It is to be regretted that the text of this valuable contribution to Newtonian studies is marred by a greater number of misprints than one would expect in a scholarly work. This blemish should not, however, impair the usefulness of the book to anyone interested in Newton and his times.

ROBERT SCHLAPP

FISCHER, G., *Mathematische Modelle: Mathematical models*, Volume 1 132 photographs, Volume 2 Commentary (Vieweg & Sohn, Braunschweig and Wiesbaden, distributed by John Wiley, Chichester, 1986) pp. xiii + 129, viii + 83, two volumes in slip-case £41.15.

Of these two volumes one is a collection of 132 photographs of mathematical models with a foreword explaining how to manufacture them; the other is a commentary. The volume of illustrations is captioned in German and English; the commentary volume is available in separate German and English editions. In this review page references are to the commentary while italic numerals refer to the photos. It is understandable that some geometers should be enthusiastic modellers because, over the real field, there are cubic surfaces with all their 27 lines and Kummer quartics with all their 16 nodes real, and the collection here reviewed can boast the zealous patronage of both Kummer and Felix Klein. By 1881 several models of cubic surfaces had been made by Klein's pupil Rodenberg.

The coloured photo *5* shows a right circular cone and circular cylinder intersecting in two equal circles in parallel planes; generators of the cone are red threads, those of the cylinder yellow. When a half-turn is imposed on either circle these threads become, in *6*, the two systems of generators of a one-sheeted hyperboloid. For the algebraic explanation see p. 3.

10, 11, 12 depict Clebsch's diagonal surface. Clebsch so named it because 15 of its 27 lines are diagonals of 5 plane quadrilaterals. The other 12 lines compose a Schläfli double-six which appears on the model with the 6 lines of one half green and those of the other half red. This surface intrigued Klein and he took a model to display at Chicago in 1894.

The commentary on cubic surfaces includes (p. 12) a list of possible isolated singularities both in classical and modern notation. Schläfli and Cayley, followed by Salmon and others, attached a suffix showing by how much the singularity diminished the class 12 of the non-singular surface; the suffixes used now for the singularities of all surfaces, whether or not cubic or even algebraic, are relevant to the appropriate Coxeter diagrams and are less by 1 than Cayley's for conic nodes and binodes but by 2 for unodes. The diagrams are given and (p. 13) are used to show what possibilities there are for different isolated singularities occurring on the same cubic surface. It is suggested that the shape of the diagonal surface may have given Klein and Rosenberg some faint inkling of all this. Photos *13–31* show cubic surfaces with singularities. *32* is the cubic scroll and *33* Cayley's special form of it.

Photos *34–39* display the Kummer surface in different affine shapes. It is curious that the references on p. 21 do not include Hudson's classical book of 1905 of which the frontispiece is an example of *34*. *35* is the quadruple tetrahedroid whose equation, on p. 93 of Hudson's book, is just that on p. 15 here homogenized. Moreover Hudson, in his eleventh chapter, obtains Kummer surfaces with 8 real nodes (*36*) as well as with 4 (*37*) and refers in a footnote to a model of the former and to a catalogue of mathematical models of date 1903. Presumably that dated 1911 in the footnote to the opening page of the foreword here is a newer edition of it.

If $\varphi=0$ is a quadric the quartic surface $\varphi^2+\lambda L_1L_2L_3L_4=0$, where each L_i is linear, has 12 nodes at the intersection of $\varphi=0$ with the six edges of the tetrahedron T. So Kummer, whose interest in quartic surfaces was by no means confined to those with 16 nodes, chose T to be regular and $\varphi=0$ to be a sphere S concentric with T (*45*). When S touches the edges of T the nodes coalesce in pairs at six binodes (*40, 41*); when S circumscribes T the nodes coalesce in threes at the vertices producing unodes (*46, 49*) where the uniplanes meet the surface in quartics with triple points.

