac{\|Y^T\sigma\|}{\varepsilon_p}\|\rho\|)$ and $\|\sigma\|\,(\hat{\omega} - \frac{\|\sigma\|}{\varepsilon_d}\hat{\omega})$ are bounded by $\varepsilon_p\frac{\|\rho\|}{4}$ and $\varepsilon_d\frac{\hat{\omega}}{4}$, respectively. Thus,

$$\dot{V} \le -x^{\mathrm{T}}Qx + \varepsilon_p\frac{\|\rho\|}{4} + \varepsilon_d\frac{\hat{\omega}}{4} \le 0 \tag{34}$$

Similar to the second and third cases, it is shown that $\dot{V} \le 0$ for $\|x\|$, where

$$\|x\| \ge \sqrt{\frac{\varepsilon_p\|\rho\|}{4\delta_{\min}(Q)} + \frac{\varepsilon_d\hat{\omega}}{4\delta_{\min}(Q)}} \tag{35}$$

5. Fuzzy Logic Adaptive Disturbance Estimation Law

Fuzzy logic control has become a very important area of research after the introduction of fuzzy logic by Zadeh [21]. This control method is very useful for a system model that is not specified exactly. A fuzzy control system is composed of a fuzzifier, a rule base, an inference engine and a defuzzifier. The rule base consists of a set of If–Then rules, such as If x_1 is A_1, x_i is A_i and x_n is A_n, then ρ is B_1, where $x = [x_1 \ldots x_i \ldots x_n]$. T and ρ are the input and output vectors of the fuzzy system, respectively. The fuzzy sets A and B are assumed to be associated with the fuzzy membership functions. For this fuzzy controller, we used two inputs representing the error values (e_1, e_2) of path tracing for the first and second links of the robot (Fig. 1), and four outputs representing control values of the constant parameters α, β, λ and b_l (Fig. 2).

By trial and error method, we found the ranges for membership functions as follows:

e_1: $[-0.05; 0.05]$; e_2: $[-0.05; 0.05]$, b_1: $0 - 500$, α: $0 - 250$, β: $-500 - +500$, λ: $-200 - 200$

Table I. *Rule table for fuzzy logic.*

α β / λ b1	e11	e12	e13	e14	e15	e16	e17	e18
e21	a1 be1	a2 be3	a6 be1	a5 be5	a7 be4	a8 be7	a1 be1	a2 be2
	te1 b11	te3 b15	te2 b15	te5 b18	te4 b18	te3 b14	te1 b13	te4 b15
e22	a1 be2	a3 be4	a3 be4	a5 be3	a8 be7	a8 be7	a2 be2	a4 be4
	te2 b13	te4 b16	te2 b18	te5 b14	te5 b13	te1 b13	te2 b14	te2 b17
e23	a2 be3	a4 be5	a6 be6	a8 be6	a7 be6	a5 be5	a3 be3	a6 be2
	te3 b14	te5 b18	te4 b15	te6 b18	te4 b13	te4 b16	te3 b15	te4 b15
e24	a3 be4	a1 be3	a2 be5	a1 be5	a1 be2	a4 be6	a4 be4	a8 be8
	te4 b15	te3 b16	te5 b16	te7 b16	te2 b18	te5 b18	te4 b16	te8 b17
e25	a2 be5	a7 be4	a1 be2	a8 be5	a2 be7	a4 be6	a5 be5	a7 be7
	te1 b16	te4 b15	te3 b17	te1 b13	te8 b11	te5 b16	te5 b17	te7 b18
e26	a5 be6	a7 be6	a1 be3	a1 be7	a8 be5	a4 be7	a8 be4	a4 be4
	te6 b17	te6 b18	te3 b12	te4 b15	te5 b14	te3 b14	te3 b18	te3 b15
e27	a6 be4	a7 be2	a8 be8	a5 be7	a4 be4	a4 be7	a2 be5	a6 be6
	te4 b18	te4 b13	te8 b18	te5 b17	te4 b17	te3 b18	te1 b13	te4 b16
e28	a8 be8	a4 be4	a5 be5	a8 be5	a8 be8	a8 be7	a8 be1	a6 be3
	te8 b15	te4 b14	te5 b11	te4 b18	te6 b18	te7 b17	te6 b18	te8 b18

Membership function of parameter α

Membership function of parameter β

Membership function of parameter λ

Membership function of parameter b₁
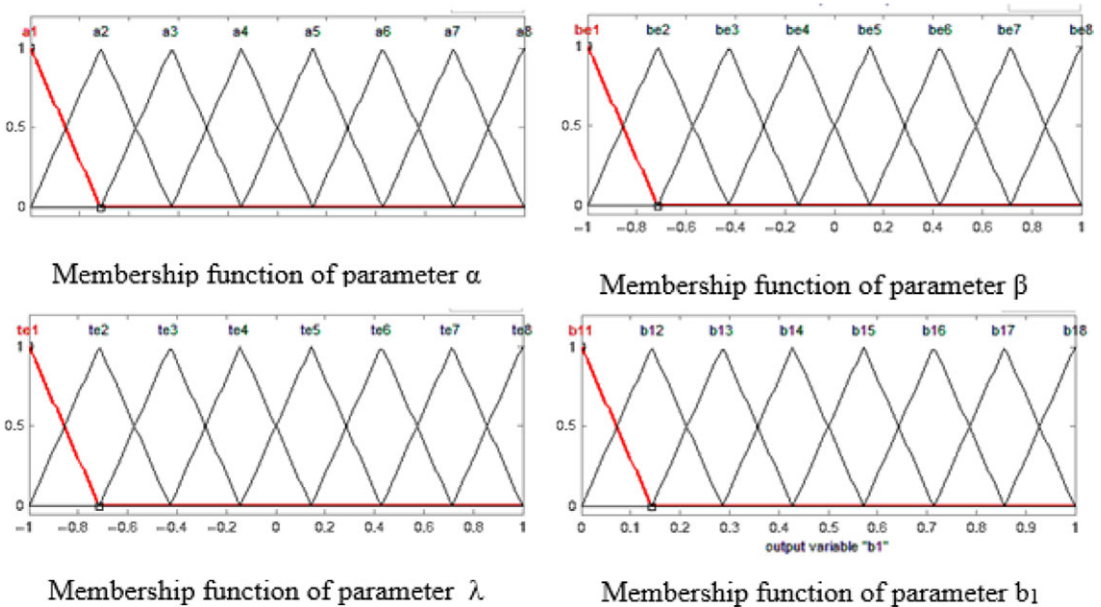
Figure 2. *Membership functions of the constant parameters α, β, λ and b₁.*

