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VOTING ONESELF INTO A CRISIS

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We suggest that voters' lack of recognition of complex economic links may give rise to economic policies that eventually lead to a crisis. We consider a two-sector economy in which a majoritarian political process determines governmental regulation in one sector: a minimum nominal wage. If voters recognize general equilibrium feedbacks, workers favoring market-clearing wages will form a majority across sectors. If voters take into account only direct effects in the regulated sector, not only workers that enjoy minimum wages but also workers in the other sector are willing to vote for wage rises in each period. The reason is that they expect higher real wages for themselves, too. The political process leads to constantly rising unemployment and tax rates. The resulting crisis may trigger new insights into economic relationships on the part of the voters and may reverse bad times.

Keywords: Awareness of General Equilibrium Effects, Unemployment, Democracies, Crises

1. INTRODUCTION

It happens frequently that governments follow fiscal policies that turn out to be unsustainable in the long run. Greece and Spain are the most recent examples. In this paper we provide an explanation for such phenomena. We argue that difficulties voters have in recognizing general equilibrium effects may yield economic policies that trigger a crisis. Moreover, a crisis may help to promote the understanding of general equilibrium effects on the voters' part, and this can reverse bad times.

The argument is developed for a two-sector, two-good economy with three types of workers. The first sector uses skilled and low-skilled workers; the second sector uses only one type of worker. The first sector is the "regulated" sector: there is a minimum wage and there are unemployment benefits for low-skilled workers. We consider the following democratic process for regulating Sector 1: Two political parties aim at maximizing their vote share and propose a minimum

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wage for low-skilled workers in Sector 1, and unemployment is financed by a tax on labor. All types of workers—low-skilled, high-skilled, and workers in Sector 2—are taxed at the same rate. As it will turn out, low-skilled workers in Sector 1 will benefit from higher minimum wages, whereas high-skilled workers in Sector 1 are hurt. The workers in Sector 2 essentially play the role of the median voter.

If workers take all direct and indirect effects into account when voting—called hereafter general equilibrium voting (GEV)—they anticipate that raising low-skilled wages in Sector 1 will affect not only Sector 1, but also wages in Sector 2 and taxes to finance unemployed individuals. The latter general equilibrium effects imply that workers in Sector 2 have single-peaked preferences regarding wages for low-skilled workers in Sector 1, with market-clearing wages as their most preferred wage. Because high-skilled workers in Sector 1 also prefer market-clearing wages over any other wage, the market-clearing wage is the Condorcet winner and thus the equilibrium platform of parties (henceforth the political equilibrium). As a consequence, there is no unemployment and hence no tax burden, as the democratic process implements the free market solution.

Suppose, however, that individuals do not take into account general feedback effects in Sector 2 connected with the minimum wage proposals in Sector 1, whereas all effects in Sector 1 are fully recognized. We refer to this as partial equilibrium voting (PEV). This will be thoroughly justified in Section 3. Voters taking this view assume that nothing will change in Sector 2, including wages and output in this sector, and also that tax rates will remain constant.

With PEV, workers in Sector 2 perceive that—from a certain wage level on—an increase in minimum wages will improve their utility. The logic can be understood by considering market clearing for the good of Sector 2. Demand of the low-skilled workers for the good of Sector 2 increases with a rising minimum wage because purchasing power of employed low-skilled workers rises and unemployed workers receive unemployment benefits. Workers in Sector 2 assume that the tax rate, their nominal wage, and hence their demand for Good 2 remains constant. Hence, all voters, and in particular workers in Sector 2, expect that demand for Good 2 from the third group—the high-skilled workers—falls to clear the market for Good 2, as the supply of Good 2 is constant. A decline of demand for Good 2 by high-skilled workers requires that the relative price of Good 1 declines, making it comparatively more expensive to buy Good 2. Finally, as workers in Sector 2 expect that the price in Sector 1 declines, they believe that their real wage increases. Therefore, under PEV, their preferred wage for low-skilled workers of Good 1 is higher than the market-clearing wage. Together with the low-skilled workers in Sector 1, Sector 2 workers will vote for an increase in wages, which results in a politically determined wage higher than the market-clearing wage.

Furthermore, we show that under PEV the economic situation deteriorates over time. After the wage has been determined in a particular period, the tax rate adjusts upward. Then, in the next period, workers in Sector 2 vote for a further minimum wage increase, because on the basis of the new situation they perceive real wage gains for themselves and no tax increase. As a consequence, the political process will lead to perpetual incremental increases of minimum wages, unemployment, and taxes until the economy collapses. One of three conceivable situations may occur: First, individuals are not willing to accept high tax rates and react by reducing labor supply or by moving into the shadow economy. Second, the tax burden approaches 100% and employed workers lapse into poverty because of the exploding welfare state. Third, at some time, voters may recognize that their PEV view is incorrect and adopt GEV.

Our main objective is to develop a coherent political-economic model that simultaneously allows for awareness of direct effects and nonawareness of general equilibrium effects. The results may explain how fiscal crises occur, as we repeatedly observe even in industrial countries such as Spain or Greece. The general argument may be also applicable to unemployment in Europe. There is a large amount of literature on European unemployment in the last decades that has stressed the interaction between shocks and labor market institutions (e.g., social protection, collective wage bargaining, minimum wages) as a potential cause of the unemployment problem.[1] This literature also identifies large heterogeneities of unemployment performance and labor market institutions across European countries, which makes it impossible to single out one overarching cause for European unemployment [see, e.g., Blanchard (2006)].

The nonawareness of general equilibrium effects offers a complementary explanation; it may have contributed to the rise and persistence of high unemployment in some countries, and that may show why such events can be reversed by a crisis. In some countries such as Sweden or the Netherlands, policy responses to a crisis have triggered a decline in unemployment, which could be interpreted as a reversal of detrimental developments due to emerging insights about economic relationships in crises.

Let us take the Netherlands as an example. In the "Wassenaar Accord" in 1982, and in the face of high unemployment, the government, unions, and employers' organizations explicitly argued that a switch from an industrial perspective to an economywide approach, i.e., taking general equilibrium feedback effects into account, requires wage moderation and more labor market flexibility to stimulate job creation. A broad-based majority in parliament supported the corresponding policy measures and wage moderation took place in some sectors. This reversal caused unemployment to fall below 5%, and this has been called the "Dutch unemployment miracle" [see Visser and Hemerijck (1997) and Nickell and van Ours (2000)].

In this paper, we compare two different awareness structures, one for GEV and one for PEV. Within each awareness structure, all agents have the same awareness and there is no uncertainty about the lack of awareness of the other agents. We relate our analysis to the unawareness models in the literature. Formalizing the concept of unawareness has turned out to be a difficult task. As shown in the seminal paper by Dekel et al. (1998), the prevailing model representing uncertainty by a state space allows only a trivial notion of unawareness: If an agent is unaware of something, then he is unaware of everything and thus knows nothing. A subsequent strand in the literature has developed important theories, using multiple state spaces to model nonawareness [Heifetz et al. (2006); Li (2009); Galanis (2013); and using a mathematical logic perspective in Halpern and Rego (2009) and Board and Chung (2009)].

We relate our work on partial equilibrium voting to this literature in the following way. In the important unawareness models of Heifetz et al. (2006) and Li (2009), each agent has a subjective state space that is less detailed than the full state space and allows for multiperson unawareness. Ozbay (2008) applies these concepts of unawareness to incomplete contracts. Galanis (2013) showed how general unawareness structures developed by Heifetz et al. (2006) can be used to model unawareness of theorems. The analogy with our model is as follows: Under partial equilibrium voting, voters assume that Sector 2 is unaffected by economic changes that occur in Sector 1 when the wage is changed. This means that when agents consider different wages in Sector 1, they are oblivious of interdependencies with Sector 2. Agents may be aware of the existence of other economic sectors, but they do not recognize interdependencies with them.[2]

When the wage in Sector 1 is changed, all changes in quantity, prices, and wages are attributed to changes in Sector 1. Among other things, this means that the relationship between employment and wages in Sector 1 is viewed as a function of wages and parameters. The parameters are objectively dependent on the outcomes in Sector 2, but subjectively they are simply perceived by the agents as fixed real numbers under partial equilibrium voting, as for them Sector 2 appears irrelevant.

Many economists have suggested that departures from rationality may be important in macroeconomics [Sargent (1993), Akerlof (2002)]. The notion that agents are unable to process all available information at once plays an important role in papers on the microfoundation of the Phillips curve [Ball (2000), Mankiw and Reis (2002), and Woodford (2003)]. In Gersbach and Schniewind (2011) it is argued that nonawareness of feedback effects by unions and employer associations may explain why unemployment is high in some European countries. These approaches view imperfect information acquisition as a device to capture the limited ability of agents to process information. We adopt a similar notion and assume that citizens may not be able to incorporate general equilibrium feedback effects when they cast their votes.

We stress in this paper that in a market economy crises may occur, because a sequence of collective decisions with agents who neglect indirect effects may lead to output-stifling regulation. Of course, bad outcomes can also occur when citizens take into account all possible feedback effects [see Gersbach and Mühe (2011) for an example] and when appropriate regulations of markets are absent [see Gersbach and Wenzelburger (2008) for an example in banking].

The paper is organized as follows: In Section 2 we set up the model and derive the market equilibrium of the economy. In Section 3 we motivate the political process and specify GEV and PEV. In Section 4 the utility functions depending on the minimum wage of the low-skilled are derived for each view and for each group of workers. This yields the political equilibria in each time period and in the long run. We compare the results from GEV with PEV and discuss how the political and economic system reacts to the emerging crisis under PEV. In Section 5 we interpret the results. We shed some light on the robustness of our results in Section 6, and set out our conclusions in Section 7. The Appendices contain proofs and supplementary material.

2. THE BASIC ECONOMIC MODEL

2.1. Production and Utility

In this section we introduce the model of the economy on which we base our examination of the process of voting on minimum wages. There are two sectors, producing Good 1 and Good 2, respectively. The only input into production is labor.[3] Each sector is assumed to consist of an identical continuum of firms and it will be sufficient to consider a representative firm in each sector. The production functions are given by

$$q_1 = L_{1l}^{\beta} L_{1h}^{(1-\beta)}, \tag{1}$$

with $0 < \beta < 1$ and

$$q_2 = L_2. \tag{2}$$

Subscripts 1 and 2 denote the first and second sector, respectively. h stands for the high-skilled workers of Sector 1, l for the low-skilled. In Sector 2 we only have one skill level for the whole workforce.

We assume perfectly competitive goods markets and immobility of workers across industries; i.e., workers can only work in one sector. Labor supply is assumed to be inelastic and is given by $\overline{L}_{1l} + \overline{L}_{1h}$ in Sector 1 and $\overline{L}_2$ in Sector 2. Firm owners are the high-skilled workers of Sector 1 and the workers of Sector 2. Each receives an equal share of the sum $\pi_1 + \pi_2$ of all the profits earned in both sectors.[4]

Furthermore, we assume that all types of workers have the same symmetric Cobb–Douglas utility function,[5]

$$u = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}, \tag{3}$$

where c_1 and c_2 denote the consumption levels of Good 1 and Good 2.

In the political process involving all workers as voters, the minimum nominal wage w_{1l} for the low-skilled workers of Sector 1 is set. In order for nominal wages to have real effects, we need a further price rigidity condition and we assume that the price in Sector 2 is constant.[6]

Thus, we can normalize p_2 to one:

$$p_2 = 1. \tag{4}$$

The appropriate consumer price index is

$$p = p_1^{\frac{1}{2}} p_2^{\frac{1}{2}} = p_1^{\frac{1}{2}}. \tag{5}$$

This price index guarantees that changes in prices do not affect workers' utility as consumers as long as real income remains constant.

As p_2 has been normalized to one, setting a minimum nominal wage for the low-skilled workers can lead to unemployment.[7] We assume that workers who have lost their jobs receive an exogenously given fraction $s \in (0, 1]$ of the minimum wage as unemployment benefits. In order to finance the benefits, labor is taxed by a percentage τ of the nominal wages paid; i.e., τ is a payroll tax. All types of labor—low-skilled, high-skilled, and workers in Sector 2—are taxed at the same rate.

