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Comparative study of dust ion acoustic Korteweg–de Vries and modified Korteweg–de Vries solitons in dusty plasmas with variable temperatures

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In this plasma model, consisting of ions and electrons with pressure variations in both the components in the presence of stationary dust, both compressive and rarefactive Korteweg–de Vries (KdV) solitons of interesting character are established. Based on high dust charge, characteristics of soliton growth are found to be amplified for various pairs of ion and electron streaming speeds. It is noteworthy to mention that for some pairs of ion and electron initial streaming speeds, only compressive KdV solitons with either decreasing or increasing growth are shown to reflect. Contrary to this, for some other pairs of ion and electron streaming speeds, the amplitudes of both rarefactive and compressive solitons are seen to be produced, changing from rarefactive to compressive growth. At the stationary background of the massive dust particles, the lighter particles suffer appreciable initial drifts (backwards streaming) which characteristically change the growth of solitons. For inclusion of higher-order nonlinearity, only compressive modified Korteweg–de Vries (MKdV) solitons of much higher amplitude are found to exist whereas for the same set of parameter values both compressive and rarefactive KdV solitons are found to exist. Smaller values of electron streaming speed are seen to produce high amplitude MKdV solitons. We also observe that due to higher-order nonlinearity, the nonlinear monotonic growth of amplitudes of MKdV solitons is supported by the almost equal streaming speed pairs of ions and electrons for relatively small values of Z_d, where Z_d is the number of charges in a dust particle

Key words: dusty plasmas, plasma nonlinear phenomena, plasma waves

1. Introduction

The study of solitons in dusty plasmas with variable temperatures starting in the last century has attained at present a high stage of investigation. Starting from the simple stage of two component plasmas, studies are extended to multi-component plasmas

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with the insertion of various characteristic effects. There are many investigations with isothermal and non-isothermal electrons without consideration of pressure variations. But the astrophysical and space environments are found to be occupied by dusty plasmas which are known from many space probes. In planetary rings (Shukla & Silin 1992), the Earth's magnetosphere (Vette 1970; Tokar & Gary 1984), neighbourhood of stars (Shukla & Mamun 2002) and in cometary tails (Chow, Mendis & Rosenberg 1993; Mendis & Rosenberg 1994), the effect of dust particles are nicely described. The presence of dust particles is found to drastically change the properties of solitary waves of space (Rao, Shukla & Yu 1990; Shukla & Silin 1992; Verheest 1992; Barkan, Marlino & D'Angelo 1995; Mamun, Cairns & D'Angelo 1996; Mamun 1999; Shukla & Mamun 2002) and laboratory (Nakamura, Bailung & Shukla 1999; Ghosh *et al.* 2000; Nakamura & Sarma 2001; Shahmansouri & Tribeche 2014) plasmas.

In most studies, the variable temperature of the plasma is found to be neglected as inclusion of temperature variation in the plasma demands the additional pressure variation equation of the state in the set of governing equations. Kalita & Barman (1995) have investigated the existence of solitary waves in multi-component un-magnetized plasmas with pressure variations for the ions, negative ions and isothermal electrons. Chatterjee & Roychoudhury (1994) have reported the role of ion temperature in the formation of solitary waves in plasmas with relativistic ions under variable pressure and non-relativistic electrons. Usually, solitary waves of small amplitude are studied through the Korteweg–de Vries (KdV) and modified Korteweg–de Vries (MKdV) equations. Das, Singh & Singh (1996) have studied KdV and MKdV ion acoustic solitons in unmagnetized plasmas with free and trapped electrons of different temperatures supported by a uniform ionization rate.

In many investigations, dust acoustic (DA) waves are studied in plasmas with the dust momentum fluid equations (Rao *et al.* 1990; Mamun 1998, 1999; Baluku & Helberg 2008; Verheest & Pillay 2008; Ghosh *et al.* 2001; Asgari, Muniandy & Wong 2013) considering a Boltzmann distribution, Cairns distribution (Cairns *et al.* 1995) of the electrons and ions or non-thermal electrons or ions and a kappa distribution of the electrons. Rosengberg (1993) has discussed the excitation of dust acoustic wave modes caused by ions and electrons streaming through dust grains which are even essential for instability studies. The existence of dust acoustic solitary waves is shown by Rouchoudhury & Mukherjee (1997) under the restriction of finite dust temperature. Rosengberg & Krall (1996) have reported that dust temperature is important for the thermalization of ions or orbital effects. The effects of finite dust temperature and dust charge fluctuation on DA waves are studied by EL-Labony & EL-Taibany (2003). Pakzad (2009) has studied DA solitary waves through the pseudo-potential method taking a non-thermal distribution of ions as variable. The dust ion acoustic (DIA) Gardner soliton is established by Masud, Asaduzzaman & Mamun (2012) in obliquely propagating magnetized dusty plasma employing the Zakharov–Kuznetsov (ZK) equation. Asgari *et al.* (2013) have studied DIA waves with a non-thermal Cairns-like distribution of the ions in the plasma. In the presence of massive stationary dust in the background plasma, the lighter particles, electrons and ions, suffer appreciable initial drift. These initial values are found to characteristically change the growth of solitons in this present investigation. In dust ion acoustic solitons with dust as a stationary background, for slow perturbations, small amplitude waves need to be described by the KdV equation, essential particularly for a multi-component plasma. To incorporate higher-order nonlinearity for an in-depth study into the plasma scenario, it is desirable to employ the MKdV equations.

Kopnin *et al.* (2005) have pointed out that the dust grain charging process is described in terms of the so-called orbit motion (Barnes *et al.* 1992). Therefore with dust as a stationary background, dust charge may be constant (Kundu & Mamun 2012) and so it is 'dust ion acoustic'. In dust acoustic solitons with mobile dust, variable dust charges are being considered although it is questionable for conservation of mass. In the presence of the massive dust particles in a space plasma, the DIA (or DA) waves are expected to form low frequency waves. A behavioural study of the properties of these waves, in particular solitary waves or solitons moving with constant but low speed, can be readily tapped for information by space probes. For complete determination of phase space (position, velocity out of any form) mapping of dust particles in the plasmas surrounding space bodies such as the Moon or Mars – through Spectrometer and 'Imager' –have been investigated in space missions such as ASPERA (automatic space plasma experiment with a rotating analyser) on the Phobos satellite. This is described in the works of Kalita & Kalita (2016).

This paper is organized as follows: the introduction is given in § 1, the basic equations in § 2, the validity condition for the neglect of charge variation in support of a conservative system in § 3, derivation of the Korteweg–de Vries and the modified Korteweg–de Vries equations in § 4, solitary wave solutions of the Korteweg–de Vries and the modified Korteweg–de Vries equations in § 5 and the discussion is in § 6. The references are included at the end.