Using (If . . . and . . . Then . . .) the structure of the rule table is created and is shown in Table I. At the defuzzification stage, we used the centroid method to obtain the output values.

6. Simulation results

For illustration, a two-link robot manipulator is given in Fig. 3 [5].

For illustration purposes, the robot's parameters are given in the following tables [5]. Tables II and III show the unloaded manipulator. Table IV shows nominal control parameters; Table V shows the

Table II. *Parameters of the unloaded arm [5].*

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

Table III. π_i *for the unloaded arm [5].*

π_1	π_2	π_3	π_4	π_5	π_6
8.33	1.67	2.5	5	5	2.5

Table IV. *Nominal parameter vector π_0 [5].*

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}
13.33	8.96	8.75	5	10	8.75

Table V. *Uncertainty bound [5].*

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
5	7.29	6. 25	0	5	6.25
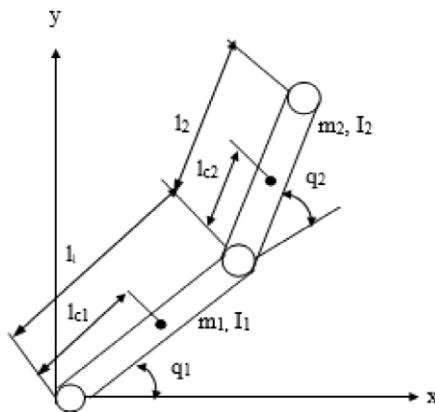
Figure 3. *Two-link planar robot [5].*

uncertainty bounds. Table V contains simply the differences between the values given in Tables IV and III [5].

In simulations, joint frictions and external disturbances are assumed to be $f + d = 10Sin(10t) + 10sin(15t) + 10$ at each joint. For comparison, the known pure robust control law is given [5]:

$$\tau = \tau_0 + Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)u(t) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\pi_0 + u(t)) - K\sigma \tag{36}$$

Control input $u(t)_i$ is

$$u_p = \begin{cases} -\dfrac{Y^T\sigma}{\|Y^T\sigma\|}\|\rho\| & if \ \|Y^T\sigma\| > \varepsilon_p \\[2ex] -\dfrac{Y^T\sigma}{\varepsilon_p}\|\rho\| & if \ \|Y^T\sigma\| \le \varepsilon_p \end{cases} \quad or \quad (u_p)_i = \begin{cases} -\dfrac{(Y^T\sigma)_i}{|(Y^T\sigma)_i|}\rho_i & if \ |(Y^T\sigma)_i| > (\varepsilon_p)_i \\[2ex] -\dfrac{(Y^T\sigma)_i}{\varepsilon_p}\rho_i & if \ |(Y^T\sigma)_i| \le (\varepsilon_p)_i \end{cases} \tag{37}$$
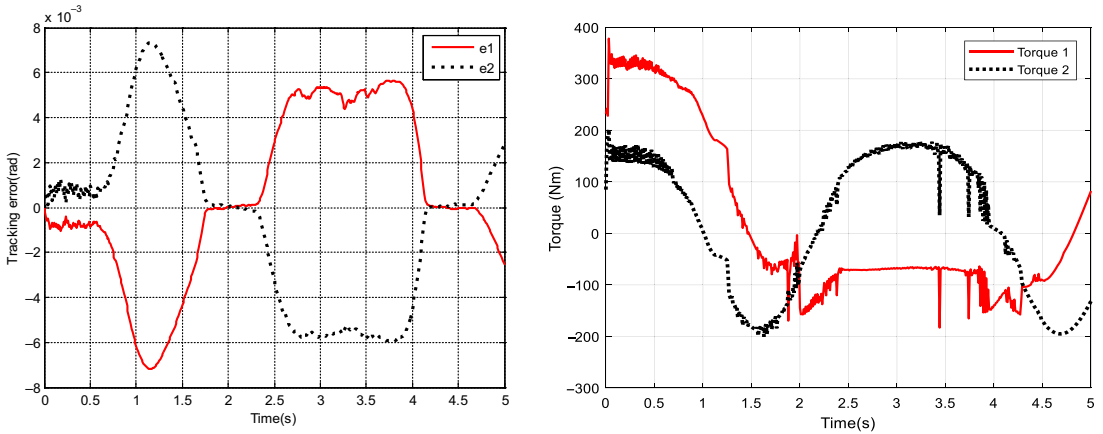
Figure 4. *Response using the pure robust control law [5] for Λ = diag([30 30]), K = diag([60 60]) without frictions and external disturbances.*
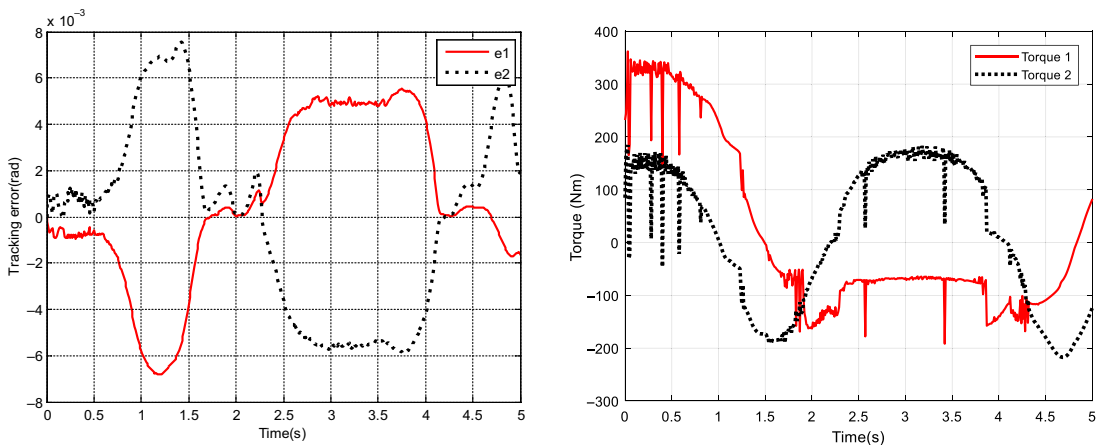
Figure 5. *Response using the pure robust control law [5] for Λ = diag([30 30]), K = diag([60 60]) with frictions and external disturbances f + d = 10Sin(10t) + 10sin(15t) + 10 at each joint.*