Finally, we assume that each of the three types of workers is a fraction of the population smaller than 50%:

$$\frac{\overline{L}_f}{\overline{L}_{1l} + \overline{L}_{1h} + \overline{L}_2} < \frac{1}{2}, \tag{6}$$

where $f = 1l, 1h, 2$. Otherwise, one type of workers could dictate policy.

2.2. Demand, Supply, and Government Budget

In a first step we derive demand and supply for goods and labor. By maximizing utility for an individual worker subject to the budget constraint $p_1 c_1^f + c_2^f \leq b_f$, we receive the following demand equations for consumption:

$$c_1^f = \frac{1}{2}\frac{b_f}{p_1}, \tag{7}$$

$$c_2^f = \frac{1}{2} b_f, \tag{8}$$

where $f = 1l, 1h, 2$ refers to the employed workers and $f = un$ refers to the unemployed. The budgets b_f are $w_f + \frac{\pi_1 + \pi_2}{L_{1h} + L_2}$ for $f = 1h, 2$. For the employed low-skilled, b_{1l} equals w_{1l}, and for $f = un$ we have

$$b_{un} = s w_{1l}. \tag{9}$$

Profits of firms are given by sales minus costs:

$$\pi_1 = p_1 q_1 - w_{1l}(1+\tau)L_{1l} - w_{1h}(1+\tau)L_{1h}, \tag{10}$$

$$\pi_2 = q_2 - w_2(1+\tau)L_2. \tag{11}$$

Firms are price-takers in both sectors. We obtain the first-order conditions for profit maximization in Sector 1 and 2 as

$$w_{1l}(1+\tau) = p_1\beta\left(\frac{L_{1h}}{L_{1l}}\right)^{(1-\beta)}, \quad \textbf{(12)}$$

$$w_{1h}(1+\tau) = p_1(1-\beta)\left(\frac{L_{1l}}{L_{1h}}\right)^{\beta}, \quad \textbf{(13)}$$

$$w_2(1+\tau) = 1. \quad \textbf{(14)}$$

Labor demand in Sector 2 is infinite as long as gross wages are below $\frac{1}{1+\tau}$ and indeterminate for $w = \frac{1}{1+\tau}$.[8]

Both unregulated labor markets clear:

$$L_{1h} = \overline{L}_{1h}, \quad \textbf{(15)}$$

$$L_2 = \overline{L}_2. \quad \textbf{(16)}$$

Because production technologies exhibit constant returns to scale, profits are zero and workers' budgets only consist of wages. The governmental budget constraint is given by

$$(w_{1l}L_{1l} + w_{1h}L_{1h} + w_2L_2)\tau = \Delta b_{un}, \quad \textbf{(17)}$$

where Δ denotes the unemployed workforce:

$$\Delta = \overline{L}_{1l} - L_{1l}. \quad \textbf{(18)}$$

Using the fact that workers exhaust their budget constraint, we can apply Walras' law to the goods markets.[9] Therefore, it suffices to clear one of the two goods markets:

$$L_{1l}c_2^{1l} + L_{1h}c_2^{1h} + L_2c_2^2 + \Delta c_2^{un} = q_2. \quad \textbf{(19)}$$

2.3. The Market Equilibrium

We obtain a system of eight equations for the eight variables $\tau, w_{1h}, w_2, p_1, L_{1l}, L_{1h}, L_2, \Delta$. The system consists of the equations for labor demand [(12),(13), (14)], the governmental budget constraint [(17),(18)], and the market-clearing conditions [(15),(16),(19)]. Solving the system yields the following unique equilibrium solution $E(w_{1l})$:

$$\tau(w_{1l}) = \frac{s(\beta\overline{L}_2 - w_{1l}\overline{L}_{1l})}{sw_{1l}\overline{L}_{1l} - 2\overline{L}_2}, \tag{20}$$

$$w_{1h}(w_{1l}) = \left(\frac{1-\beta}{1+\tau}\right)\frac{\overline{L}_2}{\overline{L}_{1h}}, \tag{21}$$

$$w_2(w_{1l}) = \frac{1}{1+\tau}, \tag{22}$$

$$p_1(w_{1l}) = \left(\frac{\overline{L}_2}{\overline{L}_{1h}}\right)^{1-\beta}\left(\frac{w_{1l}(1+\tau)}{\beta}\right)^{\beta}, \tag{23}$$

$$L_{1l}(w_{1l}) = \beta\overline{L}_2\frac{1}{w_{1l}(1+\tau)}, \tag{24}$$

$$L_{1h}(w_{1l}) = \overline{L}_{1h}, \tag{25}$$

$$L_2(w_{1l}) = \overline{L}_2, \tag{26}$$

$$\Delta(w_{1l}) = \overline{L}_{1l} - \beta\overline{L}_2\frac{1}{w_{1l}(1+\tau)}. \tag{27}$$

We note that for a given w_{1l}, the associated tax rate τ and the equilibrium are unique. An important property is that τ strictly increases in w_{1l}.[10] In the absence of regulation, the low-skilled labor market in Sector 1 also clears. Thus, we have $L_{1l} = \overline{L}_{1l}$ with $\tau = 0$ and from equation (24) we determine the lowest possible minimum wage as

$$w_{1l}^{\min} = \beta\frac{\overline{L}_2}{\overline{L}_{1l}}. \tag{28}$$

We obtain an upper bound of w_{1l}:

$$w_{1l}^{\max} = \frac{2\overline{L}_2}{s\overline{L}_{1l}}. \tag{29}$$

If w_{1l} is smaller than $w_{1l}^{\max}$ and $w_{1l} \to w_{1l}^{\max}$, we obtain $\tau \to \infty$, and thus we approach an infeasable constellation. For $w_{1l} > w_{1l}^{\max}$, we can verify that w_{1h}, w_2, and L_{1l} become negative and that p_1 becomes complex. Therefore, they represent infeasible values.

3. DYNAMICS, EXPECTATION FORMATION, AND THE POLITICAL PROCESS

In this section, we motivate and model the voting process. The two alternative views voters may hold are GEV and PEV. For that purpose, we embed the static model from the last section in a discrete dynamic framework with time indexed by $t = 0, 1, 2 \ldots$. As we will see, under GEV, the outcome in the dynamic model is simply the repetition of the outcomes in each period. With PEV, however,

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the outcome in one period impacts the outcome in subsequent periods and thus influences the time path.

GEV and PEV are connected with a certain kind of expectation formation concerning the economic effects of implementing a certain minimum wage. Workers vote accordingly. Under GEV voters hold rational expectations, whereas under PEV, they are not aware of general equilibrium feedback effects. In the following we explain these concepts in more detail.

3.1. Views

In each voting period, voters calculate their utility levels based on the view they take. In their voting decision in a particular voting period t, they derive the level of the minimum wage $w_{1l,t}$ that maximizes their utility:

$$\underset{w_{1l,t}}{\operatorname{argmax}}\, u(\tilde{E}_t^v(w_{1l,t})).$$

Here $\tilde{E}_t^v$ denotes the perceived short-run market equilibrium associated with a particular view; i.e., v = GEV or v = PEV. As discussed later in Section 3.3, *Dynamics and Crisis*, the competition of parties aimed at maximizing their votes generates the median voter's ideal choice of the minimum wage as the short-run political equilibrium $\hat{w}_{1l,t}$. As a consequence, the economy reaches market equilibrium $E(\hat{w}_{1l,t})$.

3.1.1. Perceived short-run political equilibria under general equilibrium voting (GEV). Under GEV, voters consider all general equilibrium effects represented by equations (12)–(19). Therefore, they correctly anticipate the market equilibrium $E(w_{1l,t})$. We denote the median voter's ideal choice of the minimum wage under GEV by $\hat{w}_{1l,t}^{\mathrm{GEV}}$ and the equilibrium actually achieved under GEV by $E_t^{\mathrm{GEV}} = E_t(\hat{w}_{1l,t}^{\mathrm{GEV}})$. As the voters' expected equilibrium, $\tilde{E}_t^{\mathrm{GEV}}$, equals the equilibrium E_t^{GEV} actually achieved, the optimal wage of an individual voter before voting is still optimal after the new equilibrium has been achieved and voters have no reason to change their ideal wages after casting their votes the first time. Thus, under GEV, we have $\hat{w}_{1l,t}^{\mathrm{GEV}} = \ldots = \hat{w}_{1l,1}^{\mathrm{GEV}} = \hat{w}_{1l,0}^{\mathrm{GEV}}$ as short-run political equilibria, as well as $E_t^{\mathrm{GEV}} = \ldots = E_1^{\mathrm{GEV}} = E_0^{\mathrm{GEV}}$ as short-run market equilibria.

3.1.2. Perceived short-run political equilibria under partial equilibrium voting (PEV). Under PEV, not all effects are taken into account by the voters. We assume that voters only consider changes in the regulated sector. They proceed on the assumption that the variables in Sector 2 and the tax rate τ do not change; i.e., w_2, L_2, and τ are assumed to stay constant. Therefore, voters anticipate that changing wages in Sector 1 will affect prices and output in this sector, whereas they do not take into account general equilibrium repercussions from the economy on tax rate adjustments by the government. Thus, PEV represents the plausible assumption that agents only consider the direct effects of regulatory changes when they cast their votes.

Moreover, we assume that nonawareness of general equilibrium effects persists for an extended amount of time, i.e., over many voting periods. Although we do not claim that such strong forms of nonawareness are ubiquitous, both levels of bounded rationality can be motivated well by empirical and experimental research.

Nonawareness of general equilibrium effects. With reference to nonawareness of general equilibrium effects, there are a number of studies that support this assumption. Romer (2003) develops an important theory of misconceptions where voters individually obtain misleading but correlated signals about the outcome of a certain policy. Further studies have found that ideology plays an important role in the formation of beliefs about economic policies [see Caplan (2002) and Blinder and Krueger (2004)]. The role of voter misconceptions regarding tax policy is explored in Birney et al. (2006), Krupnikov et al. (2006), and Slemrod (2006). Moreover, the nonawareness of general equilibrium effects may result from the fact that people tend to simplify decision problems—an observation that is experimentally well established [see, e.g., Rubinstein (1998)].

Persistence of the expectation formation scheme. The persistence of voter misconceptions is documented in Caplan (2002) and Blinder and Krueger (2004). We discuss some behavioral justifications for the persistence of nonawareness of general equilibrium effects. There is evidence suggesting that once people have formed an opinion, they will maintain it as long as possible.

Barberis and Thaler (2002), for instance, identify two behavioral effects supporting this. One effect, simply called "belief perseverance," induces agents to refrain from searching for new evidence and to adhere to an established opinion, even if they observe evidence to the contrary. A stronger behavioral phenomenon is called "confirmation bias." People with a confirmation bias not only ignore contrary evidence, they even interpret that evidence as supporting their original hypothesis. This is in accordance with Kahneman et al. (1982), who observe that agents are conservative in updating their beliefs, and with Brenner (1996), who observes that people are sluggish and only change their behavior when feedback is extremely negative. In our context, this sluggishness may be supported by the fact that people do not know whether erroneous expectations are due to their own misconceptions or to exogenous effects on the economy. For example, when unemployment is higher than expected, agents may presume that this is due to poor economic performance in other countries, leading to a fall in exports. They may not consider the fact that they have neglected general equilibrium effects.

Furthermore, one psychological explanation for conservatism may be "cognitive dissonance" in voting behavior. Mullainathan and Washington (2005) find empirical evidence that previous voting decisions may influence preferences and hence future voting decisions. The reason is that people feel a need for consistency; i.e., they want their behavior to be in line with their beliefs. Once people have cast their vote, they want to believe that their decision was correct, so they stick with their past decisions because they would otherwise feel uneasy.