2. Basic equations

In this model, we consider variable pressure in both the ion and electron components in the presence of immobile negatively charged dust particles in the plasma. The dynamics of dust ion acoustic (DIA) waves propagating in such a system of dusty plasma is described by means of the equations for conservation of mass, conservation of momentum and pressure variations (or temperatures) of both the ion and electron species. They are supplemented by the Poisson equation with immobile dust.

For ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{2.1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\alpha}{n_i}\frac{\partial p_i}{\partial x} = -\frac{\partial \varphi}{\partial x} \tag{2.2}$$

$$\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial v_i}{\partial x} = 0 \tag{2.3}$$

for electrons,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \tag{2.4}$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{1}{Q}\left(\frac{\partial \varphi}{\partial x} - \frac{1}{n_e}\frac{\partial p_e}{\partial x}\right), \tag{2.5}$$

where $Q = m_e/m_i$, m_e and m_i being the masses of electrons and ions respectively

$$\frac{\partial p_e}{\partial t} + v_e \frac{\partial p_e}{\partial x} + 3p_e \frac{\partial v_e}{\partial x} = 0 \tag{2.6}$$

and the Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e + Z_d n_d - n_i, \tag{2.7}$$

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where v_i, v_e; n_i, n_e and p_i, p_e are velocities, densities and pressures of the positive ions and electrons respectively, Z_d is the number of charges contained in a dust grain, n_d is the density of dust charges and φ is the electrostatic potential.

3. Validity condition for the neglect of charge variation in support of a conservative system

In most of the dynamical scenarios, we are concerned with a conservative system. Kundu & Mamun (2012) have reported that the omission of dust charge fluctuations and the ion–neutral collision frequency is valid as long as the DIA wave frequency is not comparable to the dust charging frequency and ion neutral collision frequency. Here 'comparable to' means 'remaining always smaller' because the DIA is a wave but charges are particles. The investigation of an actual comparable stage is beyond the scope of the paper. Under this assumption, the charge variation in DIA waves for variation of pressures with time can be ignored for the reason mentioned below.

An essential feature of dust ion acoustic waves is that they can exist at $T_e \approx T_i$ (Shukla & Silin 1992; Nakamura *et al.* 1999; Popel, Golubo' & Losseva 2001; Popel *et al.* 2004).

From (2.3) of the ions in this model of plasma, we get

$$\frac{\partial}{\partial t}(n_i T_i) + v_i \frac{\partial}{\partial x}(n_i T_i) + 3 n_i T_i \frac{\partial v_i}{\partial x} = 0, \quad \text{putting } p_i = n_i K T_i. \tag{3.1}$$

Using the equation of continuity (2.1), it can be simplified to give

$$\frac{1}{T_i}\frac{\partial T_i}{\partial t} + \frac{v_i}{T_i}\frac{\partial T_i}{\partial x} = -2\frac{\partial v_i}{\partial x}. \tag{3.2}$$

Also, from the pressure variation (2.6) of the electrons and the equation of continuity (2.4), we can similarly get

$$\frac{1}{T_e}\frac{\partial T_e}{\partial t} + \frac{v_e}{T_e}\frac{\partial T_e}{\partial x} = -2\frac{\partial v_e}{\partial x}. \tag{3.3}$$

Equations (3.2) and (3.3) gives for $T_e \approx T_i = T$ (approx.)

$$\frac{1}{T}(v_i - v_e)\frac{\partial T}{\partial x} = -2\frac{\partial}{\partial x}(v_i - v_e). \tag{3.4}$$

After integration, it gives

$$\frac{(v_e - v_i)}{(v_{e0} - v_{i0})} = \sqrt{\frac{T_0}{T}} \quad \text{so that } v_e - v_i = \frac{\lambda}{\sqrt{T}}, \tag{3.5}$$

where $\lambda = (v_{e0} - v_{i0})\sqrt{T_0} \neq 0$ with initial streaming velocity v_{e0} of the electrons, v_{i0} of the ions (shown in the next section) and pressure T_0 at that instant. In (3.5), $(v_e - v_i)$ represents relative velocity of the electrons with respect to the ions at any instant.

Kopnin *et al.* (2005) have reported that for negative dust charge $q_d = -Z_d e$, only electrons with velocities such that $|\boldsymbol{v}| > [(2(-e)q_d)/am_e]^{1/2} = [(2e^2 Z_d)/am_e]^{1/2}$ ('a' is the grain size and 'm_e' is the electron mass) can reach the grain surfaces, while the ions are free of this constraint. Under this circumstance, the ions are free from

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recombination with the grain surfaces. Besides, there is every possibility to admit the condition $(|\boldsymbol{v}| =)v_e = v_i + \lambda/\sqrt{T} < [(2e^2 Z_d)/am_e]^{1/2}$ for smaller relative velocity of electrons v_e and higher values of T (both feasible) so that the electrons also cannot reach the grain surfaces. Therefore, in this sense, electrons also are free of recombination with the grain surfaces. Hence, the general conservative system will continue to hold good in such a system of dusty plasmas under the consideration $T_e \approx T_i$.

4. Derivation of the Korteweg–de Vries and the modified Korteweg–de Vries equations

To derive the KdV equation from the set of (2.1)–(2.7), we use the stretched variables,

$$\xi = \varepsilon^{1/2}(x - Vt), \quad \tau = \varepsilon^{3/2} Vt, \tag{4.1a,b}$$

with the phase velocity V of the waves. We expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter ε as

$$\left.\begin{aligned} n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \cdots \\ n_e &= n_{e0} + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \cdots \\ v_i &= v_{i0} + \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \cdots \\ v_e &= v_{e0} + \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \cdots \\ \varphi &= \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \cdots \\ p_i &= p_{i0} + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \cdots \\ p_e &= p_{e0} + \varepsilon p_{e1} + \varepsilon^2 p_{e2} + \cdots \end{aligned}\right\}. \tag{4.2}$$

Following the standard perturbation method with the use of transformation (4.1), expansions (4.2) in the normalized set of (2.1)–(2.6), we equate the coefficients of ε-order equations and after integration use the boundary conditions,

$$n_{i1} = 0, \quad n_{e1} = 0; \quad v_{i1} = 0, \quad v_{e1} = 0 \quad \text{and} \quad \varphi_1 = 0 \quad \text{at } |\xi| \to \infty \tag{4.3a–e}$$

to get the following first-order perturbed quantities:

$$\left.\begin{aligned} v_{i1} &= -\frac{(v_{i0} - V)}{(v_{i0} - V)^2 - 3\alpha}\varphi_1, \quad p_{i1} = \frac{3p_{i0}}{(v_{i0} - V)^2 - 3\alpha}\varphi_1, \\ n_{i1} &= \frac{n_{i0}}{(v_{i0} - V)^2 - 3\alpha}\varphi_1, \quad v_{e1} = \frac{(v_{e0} - V)}{Q\,(v_{e0} - V)^2 - 3}\varphi_1, \\ n_{e1} &= -\frac{n_{e0}\,(v_{e0} - V)}{Q\,(v_{e0} - V)^2 - 3}\varphi_1, \quad p_{e1} = -\frac{3p_{e0}}{Q\,(v_{e0} - V)^2 - 3}\varphi_1 \end{aligned}\right\}. \tag{4.4}$$