The desired trajectory is selected for each joint as $q_1 = q_2 = 1.5\ cost − 1.5$. The control parameters K and $Λ$ are chosen as $K = diag(30\ 30)$ and $Λ = diag(60\ 60)$. The same model, same trajectories and same control parameters are used for simulations, and the results obtained are given in Figs. 4–11.

As shown in Fig. 4, the tracking error is large and chattering, and poor steady-state performance is observed for the known robust controller [5]. As shown in Figs. 6–11, the results with external disturbances are close to the results without external disturbances. Tracking performance varies according to the control parameters $α$, $β$, $λ$ and b_1. As shown in Fig. 9, the maximum tracking error is $4.6 × 10^{−3}$ rad for the first joint and $5.7 × 10^{−3}$ for the second joint in the transient state. The tracking error is reduced fast after 0.2 s and becomes $1 × 10^{−3}$ rad in steady state with the control parameters $b_1 = 120$, $β = 100$, $α = 100$ and $λ = −40$. The maximum tracking error is reduced to $2.8 × 10^{−3}$ rad in Fig. 11 with the control parameters $b_1 = 120$, $β = 10$, $α = 100$ and $λ = −40$. These results show that the desired behaviour of the system is achieved by adjusting the controllers $α$, $β$, $λ$ and b_1 to appropriate values with the same feedback gains. However, it is difficult to find the best values of the parameters for $α$, $β$, $λ$ and b_1 by trial and error method. Therefore, fuzzy logic rules find better values of the control parameters, and the result is given in Figs. 12 and 13.

