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Observer-Based Consensus of Higher-Order Nonlinear Heterogeneous Multiagent Systems with Unmatched Uncertainties: Application on Robotic Systems

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SUMMARY
The consensus of higher-order nonlinear heterogeneous multiagent systems with matched and unmatched uncertainties is studied in this paper. The distributed observer-based controllers for multiagent systems are achieved using a fixed-time sliding mode controller based on the disturbance observer. For this purpose, the disturbance observers are designed for finite-time estimation of matched and unmatched uncertainties. Using the estimated values, the fixed-time distributed sliding mode controllers are designed and the consensus protocol is achieved. In this regard, a theorem is proved, which guarantees the fixed-time convergence of consensus errors. The effectiveness of the proposed distributed controllers has been validated through simulations for two robotic multiagent systems and a numerical example.

KEYWORDS: Fixed/finite-time stability; Consensus; Nonlinear heterogeneous multiagent systems; Disturbances observer; Matched and unmatched uncertainties; Robotic multiagent systems.

1. Introduction

A multi agent system (MAS) consists of several agents, and each agent is linked to other agents through a communication topology. Agents work together to solve a problem that is impossible or difficult to solve by a single agent. In recent years, multiagent systems have been widely considered. One of the reasons for paying attention to multiagent systems is their abilities to describe many practical systems and their applications in various fields such as mathematics, physics, computer sciences, social sciences, aerospace, etc. In addition, many complex systems can be considered as a network of several agents such as transportation networks, power networks, and mobile networks.[1]

Many types of research, such as cooperation, coordination, solving distributed problems, consensus, and formation control, have been studied for multiagent systems. One of the important and significant discussions regarding multiagent systems is the issue of consensus. The consensus problem can be applied in many areas, such as unmanned air vehicles, autonomous underwater vehicles, sensor networks, and robotics.[3–5] In fact, when all agents in a multiagent system converge to a certain value, despite unforeseen changes, the consensus is achieved.[6] To reach consensus, an algorithm is needed to guarantee the agreement between the agents. The design of distributed control laws to achieve consensus depends on a variety of factors like dynamics of agents, consensus rate, and agreed value.

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In terms of consensus state, the multiagent systems have been divided into two groups of leader–follower and leaderless consensus. For leaderless consensus, the final agreed value of agents usually depends on the initial values of the agents,[7] and in the leader–follower consensus, the final agreed value is the leader's mode. The leader can be virtual or real. If one of the agents is considered to be a leader, the leader is real. In this case, the rest of the agents are followers. Moreover, the leader may be virtual. For example, a reference signal can be considered as a leader.[8] The leader's behavior affects the performance of the followers; however, it is not influenced by the follower's behavior.

One of the critical elements in solving the consensus problem is the consensus rate. Considering this, the consensus problem is classified into two groups: asymptotic and finite/fixed-time consensus. In an asymptotic consensus, the error between the system states and the agreed value decreases as time goes to infinity. In the past decades, much progress has been made on the asymptotic consensus problem of multiagent systems with different dynamics and different topologies.[9,10] In practical systems, it is desirable to achieve consensus in the shortest time and with the least error. Therefore, the finite-time consensus is preferable to the asymptomatic consensus. In a finite-time consensus, the error between the system states and the agreed value is exactly zero after a finite time and the multiagent system is more robust against disturbances. Due to these characteristics, the finite-time consensus has progressed in many aspects in recent years. Finite-time consensus for the nonlinear higher-order multiagent system with uncertainties has been addressed in ref. [11]. Finite-time observer-based consensus for the second-order multiagent system with external disturbances was presented in ref. [12]. The authors of ref. [13] investigated the finite-time consensus problem for event-triggered multiagent systems.

In the finite-time consensus, the settling time depends on the initial value of states. In order to conquest this problem, the fixed-time consensus was studied. The fixed-time consensus problem with switching topology was addressed in ref. [14]. The robust fixed-time consensus problem for nonlinear multiagent systems with external disturbances, under a weighted undirected topology, was presented in ref. [15]. The adaptive fixed-time consensus problem for the second-order multiagent system is discussed in ref. [16]. In ref. [17], this issue was studied for integrator-type multiagent systems.

The existence of uncertainties, due to model uncertainties and external disturbances in the dynamical model of the agents, is one of the challenging issues in solving the consensus problem. In order to achieve robust performance, uncertainties should be considered in the design procedure. Many references have been made to resolve the consensus problem in the presence of uncertainties; however, in most of the references, the authors considered only the matched uncertainties.[18–20] Matched uncertainty enters in the channel where the input is located. In many practical systems, uncertainties can appear on other channels, which are referred to as unmatched uncertainties. Different approaches have been proposed to solve the consensus problem in the presence of unmatched uncertainties.[21,22] One of these approaches is the use of disturbance observers.[23–27] The authors of refs. [23, 24] studied the asymptotic consensus problem with unmatched uncertainties for linear multiagent systems. The finite-time consensus for linear multiagent systems with unmatched uncertainties was discussed in ref. [25]. The asymptotic consensus for a nonlinear multiagent system subjected to unmatched uncertainties has been presented in ref. [26]. The finite-time consensus for nonlinear multiagent systems in the presence of matched and unmatched uncertainties was proposed in ref. [27].

On the other hand, most of the researches have been devoted to the consensus problem in multiagent systems with identical agents or homogeneous multiagent systems. While in practical applications, agents usually have different dynamics and so-called heterogeneous multiagent systems. Some types of research have addressed the consensus problem for heterogeneous multiagent systems. The consensus problem for linear heterogeneous multiagent systems was investigated in refs. [28, 29]. References[30–33] addressed the consensus problem for nonlinear heterogeneous multiagent systems. However, none of these references have considered unmatched disturbance in the dynamical model of agents. There are few works investigating the consensus problem for higher-order nonlinear heterogeneous multiagent systems with unmatched disturbance. For instance, in ref. [34], the authors addressed the consensus problem for these systems; however, it can only guarantee the asymptotical convergence of consensus error.

As stated, the settling time in the fixed-time consensus does not depend on the initial value of states and consensus occurs at a specific time. In some practical applications, we tend to achieve a precise settling time that is independent of the initial conditions. Furthermore, most practical systems are subject to matched and unmatched disturbances. Therefore, it is necessary to solve the consensus problem for nonlinear heterogeneous multiagent systems in the presence of matched and unmatched disturbances.

To the best of the author's knowledge, design of distributed controllers based on the *disturbance observers* to achieve the *fixed-time* consensus in the presence of both *matched* and *unmatched* uncertainties has not been studied in the literature for higher-order nonlinear heterogeneous multiagent systems. This paper considers this issue.

In this paper, for solving the fixed-time consensus problem, two steps are considered. The first step is to estimate the matched and unmatched uncertainties of agents using the distributed finite-time disturbance observers. Then, the robust distributed controllers are designed based on the sliding mode approaches such that robust consensus is achieved in fixed time in spite of unknown terms, nonlinear, and heterogeneous dynamics of agents. Since the traditional sliding mode controller has robust manner facing matched uncertainties, the sliding mode technique is modified using the estimated values of uncertainties such that the new sliding mode controller is robust against both matched and unmatched uncertainties. For this purpose, new sliding manifolds are suggested and a theorem, which guarantees the fixed-time convergence of consensus errors, is given and then is proved. Finally, simulations are presented for two robotic multiagent systems and a numerical example to show the effectiveness of the designed distributed controllers.

The rest of this paper is organized as follows: the preliminaries and some definitions are provided in Section 2. The problem statement is given in Section 3. The main ideas are proposed in Section 4. Computer simulations are provided in Section 5 to verify the theoretical results. Finally, conclusions are given in Section 6.

2. Preliminaries

2.1. Graph theory

Consider an MAS consisting of a leader and N followers. A graph $G(V, E, A)$ can be used to show the relation in the MAS where $V = \{0, 1, \ldots, N\}$ is the collection of nodes, $E \subseteq V \times V\{(i, j) \in V \times V\}$ denotes the set of edges (where $(i, j) \in E$ if the agent can obtain data from the agent j else $(i, j) \notin E$). Moreover, $A = \lfloor a_{ij} \rfloor \in R^{N\times N}$ is the weighted adjacency matrix. If $(i, j) \in E$, then $a_{ij} = 1$ else $a_{ij} = 0$. Besides, it is supposed that $a_{ii} = 0$. Also, $L = \lfloor l_{ij} \rfloor \in R^{N\times N} = D - A$ is the Laplacian matrix, where $D = diag\{d^1, d^2, \ldots, d^N\}$ and $d^i = \sum_{j=1}^{N} a_{ij}$. The diagonal matrix $B = diag\{b^1, b^2, \ldots, b^N\}$ denotes the relationship between the leader and followers. If the agent i is linked to the leader then $b^i = 1$ otherwise $b^i = 0$. It is supposed that at least one follower node has linked to the leader node.

Definition 1.[35] Consider a dynamical system $\dot{x} = f(x)$, where $f(0) = 0$ and $f : D \to R^n$ is continuous on an open neighborhood $D \subseteq R^n$ of the origin. The zero solution of this system is said to be finite time stable if the solutions of $x(t)$ reach to zero in finite time T (i.e., $\lim_{t\to T} x(t) = 0$, where T is the settling time).

Lemma 1.[35] *Consider a system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{f}(0) = 0$. If there exist a positive definite function $V(\boldsymbol{x}) : D \to R$ and positive constants ω and $0<q<1$ such that $\dot{V}(\boldsymbol{x}) \le -\omega V^q$, then the zero solution of this system is finite-time stable and the settling time T satisfies $T(\boldsymbol{x}_0) \le \frac{1}{\omega(1-q)} V(\boldsymbol{x}_0)^{(1-q)}$. If $V(\boldsymbol{x})$ is radially unbounded, then the zero solution is globally finite-time stable.*

Definition 2.[36] The zero solution of system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ with $\boldsymbol{f}(0) = 0$ is said to be fixed-time stable, if it is finite-time stable and the settling time function T is bounded and independent from the initial states.

Lemma 2.[36] *Consider a system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{f}(0) = 0$. If there exist a positive definite function $V(\boldsymbol{x}) : D \to R$ and constants $\varsigma, \phi, \chi, \kappa, \iota > 0$, where $\chi k < 1$, $\kappa\iota > 1$ such that $\dot{V}(\boldsymbol{x}) \le -[\varsigma V^{\chi} + \phi V^{\kappa}]^{\iota}$, then the zero solution of this system is fixed-time stable and the settling time T satisfies $T \le \frac{1}{\varsigma^{\iota}(1-\chi\iota)} + \frac{1}{\phi^{\iota}(\kappa\iota-1)}$. If $V(\boldsymbol{x})$ is radially unbounded, then the zero solution is globally finite-time stable.*

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Lemma 3.[37] *Consider the following system*

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = u(x) \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ *and* $u(x) \in R$. *The considered system is fixed-time stable if the controller* u *is designed as below:*

$$u(x) = -\sum\nolimits_{m=1}^{n} \lambda_m \left(sgn^{\mu_m}(x_m) + sgn^1(x_m) + sgn^{\acute{\mu}_m}(x_m) \right) \tag{2}$$

where μ_m *and* $\acute{\mu}_m$ *are satisfying the following relations:*

$$\mu_{n-k} = \frac{\mu}{(k+1) - k\mu} \qquad \mu'_{n-k} = \frac{2-\mu}{k\mu - (k-1)} \qquad k = 0, \ldots, n-1 \tag{3}$$

where $\mu \in (\alpha, 1)$ *and* $\alpha \in \left(\frac{n-2}{n-1}, 1\right)$. *Furthermore, the parameters* λ_m *for* $m = 1, \ldots, n$ *are chosen such that the polynomials* $\left(s^n + \lambda_n s^{n-1} + \cdots + \lambda_2 s + \lambda_1\right)$ *and* $\left(s^n + 3\lambda_n s^{n-1} + \cdots + 3\lambda_2 s + 3\lambda_1\right)$ *both be Hurwitz.*

Notation 1. In this paper, $sgn^p(x)$ is defined as $|x|^p sgn(x)$.