Also, using expansion (4.2) in (2.7), we get

$$n_{e0} + Z_d n_{d0} - n_{i0} = 0 \quad \text{or,} \quad \frac{n_{e0}}{n_{i0}} = 1 - Z_d\sigma, \quad \text{where } \sigma = \frac{n_{d0}}{n_{i0}}, \frac{p_{i0}}{n_{i0}} = 1 \tag{4.5a,b}$$

and

$$n_{e1} - n_{i1} = 0. \tag{4.6}$$

Using the first-order quantities in (4.6), the phase velocity equation for V is obtained as

$$(1 - Z_d\sigma + Q)\,V^2 - 2\{v_{i0}(1 - Z_d\sigma) + Qv_{e0}\}V + \{(1 - Z_d\sigma)(v_{i0}^2 - 3\alpha) + Qv_{e0}^2 - 3\} = 0 \tag{4.7}$$

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Using relation (4.4) in the ε^2-order equations (not shown here), we get the KdV equation

$$\frac{\partial \varphi_1}{\partial \tau} + p\varphi_1 \frac{\partial \varphi_1}{\partial \xi} + q\frac{\partial^3 \varphi_1}{\partial \xi^3} = 0, \tag{4.8}$$

where

$$\left.\begin{aligned} p &= \frac{BL_1(1-Z_d\sigma)(B_1L_1+4K_1M_1+2K_1) - B_1L(BL+4\alpha KM+2K)}{VBL_1(1-Z_d\sigma)(1+M_1+QB_1K_1)+VB_1L(1+\alpha M+BK)} \\ q &= -\frac{BB_1}{n_{i0}\{VBL_1(1-Z_d\sigma)(1+M_1+QB_1K_1)+VB_1L(1+\alpha M+BK)\}} \end{aligned}\right\}, \tag{4.9}$$

with $K = B/F, L = 1/F, M = 3/F, K_1 = B_1/F_1, L_1 = 1/F_1, M_1 = 3/F_1, F = B^2 - 3\alpha, F_1 = QB_1^2 - 3, B_1 = v_{eo} - V, B = v_{io} - V$.

For higher-order nonlinearity and to derive the MKdV equation from the set of (2.1)–(2.6), we introduce the new stretched variables,

$$\xi = \varepsilon(x - Vt), \quad \tau = \varepsilon^3 Vt, \quad \text{where } V \text{ is phase velocity of the waves.} \tag{4.10a,b}$$

Using these new stretched variables (4.10) and expansions (4.2) in (2.1)–(2.6) and after putting the first-order quantities in (4.6), we get the same phase velocity (4.7) for the phase velocity V. The new second-order equations subject to the boundary conditions

$$n_{i2} = n_{e2} = 0; \quad v_{i2} = v_{e2} = 0, \quad p_{i2} = p_{e2} = 0 \quad \text{and} \quad \varphi_2 = 0 \quad \text{at } |\xi| \to \infty \tag{4.11a–d}$$

after integration, produce the following results

$$\left.\begin{aligned} v_{i2} &= -R\varphi_1^2 - K\varphi_2, \quad n_{i2} = n_{i0}S\varphi_1^2 + n_{i0}L\varphi_2, \quad p_{i2} = n_{i0}T\varphi_1^2 + n_{i0}M\varphi_2, \\ v_{e2} &= -R_1\varphi_1^2 + K_1\varphi_2, \quad n_{e2} = n_{e0}S_1\varphi_1^2 - n_{e0}L_1\varphi_2, \quad p_{e2} = n_{e0}T_1\varphi_1^2 - n_{e0}M_1\varphi_2, \end{aligned}\right\} \tag{4.12}$$

where

$$\left.\begin{aligned} R &= \frac{KL}{2} + \frac{2\alpha KM}{F}, \quad S = \frac{n_{i0}L}{2F} + \frac{n_{i0}K}{B}\left(\frac{2\alpha M}{F} + L\right), \\ T &= \frac{3p_{i0}L}{2F} + \frac{3p_{i0}KM}{B}\left(\frac{3\alpha}{F} + 1\right), \quad R_1 = \frac{K_1L_1}{2} + \frac{2K_1M_1}{F_1}, \\ S_1 &= \frac{n_{e0}L_1}{2F_1} + \frac{n_{e0}K_1}{B_1}\left(\frac{2M_1}{F_1} + L_1\right), \quad T_1 = \frac{3p_{e0}L_1}{2F_1} + \frac{3p_{e0}K_1M_1}{B_1}\left(\frac{3}{F_1} + 1\right) \end{aligned}\right\} \tag{4.13}$$

Putting the values of n_{i2}, n_{e2} in the second-order Poisson equation $n_{i2} - n_{e2} = 0$, we get the following equation

$$(1 - Z_d\sigma)S_1 - S = 0. \tag{4.14}$$

From the third-order equations in ε, we get

$$B\frac{\partial n_{i3}}{\partial \xi} + n_{i0}\frac{\partial v_{i3}}{\partial \xi} + n_{i0}VL\frac{\partial \varphi_1}{\partial \tau} - 3n_{i0}(RL + KS)\varphi_1^2\frac{\partial \varphi_1}{\partial \xi} - 2n_{i0}KL\frac{\partial(\varphi_1\varphi_2)}{\partial \xi} = 0, \tag{4.15}$$

$$n_{i0}B\frac{\partial v_{i3}}{\partial \xi} + \alpha\frac{\partial p_{i3}}{\partial \xi} - n_{i0}VK\frac{\partial \varphi_1}{\partial \tau} + n_{i0}L\frac{\partial(\varphi_1\varphi_2)}{\partial \xi} + n_{i0}\frac{\partial \varphi_3}{\partial \xi} = 0, \tag{4.16}$$

$$B\frac{\partial p_{i3}}{\partial \xi}+3p_{i0}\frac{\partial v_{i3}}{\partial \xi}+p_{i0}VM\frac{\partial \varphi_1}{\partial \tau}-n_{i0}(5KT+7RM)\varphi_1^2\frac{\partial \varphi_1}{\partial \xi}-4p_{i0}KM\frac{\partial (\varphi_1\varphi_2)}{\partial \xi}=0, \tag{4.17}$$

$$B_1\frac{\partial n_{e3}}{\partial \xi}+n_{e0}\frac{\partial v_{e3}}{\partial \xi}+n_{e0}VL_1\frac{\partial \varphi_1}{\partial \tau}-3n_{e0}(R_1L_1+K_1S_1)\varphi_1^2\frac{\partial \varphi_1}{\partial \xi}-2n_{20}K_1L_1\frac{\partial (\varphi_1\varphi_2)}{\partial \xi}=0, \tag{4.18}$$