The control parameters $α$, $β$, $λ$ and b_1 according to error values and their changes with time can be seen in Figs. 14 and 15.
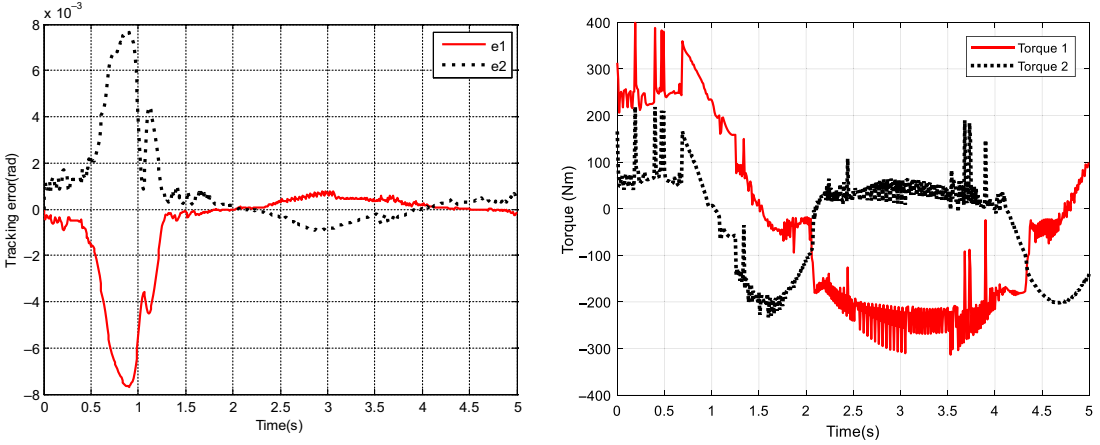
Figure 6. *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$ $b_1 = 100$, $\beta = 20$, $\alpha = 10$, $\lambda = -80$ *without frictions and external disturbances.*
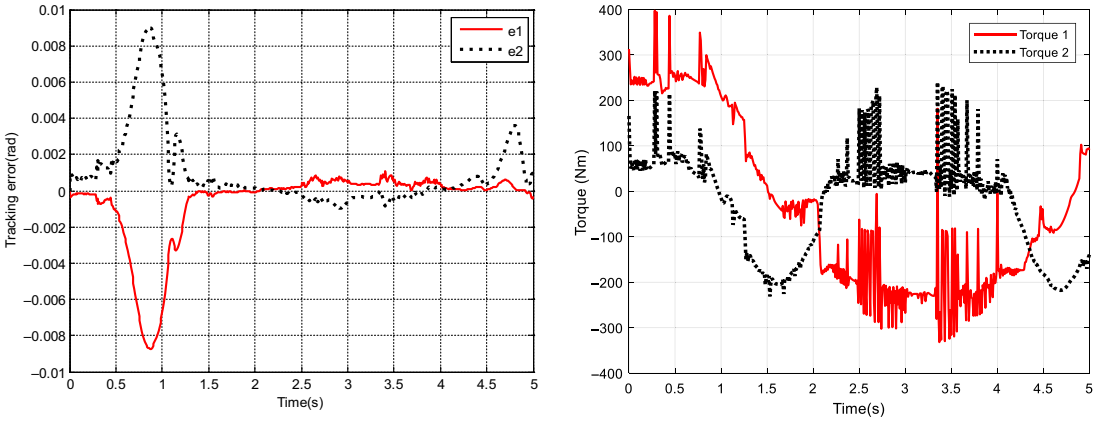
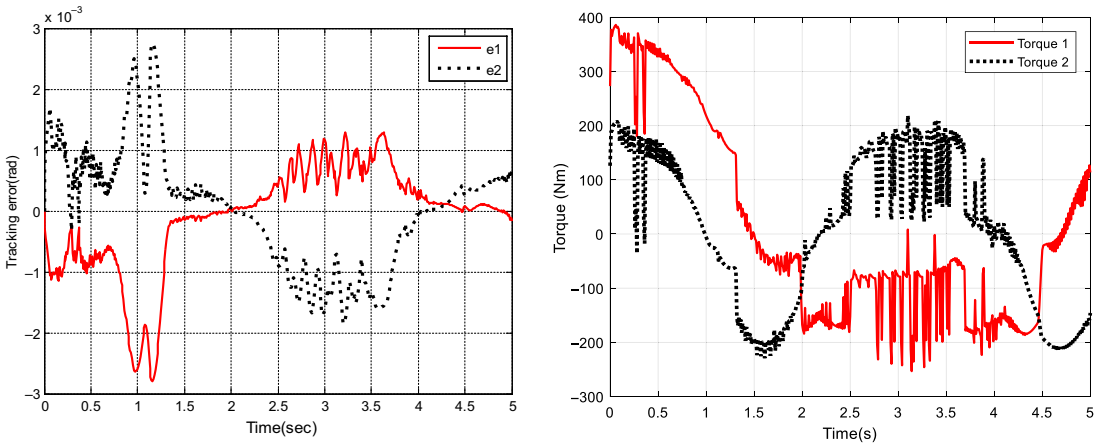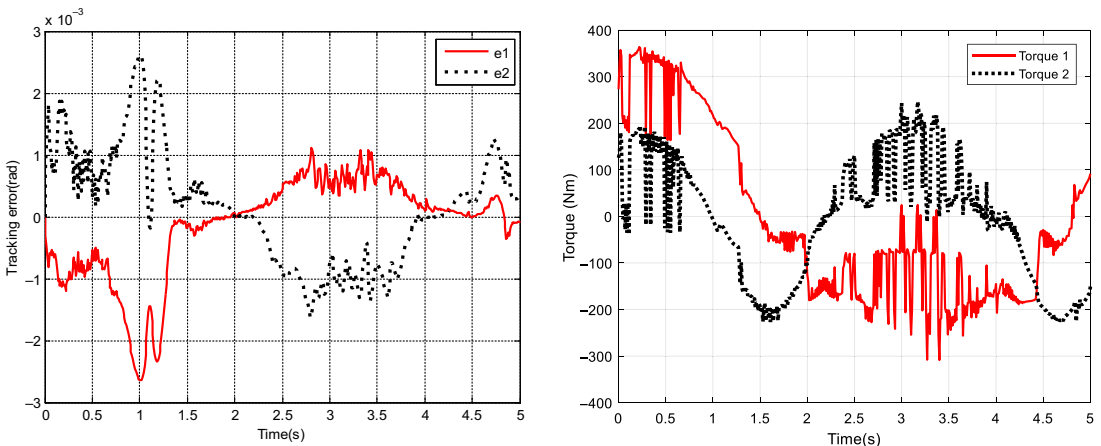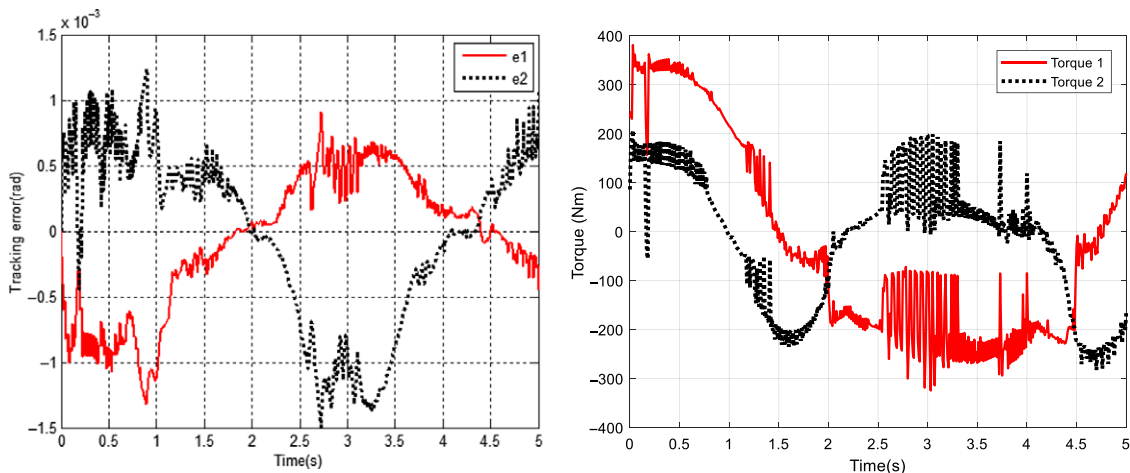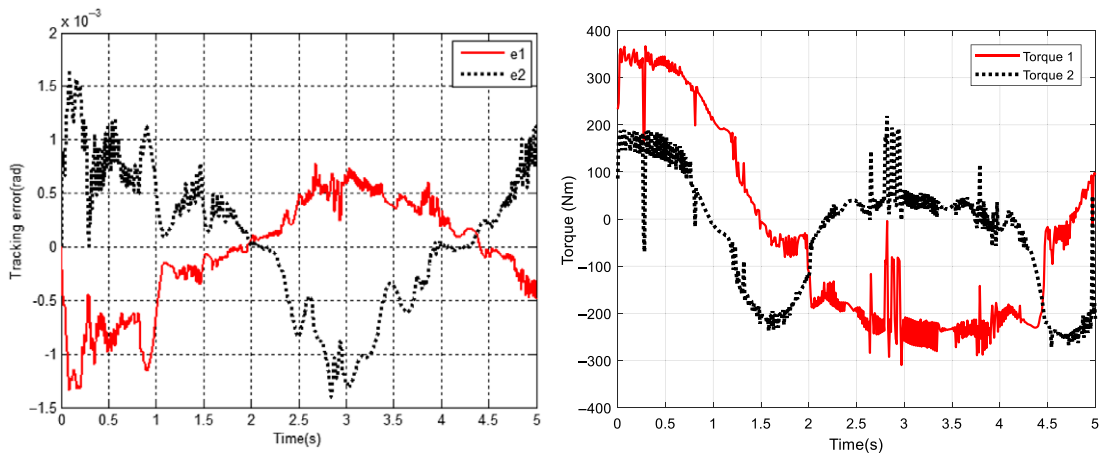**Figure 7.