3. Problem Statement

Consider a multiagent system with $N + 1$ agents consisting of one leader and N followers labeled by 0 and $i = 1, \ldots, N$, respectively. The dynamical equation of the leader is considered as follows:

$$\begin{cases} \dot{x}_m^0 = x_{m+1}^0 & m = 1, 2, \ldots, n-1 \\ \dot{x}_n^0 = u^0 \\ y^0 = x_1^0 \end{cases} \tag{4}$$

where $\boldsymbol{x}^0 = [x_1^0, x_2^0, \ldots, x_n^0]^T \in D \subseteq R^n$ and $y^0 \in R$ are the state vector and the output of the leader, respectively. Moreover, $u^0 \in R$ is the control input of the leader. The dynamical equations of the ith follower are described as:

$$\begin{cases} \dot{x}_m^i = x_{m+1}^i + \delta_m^i(\boldsymbol{x}^i, t) & m = 1, 2, \ldots, n-1 \\ \dot{x}_n^i = f^i\left(\boldsymbol{x}^i, t\right) + \delta_n^i(\boldsymbol{x}^i, t) + h^i u^i(t) & i = 1, 2, \ldots, N \\ y^i = x_1^i \end{cases} \tag{5}$$

where $\boldsymbol{x}^i = [x_1^i, x_2^i, \ldots, x_n^i]^T \in D \subseteq R^n$ is the state vector, $u^i(t) \in R$ is the control input, h^i is a constant, and $f^i \in R$ is the nonlinear known function of ith follower, which may be different in each follower. This leads to multiagent systems with nonlinear and heterogeneous dynamics of agents. The unknown parameters δ_m^i for $(m = 1, 2, \ldots, n)$ have been lumped together with various uncertainties due to model simplification, parameter uncertainties, and/or external disturbances. Moreover, $\boldsymbol{\delta}^i(\boldsymbol{x}^i, t) = [\delta_1^i(\boldsymbol{x}^i, t), \ldots, \delta_{n-1}^i(\boldsymbol{x}^i, t)]^T \in R^{n-1}$ and $\delta_n^i(\boldsymbol{x}^i, t) \in R$ represent the unmatched and matched model uncertainties, respectively. Furthermore, $y^i \in R$ denotes the output of the ith follower.

In this paper, the goal is to design distributed control laws based on the sliding mode approach for the considered multiagent system, such that the outputs of the followers track the output of the leader in the presence of the matched and unmatched uncertainties and external disturbances in a fixed time.

4. Main Result

In this section, the robust distributed controllers are proposed such that the error vectors converge to zero in fixed time and the consensus protocol has a robust manner. For this purpose, disturbance observers are designed for each follower to estimate the uncertainties. Then, the robust control

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strategy is proposed to achieve the fixed-time consensus. In this regard, a theorem is presented and proved.

The consensus error vectors, $\boldsymbol{\varrho}^i = [\varrho_1^i, \varrho_2^i, \ldots, \varrho_n^i]^T$ for $i = 1, 2, \ldots, N$, are defined as follows based on the given information in the connected graph $G(V, E, A)$.

$$\begin{bmatrix} \varrho_1^i \\ \varrho_2^i \\ \vdots \\ \varrho_n^i \end{bmatrix} = \sum\nolimits_{j=1}^{N} a_{ij} \begin{bmatrix} x_1^i - x_1^j \\ x_2^i - x_2^j \\ \vdots \\ x_n^i - x_n^j \end{bmatrix} + b^i \begin{bmatrix} x_1^i - x_1^0 \\ x_2^i - x_2^0 \\ \vdots \\ x_n^i - x_n^0 \end{bmatrix} \tag{6}$$

It can be rewritten as below:

$$\boldsymbol{\varrho}^i = \sum\nolimits_{j=1}^{N} a_{ij} \left(\boldsymbol{x}^i - \boldsymbol{x}^j\right) + b^i (\boldsymbol{x}^i - \boldsymbol{x}^0) \tag{7}$$

Therefore, according to Eqs. (4) and (5), the dynamical equations of the ith vector of the consensus errors are achieved as:

$$\begin{bmatrix} \dot{\varrho}_1^i \\ \dot{\varrho}_2^i \\ \vdots \\ \dot{\varrho}_n^i \end{bmatrix} = \sum\nolimits_{j=1}^{N} a_{ij} \begin{bmatrix} x_2^i + \delta_1^i - x_2^j - \delta_1^j \\ x_3^i + \delta_2^i - x_3^j - \delta_2^j \\ \vdots \\ f^i + \delta_n^i + h^i u^i - f^j - \delta_n^j - h^j u^j \end{bmatrix} + b^i \begin{bmatrix} x_2^i + \delta_1^i - x_2^0 \\ x_3^i + \delta_2^i - x_3^0 \\ \vdots \\ f^i + \delta_n^i + h^i u^i - u^0 \end{bmatrix} \tag{8}$$

Eq. (8) can also be described as follows:

$$\begin{cases} \dot{\varrho}_m^i = \varrho_{m+1}^i + \bar{\delta}_m^i \qquad m = 1, 2 \ldots, n-1 \\ \dot{\varrho}_n^i = \left(b^i + \sum_{j=1}^{N} a_{ij}\right)\left(f^i + \delta_n^i + h^i u^i\right) - \sum_{j=1}^{N} a_{ij} \left(f^j + \delta_n^j + h^j u^j\right) - b^i\, u^0 \end{cases} \tag{9}$$

where $\bar{\delta}_m^i = \delta_m^i \left(b^i + \sum_{j=1}^{N} a_{ij}\right) - \sum_{j=1}^{N} a_{ij} \delta_m^j$.

The following finite-time nonlinear disturbance observers are designed for each agent to estimate their uncertainties:

$$\begin{cases} \dot{\xi}_{m0}^i = \eta_{m0}^i + g_m^i(\boldsymbol{x}^i, u) \\ \dot{\xi}_{m1}^i = \eta_{m1}^i \\ \quad \vdots \\ \dot{\xi}_{m(n-m+1)}^i = \eta_{m(n-m+1)}^i \end{cases} \tag{10a}$$

$$\begin{cases} \eta_{m0}^i = -\tau_{mo}^i \left(\varpi_m^i\right)^{\frac{1}{n-m+2}} \left(\left|\xi_{m0}^i - x_m^i\right|^{\frac{n-m+1}{n-m+2}}\right) sgn\left(\xi_{m0}^i - x_m^i\right) + \xi_{m1}^i \\ \eta_{mk}^i = -\tau_{mk}^i \left(\varpi_m^i\right)^{\frac{1}{n-m+2-k}} \left(\left|\xi_{mk}^i - \eta_{m(k-1)}^i\right|^{\frac{n-m+1-k}{n-m+2-k}}\right) sgn\left(\xi_{mk}^i - \eta_{m(k-1)}^i\right) + \xi_{m(k+1)}^i \\ \eta_{m(n-m+1)}^i = -\tau_{m(n-m+1)}^i \varpi_m^i sgn\left(\xi_{m(n-m+1)}^i - \eta_{m(n-m)}^i\right) \end{cases} \tag{10b}$$

$$\begin{cases} \hat{x}_m^i = \xi_{m0}^i \\ \hat{\delta}_m^i = \xi_{m1}^i \\ \hat{\dot{\delta}}_m^i = \xi_{m2}^i \\ \quad \vdots \\ \hat{\delta}_m^{i[n-m]} = \xi_{m(n-m+1)}^i \end{cases} \tag{10c}$$

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where $i = 1, 2, \ldots N$, $m = 1, 2, \ldots n$ and $k = 0, 1, \ldots, n - m + 1$. Moreover, $g_m^i\left(\boldsymbol{x}^i, u\right) = x_{m+1}^i$ for $m = 1, 2, \ldots n - 1$, and $g_n^i\left(\boldsymbol{x}^i, u\right) = f^i\left(\boldsymbol{x}^i, t\right) + h^i u^i(t)$. Also, the parameters $\tau_{mk}^i > 0$ are the observer coefficients, which are positive constants. Additionally, $\hat{x}_m^i$, $\hat{\delta}_m^i$, $\hat{\dot{\delta}}_m^i$, and $\hat{\delta}_m^{i[n-m]}$ are the estimates of x_m^i, δ_m^i, $\dot{\delta}_m^i$, and $\delta_m^{i[n-m]}$, respectively. Now, the observer errors $(\psi_{m0}^i, \psi_{mk}^i)$ are defined as:

$$\psi_{m0}^i = \hat{x}_m^i - x_m^i = \xi_{m0}^i - x_m^i \tag{11}$$

$$\psi_{mk}^i = \hat{\delta}_m^{i[k-1]} - \delta_m^{i[k-1]} = \xi_{mk}^i - \delta_m^{i[k-1]} \tag{12}$$

where $\hat{x}_m^i$ is the estimated value of x_m^i, which is defined in Eq. (10c). Moreover, $\hat{\delta}_m^{i[k-1]}$ is the estimate of $\delta_m^{i[k-1]}$, which is stated in Eq. (10c). The superscript term, which is in the bracket, is the order of derivation.

Using Eqs. (5), (10), and (11), the dynamical equations of observer errors are obtained as follows:

$$\begin{aligned}
&\dot{\psi}_{m0}^i = -\tau_{mo}^i\left(\varpi_m^i\right)^{\frac{1}{n-m+2}}\left(\left|\psi_{m0}^i\right|^{\frac{n-m+1}{n-m+2}}\right) sgn\left(\psi_{m0}^i\right) + \psi_{m1}^i \\
&\dot{\psi}_{mk}^i = -\tau_{mk}^i\left(\varpi_m^i\right)^{\frac{1}{n-m+2-k}}\left(\left|\psi_{mk}^i - \dot{\psi}_{m(k-1)}^i\right|^{\frac{n-m+1-k}{n-m+2-k}}\right) sgn\left(\psi_{mk}^i - \dot{\psi}_{m(k-1)}^i\right) + \psi_{m(k+1)}^i \\
&\dot{\psi}_{m(n-m+1)}^i \in -\tau_{m(n-m+1)}^i \varpi_m^i sgn\left(\psi_{m(n-m+1)}^i - \dot{\psi}_{m(n-m)}^i\right) + \left[-\varpi_m^i, \varpi_m^i\right]
\end{aligned} \tag{13}$$

It is shown in ref. [38] that the observer error dynamics (13) is finite-time stable and there is a finite time t_f such that $\psi_{mk}^i = 0$ for $t > t_f$.