$$n_{e0}B_1\frac{\partial v_{e3}}{\partial \xi}+\frac{\partial p_{e3}}{\partial \xi}-n_{e0}QVK_1\frac{\partial \varphi_1}{\partial \tau}+n_{e0}L_1\frac{\partial (\varphi_1\varphi_2)}{\partial \xi}-n_{e0}\frac{\partial \varphi_3}{\partial \xi}=0, \tag{4.19}$$

$$\begin{aligned}&B_1\frac{\partial p_{e3}}{\partial \xi}+3p_{e0}\frac{\partial v_{e3}}{\partial \xi}-p_{e0}VM_1\frac{\partial \varphi_1}{\partial \tau}\\&\quad+n_{e0}(5K_1T_1+7R_1M_1)\varphi_1^2\frac{\partial \varphi_1}{\partial \xi}-4p_{e0}K_1M_1\frac{\partial (\varphi_1\varphi_2)}{\partial \xi}=0,\end{aligned} \tag{4.20}$$

$$\frac{\partial^2 \varphi_1}{\partial \xi^2}=n_{e3}-n_{i3}. \tag{4.21}$$

On differentiation of (4.21), we get

$$\frac{\partial^3 \varphi_1}{\partial \xi^3}=\frac{\partial n_{e3}}{\partial \xi}-\frac{\partial n_{i3}}{\partial \xi}. \tag{4.22}$$

Putting the values of $\partial n_{e3}/\partial \xi$ and $\partial n_{i3}/\partial \xi$ determined from relations (4.15)–(4.20), in (4.22) and using (4.14), we get the MKdV equation

$$\frac{\partial \varphi_1}{\partial \tau}+p'\varphi_1^2\frac{\partial \varphi_1}{\partial \xi}+q'\frac{\partial^3 \varphi_1}{\partial \xi^3}=0, \tag{4.23}$$

where

$$\left.\begin{aligned}p'&=-\frac{[(1-Z_d\sigma)\{BB_1L_1S_1+BL_1(5K_1T_1+7M_1R_1)+3B(K_1S_1+L_1R_1)\}]}{(1-Z_d\sigma)\{BVL_1(QB_1K_1+M_1)+BVL_1\}+B_1VL(BK+\alpha M)+B_1VL}\\&\quad+\frac{[\{BB_1LS+\alpha B_1L(5KT+7MR)+3B_1(RL+KS)\}]}{(1-Z_d\sigma)\{BVL_1(QB_1K_1+M_1)+BVL_1\}+B_1VL(BK+\alpha M)+B_1VL}\\q'&=-\frac{BB_1}{n_{i0}[(1-Z_d\sigma)\{BVL_1(QB_1K_1+M_1)+BVL_1\}+B_1VL(BK+\alpha M)+B_1VL]}\end{aligned}\right\}. \tag{4.24}$$

5. Solitary wave solutions of the Korteweg–de Vries and the modified Korteweg–de Vries equations

Introducing the transformation $\eta=\xi-C_1\tau$ where C_1 is the soliton speed in the linear η-space and using the boundary conditions $\varphi_1=\partial\varphi_1/\partial\eta=\partial^2\varphi_1/\partial\eta^2=0$ as $|\eta|\rightarrow 0$, we get the solution of the KdV (4.8) as

$$\varphi_1=\varphi_0 \sec h^2\left(\frac{\eta}{\Delta}\right). \tag{5.1}$$

The amplitude and the width of the solitary waves are respectively given by $\varphi_0=3C_1/p$ and $\Delta=\sqrt{4q/C_1}$, C_1 is soliton speed.

Using the same transformation and integrating the MKdV (4.23) subject to the boundary conditions $\varphi_1 = \partial^2\varphi_1/\partial\eta^2 = 0$ as $|\eta| \to \pm\infty$, the solution is obtained as

$$\varphi_1' = \varphi_0' \sec h \left(\frac{\eta}{\Delta_1}\right). \tag{5.2}$$

The amplitude and the width of the solitary waves represented by the MKdV equations (4.23) are respectively given by $\varphi_0' = \sqrt{6C_1/p'}$ and $\Delta_1 = \sqrt{q'/C_1}$, C_1 being the soliton speed.

6. Discussion

In this model of a dusty plasma with variable pressure in both the ions and electrons, compressive solitons in most cases and rarefactive solitons in some cases based on parametric domains are shown to exist in the presence of stationary dust particles in the plasma. To trace out a curve, the points for a graph are plotted corresponding to each v_{e0} (as for example in figure 1) and for all the values of v_{i0} to yield the unique value of the amplitude $\varphi_0 = 3C_1/p$. This principle is adopted for all cases of the graphs.

The amplitudes of the compressive solitons for small $v_{i0} < 5$ starting with reasonable magnitudes sharply decrease to smaller values for each streaming v_{e0} of electrons and increase thereafter to a critical $v_{i0} \approx 12$ for $Z_d = 270$ (figure 1). The graphs of the soliton amplitudes reflect an interesting characteristic behaviour. The soliton of smallest amplitude for $v_{e0} = 25$ further increases to a maximum after crossing the critical position of v_{i0} just to decrease thereafter. Contrary to this, the soliton of highest amplitude attained before the critical value of $v_{i0} \approx 12$ at $v_{e0} = 5$ decreases after the critical v_{i0} without growing to a maximum. Similar characteristic behaviour of amplitudes is reflected corresponding to two other values of $v_{e0} = 7.5$, 10 also. Computationally, it is found that all φ_{max} for all electron streaming velocities ($5 < v_{e0} < 25$) lie between $12 \leqslant v_{i0} \leqslant 17.5$. In conformity with the decrease and increase in amplitudes of compressive and decrease in rarefactive solitons, the growth pattern is reflected in figure 1(*b*) corresponding to figure 1(*a*). Further, the dependence of the amplitude growth pattern of compressive solitons on electron and ion streaming can also be observed in figure 1(*b*).

The widths of the compressive KdV solitons (corresponding to the amplitudes of figure 1) become very great (i.e. waves are flat) for solitons in the upper range of $v_{e0} \approx 25$ lying within the lower range of v_{i0}. Of course, these flat waves are seen to be produced within the vicinity of $v_{i0} = 5$. Widths of sharp amplitudes mostly of rarefactive character related to the corresponding v_{i0} of figure 1(*a*) are found in the vicinity of $v_{i0} = 25$. The flatness of the width shown in figure 2(*a*) is convincingly represented in figure 2(*b*). Besides, the clear flatness of waves reflecting bigger widths are shown in figure 2(*b*) for smaller values of v_{i0} corresponding to higher values of v_{e0}.