** *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$ $b_1 = 100$, $\beta = 20$, $\alpha = 10$, $\lambda = -80$ *with frictions and external disturbances* $f + d = 10Sin(10t) + 10sin(15t) + 10$ *at each joint.*
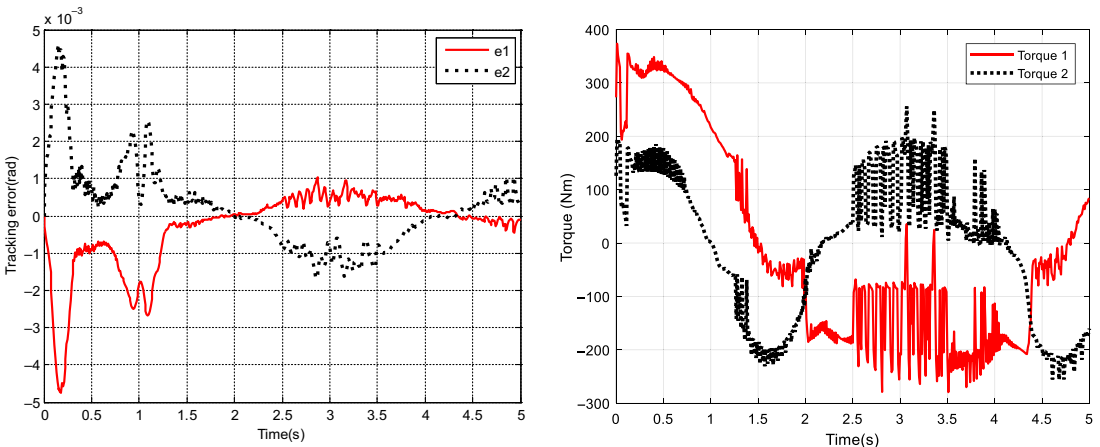
Figure 8. *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$ $b_1 = 120$, $\beta = 100$, $\alpha = 100$, $\lambda = -40$ *without frictions and external disturbances.*
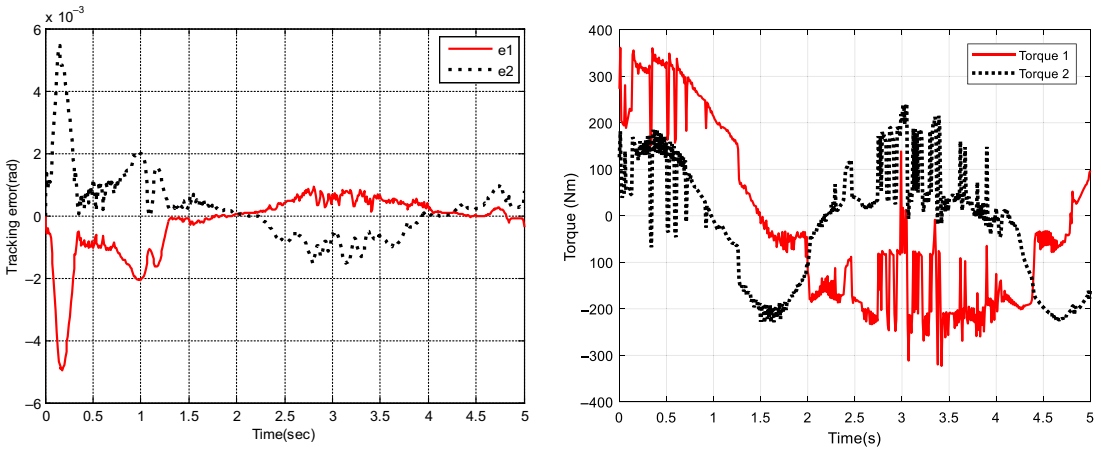
Figure 9. *Response using the proposed control law for* Λ = *diag([30 30]), K = diag([60 60]) b₁ = 120, β = 100, α = 100, λ = −40 with frictions and external disturbances f + d = 10Sin(10t) + 10sin(15t) + 10 at each joint.*