Formalization of PEV. In period t under PEV, voters apply equations (12), (13), (15), and (18), which describe the behavior of agents in Sector 1:

$$w_{1l,t}(1+\tau_t) = p_{1,t}\beta\left(\frac{L_{1h,t}}{L_{1l,t}}\right)^{(1-\beta)},$$

$$w_{1h,t}(1+\tau_t) = p_{1,t}(1-\beta)\left(\frac{L_{1l,t}}{L_{1h,t}}\right)^{\beta},$$

$$L_{1h,t} = \overline{L}_{1h},$$

$$\Delta_t = \overline{L}_{1l} - L_{1l,t}.$$

From the voters' point of view, Sector 2 is not affected at all. Therefore, they assume clearance of the market for Good 2 (19):

$$L_{1l,t}c^{1l}_{2,t} + L_{1h,t}c^{1h}_{2,t} + L_{2,t}c^{2}_{2,t} + \Delta_t c^{un}_{2,t} = q_{2,t}.$$

Voters base their considerations in period t on the realization of the variables in Sector 2 and on the tax rate in $t-1$, which are presumed to stay constant.

We use $\hat{w}^{\mathrm{PEV}}_{1l,t}$ to denote the Condorcet winner under PEV in period t. A minimum wage for the first sector is a Condorcet winner if it receives a majority of votes in a vote against an arbitrary alternative wage level. A priori it is not clear that a Condorcet winner always exists. It will be shown in Section 4 that it does exist. The Condorcet winner will depend on E_{t-1}, i.e., $\hat{w}^{\mathrm{PEV}}_{1l,t}(E^{\mathrm{PEV}}_{t-1})$, where E^{PEV}_{t-1} is the equilibrium realized under PEV in period $t-1$. Because voters only partially anticipate the resulting equilibrium under PEV, we use $\tilde{E}^{\mathrm{PEV}}_t(w_{1l,t})$ to denote the equilibrium perceived by voters when they determine $\hat{w}^{\mathrm{PEV}}_{1l,t}$. To derive $\tilde{E}^{\mathrm{PEV}}_t(w_{1l,t})$, we solve the system of five equations (12),(13),(15),(18),(19) for the perceived equilibrium values denoted by $\tilde{w}_{1h,t}$, $\tilde{p}_{1,t}$, $\tilde{L}_{1l,t}$, $\tilde{L}_{1h,t}$, and $\tilde{\Delta}_t$:

$$\tilde{\tau}^{\mathrm{PEV}}_t(w_{1l,t}) = \tau^{\mathrm{PEV}}_{t-1}, \tag{30}$$

$$\tilde{w}^{\mathrm{PEV}}_{1h,t}(w_{1l,t}) = (1-\beta)\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}, \tag{31}$$

$$\tilde{w}^{\mathrm{PEV}}_{2,t}(w_{1l,t}) = \frac{1}{1+\tau^{\mathrm{PEV}}_{t-1}}, \tag{32}$$

$$\tilde{p}^{\mathrm{PEV}}_{1,t}(w_{1l,t}) = \left(1+\tau^{\mathrm{PEV}}_{t-1}\right)\left(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\right)^{1-\beta}\left(\frac{w_{1l,t}}{\beta}\right)^{\beta}, \tag{33}$$

$$\tilde{L}^{\mathrm{PEV}}_{1l,t}(w_{1l,t}) = \beta\frac{\epsilon_t(w_{1l,t})}{w_{1l,t}}, \tag{34}$$

$$\tilde{L}^{\mathrm{PEV}}_{1h,t}(w_{1l,t}) = \overline{L}_{1h}, \tag{35}$$

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$$\tilde{L}_{2,t}^{\mathrm{PEV}}(w_{1l,t}) = \overline{L}_2, \tag{36}$$

$$\tilde{\Delta}_t^{\mathrm{PEV}}(w_{1l,t}) = \overline{L}_{1l} - \beta \frac{\epsilon_t(w_{1l,t})}{w_{1l,t}}, \tag{37}$$

where

$$\epsilon_t(w_{1l,t}) = \frac{\overline{L}_2 + \tau_{t-1}^{\mathrm{PEV}} w_{2,t-1}^{\mathrm{PEV}} \overline{L}_2 - s w_{1l,t} \overline{L}_{1l}}{1 - s\beta} \tag{38}$$

and $\tau_{t-1}^{\mathrm{PEV}}$ and $w_{2,t-1}^{\mathrm{PEV}}$ are the actual realized values of τ and w_2 under PEV in period $t-1$.

Note that $\epsilon_t(w_{1l,t})$ strictly decreases in $w_{1l,t}$ and that $\epsilon_t(w_{1l,t})$ has to be nonnegative for the solution to be meaningful. Therefore, under PEV, the perceived maximum wage for the low-skilled in Sector 1 is

$$\tilde{w}_{1l,t}^{\mathrm{PEV,max}} = \frac{\overline{L}_2 + \tau_{t-1}^{\mathrm{PEV}} w_{2,t-1}^{\mathrm{PEV}} \overline{L}_2}{s \overline{L}_{1l}}. \tag{39}$$

If $w_{1l,t} = \tilde{w}_{1l,t}^{\mathrm{PEV,max}}$, then voters perceive that all low-skilled workers of Sector 1 are unemployed, so output in this sector is zero. Finally, we note that the minimal wage $w_{1l}^{\min}$ is the same as in the GEV case.

As can be seen from equations (30) to (38), the perceived equilibrium $\tilde{E}_t^{\mathrm{PEV}}(w_{1l,t})$ in period t depends on the tax rate $\tau_{t-1}^{\mathrm{PEV}}$ actually realized in the previous period. Consequently, the minimum wage each voter group prefers to be implemented depends on the political equilibrium $\hat{w}_{1l,t-1}^{\mathrm{PEV}}$ in the previous period.

3.2. Main Question

The main question we want to analyze is whether repeated PEV will converge to the equilibrium under GEV. As will be discussed in detail in Section 4, if agents had rational expectations, the high-skilled workers in Sector 1 and the workers in Sector 2 would always vote for the market-clearing wage. Because two worker groups always form a majority of voters, we can identify the free-market solution as a political equilibrium with rational expectations. In contrast, we will show that the process involving PEV in each period does not lead to the free-market solution, as two groups of workers vote for higher wages.

As a consequence, a crisis will occur in the long run with PEV, because unemployment among the low-skilled workers will rise steadily and the real wages of the high-skilled workers and workers in Sector 2 will decline steadily.

3.3. Dynamics and Crisis

In this section, we provide a detailed introduction to the political process itself. For this purpose, we develop a dynamic framework. There are an infinite number of time periods, indexed by $t = 0, 1, 2, \ldots$. In each period, the static economy from

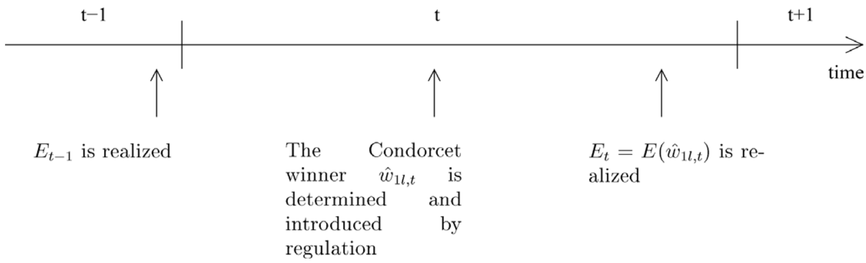

FIGURE 1. The political and economic process.

the last section is at work and we use $E(w_{1l,t})$ or E_t to denote the equilibrium realized in period t after $w_{1l,t}$ has been determined. Within this framework the political process unfolds as follows: In every period each agent acts as a voter. Voters determine the minimum wage $w_{1l,t}$ through majority rule. Although we work directly with the Condorcet winner,[11] we have the standard model of two-party competition in mind, which generates the median-voter result.[12] In every period, the minimum wage preferred by the median voter, denoted by $\hat{w}_{1l,t}$, is implemented. We use $\hat{w}_{1l,t}$ to refer to the short-run political equilibrium. Because we have three different types of workers, we will in general also have three different ideal wage levels. The political and economic process is summarized in Figure 1.

The long-run behavior of the equilibrium can exhibit two patterns. First, if at some point in time a wage $\hat{w}_{1l,t}$ reaches $w_{1l}^{\max}$, the situation is no longer economically feasible, and the economy collapses as output in Sector 1 is zero and the tax rate infinitely high. This is bound to lead to a political crisis where voters as consumers and taxpayers are no longer willing to accept the economic situation. Therefore, they would wish to return to former values of the minimum wage, or they may recognize that their view was mistaken (see Section 4.4).

Second, no economic collapse occurs, i.e., $\hat{w}_{1l,t} < w_{1l}^{\max}$ in all periods. If $\lim_{t\to\infty} \hat{w}_{1l,t}$ and $\lim_{t\to\infty} E(\hat{w}_{1l,t})$ exist, we denote them by $\hat{w}_{1l}^*$ and E^* respectively and use $\hat{w}_{1l}^*$ to refer to the long-run political equilibrium of the process.[13]

Overall, we consider three scenarios for how crises can happen: In the first scenario, the sequence of $\hat{w}_{1l,t}$ converges to or reaches $w_{1l}^{\max}$, τ becomes infinitely large, and we observe a political and economic crisis. This is because the real wages of the high-skilled of Sector 1 and the workers of Sector 2 are zero, as is output in Sector 1. Furthermore, all low-skilled workers have lost their jobs. We call this a crisis with unlimited tax tolerance (CUTT), because such a situation can only arise if voters accept any tax rate imposed by the government.

In the second scenario, we assume that voters will not tolerate arbitrarily high tax rates and that a political-economic crisis will occur when the equilibrium tax rate exceeds a value $\tau_{\max} < \infty$. We call such a crisis a crisis with limited tax tolerance (CLTT). The existence of an upper limit $\tau_{\max}$ can be justified in several ways. For instance, taxpayers may reduce labor supply or try to avoid taxes by moving into the shadow economy. Strictly speaking, to rationalize the reduction

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of labor supply we need to assume that workers receive utility from consuming leisure time. This can be integrated into our model in a simple way by assuming that the elasticity of labor supply is zero for $\tau \leq \tau_{\max}$ and (extremely) high for $\tau > \tau_{\max}$. As a consequence, the state's budget constraint cannot be satisfied with a tax rate exceeding $\tau_{\max}$, as a crisis emerges immediately in such circumstances.

Third, it could happen that voters, after experiencing a discrepancy between expected and realized utility levels for a certain time, recognize that the PEV view is incorrect and switch to GEV. Because this third scenario is qualitatively similar to the second scenario, we focus on the first two cases.

To summarize our concept of a crisis we start from the following condition:

Suppose the sequence of short-run political equilibria $\hat{w}_{1l,t}$ converges to a long-run equilibrium $\hat{w}^*_{1l}$. Suppose further that all short-run equilibria are economically feasible, i.e., $\hat{w}_{1l,t} < w^{\max}_{1l}$, where $w^{\max}_{1l}$ denotes the maximal feasible wage level. Beyond this maximum wage level the economy collapses, with output zero in Sector 1.

Then we define

DEFINITION 1 [Crisis with Limited Tax Tolerance (CLTT)]. *In period T, the tax rate associated with the short-run political equilibrium exceeds a level $\tau_{\max} < \infty$. Tax payers at large are not willing to accept a tax rate higher than $\tau_{\max}$ and will reduce labor supply or move into the shadow economy. The state's budget constraint cannot be satisfied any longer.*

DEFINITION 2 [Crisis with Unlimited Tax Tolerance (CUTT)]. *The sequence $\hat{w}_{1l,t}$ of short-run political equilibria converges to $w^{\max}_{1l}$. Voters accept any tax rate imposed by the government. As a consequence, the crisis realized in the long-run equilibrium $w^{\max}_{1l}$ is associated with the fact that all low-skilled workers in Sector 1 have lost their jobs, so output is zero in Sector 1.*

4. LONG-RUN POLITICAL EQUILIBRIA

We can now derive the political equilibria under GEV and PEV. For this, we need to identify the utility functions of voter groups, their optimal minimum wages, and the Condorcet winners.