The characteristic of slightly decreasing convex growth of only compressive KdV solitons tends to zero as v_{e0} increases for $v_{i0} = 2$(i), 3(ii), 4(iii), 5(iv), 6(v), $Z_d = 270$ (fixed) and for some value of v_{e0} in its range (figure 3*a*). Of course in all cases, the amplitudes diminish with v_{e0}. Contrary to this, the amplitudes of only compressive KdV solitons grow quite linearly to high amplitudes (figure 3*a*) subject to the limitation of KdV solitons. Interestingly, the dual linear growth of rarefactive turning to compressive KdV solitons follows the same pattern with the exception that the

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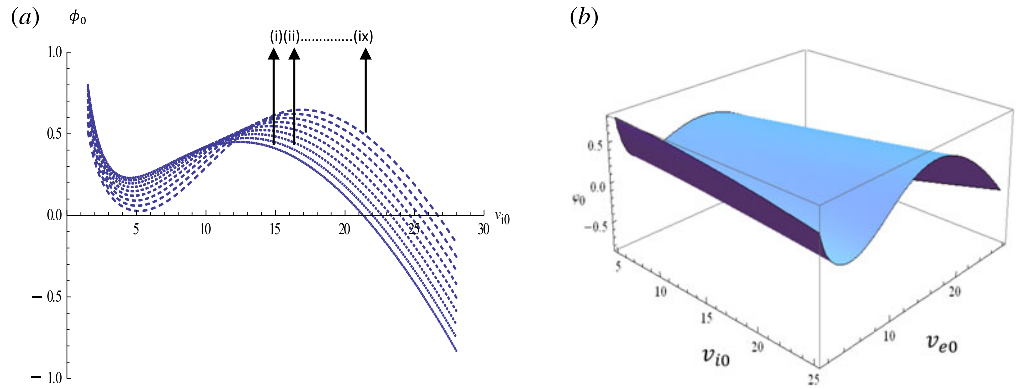

FIGURE 1. Amplitudes of compressive and rarefactive KdV solitons versus ion streaming speed v_{i0} for fixed $v_{e0} = 5$(i), 7.5(ii), 10(iii), 12.5(iv) . . . 25(ix), $\sigma = 0.001$, $Z_d = 270$ and $\alpha = 0.1$.

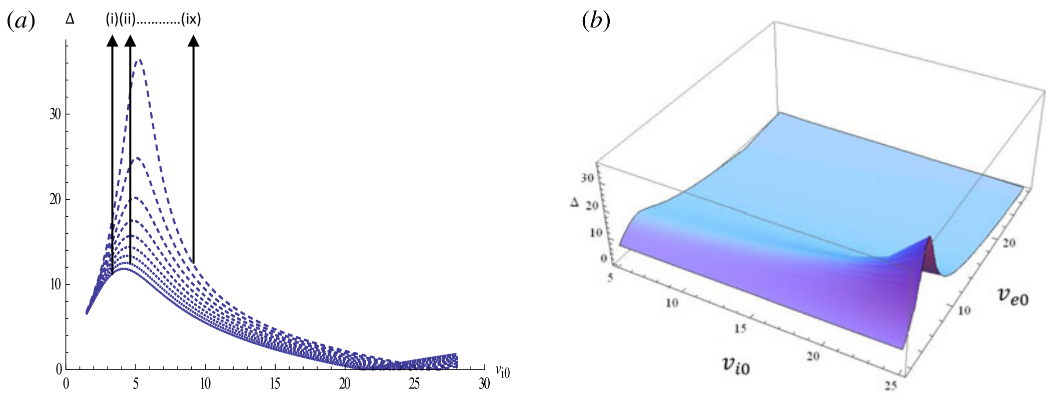

FIGURE 2. Width of compressive and rarefactive KdV solitons versus ion streaming speed v_{i0} for fixed $v_{e0} = 5$(i), 7.5(ii), 10(iii), 12.5(iv) . . . 25(ix), $\sigma = 0.001$, $Z_d = 270$ and $\alpha = 0.1$.

amplitudes of rarefactive solitons decreases from higher values to lower values (figure 3(*a*), xi, xii, xiii) but for the compressive part it reflects just the opposite character. The dual characteristic representations of compressive and rarefactive solution amplitudes are shown in figure 3(*b*). The amplitudes of the corresponding MKdV solitons (figure 3*c*) are found to decrease concavely (unlike KdV solitons figure 3*a*) with v_{e0} for different $v_{i0} = 0.5$(i), 1(ii), 1.5(iii), 2(iv), 2.5(v). The range of v_{e0} for the existence of MKdV soliton in this case is reduced to $v_{e0} = 18$ for $v_{i0} = 2.5$ when $\alpha = 0.8$ which does not exist for $\alpha 0.1$ within the limit.

For small streaming speed values of ions $v_{i0} = 0.1$(i) . . . 1.5(viii), the small amplitude compressive KdV solitons are found to convexly diminish to zero with v_{e0} at $v_{e0} \leqslant 70$ for $Z_d = 500$, $\sigma = 0.001$, $\alpha = 0.1$ (figure 4*a*). After diminishing to zero, the solitons become rarefactive when the electron streaming speed exceeds ($v_{e0} >$) 80. The small amplitude KdV compressive solitons have attained relatively high amplitudes against each small value of $v_{i0} = 0.1$(i), 0.3(ii), 0.5(iii), 0.7(iv) . . . 1.3(vii), 1.5(viii) corresponding to respective values of v_{e0} but in its lower regimes (figure 4*a*). The compressive character of the KdV solitons is rightly forced to extinction when the