Figure 10. *Response using the proposed control law for* Λ = *diag([30 30]), K = diag([60 60]) b₁ = 120, β = 10, α = 100, λ = −40 without frictions and external disturbances.*

Figure 11. *Response using the proposed control law for* Λ = *diag([30 30]), K = diag([60 60]) b₁ = 120, β = 10, α = 100, λ = −40 with frictions and external disturbances f + d = 10Sin(10t) + 10sin(15t) + 10 at each joint.*

Figure 12. *Response using the proposed control law for Λ = diag([30 30]), K = diag([60 60]) for fuzzy-logic control without frictions and external disturbances.*

**Figure 13.** *Response using the proposed control law for Λ = diag([30 30]), K = diag([60 60]) for fuzzy-logic control with frictions and external disturbances f + d = 10Sin(10t) + 10sin(15t) + 10 at each joint.*

By using the fuzzy logic robust adaptive controller, the values of α, β, λ and b_l will change with time, and better values can be extracted according to error, which varies with time and input in the fuzzy system. Thus, better tracking performance is obtained.

In order to investigate the tracking performance of the controller, the more complicated desired trajectory is selected for each joint as $q_1 = q_2 = 2 \cos 2t - 2$. The control parameters are chosen the same as those in Figs. 10 and 11 as $K = diag(30\ 30)$, $\Lambda = diag(60\ 60)$, $b_l = 120$, $\beta = 10$, $\alpha = 100$, $\lambda = -40$, and the results are given in Figs. 16–19.

As seen in Figs. 16–19, the tracking error is increased in the case of complicated trajectory. However, it is seen that the results obtained are similar in the case of all trajectories. Tracking performance varies with respect to the control parameters α, β, λ and b_l. The proposed adaptive robust controller compensates for all of the unmodelled disturbances; however, better compensation is achieved by fuzzy logic rules. As seen in Figs. 11, 13, 17 and 19, results with and without unmodelled parameter disturbances are almost the same by the addition of fuzzy logic rules and better compensation for the unmodelled parameters can be achieved.
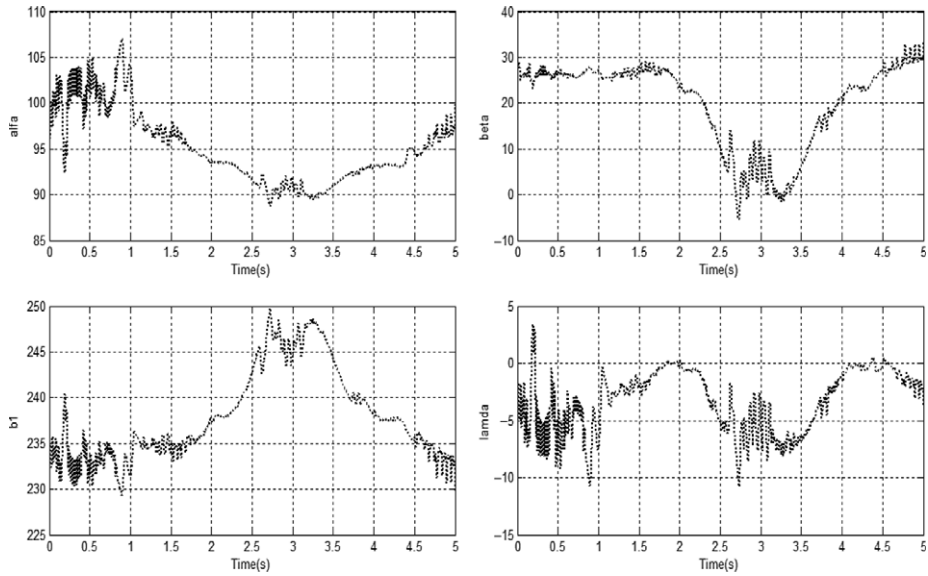
**Figure 14.** *Control parameters α, β, λ and b₁ according to error values without friction and external disturbances.*
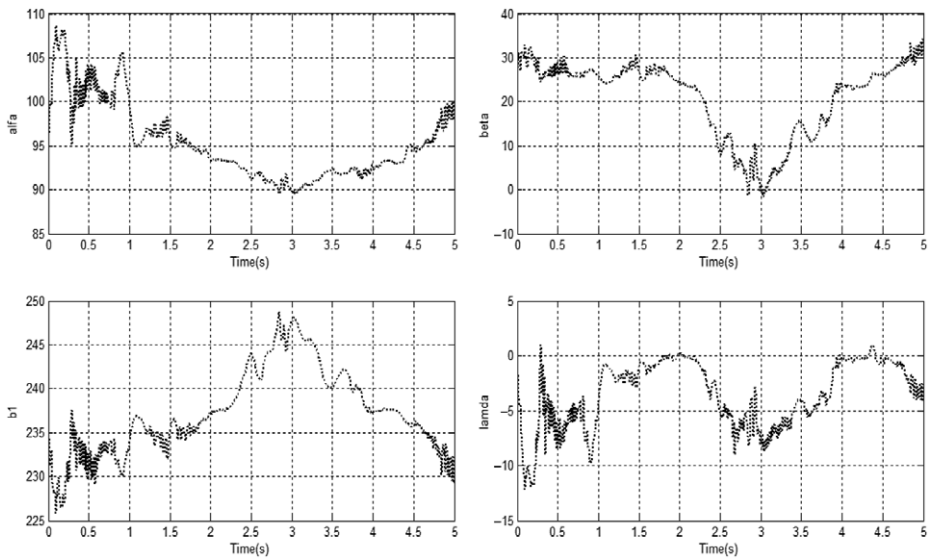
Figure 15. *Control parameters α, β, λ and b₁ according to error values with friction and external disturbances f + d = 10Sin(10t) + 10sin(15t) + 10 at each joint.*

7. Conclusion

In the present study, joint friction and external disturbance estimation laws are derived from a Lyapunov function for the control of robots to ensure robust tracking. An upper bound of unmodelled parametric uncertainty and knowledge about external disturbances and joint frictions are not required for the design of the controller. Based on the Lyapunov function, stability and uniform boundedness of tracking error are achieved.

