

Emergence and Quantum Mechanics*

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In a recent article Humphreys has developed an intriguing proposal for making sense of emergence. The crucial notion for this purpose is what he calls “fusion” and his paradigm for it is quantum nonseparability. In what follows, we will develop this position in more detail, and then discuss its ramifications and limitations. Its ramifications are quite radical; its limitations are substantial. An alternative approach to emergence that involves quantum physics is then proposed.

1. Introduction. In developing his notion of emergence, Humphreys introduces an assumption concerning the existence of distinct ontological levels (Humphreys 1997). He regards this as a simplifying assumption that will likely have to be jettisoned ultimately. Nevertheless, it plays a crucial role in his analysis, and will be used in what follows. The level-assumption, denoted “L” following Humphreys, is this:

(L) There is a hierarchy of levels of properties $L_0, L_1, \dots, L_n, \dots$ of which at least one distinct level is associated with the subject matter of each special science, and L_j cannot be reduced to L_i for any $i < j$.

A property P^i is said to be an “i-level property” if i is the lowest level at which instances of the property occur. A set of properties $\{P^i_1, \dots, P^i_m, \dots\}$ is associated with each level i , where P^i_m denotes the m -th property at i -level. The same goes for the entity x^i and the set of entities $\{x^i_1, \dots, x^i_m, \dots\}$. In general, i -level entities may have j -level properties for $i \neq j$.

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In characterizing the property-fusion operation, Humphreys introduces the cumbersome notation $P_m^i(x_r^i)(t)$, which denotes an instantiation of property P_m^i by the entity x_r^i at time t . He does so because he regards property instances as being more fundamental than properties. We use a simpler notation that does not involve explicit reference to specific individuals or times. This is done purely for the sake of convenience. It is not in any way to be construed as regarding properties as being more fundamental, from an ontological point of view, than property instances.¹ Also, the fusion operation $[\cdot * \cdot]$ is regarded by Humphreys as a process that combines two i -level properties P_m^i and P_n^i to form an $(i + 1)$ -level property $[P_m^i * P_n^i]$. It could just as well be represented as the property $P_{m,n}^{i+1}$ but we choose not to adopt this notation below.

Humphreys claims that the interactions that give rise to entangled states in quantum mechanics lend themselves to the fusion treatment described just above. He explains that the essentially relational interactions between the constituents of an entangled pair have exactly the features required for fusion. His emphasis is on the product of the interaction, a nonseparable quantum state.² The emergent phenomenon remains intact so long as the state is nonseparable, and that can be so after the interaction ceases. By contrast, we emphasize the importance of the interaction itself, and stress that the persistence of the interaction is essential for emergence. These suggestions are developed more fully below using pairs of two-state systems.

2. Aspects of Quantum States, Observables, and Evolutions. In Quantum Mechanics, the three basic elements of a quantum system are its states, its properties, and its evolution. What follows is a formal presentation of these elements for two-state systems (such as spin-1/2 systems or photon polarization), and then for pairs of two-state systems. States, properties, and evolutions of two-state systems are represented in a two-dimensional complex vector space, and those of pairs of two-state systems in a four-dimensional complex vector space.

Two-dimensional vectors (two-by-one matrices) having complex numbers as components represent states of a two-state system. Such vectors

1. We wish to remain neutral with respect to this issue, and we regard the translation to an explicit property instance notation as being relatively straightforward.

2. The entanglement of states becomes manifest in the statistics of nonlocal measurements that are made on the system. The measurements are nonlocal in the sense that they involve two spatially separated measuring devices that are configured so that the pair of measurement events on each system are spacelike separated. Entangled states exhibit correlations in nonlocal measurements in such a way that joint probabilities for the outcomes of the components of the pair of measurement events cannot be factored into single probabilities.

are normalized to unity in order to facilitate a probabilistic interpretation of states. So, the general form of a state ϕ of a two-state system is this:

$$\phi = \begin{pmatrix} a \\ b \end{pmatrix}$$

where $aa^* + bb^* = 1$ (this is the normalization condition). The asterisk denotes the complex conjugate. All vectors having this form are regarded as a possible physical state of a two-state system.

Two-by-two matrices having specific symmetry features represent properties and evolutions. To characterize these features, it is useful to introduce the notion of the adjoint of a matrix. If M is a matrix with components m_{ij} , then the adjoint of M is the matrix M^* whose components are n_{ij} , where $n_{ij} = m_{ji}^*$. The use of the complex conjugate indicates that the components of matrices representing properties and evolutions are complex numbers.

In quantum mechanics, property matrices must be self-adjoint, meaning that $a_{ij} = a_{ji}^*$ where a_{ij} is the matrix element at the intersection of row- i and column- j . So, the general form of a property matrix A for a two-state system is this:

$$A = \begin{pmatrix} a & b \\ b^* & c \end{pmatrix}$$

The occurrence of a and b in A is not in any way related to their occurrence above in ϕ . The symmetry feature of properties (self-adjointness) entails that $a, c \in \mathfrak{R}$. All matrices having the form indicated above are regarded as corresponding to a possible property of a two-state system.

An example of a property that has this form is S_z , the spin in the z -direction of a spin- $\frac{1}{2}$ system. A matrix representation of this property is $S_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Such a system can have one of two possible values for S_z : $+1$ and -1 . It can also have a probabilistic propensity to exhibit one of these two values when a measurement for this property is made on the system. If the state of the system is $\phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, then the probabilistic propensity for yielding the result $+1$ for a measurement for S_z is $\frac{1}{2}$, and that for yielding the result -1 for this measurement is also $\frac{1}{2}$.

In quantum mechanics, an evolution matrix U must be unitary, meaning that its adjoint is its inverse. V is the inverse of U if and only if $UV = VU = I$, where I is the identity matrix (which has components d_{ij} such that $d_{ij} = 1$ for $i = j$ and $d_{ij} = 0$ for $i \neq j$). The inverse of a matrix U is

denoted as U^{-1} . So, U is a unitary operator if and only if $U^* = U^{-1}$. The general form of an evolution matrix U for a two-state system is this:

$$U = e^{iAt}$$

where A is a self-adjoint operator and t is a time parameter.³ All matrices having the form indicated above are regarded as corresponding to a possible evolution of a two-state system. The inverse matrix is simply the matrix counterpart of the complex conjugate of U , meaning that $U^{-1} = e^{-iAt}$ (since A may be thought of as the matrix counterpart of a real number).