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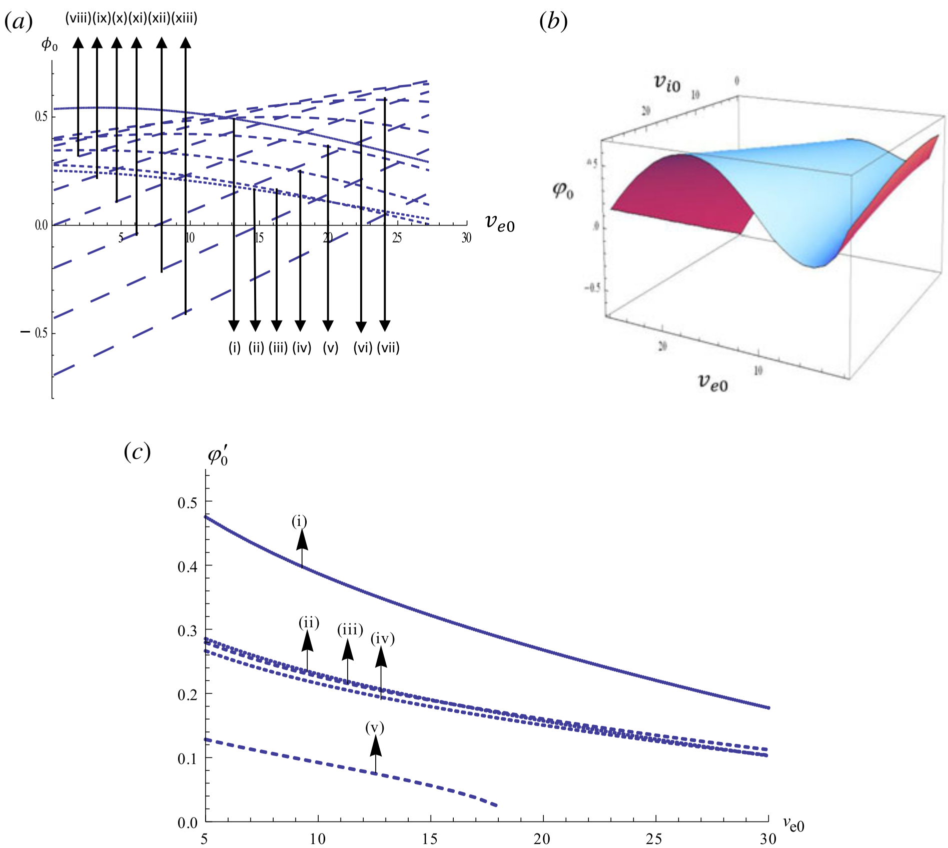

FIGURE 3. (*a*,*b*) Amplitudes of compressive and rarefactive KdV solitons versus electron streaming speed v_{e0} for fixed $v_{i0} = 2$(i), 3(ii), 4(iii), 5(iv) … 26(xiii), $\sigma = 0.001$, $Z_d = 270$ and $\alpha = 0.1$. (*c*) Amplitudes of compressive MKdV solitons versus electron streaming speed v_{e0} for fixed $v_{i0} = 0.5$(i), 1(ii), 1.5(iii), 2(iv), 2.5(v), $\sigma = 0.001$, $Z_d = 270$ and $\alpha = 0.8$.

electrons initial streaming speed attains a very high value supplemented by the high negative dust charge Z_d against very small ion initial streaming speed $v_{i0} < 1.5$. Further, it appears to generate rarefactive solitons, indicating attainment of high amplitudes. The complex representation is seen in figure 4(*b*). The KdV solitons are restricted to a small amplitude limit less than 1. For the small ion streaming speed $v_{i0} < 1.5$, the electron streaming speed v_{e0}, exceeding 100 in this case, dominates imbibing to cross the limit of existence (from computation work). Moreover, the nonlinear coefficient $p = 0$ identifies the region of non-existence of KdV solitons ($\varphi_0 = 3c/p$) in the vicinity of $v_{e0} = 100$ which keeps open the search for the MKdV solitons. This causes the sudden change of amplitudes indicating growth to infinity.

The MKdV compressive solitons are found to exist against very small values of initial streaming speed of the ions $v_{i0} = 0.1$(i), 0.2, 1.3(vii) but not for greater values of electron streaming speed (figure 5) for $Z_d = 500$, $\sigma = 0.001$, $\alpha = 0.1$ unlike the existence of KdV solitons of both kinds (figure 4). Besides, the only compressive KdV solitons appear to vanish (convexly) in a shorter interval $v_{e0} \leqslant 30$ (figure 3) but the MKdV solitons appear to vanish (concavely) at a much expanded upper limit of $v_{e0} \leqslant 70$ (figure 5). For the same set of parametric values, the amplitudes of both

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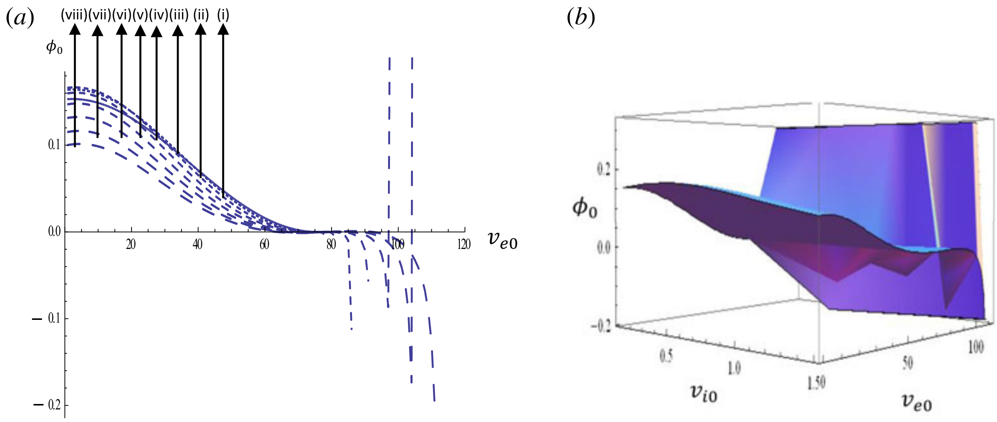

FIGURE 4. Amplitudes of compressive KdV solitons versus electron streaming speed v_{e0} for fixed $v_{i0} = 0.1$(i), 0.3(ii), 0.5(iii), 0.7(iv) . . . 1.3(vii), 1.5(viii), $\sigma = 0.001$, $Z_d = 500$ and $\alpha = 0.1$.

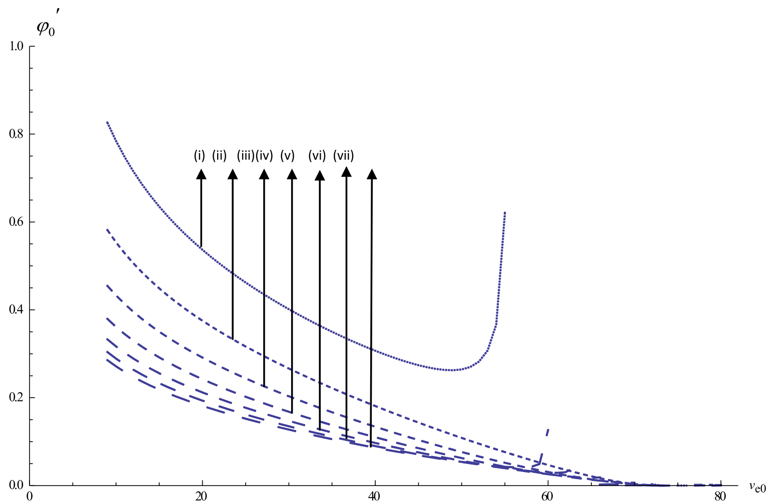

FIGURE 5. Amplitudes of compressive MKdV solitons versus electron streaming speed v_{e0} for fixed $v_{i0} = 0.1$(i), 0.3(ii), 0.5(iii), 0.7(iv) . . . 1.3(vii), $\sigma = 0.001$, $Z_d = 500$ and $\alpha = 0.1$.

compressive and rarefactive KdV solitons (figure 4) are much smaller than those of MKdV compressive solitons (figure 5).