Tracking performance can be improved by adjusting the adaptive gain parameters to appropriate values by trial and error method. However, it is difficult to find better appropriate values by trial and
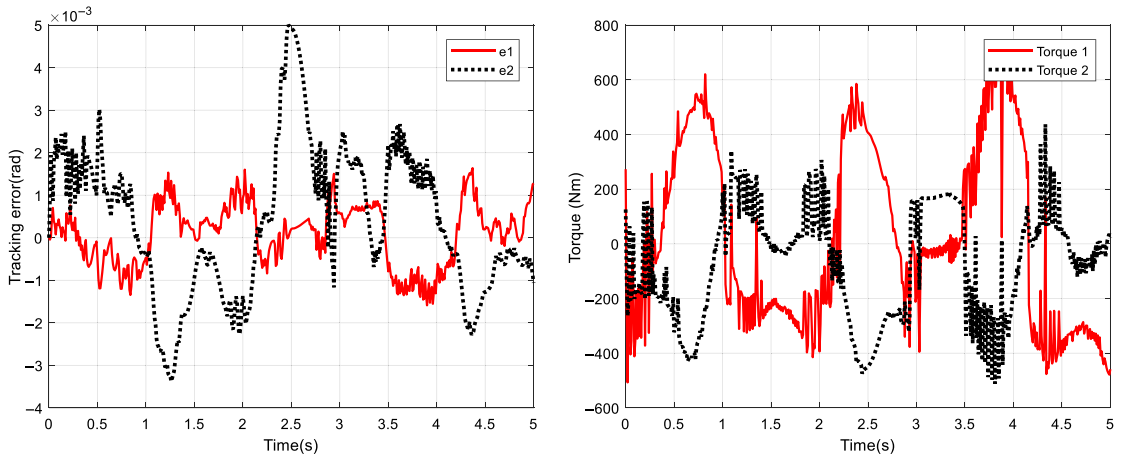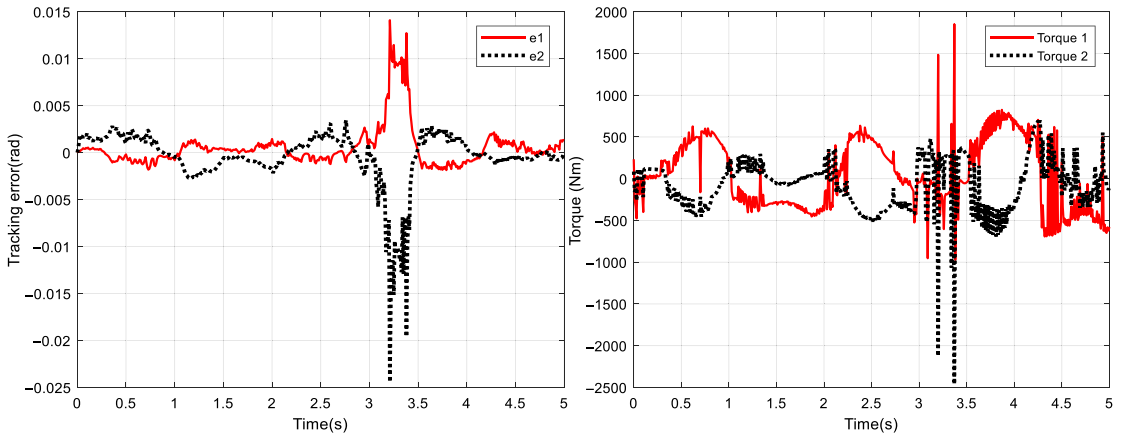
Figure 16. *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$, $b_1 = 120$, $\beta = 10$, $\alpha = 100$, $\lambda = -40$ *without frictions and external disturbances.*
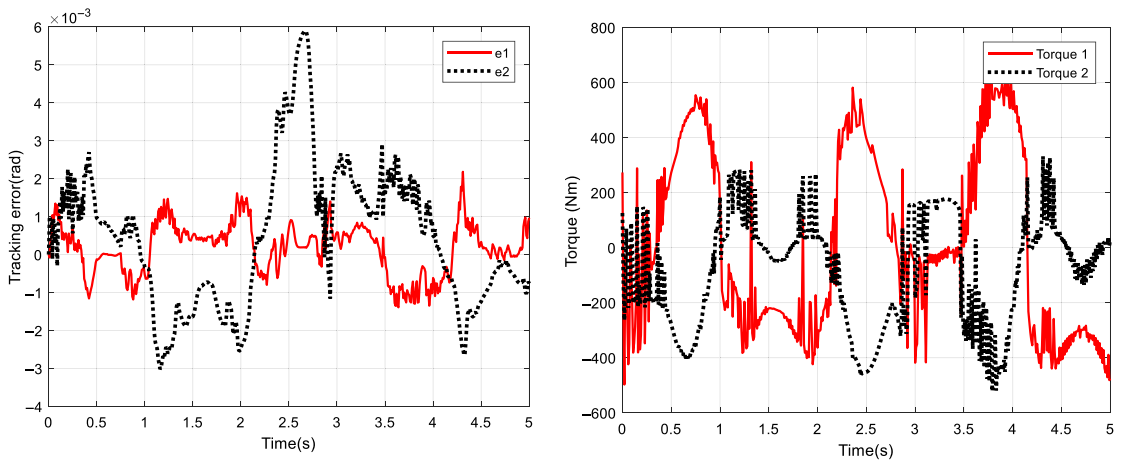
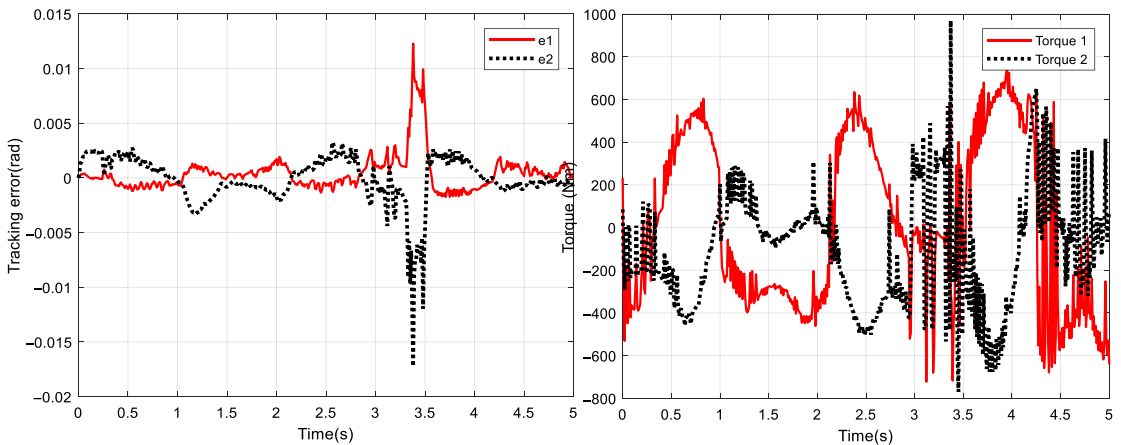**Figure 17.** *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$, $b_1 = 120$, $\beta = 10$, $\alpha = 100$, $\lambda = -40$ *with frictions and external disturbances* $f + d = 10Sin(10t) + 10sin(15t) + 10$ *at each joint.*