4.1. Long-Run Political Equilibria under General Equilibrium Voting

Using a positive monotonic transformation $U = 2 \ln u$ of the utility function u [see equation (3)], we obtain for the workers of Sector 2 in period t

$$\tilde{U}^{\mathrm{GEV}}_{2,t} = \ln\left(\frac{1}{2}\frac{\tilde{w}^{\mathrm{GEV}}_{2,t}}{\tilde{p}^{\mathrm{GEV}}_{1,t}}\right) + \ln\left(\frac{1}{2}\tilde{w}^{\mathrm{GEV}}_{2,t}\right). \tag{40}$$

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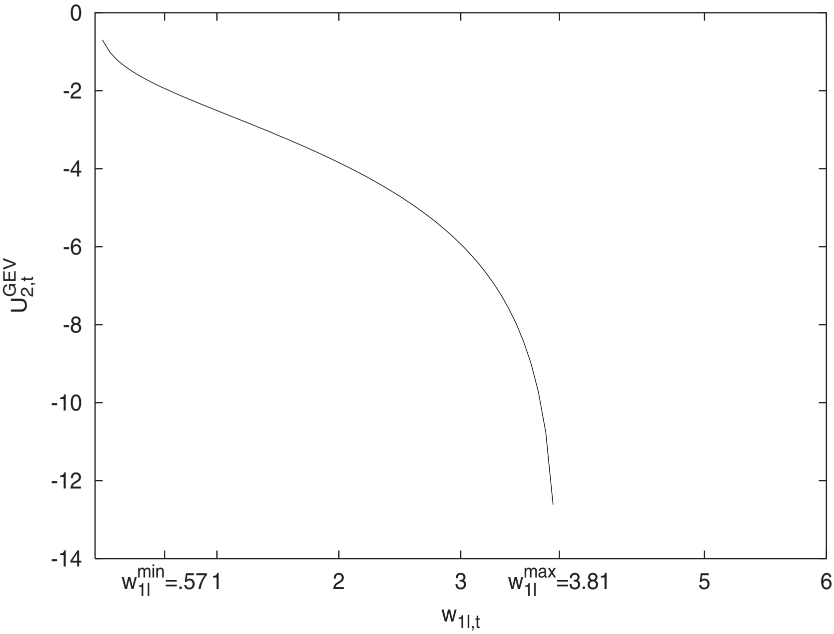

FIGURE 2. $U_{2,t}^{\text{GEV}}$ with $s = 0.75$ and $\beta = 0.4$.

Given $\tilde{E}_t^{\text{GEV}} = E_t^{\text{GEV}} = E_t$, the perceived variables equal the actual realized variables and therefore, from now on, we dispense with the tilde for variables under GEV.

Using equations (22) and (23) and the fact that τ_t^{GEV} strictly increases in $w_{1l,t}$, we find that $w_{2,t}^{\text{GEV}}$ strictly decreases and $p_{1,t}^{\text{GEV}}$ strictly increases in $w_{1l,t} \in (0, w_{1l}^{\max})$. Thus, $U_{2,t}^{\text{GEV}}$ strictly decreases in $w_{1l,t} \in (0, w_{1l}^{\max})$ and voters of Sector 2 will prefer the lowest possible wage $w_{1l}^{\min}$ for the low-skilled of Sector 1.

To illustrate this fact, we plot in Figure 2 the (transformed) utility function $\tilde{U}_{2,t}^{\text{GEV}}$ of workers of Sector 2 with the following parameter values for the economy: $s = 0.75$, $\beta = 0.4$, $\overline{L}_{1l} = 70,000$, $\overline{L}_{1h} = 50,000$, and $\overline{L}_2 = 100,000$. For these values we obtain $w_{1l}^{\min} = 0.57$ and $w_{1l}^{\max} = 3.81$. Furthermore, unless otherwise indicated, we use these parameter values for the illustrations of all other functions in this paper.

For the high-skilled of Sector 1, we obtain

$$U_{1h,t}^{\text{GEV}} = \ln\left(\frac{1}{2}\frac{w_{1h,t}^{\text{GEV}}}{p_{1,t}^{\text{GEV}}}\right) + \ln\left(\frac{1}{2}w_{1h,t}^{\text{GEV}}\right). \tag{41}$$

Because of equations (21) and (23) and the fact that τ_t^{GEV} strictly increases in $w_{1l,t}$, $w_{1h,t}^{\text{GEV}}$ strictly decreases and $p_{1,t}^{\text{GEV}}$ strictly increases in $w_{1l,t} \in (0, w_{1l}^{\max})$.

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Thus, $U_{1h,t}^{\mathrm{GEV}}$ strictly decreases in $w_{1l,t} \in (0, w_{1l}^{\max})$ and the high-skilled workers of Sector 1 will also prefer $w_{1l}^{\min}$.

We can summarize our observations in the following lemma:

LEMMA 1. *$U_{2,t}^{\mathrm{GEV}}(w_{1l,t})$ and $U_{1h,t}^{\mathrm{GEV}}(w_{1l,t})$ have the following properties in $w_{1l,t} \in (0, w_{1l}^{\max})$:*

(i) *$U_{2,t}^{\mathrm{GEV}}(w_{1l,t})$ and $U_{1h,t}^{\mathrm{GEV}}(w_{1l,t})$ strictly decrease for $w_{1l,t}$.*
(ii) *The workers of Sector 2 and the high-skilled workers of Sector 1 maximize their utilities $U_{2,t}^{\mathrm{GEV}}(w_{1l,t})$ and $U_{1h,t}^{\mathrm{GEV}}(w_{1l,t})$ if they choose the lowest possible wage $w_{1l}^{\min}$.*

As two groups of workers always have a single majority of voters, the short-run political equilibrium under GEV in each period is given by

$$\hat{w}_{1l,t}^{\mathrm{GEV}} = w_{1l}^{\min} = \beta \frac{\overline{L}_2}{\overline{L}_{1l}}, \tag{42}$$

which is the economic equilibrium wage in the unregulated case; i.e., unemployment is zero and $\tau = 0$. Thus, we obtain the following proposition:

PROPOSITION 1 (The Long-Run Political Equilibrium under General Equilibrium Voting). *Under GEV, neither CLTT nor CUTT occurs and the long-run political equilibrium of the voting process equals the short-run equilibrium in each period. It is given by*

$$\hat{w}_{1l}^{\mathrm{GEV}*} = \hat{w}_{1l,t}^{\mathrm{GEV}} = \beta \frac{\overline{L}_2}{\overline{L}_{1l}}.$$

There is no unemployment and the equilibrium corresponds to the unregulated economy.

For completeness, in Appendix B we also examine the utility of low-skilled workers in Sector 1 as a function of $w_{1l,t}$.

4.2. Long-Run Political Equilibria under Partial Equilibrium Voting

In this subsection, we derive the technical results under PEV. In Section 5 we provide intuitive explanations of the results.

Before we look at the utility functions themselves, it is useful to analyze $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ in its meaningful range; i.e., for $w_{1l,t} \in [0, \tilde{w}_{1l,t}^{\mathrm{PEV},\max}]$,

$$\tilde{p}_{1,t}^{\mathrm{PEV}} = \left(1 + \tau_{t-1}^{\mathrm{PEV}}\right) \left(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\right)^{1-\beta} \left(\frac{w_{1l,t}}{\beta}\right)^{\beta}.$$

The first derivative of $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ with respect to $w_{1l,t}$ is

$$\frac{\partial \tilde{p}_{1,t}^{\mathrm{PEV}}}{\partial w_{1l,t}} = \tilde{p}_{1,t}^{\mathrm{PEV}} \left((1-\beta) \frac{-s\overline{L}_{1l}}{\overline{L}_2 + \tau_{t-1}^{\mathrm{PEV}} w_{2,t-1}^{\mathrm{PEV}} \overline{L}_2 - s w_{1l,t} \overline{L}_{1l}} + \frac{\beta}{w_{1l,t}} \right), \tag{43}$$

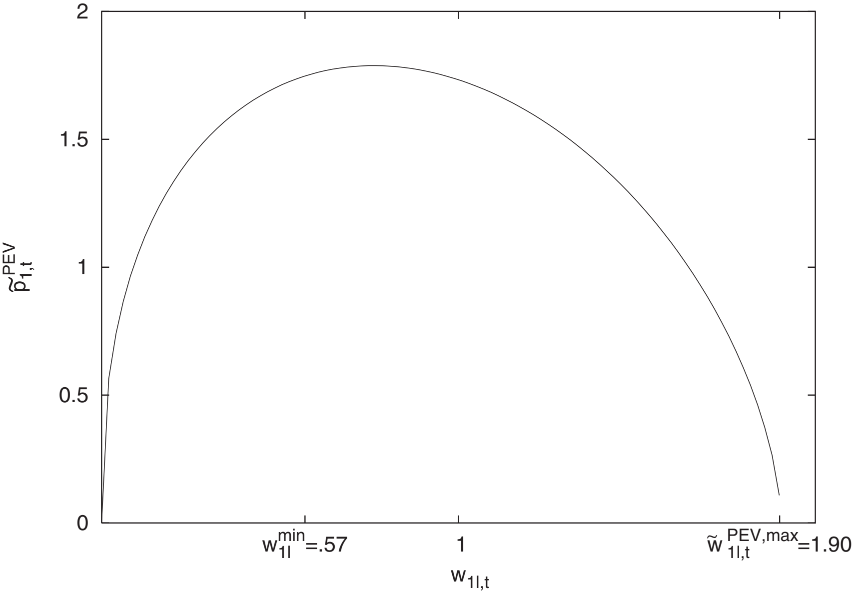

FIGURE 3. The typical shape of $\tilde{p}^{\mathrm{PEV}}_{1,t}(w_{1l,t})$.

and for $w_{1l,t} \in [0, \tilde{w}^{\mathrm{PEV,max}}_{1l,t}]$ we find one value of $w_{1l,t}$ that satisfies $\partial \tilde{p}^{\mathrm{PEV}}_{1,t}/\partial w_{1l,t} = 0$, as expressed in the next lemma.

LEMMA 2. *There exists a unique value* $\tilde{w}^{p_1}_{1l,t}$ *that maximizes* $\tilde{p}^{\mathrm{PEV}}_{1,t}$ *for* $w_{1l,t} \in [0, \tilde{w}^{\mathrm{PEV,max}}_{1l,t}]$:

$$\tilde{w}^{p_1}_{1l,t} = \beta \tilde{w}^{\mathrm{PEV,max}}_{1l,t} = \beta \frac{\overline{L}_2 + \tau^{\mathrm{PEV}}_{t-1} w^{\mathrm{PEV}}_{2,t-1} \overline{L}_2}{s\overline{L}_{1l}}. \quad \textbf{(44)}$$

The proof of Lemma 2 can be found in Appendix A.

Figure 3 shows $\tilde{p}^{\mathrm{PEV}}_{1,t}(w_{1l,t})$ for the case where $\tau^{\mathrm{PEV}}_{t-1} = 0$ and thus $w^{\mathrm{PEV}}_{2,t-1} = 1$ for the parameter values specified in Subsection 4.1.[14] Then we have $\tilde{w}^{\mathrm{PEV,max}}_{1l,t} = 1.90$ and $\tilde{w}^{p_1}_{1l,t} = 0.76$.

The utility of workers in Sector 2 is[15]

$$\tilde{U}^{\mathrm{PEV}}_{2,t}(w_{1l,t}) = \ln\left(\frac{1}{2}\frac{\tilde{w}^{\mathrm{PEV}}_{2,t}}{\tilde{p}^{\mathrm{PEV}}_{1,t}}\right) + \ln\left(\frac{1}{2}\tilde{w}^{\mathrm{PEV}}_{2,t}\right).$$

Because under PEV people consider the wage of workers in Sector 2 to be fixed, the characteristics of $\tilde{U}^{\mathrm{PEV}}_{2,t}(w_{1l,t})$ depend solely on $\tilde{p}^{\mathrm{PEV}}_{1,t}(w_{1l,t})$.