Further, with the inclusion of higher-order nonlinearity in the resulting MKdV equation, only compressive solitons of much higher amplitudes are found to exist but in a contracted range of v_{e0} (figure 5) from either side corresponding to the same set of values of v_{i0} (figure 4*a*). In growth of amplitudes, it appears to disregard the effects of higher electron streaming speed and negative dust charges Z_d in this situation.

The amplitudes of the compressive KdV solitons are found to increase uniformly but nonlinearly (figure 6) with Z_d corresponding to each value of small ion streaming speed $v_{i0} = 0.1$(i), 0.3(ii), 0.5(iii), 0.7(iv), 0.9(v), $v_{e0} = 5$, $\sigma = 0.001$, $\alpha = 0.1$. The increase of the number of dust charges Z_d is seen to contribute in intensive growth

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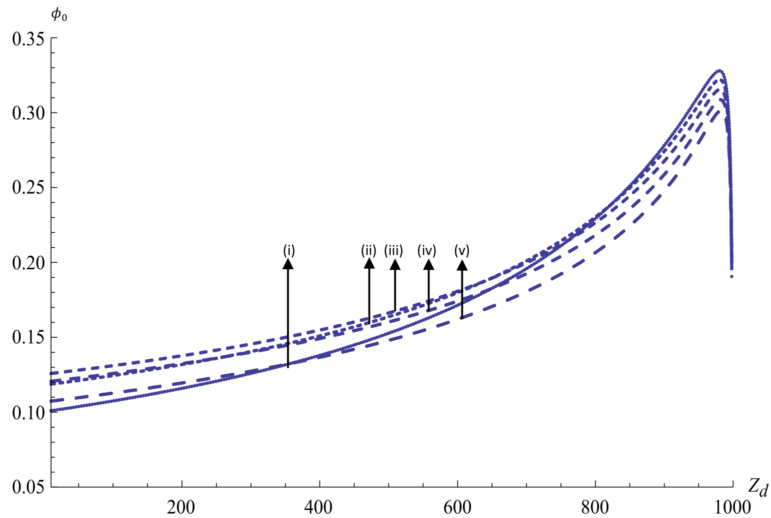

FIGURE 6. Amplitudes of compressive KdV solitons versus Z_d for fixed $v_{i0} = 0.1(\text{i}), 0.3(\text{ii}), 0.5(\text{iii}), 0.7(\text{iv}), 0.9(\text{v})$, $\sigma = 0.001$, $v_{e0} = 5$ and $\alpha = 0.1$.

of amplitudes of compressive KdV solitons. It appears that the flux of negative dust charges at large numbers to the plasma compound compresses the plasma state taking the advantage of small pairs of ions and electrons streaming. The smaller the value of the ion streaming speed the smaller is the amplitude of the compressive solitons but at its higher value supplemented by high accumulation of negative dust charges Z_d from the dust component, the growth becomes much higher turning the amplitude smaller after some stage of Z_d.

The amplitudes of the only compressive MKdV solitons are found to increase quite nonlinearly with v_{i0} (figure 7) for high values of electron streaming speeds $v_{e0} = 31(\text{i}), 33(\text{ii}) \ldots 49(x)$ and $Z_d = 90, \sigma = 0.001, \alpha = 0.1$. Smaller value of v_{e0} like 31(i) is seen to produce high amplitude MKdV compressive solitons throughout the range of v_{i0}. Due to higher-order nonlinearity, the nonlinear monotonic growth of amplitudes of MKdV solitons, we observe, is rather supported by the almost equal streaming pairs of ions and electrons for relatively small $Z_d = 90$.

For small ion streaming speed, namely $v_{i0} = 0.1(\text{i}), 0.15(\text{ii}), 0.2(\text{iii}), 0.25(\text{iv}), 0.3(\text{v})$, the amplitudes of the compressive MKdV solitons slowly increase from linear (for small Z_d) to nonlinear as Z_d increases for fixed $v_{e0} = 20, \sigma = 0.001, \alpha = 0.8$ (figure 8). Of course, the smaller the value of $v_{i0}(=0.1)$, the higher are the values of soliton amplitude in the whole lower range of Z_d, which are found to be gradually higher in the upper range (here) of Z_d.

Corresponding to the higher value of $v_{i0} = 78$ but in the higher range of electron streaming speed $v_{e0} = 176(\text{i}), 178(\text{ii}) \ldots 194(\text{x})$, the amplitudes of compressive MKdV solitons remain almost constant (figure 9) for the entire range of Z_d including its higher range (figure 9*a*). The linear growth of amplitudes is clear in figure 9(*b*). Comparing the results of (figure 8) and (figure 9*a*), it is observed, that the growth of amplitude of the compressive soliton (in case of higher-order nonlinearity) is dependent on the initial streaming speed of v_{e0}, v_{i0} and Z_d. In contrast to earlier figures of KdV soliton amplitudes, the MKdV soliton amplitudes are found not to be much dependent on Z_d and v_{e0}, rather it is submissive to higher-order nonlinearity.

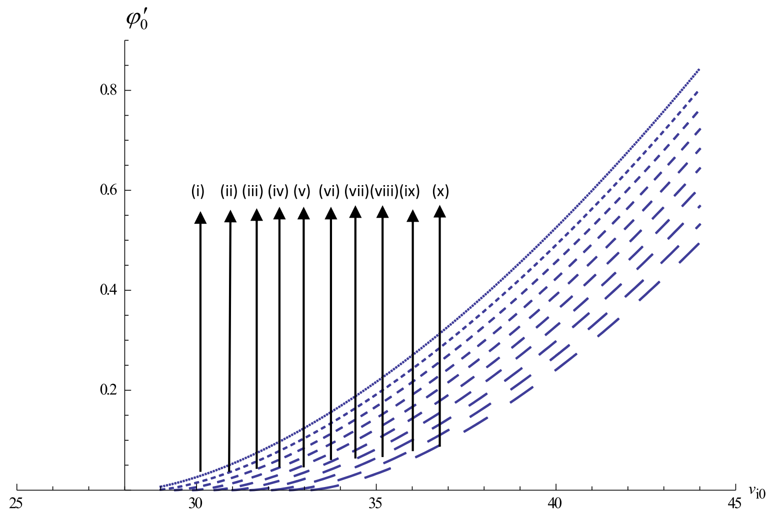

FIGURE 7. Amplitudes of compressive MKdV solitons versus electron streaming speed v_{i0} for fixed $v_{e0} = 31$(i), 33(ii), 35(iii), 37(iv) ... 49(x), $\sigma = 0.001$, $Z_d = 90$ and $\alpha = 0.1$.