Now, the task is to design the consensus protocol using the estimated values, such that the consensus errors converge to zero in fixed time. Therefore, we define new consensus errors, using the estimation of the disturbances, its derivations, and the relations of the consensus errors in (6). The equations of new consensus errors are obtained as:

$$\begin{cases}
\tilde{\varrho}_1^i = \varrho_1^i \\
\tilde{\varrho}_2^i = \varrho_2^i + \hat{\bar{\delta}}_1^i \\
\tilde{\varrho}_3^i = \varrho_3^i + \dot{\hat{\bar{\delta}}}_1^i + \hat{\bar{\delta}}_2^i \\
\tilde{\varrho}_4^i = \varrho_4^i + \ddot{\hat{\bar{\delta}}}_1^i + \dot{\hat{\bar{\delta}}}_2^i + \hat{\bar{\delta}}_3^i \\
\quad\vdots \\
\tilde{\varrho}_n^i = \varrho_n^i + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k-1]}
\end{cases} \qquad \text{for } i = 1, 2, \ldots, N \tag{14}$$

where $\hat{\bar{\delta}}_k^i = \hat{\delta}_k^i(\sum_{j=1}^N a_{ij} + b^i) - \sum_{j=1}^N a_{ij}\hat{\delta}_k^j$. Consequently, the dynamical equations of the new consensus errors are achieved as below:

$$\begin{cases}
\dot{\tilde{\varrho}}_1^i = \dot{\varrho}_1^i \\
\dot{\tilde{\varrho}}_2^i = \dot{\varrho}_2^i + \dot{\hat{\bar{\delta}}}_1^i \\
\dot{\tilde{\varrho}}_3^i = \dot{\varrho}_3^i + \ddot{\hat{\bar{\delta}}}_1^i + \dot{\hat{\bar{\delta}}}_2^i \\
\dot{\tilde{\varrho}}_4^i = \dot{\varrho}_4^i + \dddot{\hat{\bar{\delta}}}_1^i + \ddot{\hat{\bar{\delta}}}_2^i + \dot{\hat{\bar{\delta}}}_3^i \\
\quad\vdots \\
\dot{\tilde{\varrho}}_n^i = \dot{\varrho}_n^i + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]}
\end{cases} \qquad \text{for } i = 1, 2, \ldots N \tag{15}$$

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Using Eq. (9), one has

$$\begin{cases} \dot{\tilde{\varrho}}_1^i = \dot{\varrho}_1^i = \varrho_2^i + \bar{\delta}_1^i + \left(\hat{\bar{\delta}}_1^i - \hat{\bar{\delta}}_1^i\right) = \tilde{\varrho}_2^i + \bar{\delta}_1^i - \hat{\bar{\delta}}_1^i \\ \dot{\tilde{\varrho}}_2^i = \dot{\varrho}_2^i + \dot{\hat{\bar{\delta}}}_1^i = \varrho_3^i + \bar{\delta}_2^i + \left(\hat{\bar{\delta}}_2^i - \hat{\bar{\delta}}_2^i\right) + \dot{\hat{\bar{\delta}}}_1^i = \tilde{\varrho}_3^i + \bar{\delta}_2^i - \hat{\bar{\delta}}_2^i \\ \quad \vdots \\ \dot{\tilde{\varrho}}_n^i = \left(b^i + \sum_{j=1}^N a_{ij}\right)\left(f^i + \delta_n^i + h^i u^i\right) - \sum_{j=1}^N a_{ij}\left(f^j + \delta_n^j + h^j u^j\right) - b^i\, u^0 + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]} \end{cases} \tag{16}$$

Moreover, using Eq. (13), the above equations can be rewritten as the following structure:

$$\begin{cases} \dot{\tilde{\varrho}}_m^i = \tilde{\varrho}_{m+1}^i - \left(b^i + \sum_{j=1}^N a_{ij}\right)\psi_{m1}^i + \sum_{j=1}^N a_{ij}\psi_{m1}^j \qquad m = 1, 2, \ldots, n-1 \\ \dot{\tilde{\varrho}}_n^i = \left(b^i + \sum_{j=1}^N a_{ij}\right)\left(f^i + \delta_n^i + h^i u^i\right) - \sum_{j=1}^N a_{ij}\left(f^j + \delta_n^j + h^j u^j\right) - b^i\, u^0 + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]} \end{cases} \tag{17}$$

where $\psi_{m1}^i = \hat{\delta}_m^i - \delta_m^i$, which is defined in Eq. (12).

In what follows, novel distributed sliding manifolds are constructed based on the new consensus errors. Then, a theorem is given and the robust fixed-time distributed controllers are designed. Consider the following distributed sliding manifolds (for $i = 1, \ldots, N$):

$$\sigma^i = \tilde{\varrho}_n^i + \int_0^t \sum_{m=1}^n \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right)dt \tag{18}$$

where λ_m^i, μ_m^i, and $\acute{\mu}_m^i$ are chosen based on the introduced Lemma 3. Using Eq. (17), the time derivative of the ith sliding manifold is governed as below:

$$\begin{aligned} \dot{\sigma}^i &= \dot{\tilde{\varrho}}_n^i + \sum_{m=1}^n \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right) \\ &= \dot{\varrho}_n^i + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]} + \sum_{m=1}^n \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right) \\ &= \left(b^i + \sum_{j=1}^N a_{ij}\right)\left(f^i + \delta_n^i + h^i u^i\right) - \sum_{j=1}^N a_{ij}\left(f^j + \delta_n^j + h^j u^j\right) - b^i\, u^0 + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]} \\ &\quad + \sum_{m=1}^n \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right) \end{aligned} \tag{19}$$

Consequently, Eq. (17) can be rewritten as in the following format:

$$\begin{cases} \dot{\tilde{\varrho}}_m^i = \tilde{\varrho}_{m+1}^i - \left(b^i + \sum_{j=1}^N a_{ij}\right)\psi_{m1}^i + \sum_{j=1}^N a_{ij}\psi_{m1}^j \qquad m = 1, 2, \ldots, n-1 \\ \dot{\tilde{\varrho}}_n^i = -\sum_{m=1}^n \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right) + \dot{\sigma}^i \end{cases} \tag{20}$$

Theorem 1. *Consider the nonlinear heterogeneous multiagent system with Eqs. (4) and (5). The robust distributed controllers (21) guarantee the fixed-time consensus between the followers and the leader in the presence of unmatched and matched uncertainties.*

$$\begin{aligned} u^i(t) = \frac{-1}{h^i\left(b^i + \sum_{j=1}^N a_{ij}\right)} &\Bigg(\left(b^i + \sum_{j=1}^N a_{ij}\right)\left(f^i\right) - \sum_{j=1}^N a_{ij}\left(f^j + h^j u^j\right) - b^i\, u^0 \\ &+ \left(b^i + \sum_{j=1}^N a_{ij}\right)\hat{\delta}_n^i - \sum_{j=1}^N a_{ij}\hat{\delta}_n^j + \sum_{k=1}^{n-1} \hat{\bar{\delta}}_k^{i[n-k]} + \sum_{m=1}^n \lambda_m^i \Big(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) \\ &+ sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\Big) + sgn^c\left(\sigma^i\right) + sgn^1\left(\sigma^i\right) + sgn^d\left(\sigma^i\right)\Bigg), \qquad i = 1, \ldots, N \end{aligned} \tag{21}$$

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where $\hat{\hat{\delta}}_n^i = \hat{\delta}_n^i \left(b^i + \sum_{j=1}^{N} a_{ij} \right) - \sum_{j=1}^{N} a_{ij} \hat{\delta}_n^j$*. The parameters* c *and* d *are chosen to satisfy the following relations:*

$$0 < c < 1 \quad \text{and} \quad d > 1 \tag{22}$$

Proof. Substituting the controller (21) into the time derivative of the ith sliding manifold, one has

$$\dot{\sigma}^i = -sgn^c\left(\sigma^i\right) - sgn^1\left(\sigma^i\right) - sgn^d\left(\sigma^i\right) - \left(b^i + \sum\nolimits_{j=1}^{N} a_{ij}\right) \psi_{n1}^i + \sum\nolimits_{j=1}^{N} a_{ij} \psi_{n1}^j \tag{23}$$

where

$$\psi_{n1}^i = \xi_{n1}^i - \delta_n^i = \hat{\delta}_n^i - \delta_n^i$$

As seen, observer errors are entered in (23). Before preceding the proof, it should be guaranteed that the observer errors will not drive the sliding manifold dynamics (23) to infinity in finite time. For this purpose, the following Lyapunov function $V^i(\sigma^i, \tilde{\boldsymbol{\varrho}}^i)$ is defined:

$$V^i\left(\sigma^i, \tilde{\boldsymbol{\varrho}}^i\right) = \frac{1}{2}(\sigma^i)^2 + \sum\nolimits_{m=1}^{n} \frac{1}{2}(\tilde{\varrho}_m^i)^2 \quad i = 1, \ldots, N \tag{24}$$

Taking time derivation of $V^i(\sigma^i, \tilde{\boldsymbol{\varrho}}^i)$ yields to:

$$\begin{aligned}
\dot{V}^i &= \sigma^i \dot{\sigma}^i + \sum_{m=1}^{n-1} \tilde{\varrho}_m^i \left(\varrho_{m+1}^i - \left(b^i + \sum_{j=1}^{N} a_{ij} \right) \psi_{m1}^i + \sum_{j=1}^{N} a_{ij} \psi_{m1}^j \right) \\
&\quad - \tilde{\varrho}_n^i \sum_{m=1}^{n} \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right) \right) + \tilde{\varrho}_n^i \dot{\sigma}^i \\
&\leq \frac{(\sigma^i)^2 + \left(b^i + \sum_{j=1}^{N} a_{ij}\right)^2 (\psi_{n1}^i)^2}{2} + \frac{(\sigma^i)^2 + \left(\sum_{j=1}^{N} a_{ij} \psi_{n1}^j\right)^2}{2} + \sum_{m=1}^{n-1} \frac{(\tilde{\varrho}_m^i)^2 + (\tilde{\varrho}_{m+1}^i)^2}{2} \\
&\quad + \sum_{m=1}^{n-1} \frac{(\tilde{\varrho}_m^i)^2 + \left(b^i + \sum_{j=1}^{N} a_{ij}\right)^2 (\psi_{m1}^i)^2}{2} + \sum_{m=1}^{n-1} \frac{(\tilde{\varrho}_m^i)^2 + \left(\sum_{j=1}^{N} a_{ij} \psi_{m1}^j\right)^2}{2} \\
&\quad + \frac{(\tilde{\varrho}_n^i)^2 + \left(\sum_{m=1}^{n} \lambda_m^i \left((\tilde{\varrho}_m^i)^{\mu_m^i}\right)\right)^2}{2} + \frac{(\tilde{\varrho}_n^i)^2 + \left(\sum_{m=1}^{n} \lambda_m^i \left(\tilde{\varrho}_m^i\right)\right)^2}{2} \\
&\quad + \frac{(\tilde{\varrho}_n^i)^2 + \left(\sum_{m=1}^{n} \lambda_m^i \left((\tilde{\varrho}_m^i)^{\acute{\mu}_m^i}\right)\right)^2}{2} + \frac{(\tilde{\varrho}_n^i)^2 + \left((\sigma^i)^c\right)^2}{2} + \frac{(\tilde{\varrho}_n^i)^2 + (\sigma^i)^2}{2} \\
&\quad + \frac{(\tilde{\varrho}_n^i)^2 + \left((\sigma^i)^d\right)^2}{2} + \frac{(\tilde{\varrho}_n^i)^2 + \left(b^i + \sum_{j=1}^{N} a_{ij}\right)^2 (\psi_{n1}^i)^2}{2} + \frac{(\tilde{\varrho}_n^i)^2 + \left(\sum_{j=1}^{N} a_{ij} \psi_{n1}^j\right)^2}{2} \\
&\leq \Theta_v^i V^i + \Gamma_v^i
\end{aligned} \tag{25}$$

where

$$\Theta_v^i = 3 + \sum_{m=1}^{n} \lambda_m^i$$

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and

$$\Gamma_v^i = max\left[\left(b^i+\sum_{j=1}^{N}a_{ij}\right)^2(\psi_{n1}^i)^2+\left(\sum_{j=1}^{N}a_{ij}\psi_{n1}^j\right)^2\right.$$
$$+\sum_{m=1}^{n-1}\frac{\left(b^i+\sum_{j=1}^{N}a_{ij}\right)^2(\psi_{m1}^i)^2}{2}+\sum_{m=1}^{n-1}\frac{\left(\sum_{j=1}^{N}a_{ij}\psi_{m1}^j\right)^2}{2}+\frac{\left(\sum_{m=1}^{n}\lambda_m^i\left((\tilde{\varrho}_m^i)^{\mu_m^i}\right)\right)^2}{2}$$
$$\left.+\frac{\left(\sum_{m=1}^{n}\lambda_m^i\left((\tilde{\varrho}_m^i)^{\acute{\mu}_m^i}\right)\right)^2}{2}+\frac{((\sigma^i)^c)^2}{2}+\frac{\left((\sigma^i)^d\right)^2}{2}+5\frac{(\tilde{\varrho}_n^i)^2}{2}+\frac{\left(\sum_{m=1}^{n-1}(\tilde{\varrho}_{m+1}^i)\right)^2}{2}\right]$$