For pairs of two state systems, the key operation that enables the correct representation of the possible states, properties, and evolutions of such pairs is the tensor product operation. This operation corresponds to a physical relation between the states, properties, and evolutions of the component systems. The tensor product in quantum mechanics is to be contrasted with its counterpart, the direct sum, in classical mechanics. The tensor product operation is characterized in the next section; the direct sum is discussed in the section after the next one. It is the linearity of the equations of motion and the nonseparability of states, properties, and evolutions that makes it necessary to use the tensor product operation rather than the direct sum in quantum mechanics.

3. Aspects of Quantum Nonseparability (Entanglement). Consider a pair of two-state vectors $\{\phi^1, \phi^2\}$ where $\phi^1 = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\phi^2 = \begin{pmatrix} c \\ d \end{pmatrix}$. The tensor product of this pair is denoted as $\phi^1 \otimes \phi^2$, and it corresponds to the four-dimensional state vector as indicated below:

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

Each vector in the four-dimensional complex vector space is regarded as corresponding to a possible state of the associated pair of two-state systems. It is easy to find vectors in this four-dimensional space that cannot

3. This exponential matrix operation is by definition the power series for the exponential function, with ordinary powers replaced by matrix powers. This operation is perfectly well defined for all finite dimensional cases. This is so in the infinite-dimensional case only for bounded operators. An operator is bounded, if there is a positive real number such that the norm of the vector obtained when the operator is applied to any given vector in the Hilbert space is less than this number.

be expressed as a tensor product of a pair of state vectors from the associated two-dimensional vector spaces. Those that can be so expressed are said to be “separable” or “unentangled,” those that cannot are “nonseparable” or “entangled.” We use the terms “separable” and “nonseparable” in what follows. An example of a nonseparable four-dimensional state vector (nonseparable with respect to two-state pairs) is ϕ^{1+2} below:

$$\phi^{1+2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

It is clearly nonseparable since $ad = 0$ if and only if $a = 0$ or $d = 0$, but if $a = 0$ then $ac = 0$ and if $d = 0$ then $bd = 0$.⁴ It will be shown in the next section that a state vector in a four-dimensional tensor product space is either a tensor product vector, or a (nontrivial) superposition of tensor-product vectors. The significance for emergence will also be discussed in that section.

To characterize the tensor product operation for N two-state systems when $N > 2$, it will suffice to consider a triplet of two-state systems. Consider the vectors, ϕ^1 and ϕ^2 , above that are associated with a pair of two-state systems, and the state vector $\phi^3 = \begin{pmatrix} e \\ f \end{pmatrix}$ that is associated with a third two-state system. The tensor product of this triplet is denoted as $\phi^1 \otimes \phi^2 \otimes \phi^3$, and it corresponds to the eight-dimensional state vector obtained in the manner indicated by the following equation (in light of the above):

$$\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} \otimes \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ace \\ acf \\ ade \\ adf \\ bce \\ bcf \\ bde \\ bdf \end{pmatrix}$$

4. The factor of $\frac{1}{\sqrt{2}}$ does not in any way contribute to the occurrence of the nonseparability of the above state. It is present in order to satisfy the normalization condition mentioned above when quantum states were first introduced.

If the states of three two-state systems are ϕ^1 , ϕ^2 and ϕ^3 respectively, then the state of the associated composite system is $\phi^1 \otimes \phi^2 \otimes \phi^3$. Of course, it is possible for vectors in the associated eight-dimensional space to be non-separable—a case in point is the vector with $\frac{1}{\sqrt{2}}$ at the top and bottom slots with zeros in the remaining six slots (similar to ϕ^{1+2} above). Finally, it is worth noting that the state of N two-state systems corresponds to a vector in a 2^N -dimensional tensor product space, which may or may not be separable.

The considerations above suggest that for each N , there are states of the composite that cannot be represented as a tensor product of the states of its subsystems. If there is an intimate tie between emergence and non-separability, which is the main thesis of this essay, then it follows that the hierarchy of levels is actually quite large and may even be countably infinite.⁵ This would then show the shortcomings of assumption L—an assumption to which Humphreys is not ultimately committed, as noted earlier. That is to say, there are numerous levels within physics, and it is completely unclear whether there is a distinct boundary between physics and chemistry or whether there is a broad area of overlap between them. Both options leave open the possibility of a distinct domain of chemistry; but the analysis offers nothing that clarifies which is the more likely possibility or what is the distinctive characteristic of chemistry.

To discuss separability for property and evolution matrices, it is useful to begin by specifying the general condition for the separability of a four-by-four matrix into two two-by-two matrices. Consider a pair of two-by-two matrices $\{M^1, M^2\}$, where $M^1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $M^2 = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$. The tensor product of this pair is denoted as $M^1 \otimes M^2$. It corresponds to the following four-by-four matrix M^{1+2} :

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae & af & be & bf \\ ag & ah & bg & bh \\ ce & cf & de & df \\ cg & ch & dg & dh \end{pmatrix}$$

It is easy to find a four-by-four property matrix that cannot be expressed as a tensor product of a pair of two-by-two property matrices. As in the case with state vectors, property matrices that can be so expressed are said

5. This conclusion also depends on an assumption that we regard as being uncontroversial; namely, that any situation in which there is emergence is also one in which there are distinct ontological levels.

to be “separable,” those that cannot are “nonseparable.” An example of a nonseparable property matrix is A^{1+2} below:

$$A^{1+2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

A^{1+2} is a property matrix since it is self-adjoint. It is nonseparable since $ah = 0$ if and only if $a = 0$ or $h = 0$, but if $a = 0$ then $ae = 0$ and if $h = 0$ then $dh = 0$. It is not necessary to characterize the tensor product operation for properties corresponding to N two-state systems when $N > 2$, since the manner of this operation is sufficiently characterized by the corresponding case for states. It is, however, worth pointing out that A^{1+2} is a superposition of tensor product properties—the significance of which is emphasized above in connection with states and need not be repeated here—as indicated below:

$$A^{1+2} = \frac{1}{2} \left(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

For evolutions, the situation is very similar to that for properties. The only difference is that a specification of the explicit general form of a two-by-two unitary matrix in terms of an arbitrary self-adjoint matrix introduces needless complexities. It will suffice in what follows to work with diagonal matrices. A matrix is said to be diagonal if all of its elements are zero, except for those along the diagonal from the upper left-hand corner of the matrix to its lower right-hand corner. Consider the following diagonal matrix:

$$U^{1+2} = \begin{pmatrix} e^{i\alpha t} & 0 & 0 & 0 \\ 0 & e^{i\beta t} & 0 & 0 \\ 0 & 0 & e^{i\gamma t} & 0 \\ 0 & 0 & 0 & e^{i\delta t} \end{pmatrix}$$

It is easy to verify that U^{1+2} is a unitary matrix. Its adjoint is obtained simply by replacing “ i ” with “ $-i$.” It is easily shown that the adjoint of U^{1+2} is its inverse, and that U^{1+2} is nonseparable. In order for U^{1+2} to be separable into two two-by-two matrices, it must be the case that $b = 0$, $c = 0$, $f = 0$, and $g = 0$. Four conditions then follow: $ae = e^{i\alpha t}$, $ah = e^{i\beta t}$, $de = e^{i\gamma t}$, and $dh = e^{i\delta t}$, and these four can be satisfied if and only if $\alpha + \delta = \beta + \gamma$. The latter clearly need not hold in general. If U^{1+2} is not

separable into two two-by-two matrices, it cannot be separable into two such unitary matrices.