LEMMA 3. $\tilde{U}_{2,t}^{\mathrm{PEV}}(w_{1l,t})$ *has the following properties:*

(i) $\lim_{w_{1l,t}\to 0} \tilde{U}_{2,t}^{\mathrm{PEV}}(w_{1l,t}) = \infty$ *and* $\lim_{w_{1l,t}\to \tilde{w}_{1l,t}^{\mathrm{PEV,max}}} \tilde{U}_{2,t}^{\mathrm{PEV}}(w_{1l,t}) = \infty$.

(ii) *The local maximizer* $\tilde{w}_{1l,t}^{p_1}$ *for* $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ *is a local minimizer of* $\tilde{U}_{2,t}^{\mathrm{PEV}}(w_{1l,t})$ *on* $(0, \tilde{w}_{1l,t}^{\mathrm{PEV,max}})$.

(iii) *Workers in Sector 2 maximize their utility* $\tilde{U}_{2,t}^{\mathrm{PEV}}(w_{1l,t})$ *if they choose the highest possible wage,* $\tilde{w}_{1l,t}^{\mathrm{PEV,max}}$.

The last point follows from the fact that $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ is a continuous positive function on $[w_{1l}^{\min}, \tilde{w}_{1l,t}^{\mathrm{PEV,max}})$ and tends to zero as $w_{1l,t}$ tends to $\tilde{w}_{1l,t}^{\mathrm{PEV,max}}$.

Lemma 3 is the key difference between GEV and PEV, as workers in Sector 2 desire higher wages under PEV than the market-clearing wage. In Appendix C, we examine the utility of high- and low-skilled workers in Sector 1. High-skilled workers maximize their utility at the lowest possible wage $w_{1l}^{\min}$, whereas the highest possible wage, $\tilde{w}_{1l,t}^{\mathrm{PEV,max}}$, maximizes the utility of the low-skilled workers.

Now we can determine the equilibria under PEV. In each round of voting, workers in Sector 2 and the low-skilled workers in Sector 1 prefer $\tilde{w}_{1l,t}^{\mathrm{PEV,max}}$ over other wages. Thus, the short-run equilibrium in period t is $\hat{w}_{1l,t}^{\mathrm{PEV}} = \tilde{w}_{1l,t}^{\mathrm{PEV,max}}$. It depends on the tax rate that actually satisfies the state's budget constraint of the previous voting period. To derive the long-run equilibrium, we need a starting point for the economy characterized by $E(w_{1l,r})$ with the starting wage $w_{1l,r} \in [w_{1l}^{\min}, w_{1l}^{\max})$ and the corresponding tax rate τ_r. We obtain the following proposition (for a proof, see Appendix A):

PROPOSITION 2 (The Evolution of the Economy under Partial Equilibrium Voting). *Under PEV, the economy evolves according to*

$$\hat{w}_{1l,t}^{\mathrm{PEV}} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)^t(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}}, \tag{45}$$

$$\hat{w}_{2,t}^{\mathrm{PEV}} = \frac{1}{(2-s\beta)^{t+1}(1+\tau_r)}, \tag{46}$$

$$\hat{\tau}_t^{\mathrm{PEV}} = (2-s\beta)^{t+1}(1+\tau_r) - 1, \tag{47}$$

where $\tau_r < \infty$ *is the tax rate that actually satisfies the state's budget constraint before period zero starts.*

We next determine whether a crisis will occur in the long run under PEV.

For $w_{1l,r} \in [w_{1l}^{\min}, w_{1l}^{\max})$, $\hat{w}_{1l,t}^{\mathrm{PEV}}$ always remains below $w_{1l}^{\max}$, the variables $\hat{w}_{1h,t}^{\mathrm{PEV}}$, $\hat{w}_{2,t}^{\mathrm{PEV}}$, $\hat{L}_{1l,t}^{\mathrm{PEV}}$, and $\hat{p}_{1,t}^{\mathrm{PEV}}$ are always economically feasible, and no economic collapse occurs with unlimited tax tolerance in finite time. We next determine a long-run equilibrium denoted by $E^{\mathrm{PEV}*}$ by observing $\lim_{t\to\infty} \hat{\tau}_t^{\mathrm{PEV}} = \infty$ and thus $\lim_{t\to\infty} \hat{w}_{1l,t}^{\mathrm{PEV}} = w_{1l}^{\max}$.

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If we assume limited tax tolerance (and $w_{1l,r}$ as initial wage), as t increases, τ_t^{PEV} will become larger than the upper limit $\tau_{\max}$. Therefore, with a finite upper limit on taxes, $\tau_{\max}$, CLTT will occur if

$$(2 - s\beta)^{t+1}(1 + \tau_r) - 1 > \tau_{\max}$$

or if

$$t > \frac{\ln\left(\frac{1+\tau_{\max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} - 1.$$

Thus, the first voting period T where $\hat{w}_{1l,t}^{\mathrm{PEV}}$ "produces" an infeasible tax rate is

$$T = \left\lfloor \frac{\ln\left(\frac{1+\tau_{\max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} \right\rfloor, \qquad \textbf{(48)}$$

where $\lfloor\ \rfloor$ denotes the largest integer that is smaller than the expression under consideration.

We can summarize our results under the PEV view by the following proposition:

PROPOSITION 3 (The Long-Run Political Equilibrium under Partial Equilibrium Voting).

(i) *Under PEV and if there is unlimited tax tolerance, the long-run equilibrium for $w_{1l,r} \in [w_{1l}^{\min}, w_{1l}^{\max})$ is given by*

$$\hat{w}_{1l}^{\mathrm{PEV}*} = \lim_{t\to\infty} \hat{w}_{1l,t}^{\mathrm{PEV}} = w_{1l}^{\max},$$

and all low-skilled workers lose their jobs:

$$\Delta^{\mathrm{PEV}*} = \lim_{t\to\infty} \Delta_t^{\mathrm{PEV}} = \overline{L}_{1l}.$$

(ii) *If CLTT holds, the Condorcet winner of period T in which the crisis emerges is*

$$\hat{w}_{1l,T}^{\mathrm{PEV}} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)^T(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}},$$

where

$$T = \left\lfloor \frac{\ln\left(\frac{1+\tau_{\max}}{1+\tau_r}\right)}{\ln(2 - s\beta)} \right\rfloor,$$

and the number of unemployed workers is

$$\Delta_T^{\mathrm{PEV}} = \overline{L}_{1l}\frac{2(2 - s\beta)^2 - 2\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)}}{2(2 - s\beta)^2 - 2\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)} + s\beta\frac{1}{(2-s\beta)^{T-1}(1+\tau_r)}},$$

where $\tau_r < \infty$ is the tax rate that actually satisfies the state's budget constraint before period zero starts.

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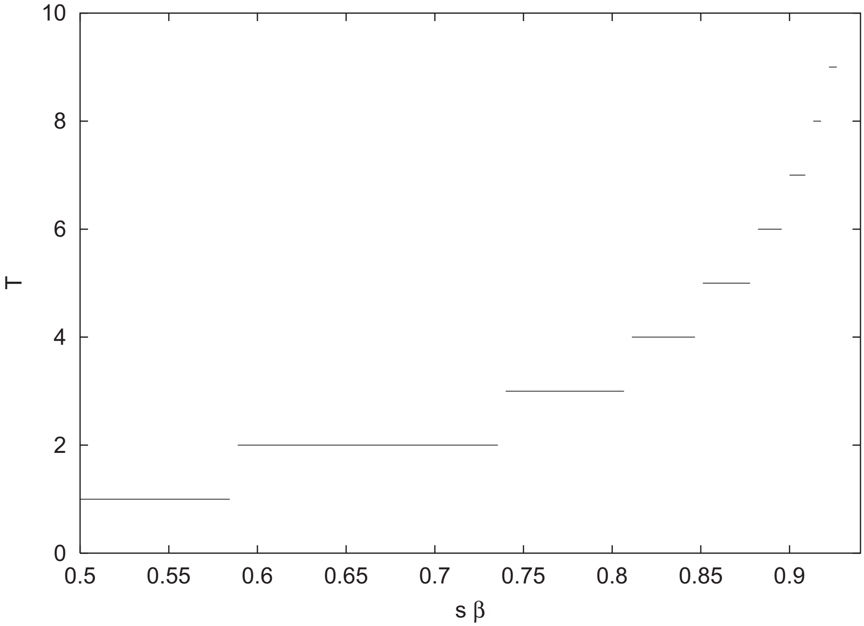

FIGURE 4. The collapse period T for $\tau_r = 0$ and $\tau_{\max} = 1$.

In Figure 4, T is plotted as a function of $s\beta$ [see equation (48)] in the range $s\beta = [0.50, 0.94]$ for the set of parameter values specified in Subsection 4.1. We assume $\tau_{\max} = 1$ and the market-clearing wage as the starting wage, which implies $\tau_r = 0$. For $s\beta \leq 0.58$, T equals 1; i.e., the implementation of the Condorcet winner in period 1 would require a tax rate that exceeds $\tau_{\max}$. As $s\beta$ increases, T also increases. The intervals for $s\beta$ in which T stays constant become smaller, and T tends to infinity as $s\beta$ approaches 1.

4.3. Comparing the Long-Run Political Equilibria under General Equilibrium Voting and Partial Equilibrium Voting

Proposition 4 summarizes our results and shows that when voters only take direct effects of regulations into account, regulations may be adjusted in a direction with increasing adverse effects, and eventually a crisis will occur.

PROPOSITION 4. *The Condorcet winner wages satisfy*

$$w_{1l}^{\min} = \hat{w}_{1l}^{\mathrm{GEV}*} < \hat{w}_{1l,T}^{\mathrm{PEV}} < \hat{w}_{1l}^{\mathrm{PEV}*},$$

where $\hat{w}_{1l}^{\mathrm{GEV}*}$ *denotes the long-run political equilibrium under GEV,* $\hat{w}_{1l,T}^{\mathrm{PEV}}$ *the critical value under PEV with limited tax tolerance (CLTT), and* $\hat{w}_{1l}^{\mathrm{PEV}*}$ *the long-run equilibrium under PEV with unlimited tax tolerance (CUTT). Accordingly,* *unemployment rates satisfy*

$$0 = \Delta^{\mathrm{GEV}*} < \Delta_T^{\mathrm{PEV}} < \Delta^{\mathrm{PEV}*};$$

i.e., there is no unemployment under GEV, whereas PEV produces unemployment.

4.4. Reaction to the Crisis

Under PEV, we assume that voters at first do not learn that their view of the economy is incorrect, although there is a discrepancy between their expected utility levels and those actually achieved. Nevertheless, at some point in time, society enters a crisis, and voters as taxpayers will recognize that there are large negative general equilibrium effects: either τ_t approaches infinity or it crosses $\tau_{\max}$. As the gap between gross wages and net wages becomes too large and real wages become too small, people will not be willing to accept the situation.

There are two conceivable reaction patterns to the crisis:

1. People perform ad hoc measures and—for the moment—give up their assumption of an unchanging tax rate, and vote for previous values of $\hat{w}_{1l,t}$ or complementary policy actions (e.g., a reduction of s). They expect a lower tax rate connected with these measures.
2. People learn that their former views are incorrect. They recognize the discrepancy between their beliefs and the actual realized values of the economy's variables. They adopt a new mental framework for thinking about the functioning of the economy and give up their PEV view in favor of the GEV view. In particular, Sector 2 workers may switch to GEV as they become aware of their tax burden and real-wage decline. If this happens, parties offering market-clearing wages and a reduction in taxes will win and the wage corresponding to the unregulated economy will emerge as the Condorcet winner.

5. INTERPRETATION OF THE RESULTS

To interpret our results, it will be useful to discuss the GEV view in detail first. Then it will become transparent how PEV differs from GEV.