The condition ($\dot{V}^i \leq \Theta_v^i V^i + \Gamma_v^i$) implies that the sliding manifolds and the state of new error dynamics will not escape to infinity in finite time.[39] Since the observer errors converge to zero in finite–time t_f^i and after that the dynamical equation of the sliding manifolds (23) will be reduced to (26)

$$\dot{\sigma}^i = -sgn^c\left(\sigma^i\right) - sgn^1\left(\sigma^i\right) - sgn^d\left(\sigma^i\right) \qquad \forall\, t \geq t_f^i \tag{26}$$

Now consider the following function as a Lyapunov function candidate for stability analysis of (26),

$$\tilde{V}^i = \frac{1}{2}\left(\sigma^i\right)^2 \tag{27}$$

Taking time derivation of $\tilde{V}^i$, one has

$$\dot{\tilde{V}}^i = \sigma^i\dot{\sigma}^i \tag{28}$$

Replacing Eq. (26) into (28):

$$\begin{aligned}\dot{\tilde{V}}^i &= \sigma^i\left(-sgn^c\left(\sigma^i\right) - sgn^1\left(\sigma^i\right) - sgn^d\left(\sigma^i\right)\right)\\ &= -\left|\sigma^i\right|^{c+1} - \left|\sigma^i\right|^2 - \left|\sigma^i\right|^{d+1}\end{aligned} \tag{29}$$

It yields

$$\begin{aligned}\dot{\tilde{V}}^i &\leq \left(\left|\sigma^i\right|^2\right)^{\frac{c+1}{2}} - \left|\sigma^i\right|^2 - \left(\left|\sigma^i\right|^2\right)^{\frac{d+1}{2}}\\ &\leq -\left(\tilde{V}^i\right)^{\frac{c+1}{2}} - \left(\tilde{V}^i\right)^{\frac{d+1}{2}}\end{aligned} \tag{30}$$

According to relation (21), one has

$$\frac{1}{2}<\frac{c+1}{2}<1 \quad and \quad \frac{d+1}{2}>1 \tag{31}$$

Comparing (30) with Lemma 2 results in the satisfaction of the relation $\dot{\tilde{V}}^i \leq -\left[\varsigma\tilde{V}^{i^{\chi}}+\phi\tilde{V}^{i^{\kappa}}\right]^{\iota}$ with

$$\chi = \frac{c+1}{2},\, \kappa = \frac{d+1}{2},\, \varsigma = \phi = \iota = 1 \tag{32}$$

Therefore, the states reach the sliding manifolds in fixed time T_s^i. Moreover, Eq. (20) suffers from disturbance error dynamic (13) and sliding manifold dynamic. Since according to the results of Eq. (25),

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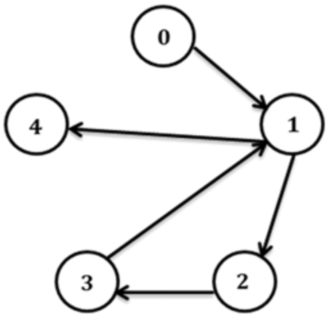

Fig. 1. Communication graph G_1.

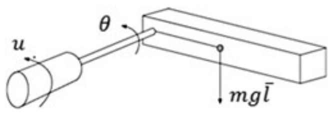

Fig. 2. Single-link robot arm.

and the convergence of the sliding manifolds to zero in fixed time T_s^i, Eq. (20) is reduced to the following equations for $T^i \geq T_s^i$:

$$\begin{cases} \dot{\tilde{\varrho}}_m^i = \tilde{\varrho}_{m+1}^i & m = 1, 2, \ldots, n-1 \\ \dot{\tilde{\varrho}}_n^i = -\sum_{m=1}^{n} \lambda_m^i \left(sgn^{\mu_m^i}\left(\tilde{\varrho}_m^i\right) + sgn^1\left(\tilde{\varrho}_m^i\right) + sgn^{\acute{\mu}_m^i}\left(\tilde{\varrho}_m^i\right)\right) & \forall\, T^i \geq T_s^i \end{cases} \tag{33}$$

According to Lemma 3, the above equation is fixed-time stable. In other words, the consensus error vectors $\tilde{\boldsymbol{\varrho}}^i = [\tilde{\varrho}_1^i, \ldots, \tilde{\varrho}_n^i]^T$ for $(i = 1, \ldots, N)$ converge to zero in fixed time. Since $\tilde{\varrho}_1^i = \varrho_1^i$, it implies that the fixed-time consensus is achieved and the outputs of the followers track the output of the leader in the fixed time in spite of the matched and unmatched uncertainties. This completes the proof.

5. Computer Simulations

In this section, two practical examples and a numerical example are provided to show the effective performance of the designed distributed controllers.

The second-order Lagrangian dynamics describe a wide variety of industrial systems, including robot manipulators, vehicle motion systems, autonomous ground vehicles, and industrial flow. In practical examples, we consider a group of single-link robots and robot manipulators as followers, and the objective is to track a prescribed motion trajectory by followers.

Example 1. Consider a multiagent system with a leader and four followers which are interconnected under the graph G_1 that is shown in Fig. 1.

The followers are single-link robot arms, and each consists of a rigid link coupled through a gear train to a dc motor, as shown in Fig. 2.

The dynamics of the robot arms is described by the following Lagrangian dynamics:[40]

$$\ddot{\theta} = -\frac{Mg\bar{l}}{J}\sin(\theta) - \frac{\overline{B}}{J}\dot{\theta} + \frac{1}{J}u(t) \tag{34}$$

where θ is the angular position of the single-link robot arm, M is the mass of payload, g is the gravity acceleration, $\bar{l}$ is the length of the arm, J is the inertia moment, $\overline{B}$ is the coefficient of viscous friction, and $u(t)$ is the control input. Consider $\theta^i = x_1^i$ and $\dot{\theta}^i = x_2^i$, therefore the dynamical equations of followers are obtained as below:

$$\begin{cases} \dot{x}_1^i = x_2^i + \delta_1^i(\boldsymbol{x}^i, t) \\ \dot{x}_2^i = f^i(\boldsymbol{x}^i, t) + \delta_2^i(\boldsymbol{x}^i, t) + h^i u^i(t) & i = 1, \ldots, 4 \\ y^i = x_1^i \end{cases} \tag{35}$$

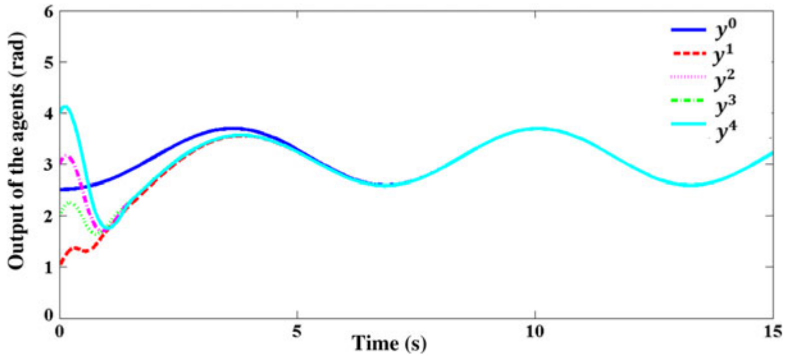

Fig. 3. Time responses of the output variables of the leader (y^0) and followers (y^i for $i = 1, \dots, 4$) in Example 1.

where $\boldsymbol{x}^i = [x_1^i, x_2^i]^T$ is the state vector, $y^i \in R$ is the output of the ith follower, $f^i\left(\boldsymbol{x}^i, t\right) = -\frac{M^i g \bar{l}^i}{J^i}\sin\left(x_1^i\right) - \frac{\bar{B}^i}{J^i}x_2^i$ is the known nonlinear function of the ith follower and $h^i = \frac{1}{J^i}$. Moreover, $\boldsymbol{\delta}^i(\boldsymbol{x}^i, t) = [\delta_1^i, \delta_2^i]^T$ represents the unmatched and matched uncertainties.

The dynamical equations of the leader are assumed as:

$$\begin{cases} \dot{x}_1^0 = x_2^0 \\ \dot{x}_2^0 = u^0 \\ y^0 = x_1^0 \end{cases} \tag{36}$$

where $\boldsymbol{x}^0 = [x_1^0, x_2^0]^T$ is the state vector, and $y^0 \in R$ is the output of the leader. Moreover, $u^0 \in R$ is the known control input of the leader and is defined as $u^0 = \frac{\sin(x_1^0)}{1+e^{-t}}$, which results in the desirable sinusoidal behaviors in the steady-state response of the leader's output (i.e., y^0 in (36)).

The goal is the robust fixed-time consensus between the outputs of the followers (y^i for $i = 1, \dots, 4$) and the output of leader (y^0) in the presence of the matched and unmatched uncertainties in the dynamical equations of the followers.