4. Additivity, Nonseparability, and Fusion. According to the British emergentists, there are two fundamental types of properties of composite systems: resultant and emergent.⁶ Resultant properties are additive, emergent properties are not. The paradigm guiding what was meant by additivity is scalar and vector addition in Newtonian mechanics. It is helpful to begin by considering the additivity of scalar quantities. Fundamental scalar quantities in Newtonian mechanics are mass, length, and time. The mass of a composite body is the sum of the masses of its components. It is well known that the additivity of mass fails in nature. Four hydrogen atoms combine to form an atom of helium in the sun. A helium atom has less mass than the sum of the masses of four hydrogen atoms. The missing mass is converted to energy in the process of nuclear fusion. In relativity theory, additivity is restored by treating mass and energy as interchangeable (as characterized by Einstein's famous formula). The associated fundamental additive scalar quantity in relativity is mass-energy.

In Newtonian mechanics, there are many vector quantities. The associated vectors have three components, and so they are often referred to as three-vectors. The best known three-vector quantities are position, velocity, acceleration, and force. All of these quantities are additive according to Newtonian mechanics. But, it is now well known that the additivity of these quantities fails in nature. For example, the velocity of massive bodies cannot be additive since that would enable massive bodies to go faster than the speed of light. Additivity is restored in relativity theory by using four-vector quantities such as position in spacetime (the corresponding four-vectors have three spatial components and one temporal component) and mom-energy (having three momentum components and one energy component).

It is instructive to consider the addition of forces in Newtonian mechanics.⁷ Suppose that a body is acted on by two forces, F and G , where $F = (f_x, f_y, f_z)$ and $G = (g_x, g_y, g_z)$. For simplicity, Cartesian coordinates are used here and in what follows. The total force acting on the body is then $F + G$, which is defined as $F + G = (f_x + g_x, f_y + g_y, f_z + g_z)$. That is to say, forces are added by adding together their respective components. The

6. For an account of these terms and their origin, see Brian McLaughlin's (1992), a superb discussion of the British emergentists, particularly section 3 (58–68).

7. Note that for classical state vectors, subscripts are used rather than superscripts to denote component and composite vector states. Also, the vectors are oriented horizontally to further distinguish classical state vectors from quantum state vectors—the latter are oriented vertically in what has preceded and in what follows.

force $F + G$ is referred to as the “resultant force” acting on the body. The British emergentists were inspired by this notion from classical mechanics. It led them to introduce the notion of a resultant property, and use that notion to characterize the notion of emergent property by way of contrast. An emergent property is any property that is not a resultant property.

It is helpful to introduce another type of additivity that occurs in classical mechanics to explain more fully the manner in which quantum non-separability is non-additive. This notion of additivity arises in connection with phase space representations of states of composite systems. The state of a single particle is represented in a six-dimensional phase space. Three of the dimensions are for position and the other three are for momentum as in the vector $V = (x \ y \ z \ p_x \ p_y \ p_z)$.

For a composite system consisting of a pair of particles, the associated state is represented by a vector V_{1+2} obtained by taking the direct sum $V_1 \oplus V_2$ of the vectors V_1 and V_2 corresponding to the states of its components. So, let $V_i = (x_i \ y_i \ z_i \ p_{x_i} \ p_{y_i} \ p_{z_i})$ for $i = 1, 2$. The direct sum $V_1 \oplus V_2$ of V_1 and V_2 may then be defined as follows:

$$V_1 \oplus V_2 = (x_1 \ y_1 \ z_1 \ x_2 \ y_2 \ z_2 \ p_{x_1} \ p_{y_1} \ p_{z_1} \ p_{x_2} \ p_{y_2} \ p_{z_2})$$

So, the state of a system consisting of a pair of point particles corresponds to a vector in a twelve-dimensional phase space. More generally, the state of a composite system consisting of N point particles corresponds to a vector in a $6N$ -dimensional phase space. This may be contrasted with the situation in quantum mechanics where the state of a composite system consisting of N two-state systems corresponds to a vector in a 2^N -dimensional tensor-product space. For N M -state systems, states correspond to vectors in a M^N -dimensional tensor-product space.

The situation above in classical mechanics will now be contrasted with that in quantum mechanics with regards to additivity and emergence. Because the direct sum is used in classical mechanics to define the states of a composite system in terms of its components, rather than the tensor product operation as in quantum mechanics, there are no nonseparable states in classical mechanics. There are nonseparable Hamiltonians in classical mechanics—the Hamiltonian corresponds to the total energy of the system and is related to the time evolution of the system. This type of nonseparability is the result of nonlinear terms in the equations of motion. Perhaps a kind of emergence can be associated with it. Some measure of plausibility is given to this claim since a classical system can exhibit chaotic behavior only if its Hamiltonian is nonseparable. This suggests that the British emergentists may have placed too much emphasis on the Newtonian formulation of classical physics in motivating their metaphysical doctrine by regarding forces as fundamental, and then developing their meta-

physics by analogy with the way in which forces behave—this is suggested by McLaughlin (1992, 53–54). If they had modeled their metaphysical doctrines on the Hamiltonian formulation of classical mechanics, in which energies (both kinetic and potential) are fundamental, the associated metaphysical doctrine may have had a very different character. Poincaré was aware of chaotic classical models, and that they involved nonseparable Hamiltonians; but it is unclear whether the British emergentists were familiar with them. In any case, these issues will not be considered further here. The focus of this discussion is emergence and additivity in quantum mechanics.

The key for discussing emergence and additivity in quantum mechanics revolves around the use of the tensor product operation to characterize states, properties, and evolutions of composite systems rather than the direct sum. It must be emphasized, however, that it is not the tensor product operation alone that gives rise to the nonseparability of states. A tensor product state vector is separable. An additive operation is also needed in order to get a nonseparable state. That is to say, each nonseparable state vector is a (nontrivial) superposition of tensor product state vectors—this claim and related claims in this paragraph will be explained below. A similar situation exists for properties. A tensor product of property matrices is not an emergent property matrix. But, each nonseparable property matrix is a (nontrivial) superposition of tensor product property matrices.