5.1. General Equilibrium Voting (GEV)

Under GEV, voters have equations (7) to (19) in mind when they contemplate the consequences of the minimum wage's value $w_{1l,t}$ for their utility levels. To achieve an economic understanding of the effects of a changing minimum wage $w_{1l,t}$ on the variables of the model, they start with some $w_{1l,t}$ and consider what happens if $w_{1l,t}$ increases by a certain amount. From this they obtain τ_t^{GEV} and $p_{1,t}^{\mathrm{GEV}}$ such that the market-clearing condition (19) and the governmental budget constraint (17) are fulfilled simultaneously:

$$L_{1l,t}^{\mathrm{GEV}} \frac{b_{1l,t}^{\mathrm{GEV}}}{2} + L_{1h,t}^{\mathrm{GEV}} \frac{b_{1h,t}^{\mathrm{GEV}}}{2} + L_{2,t}^{\mathrm{GEV}} \frac{b_{2,t}^{\mathrm{GEV}}}{2} + \Delta_t^{\mathrm{GEV}} \frac{b_{un,t}^{\mathrm{GEV}}}{2} = q_{2,t}^{\mathrm{GEV}}, \tag{49}$$

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$$\left(w_{1l,t} L_{1l,t}^{\text{GEV}} + w_{1h,t}^{\text{GEV}} L_{1h,t}^{\text{GEV}} + w_{2,t}^{\text{GEV}} L_{2,t}^{\text{GEV}}\right) \tau_t^{\text{GEV}} = \Delta_t^{\text{GEV}} b_{un,t}^{\text{GEV}}, \tag{50}$$

where

$$b_{1l,t}^{\text{GEV}} = w_{1l,t}, b_{1h,t}^{\text{GEV}} = w_{1h,t}^{\text{GEV}}, b_{2,t}^{\text{GEV}} = w_{2,t}^{\text{GEV}} \quad \text{and} \quad b_{un,t}^{\text{GEV}} = s w_{1l,t}.$$

In Appendix D, we explain in detail why workers in Sector 2 and the high-skilled workers in Sector 1 prefer the market-clearing wage, whereas low-skilled workers might prefer a higher wage. The main argument is as follows. An increasing minimum wage has two effects: a negative effect on total wealth and a redistributive effect in favor of the low-skilled. First, minimum wages increase unemployment, lower total output, and therefore reduce the total wealth of society. This is represented by an increasing price for Good 1 such that real wages become smaller and smaller not only for the high-skilled in Sector 1 and workers in Sector 2 but also—at least in some range—for the low-skilled in Sector 1. Second, setting a higher minimum wage increasingly redistributes the remaining wealth in favor of the low-skilled workers. This is represented by an increasing tax rate. In the extreme case where all wealth is allocated to the low-skilled workers, the tax rate must be infinitely high to ensure that all other groups channel all their gross earnings to the low-skilled via the state's tax regime.

The exact analytic result of voters' reasoning processes is given by equations (20) to (27). Clearly, workers in Sector 2 and the high-skilled workers in Sector 1 prefer the lowest possible minimum wage because an increase in $w_{1l,t}$ lowers their net wages and increases the price of Good 1. The low-skilled may face a trade-off between a higher $p_{1,t}^{\text{GEV}}$ and increasing unemployment on the one hand, and higher net wages and unemployment benefits on the other. For some values of s and β, they will prefer a minimum wage that exceeds $w_{1l}^{\min}$.

5.2. Partial Equilibrium Voting (PEV)

Under PEV, the same reasoning by agents occurs, but with two important differences: Both the nominal wage in Sector 2, $\tilde{w}_{2,t}^{\text{PEV}}$, and the tax rate, $\tilde{\tau}_t^{\text{PEV}}$, are assumed to stay constant. The reasoning process can be explained as follows.

Voters look at the second goods market and perform their computations concerning the price of Good 1 such that Goods Market 2 clears. From these considerations they derive not only the price of Good 1 but also their wages. This enables them to compute their Marshallian demand functions, which they assume will be satisfied. Thus, voters only indirectly observe output in Sector 1 through the assumption that their Marshallian demand resulting from perceived prices and wages can be satisfied. But under PEV this assumption is false, because the voters do not take into account general equilibrium repercussions from the economy that result from higher unemployment and the attendant change of the tax rate. This ignorance is represented by their assumption of a constant tax rate.

The key insight is the following: As voters assume that $\tilde{w}_{2,t}^{\text{PEV}}$ and $\tilde{\tau}_t^{\text{PEV}}$ remain constant, the demand of workers in Sector 2 for the second good is also perceived

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to remain constant. If $w_{1l,t}$ rises, the demand of low-skilled workers for the second good must increase above a certain value of $w_{1l,t}$. To obtain market clearing in Sector 2, the demand of high-skilled workers for the second good would have to decline in the eyes of the voters, which would require a decline of $\tilde{w}_{1h,t}^{\mathrm{PEV}}$. A lower $\tilde{w}_{1h,t}^{\mathrm{PEV}}$ would have to be accompanied in turn by a lower price for Good 1. This follows from the continuity of the price function and the arguments we present in the next paragraph. Because $\tilde{p}_{1,t}^{\mathrm{PEV}}$ would decline under PEV and workers in Sector 2 expect their nominal net wages to remain constant, they perceive that their utility increases with a rising $w_{1l,t}$. We observe that workers in Sector 2 do not anticipate that their own demand for Sector 2 goods will decline, because they assume $\tilde{w}_{2,t}^{\mathrm{PEV}}$ and $\tilde{\tau}_t^{\mathrm{PEV}}$ to be constant. This failure to recognize general equilibrium effects translates into a mistaken view about price reactions when $w_{1l,t}$ changes, based on the market clearing in Sector 2.

Under GEV, an increase in $w_{1l,t}$ leads to higher unemployment and therefore to an increasing tax rate. The increase in $\tilde{\tau}_t^{\mathrm{PEV}}$ guarantees a decrease in aggregate demand for Good 2 by the high-skilled in Sector 1 and workers in Sector 2, whereas $w_{1l,t}$ increases and leads to growing demand for Good 2 by low-skilled workers. Because under PEV both $\tilde{\tau}_t^{\mathrm{PEV}}$ and $\tilde{w}_{2,t}^{\mathrm{PEV}}$ are perceived to remain constant, an increase in aggregate demand from the low-skilled could only arise if accompanied by decreasing demand from the high-skilled in Sector 1. In an extreme case, all of Good 2 would be allocated to the low-skilled and to the workers of Sector 2. In this case, $\tilde{w}_{1h,t}^{\mathrm{PEV}}$ would have to be zero and the corresponding minimum wage would be $\tilde{w}_{1l}^{\mathrm{PEV,max}}$.

If we look at the political outcome under PEV, we find that the crisis is self-enforcing: The higher the last period's equilibrium tax rate is, the higher is the minimum wage the median voters prefer in the present period. The short-run political equilibrium under PEV, $\hat{w}_{1l,t}^{\mathrm{PEV}}$, strictly increases in the last period's tax rate $\tau_{t-1}^{\mathrm{PEV}} = (2 - s\beta)^t(1 + \tau_r) - 1$ (see Proposition 2), which in turn strictly rises in t. One possible interpretation is that with an increasing tax rate the perceived nominal wage in Sector 2, $\tilde{w}_{2,t}^{\mathrm{PEV}}$, decreases. Hence—in the perception of voters—more wealth can be redistributed to the low-skilled workers. The perceived maximum value for the minimum wage would increase and so would the value of the Condorcet winner $\hat{w}_{1l,t}^{\mathrm{PEV}}$ in the prospection period.

6. ROBUSTNESS AND ALTERNATIVE FORMULATIONS

6.1. Narrow Viewpoints of Voters

The intuition behind PEV is that voters take a narrow standpoint: They assume that regulations in Sector 1 only affect this sector. Sector 2 and the tax rate are perceived to stay constant.

There are a variety of alternative formulations of such a narrow viewpoint of voters, which we discuss briefly in this section. First, instead of clearing the second goods market in their minds, they could clear the first goods market

(View 2). Furthermore, it is conceivable that voters take PEV or View 2 but assume the price for Good 1 to be fixed instead of the price for Good 2 (View 3 and View 4, respectively).[16]

It can be shown that for all such partial equilibrium views at least two voter groups prefer a minimum wage as high as possible as long as it is economically feasible.[17] Therefore, if voters take any of these views that are narrower than GEV, long-run equilibria can occur with high unemployment that are Pareto-inefficient. Under GEV we always have full employment.

6.2. Learning General Equilibrium Voting

In the paper, we have assumed that only a crisis could reverse the PEV view in favor of the GEV view. In practice, other, less pessimistic scenarios may prevail. For instance, when voters recognize the discrepancy between their expected utility levels and those realized, they may revise their mental model before a crisis occurs. The specification and examination of such learning processes is left to future research.

7. CONCLUSIONS

In this paper, we give an explanation for the emergence of crises in democracies. In particular, we show that neglecting general equilibrium repercussions from the regulated sector on the rest of the economy (i.e., the unregulated sector and the tax rate) can lead voters to set regulations that not only are detrimental for the economy as a whole (total output) but also damage their own welfare. Even if a crisis occurs, reforms that result in efficient regulations can only take place if people anticipate general equilibrium effects correctly. However, crises can induce a better recognition of general equilibrium effects, which, in turn, can trigger a reversal of bad times. If this argument is significant enough, the question emerges whether it is possible for democracies to adopt GEV early on and thus avoid the painful cleansing effect of crises. Whether institutional frameworks for democracies exist that can trigger GEV is the fundamental and open question which we hope to answer in subsequent research.

NOTES

1. There is a significant academic debate on the impact of binding minimum wages. Whereas we focus on deteriorating economic situations when such minimum wages are set in politics, other approaches focus on the positive effects of minimum wages. Kaas and Madden (2010) and Beugnot (2013), for instance, show that binding minimum wage rules can avoid the occurrence of inefficient equilibria, and Seppecher (2012) suggests that a minimum wage can boost aggregate demand and can avoid a deflationary spiral.

2. Sector 2 may be relevant for them in other economic activities. For instance, when they act as consumers, they may buy goods produced in Sector 2.

3. Neglecting capital may be justified in the long run, provided that capacity constraints are not binding and the long-run capital stock is determined by equating the marginal product of capital with the world real interest rate.

4. This profit-splitting assumption is inessential for the results. We note that we have constant returns to scale in both production technologies. Therefore, we have zero profits as long as firms can satisfy their optimal labor demand.

5. The symmetry assumption is made solely for ease of presentation. However, the assumption of constant and equal elasticities of substitution across all individuals is important.

6. Alternatively, we could assume that real minimum wages are set directly in the political sphere.

7. Because $p_2 = 1$, w_{1l} is the price of low-skilled labor in terms of Good 2.

8. If gross wages do not exceed 1, profits are non-negative and independent of the employed labor force. If gross wages are higher than 1, profits are negative and the firm closes down.

9. As workers adjust their demand for goods under consideration of their constraints, goods markets clear in spite of unemployment in one labor market.

10. The first derivative of τ with respect to w_{1l} is $\frac{s\overline{L}_{1l}\overline{L}_2(2-s\beta)}{(sw_{1l}\overline{L}_{1l}-2\overline{L}_2)^2} > 0$.

11. This is the minimum wage that defeats all other values of $w_{1l,t}$ in pairwise majority voting.

12. As we will see in the next section, there always exists a Condorcet winner.

13. If $\hat{w}^*_{1l}$ is reached in finite time, the wages and the equilibrium of the economy remain constant thereafter.

14. This is the case when there was no regulation in $t-1$.

15. Under PEV, profits of firms are zero because firms are assumed to be price takers and do not need to worry about equilibrium feedback effects.

16. One could also imagine that voters take relative prices of Good 1 and Good 2 as constant and expect changes in both sectors.

17. A detailed analysis is available on request. Economic feasibility means that markets clear and taxes are finite.

18. This follows from the profit maximization condition with respect to L_{1l} [see equation (12)] and the fact that the high-skilled labor market always clears and therefore $L_{1h,t} = \overline{L}_{1h}$ in all periods.