For the simulation, the parameters of the systems are selected as:

$$J^1 = 6.9667,\ J^2 = 7.7,\ J^3 = 8.46,\ J^4 = 10.2$$

$$\bar{B}^1 = \bar{B}^2 = \bar{B}^3 = \bar{B}^4 = 30.5,\ g = 9.8$$

$$\bar{l}^1 = 0.8, \bar{l}^2 = 1,\ \bar{l}^3 = 1.2,\ \bar{l}^4 = 1.5$$

$$M^1 = 1,\ M^2 = 1.7,\ M^3 = 2,\ M^4 = 2.3$$

The initial conditions are also considered as:

$$\boldsymbol{x}^0 = \left[\frac{pi}{1.25}, 0\right]^T, \boldsymbol{x}^1 = [1, 0]^T, \boldsymbol{x}^2 = [2, 0]^T, \boldsymbol{x}^3 = [3, 0]^T, \boldsymbol{x}^4 = [4, 0]^T.$$

The uncertainties for each follower are supposed as:

$$\delta_1^i(\boldsymbol{x}^i, t) = 1 + \sin\left(i\frac{t}{9} + i\frac{\pi}{4}\right),\ \delta_2^i(\boldsymbol{x}^i, t) = 0.5\cos\left(x_1^i\right)x_2^i + \sin(0.1t) \text{ for } i = 1, \dots, 4.$$

Furthermore, the disturbance observer parameters are chosen as:

$$\tau_{m0}^i = 2,\ \tau_{m1}^i = 1.5,\ \tau_{m2}^i = 0.01,\ \varpi_m^i = 10 \text{ for } m = 0, 1, 2.$$

and the controller parameters are selected as below:

$$\lambda_1^i = 2,\ \lambda_2^i = 3,\ \mu_1^i = \frac{0.7}{1.3},\ \mu_2^i = 0.7,\ \acute{\mu}_1^i = \frac{1.3}{0.7},\ \acute{\mu}_2^i = 1.3,\ c = 0.7,\ d = 1.5$$

Figures 3–7 display the results of simulations. Figure 3 illustrates the time responses of the controlled outputs of each agent. It is clearly shown that the outputs of the followers track the output of the leader

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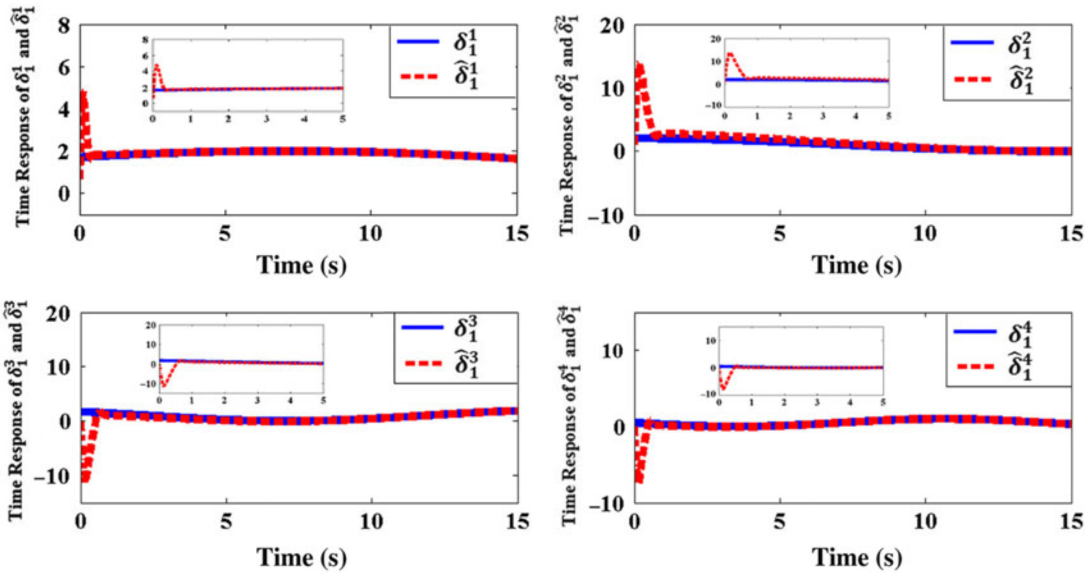

Fig. 4. Time responses of the uncertain terms $\delta_1^i(\boldsymbol{x}^i, t)$ and their estimations $\hat{\delta}_1^i(\boldsymbol{x}^i, t)$ for $i = 1, \ldots, 4$ in Example 1.

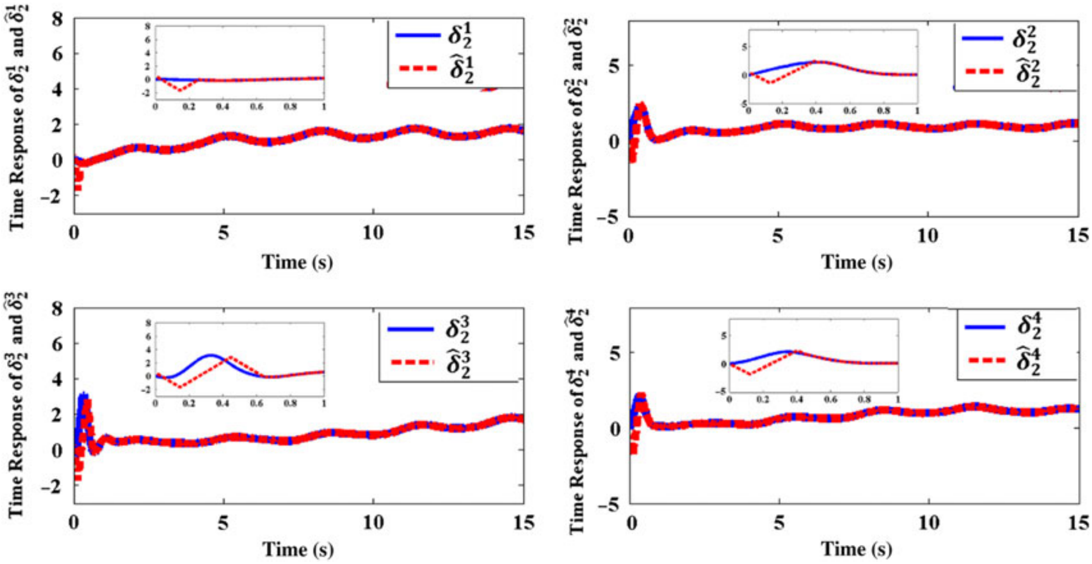

Fig. 5. Time responses of the uncertain terms $\delta_2^i(\boldsymbol{x}^i, t)$ and their estimations $\hat{\delta}_2^i(\boldsymbol{x}^i, t)$ for $i = 1, \ldots, 4$ in Example 1.

and consensus is achieved in fixed time. The time responses of uncertainties and their estimations are shown in Figs. 4 and 5. As seen, the performance of the disturbance observer is satisfactory and the estimated values of uncertainties converge to their real values in a short time. Figure 6 shows the time responses of the control inputs of each follower. As seen, the control signals are chattering free. The time responses of the sliding manifolds for each follower are also given in Fig. 7.

In the following, to illustrate the superiority of the proposed method in this paper over other techniques, a comparison is made between this paper and the recent consensus protocol presented in ref. [34]. As mentioned before, the settling time, in fixed-time consensus, is independent of the initial values of the state vector. In order to show this issue, Example 1 is simulated for two different scenarios of initial value conditions of the followers, and in each scenario, the proposed method is compared with the given method in ref. [34]. In both scenarios, the initial value of the leader is $\boldsymbol{x}^0 = \left[\frac{\pi}{1.25}, 0\right]^T$.

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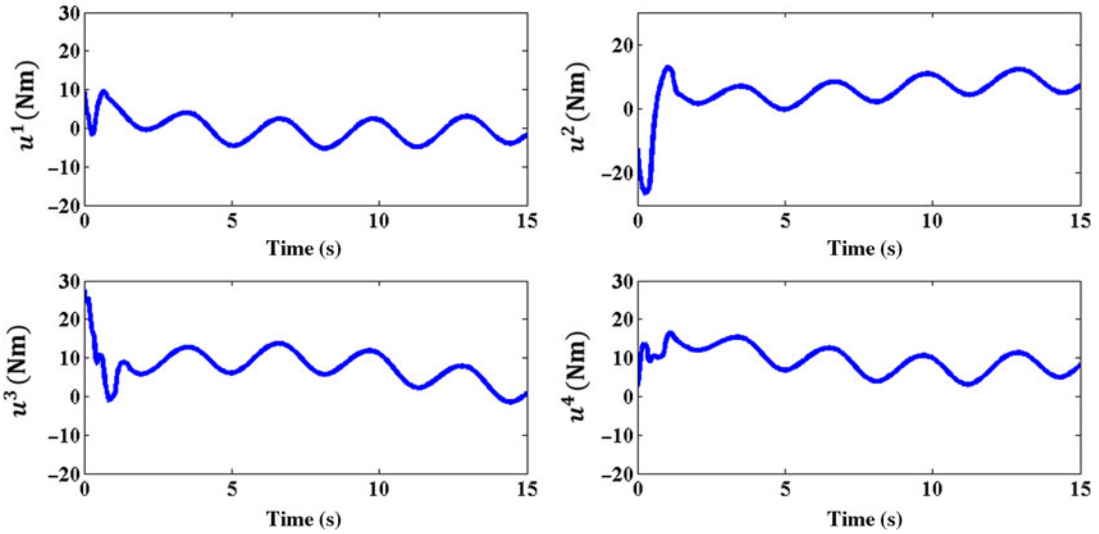

Fig. 6. Time responses of the control inputs $u^i(t)$ for $i = 1, \ldots, 4$ in Example 1.

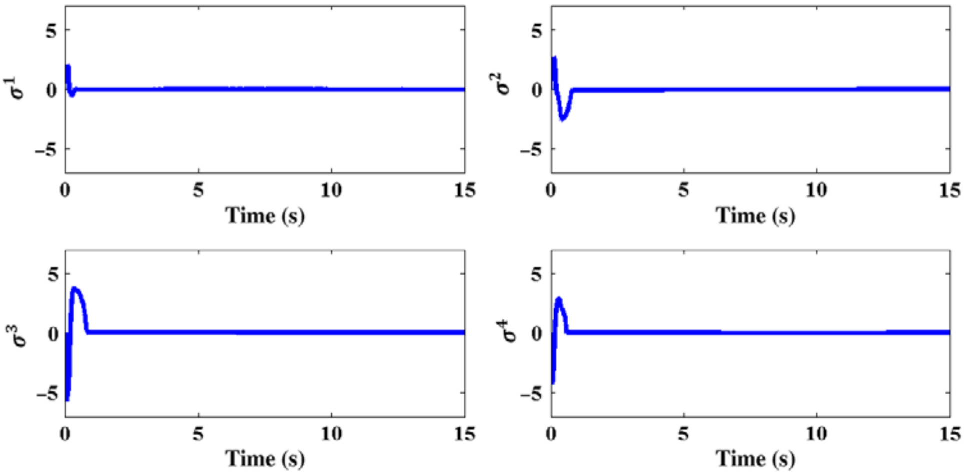

Fig. 7. Time responses of sliding manifolds σ^i for $i = 1, \ldots, 4$ in Example 1.

First scenario: $\boldsymbol{x}^1 = [1, 0]^T, \boldsymbol{x}^2 = [2, 0]^T,\ \boldsymbol{x}^3 = [3, 0]^T, \boldsymbol{x}^4 = [4, 0]^T$.

Second scenario: $\boldsymbol{x}^1 = [10, 0]^T, \boldsymbol{x}^2 = [20, 0]^T,\ \boldsymbol{x}^3 = [30, 0]^T, \boldsymbol{x}^4 = [40, 0]^T$.

The time responses of the output variables based on ref. [34] are shown in Fig. 8. Besides, Fig. 9 shows the time responses of the output variables via the proposed method. As shown in Fig. 8, when the initial conditions change, the convergence rate varies. In other words, convergence rate depends on the initial conditions of the followers. While using the given protocol presented in this paper, the convergence time is independent of the initial conditions of the followers and has similar values for both scenarios. This can be seen in Fig. 9.

Another feature of the fixed-time consensus is reaching consensus at a specific time. In other words, in the asymptotic consensus, as time goes to infinity, the tracking error converges to zero; however, in fixed-time consensus, the tracking error converges to zero at a given time. To illustrate this, we simulated Example 1 with both controllers and compared the result. Figure 10 shows the time responses of output tracking errors (i.e., $(y^i - y^0)$ for $i = 1, \ldots, 4$) via the method in ref. [34]. Moreover, the time responses of the tracking errors using the proposed consensus protocol in this paper are given in Fig. 11. As seen, the proposed method results in the convergence of output tracking errors to zero in fixed time.