In light of the above, it appears that a central claim of the British emergentists, that additivity is the mark of resultant (i.e., non-emergent) properties, is wrong. The irony is compounded by noting that the mark of a non-emergent property of composite systems in quantum mechanics crucially involves a multiplicative operation, factorizability into tensor product vectors (in the case of states) or matrices (in the case of properties), rather than an additive one. The situation is different for evolutions. A nonseparable evolution is a *product* rather than a superposition of tensor-product evolutions. This may provide a way to partially vindicate the British emergentists.

The claim above that each nonseparable state vector is a superposition of tensor product state vectors will now be illustrated for pairs of two-state systems. To facilitate doing so, let $\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. It is easily shown that $\{\alpha \otimes \alpha, \alpha \otimes \beta, \beta \otimes \alpha, \beta \otimes \beta\}$ is a basis for the four-dimensional tensor product space.⁸ That is to say, any vector in this space can be represented as a superposition of the vectors of this set. It turns out that

8. The superscripts used in the previous section to distinguish states and properties of the components of a composite is somewhat redundant—the position in the tensor product is already an indication of this—and will not be used in what follows.

the associated superposition has a particularly simple form as indicated below:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = a\alpha \otimes \alpha + b\alpha \otimes \beta + c\beta \otimes \alpha + d\beta \otimes \beta$$

So, it is the tensor product operation together with the requirement that the space of states associated with a composite system must be a vector space that gives rise to nonseparable states. That requirement entails that the space of states is closed under vector addition and scalar-vector multiplication, meaning a superposition of tensor product states must be a possible state of the pair of two-state systems. Thus, a vector sum of tensor product states gives rise to a nonseparable state, and similarly for properties. In what follows, we shall show that tensor products of Hamiltonians do not give rise to emergent properties, and that nonseparable sums of Hamiltonians do give rise to emergent properties.

The claim above that each nonseparable property matrix is a superposition of tensor product property matrices may also be illustrated for pairs of two-state systems. To facilitate doing so, let $P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. It follows that each diagonal four-by-four matrix A that has elements of a set $\{a, b, c, d\}$ of real numbers along the diagonal is a superposition of a set of $\{P_1 \otimes P_1, P_1 \otimes P_2, P_2 \otimes P_1, P_2 \otimes P_2\}$ of property matrices:

$$A = aP_1 \otimes P_1 + bP_1 \otimes P_2 + cP_2 \otimes P_1 + dP_2 \otimes P_2$$

In this case, $\{a, b, c, d\}$ is the set of eigenvalues of the property matrix A . Each four-by-four property matrix has such a set and may be represented in this manner. This is also the case for property matrices of any vector space having a finite number of dimensions. It is *not* so in general for property matrices of the infinite-dimensional vector spaces that are used in quantum mechanics. “Position” and “momentum” are key properties that are represented in such a vector space, and they have no eigenvalues.

The reason that a nonseparable evolution is a *product* of tensor-product evolutions rather than a superposition of tensor-product evolutions follows from the exponentiation operation that is used in defining evolutions in terms of property matrices (as indicated in the previous section). Using the property matrix A above, the evolution matrix $U = e^{iAt}$ may be characterized as follows:

$$\begin{aligned}
 U &= e^{iAt} \\
 &= e^{i(aP_1 \otimes P_1 + bP_1 \otimes P_2 + cP_2 \otimes P_1 + dP_2 \otimes P_2)t} \\
 &= e^{iaP_1 \otimes P_1 t} e^{ibP_1 \otimes P_2 t} e^{icP_2 \otimes P_1 t} e^{idP_2 \otimes P_2 t}
 \end{aligned}$$

The third equation holds because members of the set $\{P_1 \otimes P_1, P_1 \otimes P_2, P_2 \otimes P_1, P_2 \otimes P_2\}$ are mutually commuting. Finally, a partial vindication of the emergentists' claim concerning additivity might be developed on the grounds that nonseparable evolutions are primary, and that nonseparable states and properties are derivative upon these. This perspective would have to be developed from an ontological point of view—from a mathematical point of view evolution matrices are derivative from property matrices. Perhaps it could be suitably developed (or re-enforced) with a strong process-based metaphysics that gives primacy to process over structure. This point will not be developed further here.

So, how do the considerations above shed light on the fusion operation? They do so by serving to explain how Humphreys' schematic characterization of this operation plays out in quantum mechanics, which is the underlying basis for his characterization of it. The specific models that he mentions, superconductivity and superfluidity, are rather complicated for reasons to be discussed below. We will instead consider simpler models from chemistry. Before doing so, we continue to elaborate on the fusion operation. The explanation above has served to reveal surprising consequences involving additivity. These revelations may be connected more explicitly with Humphreys' characterization of fusion.

As indicated above, Humphreys characterizes the fusion operation $[.*]$ as a process that combines two i -level properties P_m^i and P_n^i to form an $(i+1)$ -level property $[P_m^i * P_n^i]$. We have explored how this operation might be implemented within quantum mechanics. What is revealed by the analysis of properties of composite quantum systems is that i -level properties P_m^i and P_n^i are associated with tensor product property matrices (using identity operators), the “*” corresponds to matrix sum, and the brackets indicate that the matrix sum is nonseparable.

So, if level-1 corresponds to the properties of individual two-state systems, then properties of pairs (triplets, quadruplets, etc.) of two-state systems that can be represented as a tensor product of properties of its components are also at level-1. Properties of pairs (triplets, quadruplets, etc.) that cannot be represented as a tensor product of its components are at level-2 (level-3, level-4, etc.). Now consider the property matrix A^{1+2} that was introduced above. Let P_m^1 and P_n^1 denote the tensor product properties

$$\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ respectively. Since } A^{1+2} = P_m^1 + P_n^1 \text{ and } A^{1+2} \text{ is nonseparable, we may write } [P_m^1 * P_n^1] \text{ and thereby}$$

indicate that A^{1+2} is a level-2 property. It is worth noting that there are actually six level-1 properties involved in the construction of the emergent property A^{1+2} : two properties for system-1, two for system-2, and two for the composite system-(1 + 2).

5. Exclusion and the Fine-Grained Hierarchy. In recent literature on philosophy of mind, Kim's exclusion argument has received a fair amount of discussion (Kim 1998). The following diagram, Figure 1, is an attempt to capture the essential structure of the argument. In chapter 2 of the above, Kim argues that the causal closure of the physical and the supervenience of the mental on the physical together entail that the evolution $P \rightarrow P^*$ is the only real causal process, and that the evolution $M \rightarrow M^*$ is a pseudo process (meaning that it is entirely epiphenomenal).