19. Note that $\partial^2 q_{1,t}/(\partial L_{1h,t}\partial L_{1l,t}) > 0$. If the use of $L_{1l,t}$ decreases, the marginal productivity of $L_{1h,t}$ also decreases. Because $\partial^2 q_{1,t}/\partial(L_{1h,t})^2 < 0$, the use of $L_{1h,t}$ has to decrease for a given wage level if firms want to maximize their profits.

20. For $w^{\max}_{1l}$ the demand of the low-skilled for Good 2 is equal to $q^{\mathrm{GEV}}_{2,t} = \overline{L}_2$.

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APPENDIX A: PROOFS

Proof of Lemma 2. We note that $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ is continuous and $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t}) \geq 0$, $\tilde{p}_{1,t}^{\mathrm{PEV}}(0) = 0$, and $\tilde{p}_{1,t}^{\mathrm{PEV}}(\tilde{w}_{1l,t}^{\mathrm{PEV,max}}) = 0$. We conclude that $\tilde{w}_{1l,t}^{p_1}$ must be the unique local maximizer of $\tilde{p}_{1,t}^{\mathrm{PEV}}(w_{1l,t})$ in $[0, \tilde{w}_{1l,t}^{\mathrm{PEV,max}}]$, because $\partial \tilde{p}_{1,t}^{\mathrm{PEV}}/\partial w_{1l,t} = 0$ only for $w_{1l,t} = \tilde{w}_{1l,t}^{p_1}$, and

$$\left.\frac{\partial^2 \tilde{p}_{1,t}^{\mathrm{PEV}}}{\partial (w_{1l,t})^2}\right|_{w_{1l,t}=\tilde{w}_{1l,t}^{p_1}} = \tilde{p}_{1,t}^{\mathrm{PEV}}\left((1-\beta)\frac{-(s\overline{L}_{1l})^2}{(\overline{L}_2 + \tau_{t-1}^{\mathrm{PEV}} w_{2,t-1}^{\mathrm{PEV}}\overline{L}_2 - s w_{1l,t}\overline{L}_{1l})^2} - \frac{\beta}{(w_{1l,t})^2}\right) < 0.$$

■

Proof of Proposition 2. Equation (39) gives us the general connection between the Condorcet winner in one period and the previous period's realized tax rate and Sector 2 wage values,

$$\hat{w}_{1l,t+1}^{\mathrm{PEV}} = \frac{\overline{L}_2 + \tau_t^{\mathrm{PEV}} w_{2,t}^{\mathrm{PEV}}\overline{L}_2}{s\overline{L}_{1l}}.$$

Thus, the Condorcet winner in period zero is

$$\hat{w}_{1l,0}^{\mathrm{PEV}} = \frac{\overline{L}_2 + \tau_r w_{2,r}\overline{L}_2}{s\overline{L}_{1l}}.$$

Using $w_2 = 1/(1+\tau)$ [see equation (22)], we obtain

$$\hat{w}_{1l,0}^{\mathrm{PEV}} = \frac{\overline{L}_2 + \frac{\tau_r}{1+\tau_r}\overline{L}_2}{s\overline{L}_{1l}} = \frac{\overline{L}_2 + \frac{\tau_r}{1+\tau_r}\overline{L}_2 - \frac{1+\tau_r}{1+\tau_r}\overline{L}_2 + \overline{L}_2}{s\overline{L}_{1l}} = \frac{2\overline{L}_2 - \frac{1}{1+\tau_r}\overline{L}_2}{s\overline{L}_{1l}}.$$

With equations (20) and (22) we find that, in general,

$$w_{2,t}^{\mathrm{PEV}} = \frac{2\overline{L}_2 - s w_{1l,t}^{\mathrm{PEV}}\overline{L}_{1l}}{\overline{L}_2(2-s\beta)},$$

and therefore

$$w_{2,0}^{\mathrm{PEV}} = \frac{2\overline{L}_2 - s\hat{w}_{1l,0}^{\mathrm{PEV}}\overline{L}_{1l}}{\overline{L}_2(2-s\beta)} = \frac{1}{(2-s\beta)(1+\tau_r)}.$$

Thus, the tax rate in period zero is

$$\tau_0^{\mathrm{PEV}} = (2-s\beta)(1+\tau_r) - 1.$$

Inserting $w_{2,0}^{\mathrm{PEV}}$ and τ_0^{PEV} in (39), we have

$$\hat{w}_{1l,1}^{\mathrm{PEV}} = \frac{2\overline{L}_2 - \frac{1}{(2-s\beta)(1+\tau_r)}\overline{L}_2}{s\overline{L}_{1l}},$$

and therefore

$$w_{2,1}^{\mathrm{PEV}} = \frac{1}{(2-s\beta)^2(1+\tau_r)}$$
$$\tau_1^{\mathrm{PEV}} = (2-s\beta)^2(1+\tau_r) - 1.$$

Continuing in this fashion, we obtain Proposition 2. ■

APPENDIX B: UTILITY OF LOW-SKILLED WORKERS UNDER GEV

In the following lemma, we examine the properties of the utility of the low-skilled workers in Sector 1 under GEV.

LEMMA 4. *$u_{1l,t}^{\text{GEV}}(w_{1l,t})$ has the following properties for $w_{1l,t} \in (0, w_{1l}^{\max})$:*

(i) $\lim_{w_{1l,t} \to w_{1l}^{\max}} u_{1l,t}^{\text{GEV}} = 0$.
(ii) *Depending on s and β, the optimal wage for the low-skilled workers of Sector 1 can exceed $w_{1l}^{\min}$.*

Proof of Lemma 4. We calculate the expected utility of the low-skilled workers in Sector 1:

$$u_{1l,t}^{\text{GEV}} = \frac{L_{1l,t}^{\text{GEV}}}{\overline{L}_{1l}} \left(\frac{1}{2}\frac{w_{1l,t}}{p_{1,t}^{\text{GEV}}}\right)^{\frac{1}{2}} \left(\frac{1}{2}w_{1l,t}\right)^{\frac{1}{2}} + \frac{\Delta_t^{\text{GEV}}}{\overline{L}_{1l}} \left(\frac{1}{2}\frac{sw_{1l,t}}{p_{1,t}^{\text{GEV}}}\right)^{\frac{1}{2}} \left(\frac{1}{2}sw_{1l,t}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\frac{L_{1l,t}^{\text{GEV}}}{\overline{L}_{1l}} w_{1l,t} \frac{1}{\sqrt{p_{1,t}^{\text{GEV}}}}(1-s) + \frac{1}{2}sw_{1l,t}\frac{1}{\sqrt{p_{1,t}^{\text{GEV}}}}$$

$$= \frac{1}{2}\beta\frac{\overline{L}_2}{\overline{L}_{1l}}\frac{1}{(1+\tau_t^{\text{GEV}})}\left(\frac{\overline{L}_{1h}}{\overline{L}_2}\right)^{\frac{1-\beta}{2}}\left(\frac{\beta}{1+\tau_t^{\text{GEV}}}\right)^{\frac{\beta}{2}}(1-s)(w_{1l,t})^{-\frac{\beta}{2}}$$

$$+ \frac{1}{2}sw_{1l,t}^{1-\frac{\beta}{2}}\left(\frac{\overline{L}_{1h}}{\overline{L}_2}\right)^{\frac{1-\beta}{2}}\left(\frac{\beta}{(1+\tau_t^{\text{GEV}})}\right)^{\frac{\beta}{2}}.$$

The proof of statement (i) is derived as follows. By using equations (23) and (24) and equation (20), we obtain $\lim_{w_{1l,t} \to w_{1l}^{\max}} \tau = \infty$. Hence, $\lim_{w_{1l,t} \to w_{1l}^{\max}} u_{1l,t}^{\text{GEV}} = 0$. Statement (ii) can be verified numerically.

■

APPENDIX C: UTILITIES OF WORKERS IN SECTOR 1 UNDER PEV

The utility function of the high-skilled workers of Sector 1 under PEV is given by

$$\tilde{U}_{1h,t}^{\text{PEV}}(w_{1l,t}) = \ln\left(\frac{1}{2}\frac{\tilde{w}_{1h,t}^{\text{PEV}}}{\tilde{p}_{1,t}^{\text{PEV}}}\right) + \ln\left(\frac{1}{2}\tilde{w}_{1h,t}^{\text{PEV}}\right).$$

Dividing $\tilde{w}_{1h,t}^{\text{PEV}}$ by $\tilde{p}_{1,t}^{\text{PEV}}$, we obtain

$$\frac{\tilde{w}_{1h,t}^{\text{PEV}}}{\tilde{p}_{1,t}^{\text{PEV}}} = \left(\frac{1-\beta}{1+\tau_{t-1}^{\text{PEV}}}\right)\left(\frac{\beta}{w_{1l,t}}\right)^{\beta}\left(\frac{\epsilon_t(w_{1l,t})}{\overline{L}_{1h}}\right)^{\beta}. \qquad \textbf{(C.1)}$$

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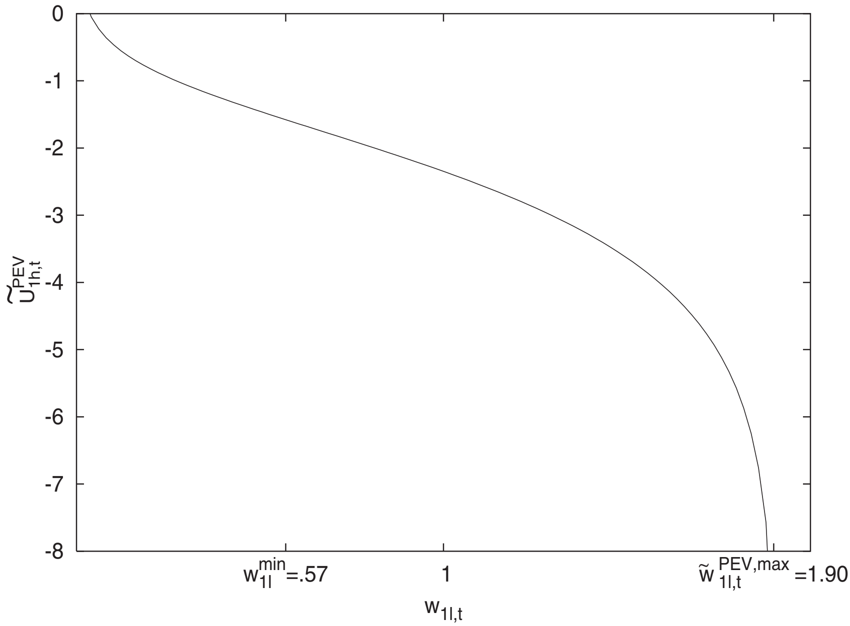

FIGURE C.1. $\tilde{U}^{\mathrm{PEV}}_{1h,t}$ with $\tau^{\mathrm{PEV}}_{t-1} = 0$.

From equations (31) and (C.1), we obtain

LEMMA 5. *$\tilde{U}^{\mathrm{PEV}}_{1h,t}(w_{1l,t})$ has the following properties:*

(i) *$\tilde{U}^{\mathrm{PEV}}_{1h,t}(w_{1l,t})$ is strictly decreasing in $w_{1l,t} \in (0, \tilde{w}^{\mathrm{PEV,max}}_{1l,t})$.*
(ii) *The high-skilled workers of Sector 1 maximize their utility $\tilde{U}^{\mathrm{PEV}}_{1h,t}(w_{1l,t})$ if they choose the lowest possible wage $w^{\min}_{1l}$.*

Figure C.1 illustrates the findings of Lemma 5 for $\tau^{\mathrm{PEV}}_{t-1} = 0$ and for the set of parameter values specified in Subsection 4.1.