Example 2. In this example, a 3-degree-of-freedom manipulator is considered. Each joint is attached to an encoder for joint position measurement. The manipulator is shown in Fig. 12.

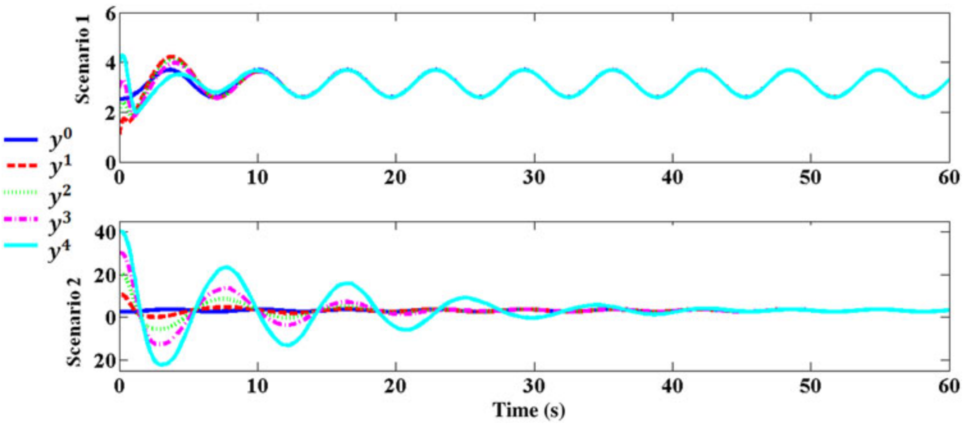

Fig. 8. Time responses of the output variables for the first and second scenarios via the method given in ref. [34].

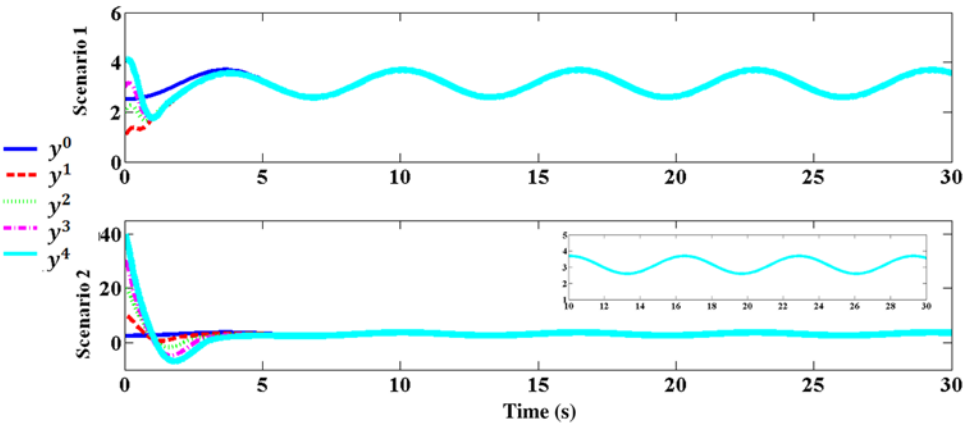

Fig. 9. Time responses of the output variables for the first and second scenarios by the proposed method in this paper.

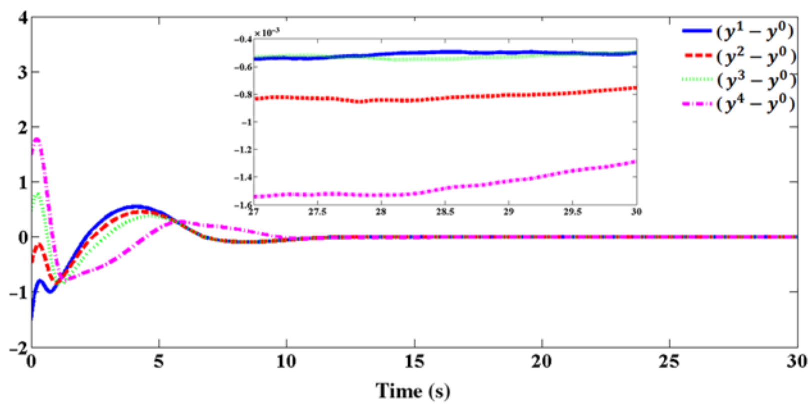

Fig. 10. Time responses of the output tracking errors $(y^i - y^0)$ for $i = 1, \ldots, 4$ via the given method in ref. [34].

The dynamical equation of motion of this manipulator is described by:[41]

$$M(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + G(\boldsymbol{q}) + \overline{D}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t, w) = U(t) \tag{37}$$

where $\boldsymbol{q} \in R^3$, $\dot{\boldsymbol{q}} \in R^3$, *and* $\ddot{\boldsymbol{q}} \in R^3$ are the angles, angular velocities, and angular accelerations of the joints, respectively. $M(\boldsymbol{q}) \in R^{3\times3}$ is the inertia matrix, $C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in R^{3\times3}$ is the Coriolis and centripetal

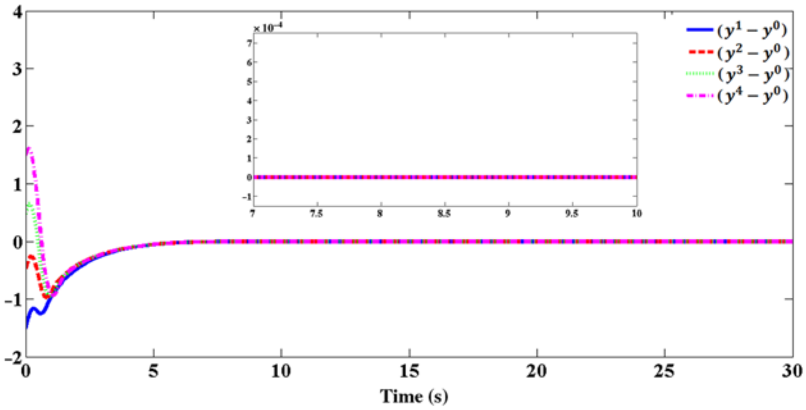

Fig. 11. Time responses of the output tracking errors $(y^i - y^0)$ for $i = 1, \ldots, 4$ via the proposed method in this paper.

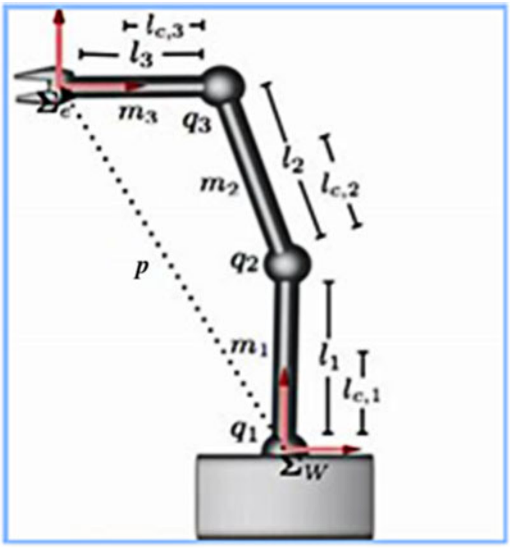

Fig. 12. 3-degree-of-freedom manipulator.

forces matrix, $G(\boldsymbol{q}) \in R^3$ represents the potential forces vector, and $\overline{D}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t, w) \in R^3$ is a vector of uncertainties and external disturbances. Moreover, $U \in R^3$ consists of the control input torques.

$$M(\boldsymbol{q}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \quad C(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$G(\boldsymbol{q}) = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}, U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \boldsymbol{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{38}$$

$$\bar{D}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t, w) = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{bmatrix}$$

where

$$m_{11} = \varphi_1 + \varphi_2 \cos^2(q_2) + (\varphi_3 + \varphi_5)\sin^2(q_3) + 2\varphi_6 \cos(q_2)\sin(q_3),\ m_{12} = 0,$$
$$m_{13} = 0,\ m_{21} = 0$$
$$m_{22} = \varphi_4 + \varphi_5 - 2\sin(q_{2-}q_3),\ m_{31} = 0,\ \ m_{23} = m_{32} = \varphi_5 + \varphi_6 - 2\sin(q_{2-}q_3),\ \ m_{33} = \theta_5$$

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$$c_{11}=-\left(\varphi_2\sin\left(q_2\right)\cos\left(q_2\right)\ +\varphi_6\sin\left(q_2\right)\sin\left(q_3\right)\right)\dot{q}_2+\left(\left(\varphi_3+\varphi_5\right)\sin\left(q_3\right)\cos\left(q_3\right)\right.$$
$$\left.+\varphi_6\cos\left(q_2\right)\cos(q_3)\dot{q}_3\right)$$
$$c_{12}=-\left(\varphi_2\sin\left(q_2\right)\cos\left(q_2\right)\ +\varphi_6\sin\left(q_2\right)\sin\left(q_3\right)\right)\dot{q}_1$$
$$c_{13}=\left(\left(\varphi_3+\varphi_5\right)\sin\left(q_3\right)\cos\left(q_3\right)\ +\varphi_6\cos\left(q_2\right)\cos(q_3)\dot{q}_3\right)$$
$$c_{21}=\left(\varphi_2\sin\left(q_2\right)\cos\left(q_2\right)\ +\varphi_6\sin\left(q_2\right)\sin\left(q_3\right)\right)\dot{q}_1$$
$$c_{22}=\varphi_6\cos\left(q_2-q_3\right),\qquad c_{23}=\varphi\cos\left(q_2-q_3\right)\left(\dot{q}_2-\dot{q}_3\right)$$
$$c_{31}=-\left(\left(\varphi_3+\varphi_5\right)\sin\left(q_3\right)\cos\left(q_3\right)\ -\varphi_6\cos\left(q_2\right)\cos\left(q_3\right)\right)\dot{q}_1,\ c_{32}=0,\ c_{33}=0$$
$$g_1=0,\ \ g_2=\varphi_7\cos\left(q_2\right),\ \ g_3=\varphi_8\sin\left(q_3\right)$$
$$\varphi_1=32,\ \varphi_2=34,\ \varphi_3=20,\varphi_4=74,\varphi_5=1,\varphi_6=2,\ \varphi_7=-926,\ \varphi_8=-685$$

Assuming that $\boldsymbol{q}=X_1$ and $\dot{\boldsymbol{q}}=X_2$, Eq. (37) can be written as the following form:

$$\begin{cases}\dot{X}_1=X_2\\ \dot{X}_2=F\left(X_1,X_2,U\right)+\mathrm{H}U(t)+Z(X_1,X_1,t,w)\\ Y=X_1\end{cases}\tag{39}$$

where $X_1=[x_1^1,x_1^2,x_1^3]^T$, $X_2=[x_2^1,x_2^2,x_2^3]^T$, and $Y=[y^1,y^2,y^3]^T$, $U=[u^1,u^2,u^3]^T$. Moreover, $Z=[\delta_2^1,\delta_2^2,\delta_2^3]^T=M^{-1}\overline{D}(X_1,X_1,t,w)$ is the vector of the unknown terms and $F=[f^1,f^2,f^3]^T=-M^{-1}C\left(X_1,X_2\right)X_2-M^{-1}G\left(X_1\right)$ and H$=[h^1,h^2,h^3]=M^{-1}$ are the known terms where

$$f^1=-\frac{g_1}{m_{11}}-\frac{c_{11}}{m_{11}}x_1^1-\frac{c_{12}}{m_{11}}x_1^2-\frac{c_{13}}{m_{11}}x_1^3$$
$$f^2=-\frac{1}{(m_{23})^2+m_{22}m_{33}}\left(-\left(c_{21}m_{33}-c_{31}m_{23}\right)x_1^1-c_{22}m_{33}x_1^2-c_{23}m_{33}x_1^3-m_{23}u^3\right)$$
$$-\frac{1}{(m_{23})^2+m_{22}m_{33}}\left(g_3m_{23}-g_2m_{33}\right)$$
$$f^3=-\frac{1}{(m_{23})^2+m_{22}m_{33}}\left(\left(c_{21}m_{23}-c_{31}m_{22}\right)x_1^1+c_{22}m_{23}x_1^2+c_{23}m_{23}x_1^3-m_{23}u^2\right)$$
$$-\frac{1}{(m_{23})^2+m_{22}m_{33}}\left(-g_3m_{22}+g_2m_{23}\right),$$
$$h^1=\frac{1}{m_{11}},\ h^2=-\frac{m_{33}}{(m_{23})^2+m_{22}m_{33}},\ h^3=-\frac{m_{22}}{(m_{23})^2+m_{22}m_{33}}.$$

As already mentioned, one of the applications of multiagent systems is to break down a complex system into several subsystems. Thus, in this example, we split the manipulator into three subsystems and each subsystem is considered as a follower.