The view of emergence developed by Humphreys is radically different from the picture characterized above. Given assumption L and the existence of emergence, the picture that follows involves causal processes occurring at various levels, with possible causal process occurring at lower levels (except, of course, in the case of physics). Levels above biology, such as psychology and sociology, are not included in the diagram below (Figure 2), in order to keep the diagram simple. Three assumptions are involved in Figure 2. They are extrapolated from the views expressed by Humphreys. First, there is for a given system at a given time a highest

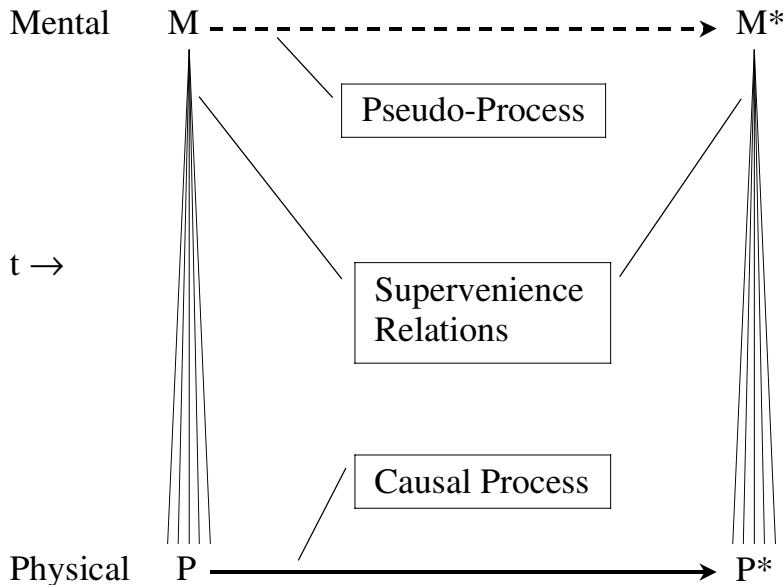


Figure 1.

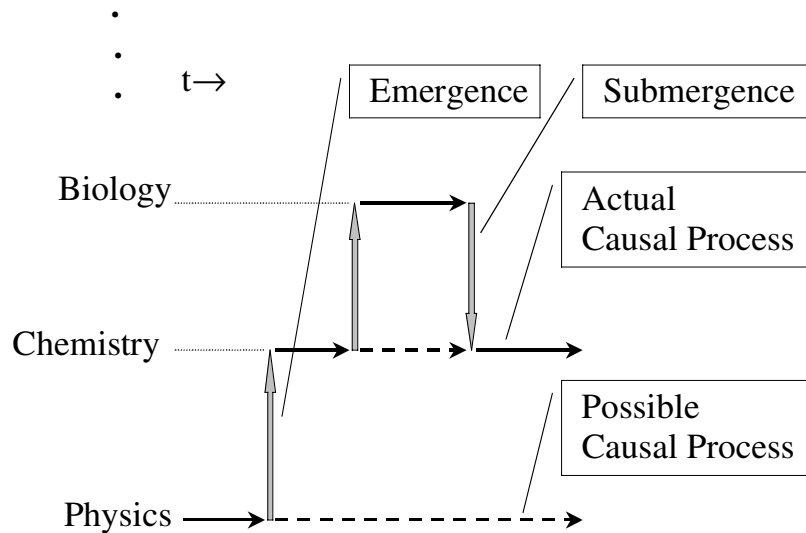


Figure 2.

level of organizational complexity. Second, this system can have subsystems that causally evolve at a lower level in a manner that is independent from other subsystems of the given system. Third, for at least some systems undergoing a causal process at a level higher than physics, there are time intervals during which there are no independently-evolving lower-level subsystems for some (and possibly all) of the lower levels.

Humphreys indicates, in characterizing assumption L, that at least one level may be associated with each of the special sciences. This is a bit of an understatement. In light of the discussion in the previous section, it seems that there will be an immense number of levels associated with each of the special sciences. Before justifying this claim, it will be helpful to consider a simple example.

A triplet of two-state systems, which was briefly described in the previous section, may be unentangled (level-1), partially entangled (level-2), or completely entangled (level-3). Suppose that the triplet is initially unentangled, they evolve independently for a time, and then become partially entangled. The partial entanglement is the result of an interaction between just two members of the triplet—the interaction corresponds to a nonseparable time-evolution operator that is generated by a nonseparable Hamiltonian. The elements of the triplet then evolve independently for a time, and then become completely entangled by way of a triply entangled Hamiltonian (or by way of an entangled Hamiltonian involving the unentangled system and one of the entangled systems). The evolution is then re-

versed. The evolution of the triple just described is characterized graphically below in Figure 3. In general, a Hamiltonian can produce emergence from level- i to level- $(i+j)$ if and only if it can produce submergence from level- $(i+j)$ to level- i .

Crystals are often represented as a lattice consisting of a large number of two-state systems. Often, nearest neighbor interactions are assumed. But such interactions are all that is needed in order for the crystal system to evolve to a nonseparable state involving a large number of its components. Is the crystal a physical system, or is it a chemical system, or is it both? If textbooks may serve as a guide, then the only reasonable conclusion can be that crystals belong to both areas of science. Physics will study lattice structures to understand matters such as phase transitions (e.g., Ising ferromagnets and magnetic critical points), and chemistry does so in order to understand chemical properties (e.g., ionic bonds and relatively high melting points). It is not unreasonable to suppose that some aspects of crystals are biological, since there are protein crystals.

6. A Possible Objection and Two Responses. One criticism that might be raised against a notion of emergence based on the nonseparability of the Hamiltonian in quantum mechanics is this. It takes too literally the notion of resultant (i.e., non-emergent) forces in classical mechanics. That is to

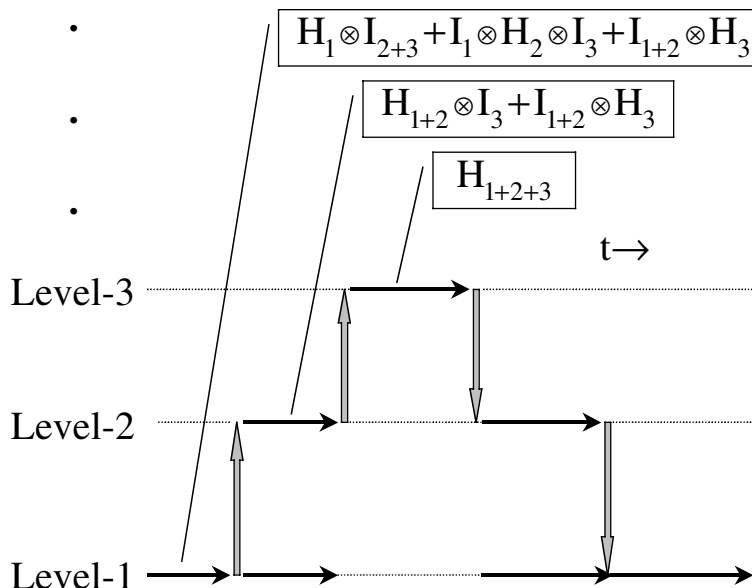


Figure 3.