For the low-skilled worker in Sector 1, we have the following lemma:

LEMMA 6. *The low-skilled workers of Sector 1 maximize their utility $\tilde{u}^{\mathrm{PEV}}_{1l,t}(w_{1l,t})$ if they choose the highest possible wage $\tilde{w}^{\mathrm{PEV,max}}_{1l,t}$.*

Proof of Lemma 6. The expected utility of low-skilled people in Sector 1 is given by

$$\tilde{u}^{\mathrm{PEV}}_{1l,t} = \frac{1}{2}\frac{\tilde{L}^{\mathrm{PEV}}_{1l,t}}{\overline{L}_{1l}} w_{1l,t} \frac{1}{\sqrt{\tilde{p}^{\mathrm{PEV}}_{1,t}}}(1-s) + \frac{1}{2} s w_{1l,t} \frac{1}{\sqrt{\tilde{p}^{\mathrm{PEV}}_{1,t}}}$$

$$= \frac{1}{2}\frac{1}{\overline{L}_{1l}} \beta \epsilon_t(w_{1l,t}) \frac{1}{(1+\tau^{\mathrm{PEV}}_{t-1})^{\frac{1}{2}}} \left(\frac{\overline{L}_{1h}}{\epsilon_t(w_{1l,t})}\right)^{\frac{1-\beta}{2}} \left(\frac{\beta}{w_{1l,t}}\right)^{\frac{\beta}{2}} (1-s)$$

$$+ \frac{1}{2} s w_{1l,t}^{1-\frac{\beta}{2}} \frac{1}{(1+\tau^{\mathrm{PEV}}_{t-1})^{\frac{1}{2}}} \left(\frac{\overline{L}_{1h}}{\epsilon_t(w_{1l,t})}\right)^{\frac{1-\beta}{2}} \beta^{\frac{\beta}{2}}.$$

we note that $\lim_{w_{1l,t}\to\tilde{w}^{\text{PEV,max}}_{1l,t}} \tilde{L}^{\text{PEV}}_{1l,t} = 0$ [see equations (34),(38),(39)]. Because $\lim_{w_{1l,t}\to\tilde{w}^{\text{PEV,max}}_{1l,t}} (w_{1l,t}/\sqrt{\tilde{p}^{\text{PEV}}_{1,t}}) = \infty$ [using equations (38),(39), (33)], $\tilde{u}^{\text{PEV}}_{1l,t}(w_{1l,t})$ goes to infinity as $w_{1l,t}$ approaches $\tilde{w}^{\text{PEV,max}}_{1l,t}$. As $\tilde{u}^{\text{PEV}}_{1l,t}(w_{1l,t})$ is a continuous function in $[w^{\min}_{1l}, \tilde{w}^{\text{PEV,max}}_{1l,t})$, and τ^{PEV}_{t-1} is taken as given, the low-skilled cannot do better with any other wage level than $\tilde{w}^{\text{PEV,max}}_{1l,t}$.

■

APPENDIX D: INTERPRETATION UNDER GEV

To explain the results with GEV, we introduce relative labor costs, which will help to explain the functioning of the economy. The tax rate, the price for Good 1 and the nominal wages determine the relative labor costs $w_{1l,t}(1+\tau_t)/p_{1,t}$ and $w_{1h,t}(1+\tau_t)/p_{1,t}$ and therefore labor demand in Sector 1.

Suppose, for example, that $w_{1l,t}(1+\tau_t)/p_{1,t}$ increases. Then labor demand for the low-skilled will decrease.[18] As the minimum wage is binding, the low-skilled labor force that will be employed also decreases. Furthermore, because low-skilled and high-skilled labor are complementary inputs, the demand for high-skilled workers in Sector 1 for a given wage level $w_{1h,t}$ decreases as well.[19] Consequently, as the high-skilled labor market in Sector 1 is not regulated, the wage level $w_{1h,t}$ declines, so that the labor market for high-skilled workers will clear. Of course, a change in $(1+\tau_t)/p_{1,t}$ itself, ceteris paribus, changes labor demand for the high-skilled. If $(1+\tau_t)/p_{1,t}$ goes down, $w_{1h,t}$ goes up and vice versa. Because $p_2 = 1$, relative labor costs in Sector 2 are $w_{2,t}(1+\tau_t)$. Again, this labor market is not regulated and thus relative labor costs remain constant; i.e., in the same proportion that $(1+\tau_t)$ changes, $w_{2,t}$ has to change, but in the opposite direction.

We can draw the conclusions of Proposition 1 mainly from equations (49) and (50) intuitively without explicitly computing the results.

In equilibrium, unemployment increases if the minimum wage $w_{1l,t}$ goes up. To see this, suppose that—starting from an equilibrium situation—unemployment would not increase if $w_{1l,t}$ increased. Then $L^{\text{GEV}}_{1l,t}$ would have to remain constant or increase. Hence, $(1+\tau^{\text{GEV}}_t)/p^{\text{GEV}}_{1,t}$ would have to fall in at least the same proportion as $w_{1l,t}$ increased. But if $(1+\tau^{\text{GEV}}_t)/p^{\text{GEV}}_{1,t}$ declined while $L^{\text{GEV}}_{1l,t}$ did not fall, $w^{\text{GEV}}_{1h,t}$ would rise as $L^{\text{GEV}}_{1h,t} = \overline{L}_{1h}$ and thus demand of the high-skilled worker for Good 2 would increase. To complete the argument, we have to distinguish two cases: First, a constant or falling tax rate, and second, an increasing tax rate. In the first case, i.e., in the case of a constant or decreasing tax rate, $w^{\text{GEV}}_{2,t}$ and therefore aggregate demand of Sector 2 workers for Good 2 would at least remain constant but never fall, because $w^{\text{GEV}}_{2,t} = 1/(1+\tau^{\text{GEV}}_t)$. Furthermore, if an increasing $w_{1l,t}$ caused constant or decreasing unemployment, aggregate demand for Good 2 of all low-skilled would go up. Hence, an increasing $w_{1l,t}$ would correspond to an increasing aggregate demand of all voter groups for Good 2 as long as τ^{GEV}_t does not increase. Given that the right-hand side of (49) always equals $q^{\text{GEV}}_{2,t} = \overline{L}_2$, it follows that a situation where unemployment decreases or remains constant while $w_{1l,t}$ increases and τ^{GEV}_t does not cannot be an equilibrium. In the second case, i.e., if τ^{GEV}_t increased, $p^{\text{GEV}}_{1,t}$ also would have to increase because $(1+\tau^{\text{GEV}}_t)/p^{\text{GEV}}_{1,t}$ would have to decline in the case of nonincreasing unemployment. If we look at market clearing in the first goods market,

$$\left(L_{1l,t}^{\mathrm{GEV}} \frac{b_{1l,t}^{\mathrm{GEV}}}{2} + L_{1h,t}^{\mathrm{GEV}} \frac{b_{1h,t}^{\mathrm{GEV}}}{2} + L_{2,t}^{\mathrm{GEV}} \frac{b_{2,t}^{\mathrm{GEV}}}{2} + \Delta_t^{\mathrm{GEV}} \frac{b_{un,t}^{\mathrm{GEV}}}{2} \right) \Big/ p_{1,t}^{\mathrm{GEV}} = q_{1,t}^{\mathrm{GEV}}, \qquad \mathbf{(D.1)}$$

we can recognize that an increasing $p_{1,t}^{\mathrm{GEV}}$ together with an increasing or constant $q_{1,t}^{\mathrm{GEV}}$ (nondecreasing employment of the low-skilled workers) would imply an increasing numerator on the left-hand side of equation (D.1). Because $q_{2,t}^{\mathrm{GEV}}$ remains constant, equation (49) would not hold, and the Goods Market 2 would not clear. Thus, a situation where a rising $w_{1l,t}$ corresponds to nonincreasing unemployment and an increasing tax rate cannot be an equilibrium either. Therefore, independent of the changes in τ_t^{GEV}, unemployment will always increase when $w_{1l,t}$ increases.

When unemployment increases as the minimum wage goes up, output in Sector 1 will decrease [see equations (1) and (15)], thus increasing the scarcity of Good 1. Hence, its price, $p_{1,t}^{\mathrm{GEV}}$, must rise when $w_{1l,t}$ increases.

Furthermore, because unemployment increases when $w_{1l,t}$ rises and thus $\Delta_t^{\mathrm{GEV}} \frac{b_{un,t}^{\mathrm{GEV}}}{2}$ also rises, the sum $L_{1l,t}^{\mathrm{GEV}} \frac{b_{1l,t}^{\mathrm{GEV}}}{2} + L_{1h,t}^{\mathrm{GEV}} \frac{b_{1h,t}^{\mathrm{GEV}}}{2} + L_{2,t}^{\mathrm{GEV}} \frac{b_{2,t}^{\mathrm{GEV}}}{2}$ has to fall to satisfy equation (49). But then $w_{1l,t} L_{1l,t}^{\mathrm{GEV}} + w_{1h,t}^{\mathrm{GEV}} L_{1h,t}^{\mathrm{GEV}} + w_{2,t}^{\mathrm{GEV}} L_{2,t}^{\mathrm{GEV}}$ also declines and therefore τ_t^{GEV} has to rise, according to equation (50). Consequently, the tax rate increases monotonically in $w_{1l,t}$. Because relative labor costs $w_{2,t}^{\mathrm{GEV}}(1 + \tau_t^{\mathrm{GEV}})$ in Sector 2 have to remain constant as the labor market clears, the nominal wage of Sector 2 workers declines when $w_{1l,t}$ increases.

The question arises of whether $w_{1l,t}$ can become infeasible. If we look at equation (49), we recognize that this must be the case from a certain value of $w_{1l,t}$ onward, denoted by $w_{1l}^{\max}$. From this point on—as $w_{1l,t}$ is increased exogenously—the demand of the low-skilled for Good 2 will exceed $q_{2,t}^{\mathrm{GEV}} = \overline{L}_2$ even if all low-skilled are unemployed, because unemployed individuals receive $sw_{1l,t}$.[20] Thus, the market for Good 2 could only clear if $L_{1l,t}^{\mathrm{GEV}}$ were negative, which is not possible. Furthermore, at the critical level $w_{1l}^{\max}$, the aggregate demand for Good 2 of the high-skilled workers and workers of Sector 2 has to be zero, because the goods market in Sector 2 clears. Thus, at $w_{1l}^{\max}$, $w_{1h,t}^{\mathrm{GEV}}$ and $w_{2,t}^{\mathrm{GEV}}$ have to be zero. For a given non-negative value of $L_{1l,t}^{\mathrm{GEV}}$, $w_{1h,t}^{\mathrm{GEV}} = 0$ can only hold if $\lim_{w_{1l,t} \to w_{1l}^{\max}} (1 + \tau_t^{\mathrm{GEV}})/p_{1,t}^{\mathrm{GEV}} = \infty$ [see equation (13)]. Hence, because of equation (12), the employment of the low-skilled is also zero. We can conclude that for $w_{1l,t} = w_{1l}^{\max}$, where low-skilled alone consume all of Good 2, all low-skilled are unemployed and $(1 + \tau_t^{\mathrm{GEV}})/p_{1,t}^{\mathrm{GEV}} = \infty$.

Thus, when $w_{1l} = w_{1l}^{\max}$, output in Sector 1 is zero as well, and for clearance of this good market, demand has to be zero, which implies that $\lim_{w_{1l,t} \to w_{1l}^{\max}} p_{1,t}^{\mathrm{GEV}} = \infty$. Because $\lim_{w_{1l,t} \to w_{1l}^{\max}} (1 + \tau_t^{\mathrm{GEV}})/p_{1,t}^{\mathrm{GEV}} = \infty$, it follows that $\lim_{w_{1l,t} \to w_{1l}^{\max}} (1 + \tau_t^{\mathrm{GEV}}) = \infty$. The latter can also been seen from the fact that $w_{2,t}^{\mathrm{GEV}}$ has to be zero and according to equation (14), $w_{2,t}^{\mathrm{GEV}} = 1/(1 + \tau_t^{\mathrm{GEV}})$.

Summarizing the analysis, we can say that an increasing minimum wage has two effects: a negative effect on total wealth and a redistributive effect in favor of the low-skilled.