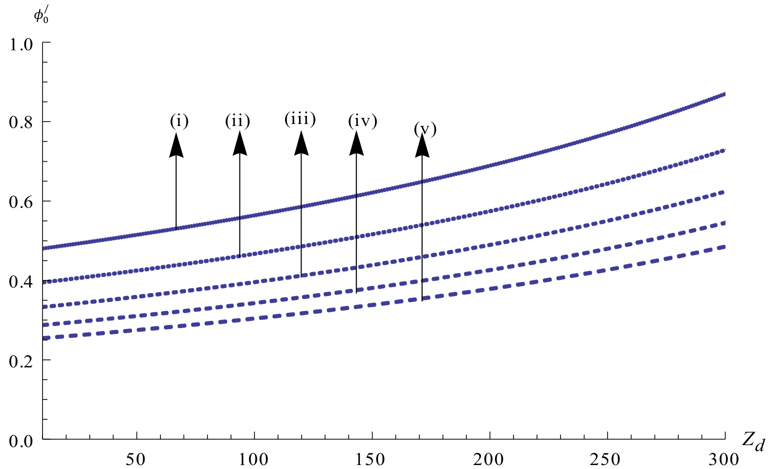

FIGURE 8. Amplitudes of compressive MKdV solitons versus Z_d for fixed v_{i0} = 0.1(i), 0.15(ii), 0.2(iii) ... 0.3(v), $\sigma = 0.001$, $v_{e0} = 20$ and $\alpha = 0.8$.

The nature of amplitude growth of KdV compressive solitons with temperature α remains almost constant for all v_{i0}. For $v_{i0} = 0.2$(i)–0.8(iv) small amplitudes of the compressive solitons are seen to increase up to some α but for $v_{i0} = 1$(v)–2(x), it is seen (figure 10) that the amplitudes of solitons remain constant, becoming smaller and smaller after some α. But the small amplitude solitons reflect increasing growth with α for all $v_{i0} \leqslant 1$.

The parametric limitations of the parameters $v_{i0}, v_{e0}, Z_d, \alpha$, required for the existence of DIA waves in plasmas under the small amplitude consideration (<1 for the perturbative method) and variable temperatures are shown almost in all figures in this investigation. These are quite new results in this direction.

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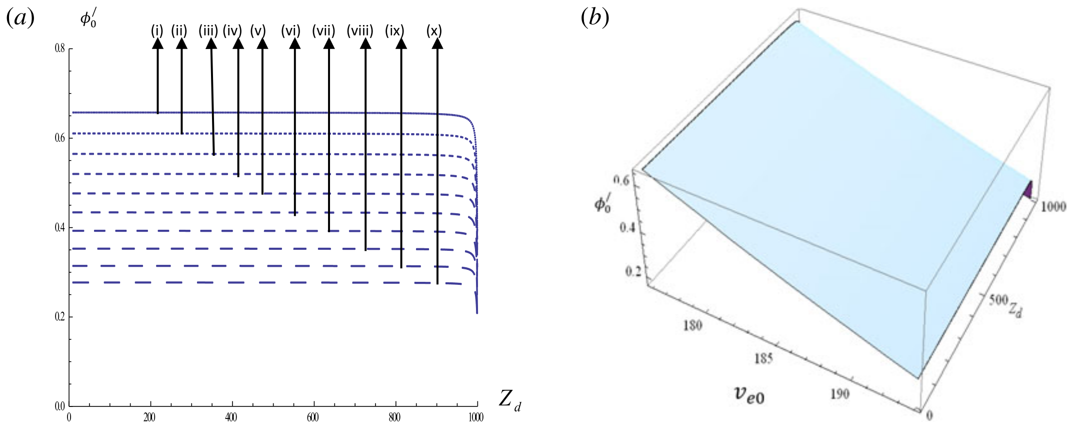

FIGURE 9. Amplitudes of compressive MKdV solitons versus Z_d for fixed v_{e0} = 176(i), 178(ii), 180(iii) . . . 194(x), $\sigma = 0.001$, $v_{i0} = 78$ and $\alpha = 0.1$.

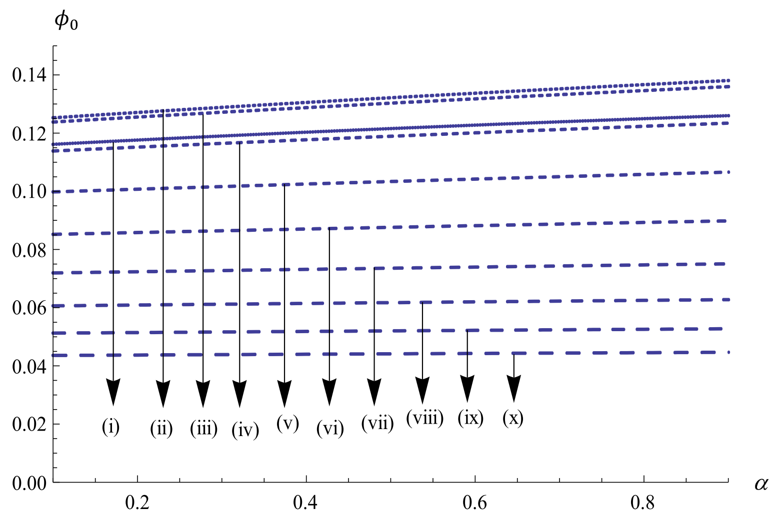

FIGURE 10. Amplitudes of compressive KdV solitons versus α for fixed v_{i0} = 0.2(i), 0.4(ii), 0.6(iii), 0.8(iv) . . . 2(x), $\sigma = 0.001$ and $Z_d = 200$.

One of the primary objectives of space probes is to investigate the properties of the dust surrounding space bodies like the Moon or Mars. The dust acoustic waves are basically low frequency waves in plasmas and for stationary background mapping of the massive dust particles, spectrometer insertion into a space vehicle is rather more convenient. For the flux of solar flares, solar winds in interplanetary space at high altitudes, the temperature of the ions and electrons vary causing release of charge (negative or positive) to stationary dust and the system turns into one of higher-order nonlinearity inviting the MKdV equation for steady waves. The temperature variations of the plasma species posing behavioural changes of the dust in DIA waves may be helpful from our studies to know the character of the dust in space probes. Further, the proposed information of the parameters may be of great help for experimentalists in using the Q-machine.

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The plasma scenario is greatly affected by frequent bursts of solar winds of different kinds and magnetic storms. The nonlinearity developed in space plasmas due to various factors creates hazards to space vehicles and probes. The study of nonlinear DIA waves in the presence of dust predicting the character of plasma parameters in space, which are a kind of slow and stable waves, may be a great help to meet these hazards in some extent.

Though not exactly similar, Verheest, Olivier & Hereman (2016) have recently shown a comparative and nice representative structure of KdV and MKdV solitons in plasmas but with quartic nonlinearity for the latter.

The characteristic change of the DIA waves shown in the discussion, subject to variations of pressure, is the important highlight of our investigation.

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