error. By using fuzzy logic, gains are made adaptive and better values can be extracted according to the error, which varies with time via the fuzzy system. Before creating the fuzzy logic rule table, the effect of α, β, λ and b_1 on the trajectory tracking error is investigated. While doing this research, α, β, λ and b_1 are taken in a wide range, and all their combinations were tried to determine the effect on the trajectory tracking error. The rule table is created by taking the intervals that have better effect on the reduction of the trajectory tracking error. Membership functions in the rule table are determined according to these intervals. Our values are made variable by choosing the best membership function containing α, β, λ and b_1 values that will minimize the error in a certain time.

The aim of the estimation laws is not to determine the actual values of the friction and external disturbance parameters. The estimation law is the dynamic compensators designed to obtain a very small tracking error, that is, to estimate the greatest estimation of dynamics compensators and to obtain a very small tracking error with small feedback gains. As shown in the figures, the tracking error is very small. These results also verify that the fundamental shortcoming of the robust controllers is remedied and a very small tracking error with small feedback gain is achieved.

Figure 18. *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$ *for fuzzy-logic control without frictions and external disturbances.*

Figure 19. *Response using the proposed control law for* $\Lambda = diag([30\ 30])$, $K = diag([60\ 60])$ *for fuzzy-logic control with frictions and external disturbances* $f + d = 10Sin(10t) + 10sin(15t) + 10$ *at each joint.*

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Appendix A

$$\pi_1 = m_1 l_{c1}{}^2 + m_2 l_1{}^2 + I_1, \pi_2 = m_2 l_{c2}{}^2 + I_2, \pi_3 = m_2 l_1 l_{c2}, \pi_4 = m_1 l_{c1}, \pi_5 = m_2 l_1, \pi_6 = m_2 l_{c2},$$

With this parameterization, the dynamic model in Eq. (1) can be written as $Y(q, \dot{q}, \ddot{q})\pi = \tau$

The component y_{ij} of $Y(q, \dot{q}, \ddot{q})$ are given as

$$y_{11} = \ddot{q}_1; y_{12} = \ddot{q}_1 + \ddot{q}_2; \ y_{13} = \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2), \ y_{14} = g_c \cos(q_1),$$

$$y_{15} = g_c \cos(q_1), y_{16} = g_c \cos(q_1 + q_2), y_{21} = 0, y_{22} = \ddot{q}_1 + \ddot{q}_2, y_{23} = \cos(q_2)\ddot{q}_1 + \sin(q_2)(\dot{q}_1^2)$$

$$y_{24} = 0, y_{25} = 0, y_{26} = gc \cos(q_1 + q_2).$$

$Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ in Eq. (7) have the components

$$y_{11} = \ddot{q}_{r1}; y_{12} = \ddot{q}_{r1} + \ddot{q}_{r2}, y_{13} = \cos(q_2)(2\ddot{q}_{r1} + \ddot{q}_{r2}) - \sin(q_2)(\dot{q}_1\dot{q}_{r2} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2})$$

$$y_{14} = g_c \cos(q_1); y_{15} = g_c \cos(q_1); y_{16} = g_c \cos(q_1 + q_2); y_{21} = 0; y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2},$$

$$y_{23} = \cos(q_2)\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1}); y_{24} = 0, y_{25} = 0, y_{26} = g_c \cos(q_1 + q_2).$$