Therefore, the dynamical equation of the manipulator system can be divided into the following three subsystems, and their dynamic equations can be described as below:

$$\begin{cases}\dot{x}_1^i=x_2^i\\ \dot{x}_2^i=f^i\left(x_1^i,x_2^i,u^i\right)+h^iu^i(t)+\delta_2^i(\boldsymbol{x}^i,t)\\ y^i=x_1^i\end{cases}\qquad i=1,2,3\tag{40}$$

where $\boldsymbol{x}^i=[x_1^i,x_2^i]^T$ is the state vector and $y^i\in R$ is the output of the ith follower. All the notations in the above relation are described after (39). To fit Eq. (40) to the model presented in this paper, we add the unmatched perturbation $(\delta_1^i(\boldsymbol{x}^i,t))$ to Eq. (40), therefore one has

$$\begin{cases}\dot{x}_1^i=x_2^i+\delta_1^i(\boldsymbol{x}^i,t)\\ \dot{x}_2^i=f^i\left(x_1^i,x_2^i,u^i\right)+h^iu^i(t)+\delta_2^i\left(\boldsymbol{x}^i,t\right)\\ y^i=x_1^i\end{cases}\qquad i=1,2,3\tag{41}$$

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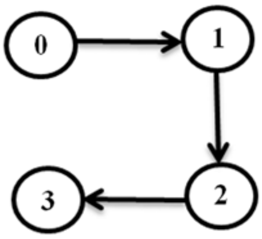

Fig. 13. Communication graph G_2.

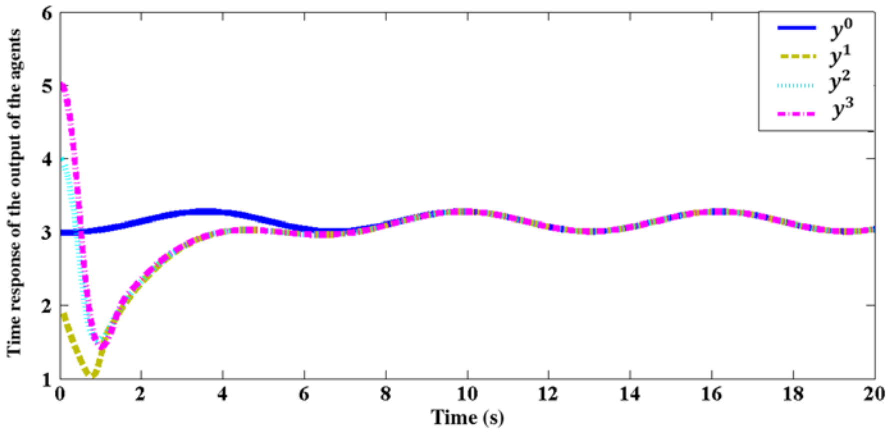

Fig. 14. Time responses of the output variables of the leader (y^0) and followers (y^i for $i = 1, 2, 3$) in Example 2.

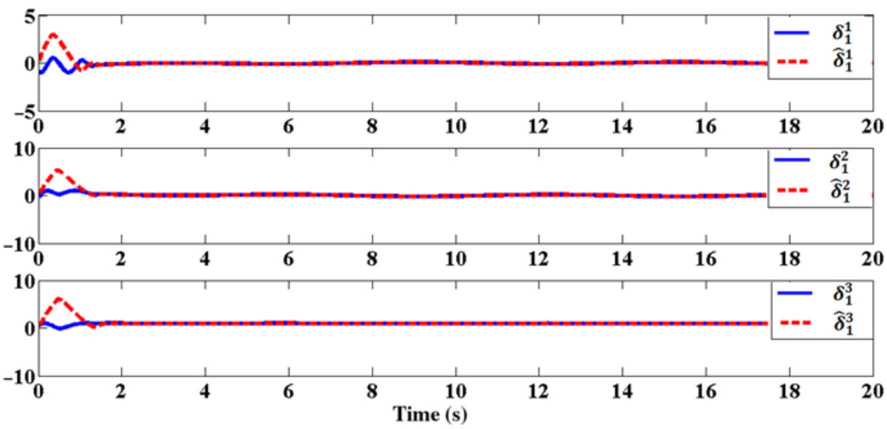

Fig. 15. Time responses of the uncertain terms $\delta_1^i(\boldsymbol{x}^i, t)$ and their estimations $\hat{\delta}_1^i(\boldsymbol{x}^i, t)$ for $i = 1, 2, 3$ in Example 2.

$\delta_1^i(\boldsymbol{x}^i, t)$ and $\delta_2^i(\boldsymbol{x}^i, t)$ represent the unmatched and matched uncertainties, respectively. The communication graph G_2 is given in Fig. 13. The unmatched and matched uncertainties are assumed as $\delta_1^i(\boldsymbol{x}^i, t) = \sin(x_1^i + x_2^i + \frac{\pi}{i})$ and $\delta_2^i(\boldsymbol{x}^i, t) = i + i\sin\left(0.5it + \frac{\pi}{i}\right) + 0.5\cos(x_1^i x_2^i)$. The leader dynamics, observer parameters, and control parameters have been considered the same as Example 1.

The simulation results are given in Figs. 14–18. The time responses of the output variables of all agents are illustrated in Fig. 14. It is obviously shown that the followers track the leader in spite of heterogeneous agents' dynamics and uncertainties, and the systems reach consensus in fixed time. Figures 15 and 16 show the time responses of uncertainties and their estimations and verify the effective performance of the disturbance observers. Figure 17 plots the time responses of the control input of each follower. The time responses of the sliding manifolds are depicted in Fig. 18. It can be seen clearly that the sliding manifolds converge to zero in a short time.

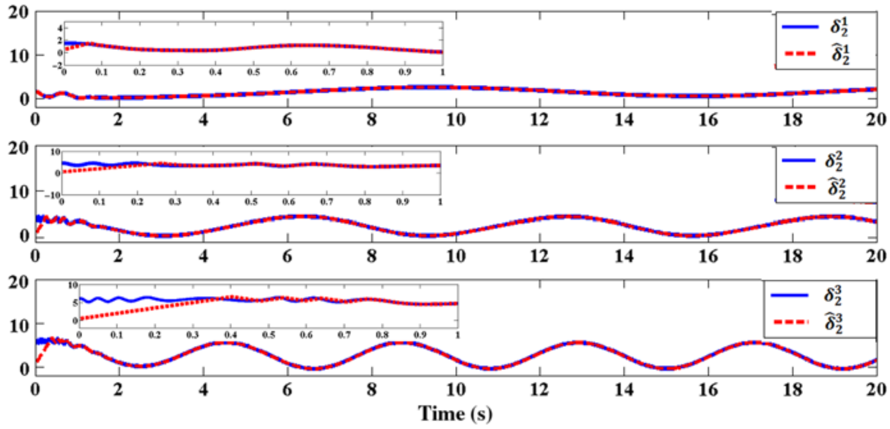

Fig. 16. Time responses of the uncertain terms $\delta_2^i(\boldsymbol{x}^i, t)$ and their estimations $\hat{\delta}_2^i(\boldsymbol{x}^i, t)$ for $i = 1, 2, 3$ in Example 2.

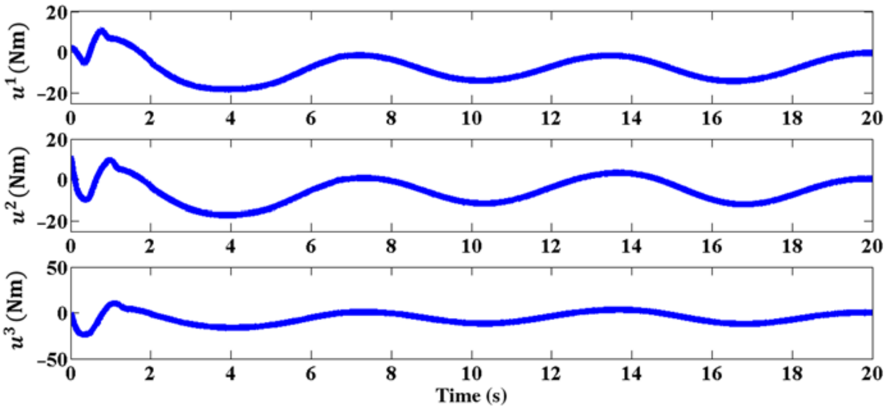

Fig. 17. Time responses of the control inputs $u^i(t)$ for $i = 1, 2, 3$ in Example 2.

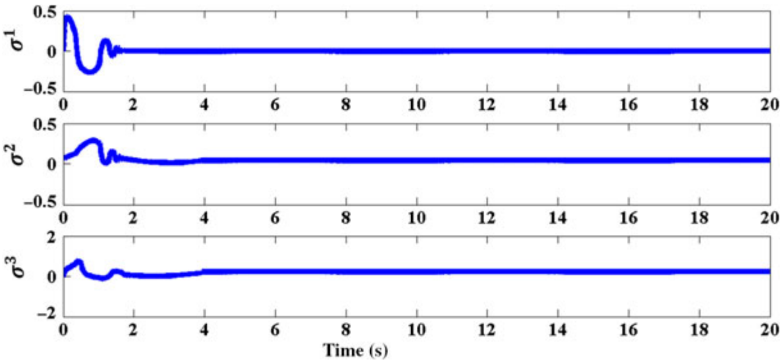

Fig. 18. Time responses of sliding manifolds σ^i for $i = 1, 2, 3$ in Example 2.

Example 3. In this section, a numerical example is provided to show the effectiveness of the designed distributed controllers. Consider a group of agents that include a leader and four followers which are interconnected under the graph G_1, which is shown in Fig. 1. The dynamical equations of the leader are as follows:

$$\begin{cases} \dot{x}_1^0 = x_2^0 \\ \dot{x}_2^0 = x_3^0 \\ \dot{x}_3^0 = u^0 \\ y^0 = x_1^0 \end{cases} \tag{42}$$

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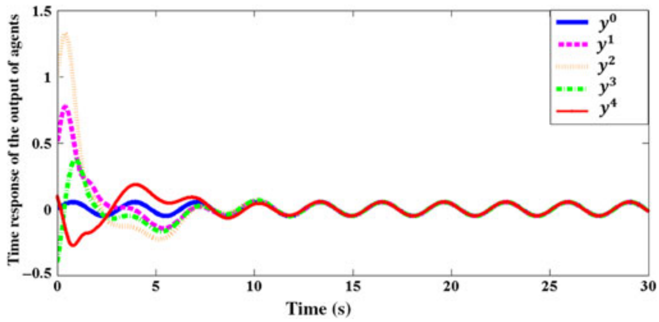

Fig. 19. Time responses of the output variables of the leader (y^0) and followers (y^i for $i = 1, 2, 3, 4$) in Example 3.

