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ANALYSIS OF THE NETWORK WITH MULTIPLE CLASSES OF POSITIVE CUSTOMERS AND SIGNALS AT A NON-STATIONARY REGIME

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The object of research is G-network with positive customers and signals of multiple classes. The present paper describes an analysis of this network at a non-stationary regime, also provided a description of method for finding non-stationary state probabilities.

At the beginning of the article, a description of the network with positive customers and signals is given. A signal when entering the system destroys a positive customer of its type or moves the customer of its type to another system. Streams of positive customers and signals arriving to each of the network systems are independent. Selection of positive customers of all classes for service – randomly. For non-stationary state probabilities of the network, the system of Kolmogorov difference-differential equations (DDE) has been derived. It is solved by a modified method of successive approximations, combined with the method of series. The convergence of successive approximations with time has been proved to the stationary distribution of probabilities, the form of which is indicated in the article, and the sequence of approximations converges to the unique solution of the DDE system. Any successive approximation is representable in the form of a convergent power series with an infinite radius of convergence, the coefficients of which satisfy recurrence relations, which is convenient for computer calculations.

The obtained results can be applied for modeling behavior of computer viruses and attack in computer systems and networks, for example, as model impact of some file viruses on server resources.

Keywords: classes, combined with the method of series, g-network, method of successive approximations, non-stationary regime, positive customers and signals of multiple

1. INTRODUCTION

G-networks were first introduced in an article [1] by Gelenbe in 1991. The transient regime of this network has been considered in [2]. These networks have wide practical application: as a model of a neural networks [3], gene regulatory networks [4], models of the behavior of

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computer viruses [5]. The concept of a trigger was introduced in [6]; in contrast to a negative customer, it does not destroy a positive customer, but it moves it to another queue or sub-system. Combining the trigger and the negative customer into one object, the "signal" was introduced in [7], and the transient regime of such systems was first discussed in [8]. The work in [2,8] discusses the non-stationary probabilities of G-network states using the method of multidimensional generating functions. In [9], G-networks with positive and negative customers of multiple classes were introduced, where a negative customer of one type could destroy with a given probability a negative customer of another type, and expressions were obtained for the stationary state probabilities, and they were shown to have product form. In [10], the same network was considered at a transient regime under the assumption that a negative customer can destroy a positive customer of only its own type. Using the modified method of successive approximations, combined with the method of series, the non-stationary state probabilities were found. In [11], the steady-state probabilities of states for a network with positive customers and signals of multiple types were also obtained.

This article is devoted to finding the non-stationary probabilities of network states [11], by the modified method of successive approximations, combined with the method of series.

G-networks with revenues were introduced in [12]. Revenues in a G-network are used to calculate the losses by the modified method of successive approximations, combined with the series method in [13] and z-transforms method were used in [14], while revenues from transitions between states are considered to be random values with first and second moments. Simulation modeling of this network was carried out in [15]. Expected revenues for G-networks with signals were found in [16]. Revenues for G-networks with positive and negative customers of multiple classes were found by the method of successive approximations, combined with the method of series in information systems and the penetration of variable computer viruses, for example, DDoS attack [17]. For finding expected revenues, a method of successive approximations was applied, combined with the method of series in the assumption that a negative customer can destroy a positive customer of only its type has been found in paper [18]. We also note that transient response is relevant to the bioinformatics applications of G-networks [19,20].

This paper is also devoted to finding the expected revenues of network systems described in [11] by the modified method of successive approximations, combined with the method of series.