say, the British emergentists intended vector addition as merely an example of what is required to show that a combination of causal components is non-emergent; namely, the existence of a simple algorithm for combining causal factors. Thus, vector additivity of forces should not be regarded as the definitive characterization of resultant causal features; rather, it is merely a paradigm for showing what must be provided in order to undermine the claim of emergence. This means that despite the substantial differences between the quantum and classical algorithms for combining causal factors of component systems, the crucial feature is present in the quantum case: there is a relatively simple algorithm for combining causal factors.⁹ The kinetic and potential energy terms of the components and the interaction terms involving those components are the causal elements in the quantum context, and they are combined using the tensor product operation and the addition of linear operators—matrix addition is a case in point. This is a relatively simple mathematical operation and its simplicity, one might conclude, is sufficient to regard the corresponding combination of quantum causal factors as non-emergent.

The argument above for not regarding the nonseparability of the Hamiltonian as emergent seems to be rather persuasive. But, there are two serious problems with it. It turns out that the second is the more substantial of the two. First, although the algorithm for forming a nonseparable Hamiltonian is simple, the task of providing a solution to the associated equations of motion is usually an intractable problem. That is to say, the toy models introduced above are rather misleading with regards to the more general situation. One might take them to imply that there is a corresponding simplicity with regards to providing a solution to the resulting equations of motion. The toy models may be characterized as having a finite number of degrees of freedom for which each degree of freedom has a finite dimension. For such models, it is a simple matter to solve Schrödinger's equation (an eigenvalue problem involving finite matrices) and to demonstrate the nonseparability of the Hamiltonian. This is not so in general. Most models in quantum mechanics have either a finite number of degrees of freedom where at least some of those degrees are infinite dimensional, or an infinite number of degrees of freedom—each degree may be either finite or infinite dimensional. This goes for most physical systems modeled in quantum mechanics that are more complex than the hydrogen atom. Such models cannot be solved exactly (or even quasi-

9. Force is regarded as a paradigm causal factor. Energy is as well, if not more so; indeed, in fundamental theories, such as quantum field theory, it is the Lagrangian and Hamiltonian formulations (in which kinetic and potential energies play the primary roles in characterizing the structure and dynamics of physical systems) that predominate.

exactly) due to the nonseparability of the Hamiltonian. Numerical methods can be used to make some headway with these models, but these methods usually involve very significant (and rather questionable) ad hoc assumptions.

Second, the simplicity of the algorithm and the complexity of solving the resulting equations of motion are primarily epistemic considerations, and focusing on them directs attention away from crucial ontological features of quantum models that have nonseparable Hamiltonians. Ultimately, it is not the simplicity or complexity of the algorithm that is crucial. What is crucial is what the resulting structure shows about the relationships existing between the causal features of the components. More to the point, the causal features remain independent in their action in the classical case involving forces. By contrast, the causal features in the quantum models discussed above become inextricably linked due to a continual essential interaction. It may be that such links are so robust that it is no longer meaningful to talk about components that are parts of a compound, meaning that there are components only with reference to what existed prior to the interaction and what exists after the interaction has ceased. This extreme view is akin to Humphreys' own position. We adopt a mitigated position in what follows—see the concluding section.

7. Elaboration of the First Response: The Intractability Issue. It is possible to solve Schrödinger's equation exactly for the hydrogen atom. What this means is that it is possible to find all of the eigenvalues and eigenvectors of the Hamiltonian for the hydrogen atom. For slightly more complicated systems, such as the helium atom and the hydrogen molecule H_2 , this is no longer the case. Approximation methods can be used rather effectively for these simple systems to calculate eigenvalues and eigenvectors, but their effectiveness in that regard quickly diminishes for multi-electron atoms and simple polyatomic molecules. The key mathematical feature that serves to explain these limited successes and ultimate failures is the nonseparability of the Hamiltonian.

Consider first multi-electron atoms.¹⁰ Let Z be the charge of the nucleus, e be the proton charge in Gaussian units, r_k the distance from the nucleus to k^{th} electron, and $r_{ij} = |r_i - r_j|$. Rather than introduce the complete Hamiltonian H , an approximate Hamiltonian \hat{H} is typically formulated. The nucleus is assumed to be infinitely heavy, which is justified on the grounds that electron mass is at least three orders of magnitude smaller

10. This brief discussion of the standard treatment of many electron atoms in quantum chemistry is abstracted from Chapter 11 of the fifth edition of Levine's superb quantum chemistry textbook (Levine 2000). There are many other texts worth consulting including Atkins and Friedman 1997, Bader 1990, McQuarrie 1983, and Gasiorowicz 1974.

than nucleon mass. Interactions other than the interelectron repulsion terms (such as spin-orbit interactions and relativistic interactions, which are typically very small) are also omitted. The approximate Hamiltonian \hat{H} for an n -electron atom then takes the following form:

$$\hat{H}_{at} = \underbrace{\frac{-\hbar^2}{2m_e} \sum_{i=1}^n \nabla_i^2}_{\text{electron kinetic energies}} - \underbrace{\sum_{i=1}^n \frac{Ze^2}{r_i}}_{\text{electron/nucleus interactions}} + \underbrace{\sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{e^2}{r_{ij}}}_{\text{electron/electron interactions}}$$

The first term on the right is the sum of kinetic energy terms associated with each electron. The second is the sum of potential energy terms due to the attraction between the nucleus and the electrons, and the third is the sum of interelectron repulsion terms—an n -electron atom has $n(n-1)/2$ interelectronic repulsion terms. It is the interelectron repulsion terms that make the \hat{H} nonseparable, and it is the nonseparability of \hat{H} that necessitates the use of approximation methods to solve Schrödinger's equation. The standard approximation method is the Hartree SCF (self-consistent field) method. The electron state function is assumed to be a tensor product of spatial state functions, one per electron. Spatial state functions of single electrons are referred to as "orbitals." The orbital associated with an electron is "improved upon" by averaging the associated state functions of the other electrons into a static charge distribution—this yields a sum of Coulomb integrals. The resulting one-electron Schrödinger equation may then be solved to obtain an improved orbital for the chosen electron. This orbital provides a new and improved charge distribution for the electron when the next electron is treated. This process is done for each electron in the atom, and then the process is repeated until a fixed point is reached. The Hartree method may be improved upon by introducing spin explicitly and then anti-symmetrizing the state with respect to electron exchange. This is necessary in order to satisfy Pauli's exclusion principle. The standard anti-symmetrization technique is to use Slater determinants of one-electron spin orbitals. This improvement is known as the Hartree-Fock SCF method. As before, one obtains a one-electron Schrödinger equation, but it is more complicated than that obtained using Hartree's method: there are exchange integrals in the effective Hamiltonian in addition to the Coulomb integrals. Moreover, if the atom is not a closed subshell atom or an atom with a single electron outside the closed subshells, then the Hartree-Fock wave function contains more than one Slater determinant, and the effective Hamiltonian is even more complicated. Standard numerical and algebraic methods—especially the Roothan expansion procedure—are used to complement the Hartree-Fock method. Errors associated with energies calculated using the Hartree-Fock method