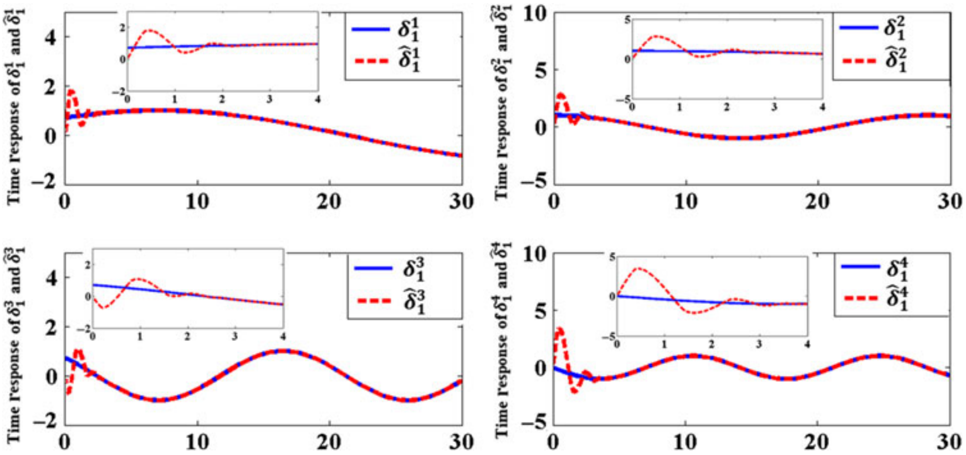

Fig. 20. Time responses of the uncertain terms δ_1^i and their estimations $\hat{\delta}_1^i$ for $i = 1, \dots, 4$ in Example 3.

where $\boldsymbol{x}^0 = [x_1^0, x_2^0, x_3^0]^T$ is the state vector, and $y^0 \in R$ is the output of the leader. Moreover, $u^0 \in R$ is the known control input of the leader and is defined as $u^0 = -80x_1^0 - 40x_2^0 - 20x_3^0$, which results in the desirable sinusoidal behaviors in the steady-state response of the output of (42).

The dynamical equations of the followers are considered as follows:

$$\begin{cases} \dot{x}_1^i = x_2^i + \delta_1^i \\ \dot{x}_2^i = x_3^i + \delta_2^i \\ \dot{x}_3^i = f^i(\boldsymbol{x}^i, t) + \delta_3^i + u^i \\ y^i = x_1^i \end{cases} \qquad i = 1, 2, 3, 4 \tag{43}$$

where $\boldsymbol{x}^i = [x_1^i, x_2^i, x_3^i]^T$ is the state vector, $\boldsymbol{\delta}^i = [\delta_1^i, \delta_2^i, \delta_3^i]^T$ includes the unmatched and matched uncertain terms, $y^i \in R$ is the output, and $f^i \in R$ is the known nonlinear function of the ith follower. Moreover, $u^i \in R$ is the control input of the ith follower.

For simulations, the uncertainties for each follower are supposed as: $\delta_1^i = \sin(\frac{i}{9}t + i\frac{\pi}{4})$, $\delta_2^i = \cos(x_1^i)$ and $\delta_3^i = x_2^i \sin(x_3^i)$, (for $i = 1, 2, 3, 4$). Moreover, the known nonlinear functions $f^i = x_1^i x_2^i x_3^i$ for $i = 1, 2$ and $f^i = x_1^i (x_2^i)^2$ for $i = 3, 4$ are considered. Also, the disturbance observer parameters are chosen as:

$$\tau_{m0}^i = 2, \tau_{m1}^i = 1.5, \tau_{m2}^i = 1.1, \tau_{m3}^i = 0.01, \varpi_m^i = 10, \text{ for } m = 0, 1, 2, 3$$

and the controller parameters are considered as follows:

$$\lambda_1^i = 2, \lambda_2^i = 3, \lambda_3^i = 5 \;\; \mu_1^i = \frac{0.7}{1.6}, \mu_2^i = \frac{0.7}{1.3},$$

$$\mu_3^i = 0.7, \acute{\mu}_1^i = \frac{1.3}{0.4}, \acute{\mu}_2^i = \frac{1.3}{0.7}, \acute{\mu}_3^i = 1.3, c = 0.7, d = 1.5$$

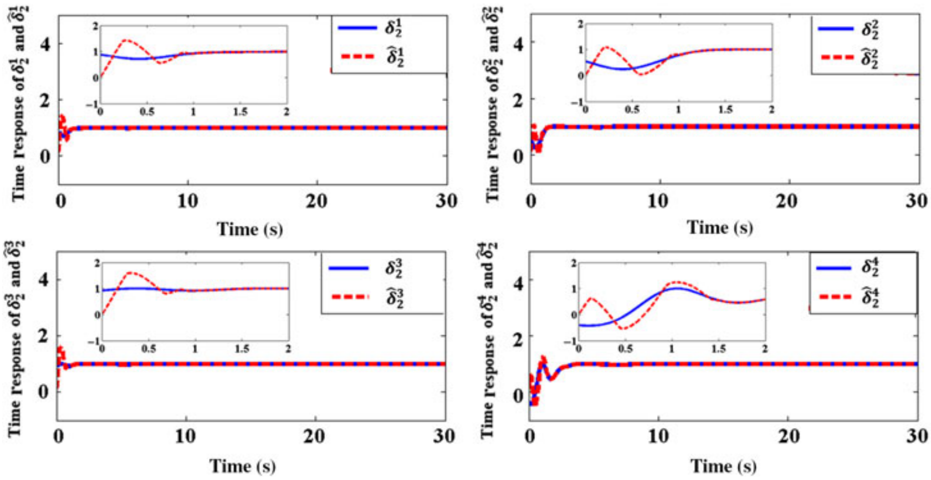

Fig. 21. Time responses of the uncertain terms δ_2^i and their estimations $\hat{\delta}_2^i$ for $i = 1, \ldots, 4$ in Example 3.

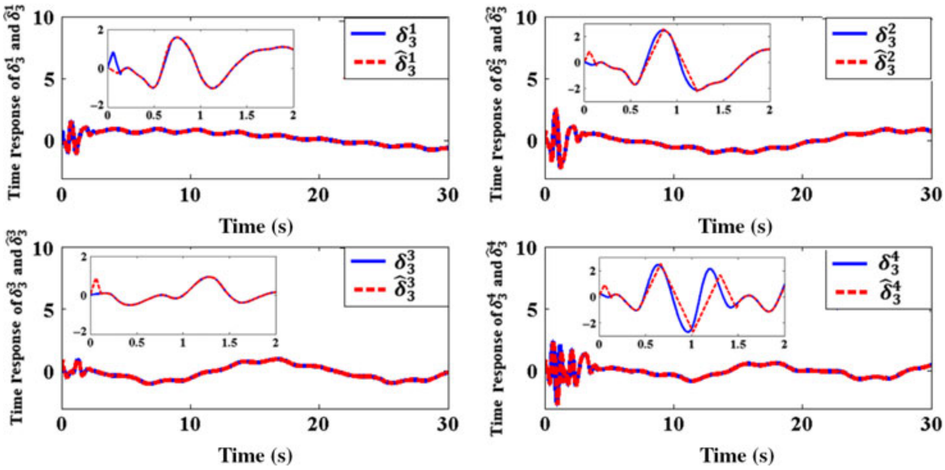

Fig. 22. Time responses of the uncertain terms δ_3^i and their estimations $\hat{\delta}_3^i$ for $i = 1, \ldots, 4$ in Example 3.

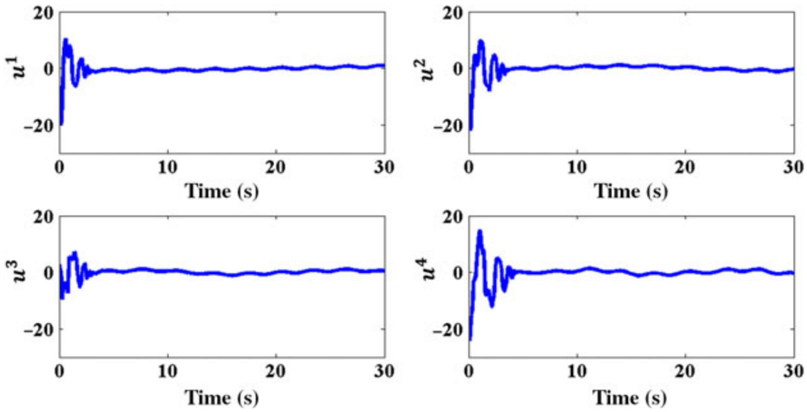

Fig. 23. Time responses of the control inputs of followers in Example 3.

Figures 19–24 display the results of simulations. Figure 19 illustrates the time responses of the output of each agent. As can be seen, the outputs of the followers track the output of leader and consensus is achieved in a fixed time. The time responses of uncertain terms and their estimations are shown in Figs. 20–22. These figures show that the performance of the disturbance observer is satisfactory and the estimated values of uncertain terms converge to their real values in a short time. Figure 23 displays

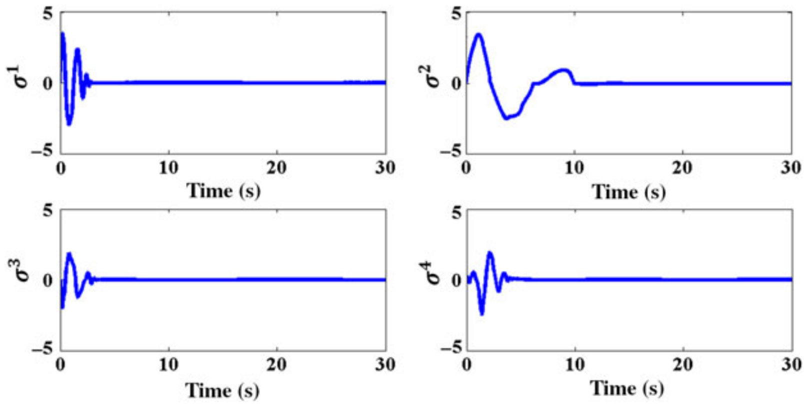

Fig. 24. Time responses of sliding manifolds in Example 3.

the time responses of the control inputs of each agent. As seen, the control signals are chattering free. The time responses of the sliding manifolds for each agent are also given in Fig. 24.

6. Conclusion

In this paper, distributed observer-based fixed-time control laws were presented to guarantee the consensus between the agents in nonlinear heterogeneous multiagent systems of higher-order with matched and unmatched uncertainties. For this purpose, fixed-time distributed sliding mode controllers were designed based on the disturbance observer to conquer both the matched and unmatched uncertainties. In this regard, a theorem has been presented and proved which guarantees that the consensus errors converge to zero in a short time and the follower tracks the leader in a robust manner. Finally, the simulation results show the effectiveness of the proposed controller for two practical examples and a numerical example.

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