2. NETWORK DESCRIPTION

We will consider an open G-network with n single-queue systems (QS). Simple flow of customers arrives to the network from external environment (QS S_0) with the rate λ^+ and additional simplest signal flow also arrives with the rate $\lambda^{(1)}$. Streams arriving in all systems of the network are independent. All positive customers arrive the stream of class c independently of other customers which move in the queue S_i with a probability $p_{0ic}^+, \sum_{i=1}^n \sum_{c=1}^r p_{0ic}^+ = 1$. The service durations of positive customers of class c in i-th QS are distributed exponentially with the rate μ_{ic}. We assume that customers are randomly selected on service: if in i-th QS are k_{is} customers of class s, then the probability of select for service customer of class c is $(k_{ic}/\sum_{s=1}^r k_{is}),\ i=\overline{1,n},\ c=\overline{1,r}$.

The positive customer of class c processed by S_i moves to QS S_j as a positive customer of class s with a probability p_{icjs}^+, moves as a signal of class s with a probability p_{icjs}^-, and moves out of the network to the external environment with a probability $p_{ic0} = 1 - \sum_{j=1}^n \sum_{s=1}^r (p_{icjs}^+ + p_{icjs}^-)$.

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The signal of class c arriving to the system, in which there are no positive customers of class c, does not exert any influence on the queueing network and immediately disappears from it. Otherwise, while in this system arriving to signal of class c, the following can occur: arriving signal moves positive customer of class $c$from this system into queue S_j as a positive customer of class s with a probability q_{icjs}; in this case, signal is referred to as a trigger; signal acts as a negative customer of class c with a probability $q_{ic0} = 1 - \sum_{j=1}^{n}\sum_{s=1}^{r} q_{icjs}$ and destroys one positive customer of class c.

The network state at time t is described by the vector $(\vec{k}, t) = (k_{11}, k_{12}, \ldots, k_{1r}, k_{21}, k_{22}, \ldots, k_{2r}, \ldots, k_{n1}, k_{n2}, \ldots, k_{nr}, t)$, where k_{ic} - number positive customer of i-th QS of class c, which forms a homogeneous Markov process with a countable number of states. There is a need to find time-dependent state probability in non-stationary mode.

3. FINDING STATE PROBABILITIES

Denoted by I_{ic} vector of dimension $n \times r$, consisting of zeros, with the exception of the component with a number $r(i-1)+c$, which is equal to 1, $I_{00} - n \times r$ zero-vector, and $P(\vec{k}, t)$ – probability of state $\vec{k}(t)$; $u(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}$ – the Heaviside function. The following transitions of the Markov chain to the state $(\vec{k}, t)$ during the time Δt are possible:

(1) From the state $(\vec{k} - I_{js}, t)$ in this case in the j-th QS in time Δt, a positive customer of class s will arrive with a probability $\lambda^{+} p_{0js}^{+} u(k_{js}) \Delta t + o(\Delta t),\ j = \overline{1, n}, s = \overline{1, r}$;

(2) From the state $(\vec{k} + I_{ic}, t)$ in this case in the i-th QS in time Δt, a signal of class s will arrive and the signal acts as a negative customer of class $c$and destroys positive customer this class or positive customer and moves out of the network to the external environment or moves in j-th QS as a signal of class $s$and if there were no customers of this class in it with a probability
$(\lambda^{(1)} p_{0ic}^{-} q_{ic0} + \mu_{ic}(k_{ic} + 1/\sum_{s=1}^{r} k_{is} + 1) p_{ic0} + \mu_{ic}(k_{ic} + 1/\sum_{s=1}^{r} k_{is} + 1) p_{icjs}^{-}$
$(1 - u(k_{ic})))\Delta t + o(\Delta t),\ i = \overline{1, n},\ c = \overline{1, r}$;

(3) From the state $(\vec{k} + I_{ic} + I_{js}, t)$ in this case, after the end of the service of the positive customer of class c in the i-th QS, it is moved to the j-th QS again as a positive customer of class s or arrived to the i-th QS signal of class c and instantly moves the positive customer of class c from the i-th QS to the j-th QS as a positive customer of class $s$with a probability
$(\mu_{ic}(k_{ic} + 1/\sum_{s=1}^{r} k_{is} + 1) p_{icjs}^{+} + \lambda^{(1)} p_{0ic} q_{icjs}) u(k_{js}) \Delta t + o(\Delta t),\ i, j = \overline{1, n}$;