are typically about $\frac{1}{2}\%$, which is too large for chemists. A standard technique for going beyond the Hartree-Fock method is to suppose that there are “configuration-interactions,” meaning that the electron state function is assumed to be a superposition of configuration functions corresponding to various excited electron configurations. It takes many configurations to give a truly accurate wave function. Unfortunately, configuration-interaction calculations are very time consuming, even on supercomputers, for systems with more than a few electrons.

Now consider small molecules.¹¹ The notation introduced above for multi-electron atoms is supplemented as follows: α , β are nuclei indices, Z_α is the charge of nucleus α , $r_{\alpha\beta}$ is the distance between nuclei α and β , and $r_{i\alpha}$ is the distance between electron i and nucleus α . As before, an approximate Hamiltonian is formulated, meaning in this case that interactions other than the interelectron repulsion, internucleon repulsion, and nucleon/electron attraction are ignored. The approximate molecular Hamiltonian \hat{H} then takes the following form:

$$\hat{H}_{mol} = \underbrace{\frac{-\hbar^2}{2} \sum_{\alpha} \frac{1}{m_{\alpha}} \nabla_{\alpha}^2}_{\text{nucleon kinetic energies}} + \underbrace{\frac{-\hbar^2}{2m_e} \sum_{i=1}^n \nabla_i^2}_{\text{electron kinetic energies}} + \underbrace{\sum_{\alpha} \sum_{\beta > \alpha} \frac{e^2}{r_{\alpha\beta}}}_{\text{nucleon/nucleon interactions}} - \underbrace{\sum_{\alpha} \sum_i \frac{e^2}{r_{i\alpha}}}_{\text{nucleon/electron interactions}} + \underbrace{\sum_j \sum_{i > j} \frac{e^2}{r_{ij}}}_{\text{electron/electron interactions}}$$

For the purposes of calculating the electron configurations and energies, it is assumed that the nuclei are fixed. The nucleon kinetic energies disappear, and the nucleon repulsion term becomes a constant. This means that it is only necessary to solve the Schrödinger equation for the purely electronic Hamiltonian:

$$\hat{H}_{el} = \underbrace{\frac{-\hbar^2}{2m_e} \sum_{i=1}^n \nabla_i^2}_{\text{electron kinetic energies}} - \underbrace{\sum_{\alpha} \sum_i \frac{e^2}{r_{i\alpha}}}_{\text{electron/nucleus interactions}} + \underbrace{\sum_j \sum_{i > j} \frac{e^2}{r_{ij}}}_{\text{electron/electron interactions}}$$

The constant does not substantially affect the solution of the Schrödinger equation—the wave functions are the same, the eigenvalues are reduced by that constant. The resulting approximation is known as the Born-Oppenheimer approximation, and it is justified on the grounds that the electrons move much faster than the nuclei (since the nuclei are much heavier than the electrons). The upshot is that electron and nucleon motions are treated as separable (meaning that the wave function for a molecule is a tensor product of a wave function for electronic motion and another for nuclear motion). It should be clear, however, that the (molec-

11. This discussion of molecules in quantum chemistry is abstracted from Chapters 13–17 of Levine 2000.

ular) electron Hamiltonian is more nonseparable than the multi-electron atom Hamiltonian. This means that the Hartree-Fock approximation (which comes into play after the Born-Oppenheimer approximation) and more sophisticated variants of Hartree-Fock lose their efficacy more quickly than the atomic case (with increasing electron number). Semi-empirical methods help, but this is quite far from first principles.

Substantial complaints have been voiced about the Born-Oppenheimer approximation on similar grounds. Much structure has to be put in before structure comes out. That is to say, the Born-Oppenheimer approximation elucidates structure, but does not predict it. Moreover, it is a gross exaggeration to claim that molecular structure may be derived from first principles. For example (one of numerous examples), no one has a clue as to how to use quantum mechanics to explain the different isomers (different molecular structure that correspond to the same chemical formula) of C_6H_6 (Woolley 1991). At best, this is a semi-classical theory, meaning that a good bit of classical structure of the nuclei (bond lengths, bond angles, dihedral angles of rotation about single bonds) must be put in before quantum structure (of energy levels and orbitals) comes out.¹²

8. Elaboration of the Second Response: The Inextricability Issue. Given the state of a compound system, one obtains a density operator for one of its components by tracing out the degrees of freedom associated with its other components. That density operator provides the most complete information possible concerning any measurement that can be made on that component, if the state of the compound system is a pure state. If the state of the compound system is a nonseparable pure state, then its components are entangled. In that case the density operator associated with an entangled component is a mixed density operator, which is not a pure state. Moreover, the mixed density operator cannot in general be interpreted as a mixture, meaning that one cannot suppose that the component system is in some unknown pure state; for this reason, such density operators are sometime referred to as “improper mixtures.” The upshot is that there is no pure state that can be associated with the component of an entangled pair, according to quantum mechanics.

The nonseparability of the state of a composite system is one degree of inextricability, but it is not the most robust form to be found in quantum mechanics. A greater degree is to be found when the Hamiltonian of the compound systems is nonseparable. In that case, the time evolution of the density operator that is associated with a part of a composite system can-

12. For further discussion of these issues and related issues, see (Hendry forthcoming, 1998; Amann and Müller-Herold 1999; Ogilvie 1994; Bader 1990; Primas 1983; and Pfeifer 1980).

not in general be characterized in a way that is independent of the time evolution of the whole. If the Hamiltonian is separable, then the time evolution of the density operator associated with a part can in general be characterized independently of the time evolution of the whole.