Photos *42, 43, 44* are of Steiner's surface modelled, if not by Kummer himself, at least under his direction; it has three nodal lines concurring at a triple point. Each line, so far as *real* pairs of

https://doi.org/10.1017/S0013091500028522 Published online by Cambridge University Press

tangent planes are concerned, is limited by two pinch points so that the whole model is in a finite space. As it happens also to be one of the models of the real projective plane it duly appears later; *105–114* show various plane sections.

This brings us to the cyclides which, although algebraic, indeed quartic, surfaces figure in the section of the commentary concerned with differential geometry presumably because the circles that are their lines of curvature are so prominent. The standard 1822 reference to Dupin is given but, as this review is edited from headquarters named after James Clerk Maxwell, one may be allowed to regret that there is no reference to him: he gave stereoscopic diagrams of horned, parabolic, ring and spindle cyclides; see *71–76*. H. F. Baker, in his fourth volume (1925) on *Principles of Geometry*, gave the cyclides full treatment, using projection from four dimensions to derive many of their properties as envelopes of spheres, inverses of tori, and so on, and duly referred to Maxwell who arrived at his conclusions by optical considerations. The reference was adequate, but in his 1940 reprint he was moved to supply a postscript detailing some of Maxwell's calculations. What Baker, perhaps writing "off duty", describes as "the surface of two bananas placed with the ends of either coincident with the ends of the other" is *74*, while that "of the shape of a distorted anchor ring" is *71* which it is easy to visualize as the envelope of spheres touching it internally along the circles shown. Clear diagrams of ring, spindle and horned tori, with the two orthogonal families of circles marked on each, are on p. 28.

Photos *79–88* are of surfaces of constant Gaussian curvature; *90–96* are of minimal surfaces; *98–102* of convex bodies of constant width. To derive these, two plane curves of constant width are used: (a) an involute of the 3-cusped hypocycloid, (b) the Reuleaux triangle of three circular arcs: each joins two vertices of an equilateral triangle and is centred at the third. These provide surfaces of revolution of constant width (*98, 99*). A surface not of revolution that has this property (*100–102*) is not so simple, but its construction is fully described (pp. 54–55). The next photos are of regular star polyhedra (*103–106*) and of models of the real projective plane (*107–120*).

From now onwards the photos illustrate topics in analysis, the first two being almost too familiar to university teachers. The surface $z(x^2+y^2)=xy(x^2-y^2)$ of *121* would seem visually non-singular at $x=y=0$ with a unique tangent plane meeting it in four equally spaced lines; but although the mixed second partial derivatives of z with respect to x and y both exist at $x=y=0$ they are not equal. But on $z=(2x^2-y)(y-x^2)$ pictured in *122* $x=y=0$ is genuinely non-singular, the tangent plane $z=0$ meeting it in two parabolas, both clear on the model, with a common tangent. This phenomenon of a Peano saddle point, signalized by him in 1884, occurs when the section of a surface by its tangent plane at a non-singular point P has a tacnode at P, the two touching branches being real (not complex conjugates). The distance of a point on the surface from the tangent plane at P has then neither a maximum nor a minimum at P.

Later photos depict real and imaginary parts of Riemann surfaces (*123–125*) and of the Weierstrass elliptic function and its derivative (*129–131*).

The photos are a pleasure to contemplate, the commentary a pleasure to read, the books a pleasure to handle. A table at the close of the commentary gives, so far as they are known, the authors of the models, the dates of their construction and their present location.

W. L. EDGE

RUSTON, A. F., *Fredholm theory in Banach spaces* (Cambridge Tracts in Mathematics 86, Cambridge University Press, 1986), x + 293 pp. £30.

Books by pioneers are always valuable and the author is one of the pioneers of the important theory of determinantal expansions of the resolvent of certain operators in Banach spaces associated with Fredholm type solutions of integral equations. In the L^2-case if the underlying operator is compact the technique consists of finding determinantal expansions (using tensor product notation) for resolvents of finite rank operators and then approximating. A stylish exposition of this theory was given by Smithies in his Cambridge Tract *Integral Equations* in

https://doi.org/10.1017/S0013091500028522 Published online by Cambridge University Press