(4) From the state $(\vec{k} + I_{ic} + I_{js}, t)$ in this case, after the end of the customer of class c service in the i-th QS, it is sent to the j-th QS as a signal of class s that destroys positive customers of class s; probability of this event is equal to $\mu_{ic}(k_{ic} + 1/\sum_{s=1}^{r} k_{is} + 1) p_{icjs}^{-} q_{js0} \Delta t + o(\Delta t)$;

(5) From the state $(\vec{k} + I_{ic} + I_{js} - I_{ml}, t)$ in this case, after the end of the service of the customer of class c in the i-th QS, it is sent to the j-th QS as a signal of class s, which instantly moves the positive customer of class s from the j-th QS with a number m as a customer of class l; the probability of this event is equal to $\mu_{ic}(k_{ic} + 1/\sum_{s=1}^{r} k_{is} + 1) p_{icjs}^{-} q_{jm} u(k_{ml}) \Delta t + o(\Delta t)$;

(6) From the state $(\vec{k}, t)$ in this case, in time Δt no positive customers of all classes or signals of all classes are received in each, and no positive customers of all

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classes have been served in them for a time; the probability of this event is $1-\sum_{i=1}^{n}\sum_{c=1}^{r}[\lambda^{+}p_{0ic}^{+}+\lambda^{-}p_{0ic}^{-}+\mu_{ic}]\Delta t+o(\Delta t);\ i=\overline{1,n}$.

(7) From the remaining states with a probability $o(\Delta t)$.

Then, using the formula of total probability and taking the limit $\Delta t\to 0$, we obtain the system of difference-differential equations (DDE) for the state probabilities of the network:

$$\begin{aligned}\frac{dP(\vec{k},t)}{dt} &= -\sum_{i=1}^{n}\sum_{c=1}^{r}\left[\lambda_{0ic}^{+}+\lambda_{0ic}^{-}+\mu_{ic}\right]P(\vec{k},t)+\sum_{i=1}^{n}\sum_{c=1}^{r}\Bigg\{\lambda_{0ic}^{+}u(k_{ic})P(\vec{k}-I_{ic},t)\\
&\quad+\Bigg\{\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{ic0}+\lambda_{0ic}^{-}q_{ic0}\right)\\
&\quad+\sum_{j=1}^{n}\sum_{s=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}(1-u(k_{js}))p_{icjs}^{-}\Bigg\}P(\vec{k}+I_{ic},t)\\
&\quad+\sum_{j=1}^{n}\sum_{s=1}^{r}\Bigg[\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{icjs}^{+}+\lambda_{0ic}^{-}q_{icjs}\right)u(k_{js})P(\vec{k}+I_{ic}-I_{js},t)\\
&\quad+\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{icjs}^{-}q_{js0}P(\vec{k}+I_{ic}+I_{js},t)\\
&\quad+\sum_{m=1}^{n}\sum_{l=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{icjs}^{-}q_{ml}u(k_{ml})P(k+I_{ic}+I_{js}-I_{ml},t)\Bigg]\Bigg\}. \qquad \textbf{(1)}\end{aligned}$$

The system of DDE (1) is represented in the form:

$$\begin{aligned}\frac{dP(\vec{k},t)}{dt} &= -\Lambda(\vec{k})P(\vec{k},t)+\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi_{icjs}^{+-}(\vec{k})P(\vec{k}+I_{ic}-I_{js},t)\\
&\quad+\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi_{icjs}^{++}(\vec{k})P(\vec{k}+I_{ic}+I_{js},t)\\
&\quad+\sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi_{icjsml}^{++}(\vec{k})P(\vec{k}+I_{ic}+I_{js}-I_{ml},t), \qquad \textbf{(2)}\end{aligned}$$