The statements made in the previous paragraph may be made explicit as follows. Let S_1, S_2 denote two components of a compound system S , and suppose that S is initially in the state $\rho = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \sum_i c_i |\alpha_i \beta_i\rangle$, and $c_i = \langle\alpha_i \beta_i | \psi\rangle$. Let $U(t)$ be the time evolution operator for S during the time interval $[0, t]$. It follows that S is in the state $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ at time t , where $|\psi(t)\rangle = U(t)|\psi\rangle$. If the Hamiltonian of S is separable, then $U(t)$ can be placed in tensor product form, $U(t) = U_1(t) \otimes U_2(t)$. In that case, the density operator $\rho_i(t)$ corresponding to S_i takes the following form:

$$\begin{aligned}\rho_i(t) &= \text{Tr}^{(2)}(\rho(t)) \\ &= \sum_i p_i |\alpha_i(t)\rangle\langle\alpha_i(t)| \\ &= \sum_i p_i(t) |\alpha_i\rangle\langle\alpha_i|\end{aligned}$$

where $p_i = |c_i|^2$, $|\alpha_i(t)\rangle = U_i(t)|\alpha_i\rangle$, and $p_i(t) = |\langle\alpha_i(t)\beta_i | \psi\rangle|^2$. Similarly for S_2 . If the Hamiltonian of S is nonseparable, then $U(t)$ cannot be placed in tensor product form and $\rho_i(t)$ takes the following form:

$$\begin{aligned}\rho_i(t) &= \text{Tr}^{(2)}(\rho(t)) \\ &= \sum_i q_i(t) |\alpha_i\rangle\langle\alpha_i|\end{aligned}$$

where $q_i(t) = |\langle\alpha_i \beta_i | \psi(t)\rangle|^2$. Again, similarly for S_2 . The reason that the nonseparable Hamiltonian provides a greater degree of inextricability is now evident. If the Hamiltonian is separable, the nonseparable state is only needed at one time in order to describe the time evolution of the component systems, as indicated in $p_i(t)$; whereas, the nonseparable state is required at each instant of time to describe the time evolution of the component systems, as indicated in $q_i(t)$.

9. Conclusion. There are at least three ways in which philosophers could develop a metaphysical account of emergence in mereological terms. To specify these accounts it is useful to introduce two notions: that of an independent characterization of an entity, and that of contemporaneous parts. A characterization of an entity is an exhaustive list of the properties that are instantiated by the entity. A characterization is independent if the elements of the list make no essential reference to some other entity. An entity is said to be a contemporaneous part of some whole if that part exists while the whole does.¹³ With these notions in mind, we can now list three possible metaphysical accounts of emergence.

13. In relativistic contexts, this relation is to be understood as being reference-frame dependent due to the relativity of simultaneity.

Prototypical Emergence: Every whole consists entirely of contemporaneous parts that have independent characterizations. There is some suitable criterion for distinguishing between part-whole relations that are emergent from those that are resultant. As an example, the British Emergentists introduced the criterion of additivity of forces to distinguish between emergent and resultant wholes. In their view, wholes involving additive forces are resultant, and those involving non-additive (or “configurational”) forces are emergent. A problem facing proponents of this line is that it is difficult to come up with an appropriate criterion that avoids trivializing the notion of emergence by countenancing either too wide (virtually everything) or too narrow (nothing) as the range of emergent phenomena.¹⁴

Radical Emergence: Only resultant wholes have contemporaneous parts, emergent wholes do not. Emergent wholes are produced by a fusion of certain entities that can be likened to parts. However, these part-like entities cease to exist upon fusion—they only exist when the whole does not, and vice versa. The view that Humphreys sketches can perhaps be regarded as a case in point. He attempts to implement this view by regarding fusion as the formation of a nonseparable state, though it is not clear at all that this is a suitable example.¹⁵ Indeed, the problem facing proponents of this line is that it is difficult to come up with any example of a natural phenomenon that actually satisfies this model of emergence.

Dynamic Emergence: Emergent wholes have contemporaneous parts, but these parts cannot be characterized independently from their respective wholes. Emergent wholes are produced by an essential, ongoing, interaction of its parts. These are the central features of the new view sketched above; the nonseparable Hamiltonian constitutes an essential ongoing interaction.

By adopting the third metaphysical view, we can say that it does not make sense to talk about reducing an emergent whole to its parts, since the parts are in some sense constructs of our characterization of the whole. There

14. McLaughlin (1992) makes a good case that the British Emergentists fall victim to the second of these problems.

15. Humphreys says “. . . when emergence occurs, the lower level property instances go out of existence in producing the higher level emergent instances” (1997, 10). It is not at all clear that his paradigm example involving nonseparable states requires this extreme view, and at times he seems to suggest a less radical view that is more akin to the third view that follows in the text above. He says just before the passage cited above that “. . . i-level property instances no longer have an independent existence within the fusion.” Paul Teller is a proponent of this less radical view with regards to nonseparable states (Teller 1989). Thanks to an anonymous referee for suggesting this reference to us.

is no genuine explanation produced by referring to a drug's dormative powers when trying to account for how that drug puts patients to sleep. Similarly, there is no genuine mereological reduction of X produced by a description of the parts of X that makes an ineliminable reference to X.

The third view falls somewhere between the first and the second. Emergent wholes do have parts, as in the first view but not in the second; however, there is no characterization of these parts that is independent of that of the whole to which it belongs, unlike the first view. The third view is less extreme than the second in that an essential interaction of the parts causes them to go out of existence, as in the second view; but new parts arise that are dependent on the whole, unlike the second view.

In almost every case of relevance, "identical particles" are involved.¹⁶ Special care must be taken whenever this is so. Emergent wholes are produced by an essential ongoing interaction of its parts, and when that interaction ensues the independent particles become dependent. But, if some of those parts are identical particles, then they cannot be identified with those that existed prior to the interaction, as a result of Pauli's exclusion principle. That is to say, the independent parts cease to exist and the dependent parts come into existence. The density operators for identical parts will be identical, except for the label. The label assists in indicating how many identical particles are present in the system, but it does not denote a particular particle in the set of identical particles. Margenau puts it this way: ". . . number becomes an observable despite the indistinguishability of the numbered entities" (1950, 441).¹⁷

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16. The term "identical particles" is being used in the physicists' sense. There are two distinct types of identical particles: fermions (spin-1/2 particles such as electrons) and bosons (spin-1 particles such as photons). There is an ongoing controversy in the philosophical literature as to whether the existence of such particles constitutes a violation of Leibniz law.

17. It is not necessary to adopt such a radical ontological thesis with regards to identical particles. Van Fraassen explains that each boson may be individuated by its history (an empirically vacuous non-quantum characteristic), and that fermions may be individuated by value-states that involve only quantum quantities (meaning that the eigenvector-eigenvalue link is rejected). See §§11–12 of van Fraassen 1991. This interpretive approach (involving hidden-variables) runs against the grain of most philosophers of physics.

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