where

$$\begin{aligned}\Lambda(\vec{k}) &= \sum_{i=1}^{n}\sum_{c=1}^{r}\left[\lambda_{0ic}^{+}+\lambda_{0ic}^{-}+\mu_{ic}\right],\\
\Phi_{icjs}^{+-}(\vec{k}) &= \delta_{0i}\delta_{0c}\lambda_{0js}^{+}u(k_{js})+\delta_{0j}\delta_{0s}\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{ic0}+\lambda_{0ic}^{-}q_{ic0}\right)\\
&\quad+\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}(1-u(k_{js}))p_{icjs}^{-}\\
&\quad+\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r}k_{is}+1}p_{icjs}^{+}+\lambda_{0ic}^{-}q_{icjs}\right)u(k_{js}),\end{aligned}$$

$$\Phi^{++}_{icjs}(\vec{k}) = \mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r} k_{is}+1} p^{-}_{icjs} q_{js0},$$

$$\Phi^{++}_{icjsml}(\vec{k}) = \mu_{ic}\frac{k_{ic}+1}{\sum_{s=1}^{r} k_{is}+1} p^{-}_{icjs} q_{ml} u(k_{ml}).\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases}.$$

The solution of the system (2) has the form:

$$P(\vec{k},t) = e^{-\Lambda(\vec{k})t}\left(P(\vec{k},0) + \int_0^t e^{\Lambda(\vec{k})x}\left(\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{+-}_{icjs}(\vec{k})P(\vec{k}+I_{ic}-I_{js},x)\right.\right.$$
$$+\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{++}_{icjs}(\vec{k})P(\vec{k}+I_{ic}+I_{js},x)$$
$$\left.\left.+\sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi^{++}_{icjsml}(\vec{k})P(\vec{k}+I_{ic}+I_{js}-I_{ml},x)\right)dx\right). \quad \textbf{(3)}$$

Let $P_q(\vec{k},t)$ – be approximation of $P(\vec{k},t)$ at the q-th iteration, $P_{q+1}(k,t)$ – the solution of (2) obtained by successive approximations. Then it follows from (3):

$$P_{q+1}(\vec{k},t) = e^{-\Lambda(\vec{k})t}\left(P(\vec{k},0) + \int_0^t e^{\Lambda(\vec{k})x}\left(\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{+-}_{icjs}(\vec{k})P_q(\vec{k}+I_{ic}-I_{js},x)\right.\right.$$
$$+\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{++}_{icjs}(\vec{k})P_q(\vec{k}+I_{ic}+I_{js},x)$$
$$\left.\left.+\sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi^{++}_{icjsml}(\vec{k})P_q(\vec{k}+I_{ic}+I_{js}-I_{ml},x)\right)dx\right). \quad \textbf{(4)}$$

As an initial approximation, we take the stationary distribution $P_0(\vec{k},t) = P(\vec{k}) = \lim_{t\to\infty} P(\vec{k},t)$, which satisfies the relation

$$\Lambda(\vec{k})P(\vec{k}) = \sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{+-}_{icjs}(\vec{k})P_q(\vec{k}+I_{ic}-I_{js})$$
$$+\sum_{i,j=1}^{n}\sum_{c,s=1}^{r}\Phi^{++}_{icjs}(\vec{k})P_q(\vec{k}+I_{ic}+I_{js})$$
$$+\sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi^{++}_{icjsml}(\vec{k})P_q(\vec{k}+I_{ic}+I_{js}-I_{ml}). \quad \textbf{(5)}$$

The following theorems are valid for successive approximations.

THEOREM 1: *Sequential approximations $P_q(\vec{k},t)$, $q=0,1,2,\ldots,$ converge for $t\to\infty$ to a stationary solution of the system of equation (2), and the sequence constructed according to scheme (4), for any zeroth approximation bounded in $\{P_q(\vec{k},t)\}$, $q=0,1,2,\ldots,$ converges for $q\to\infty$ to a unique solution of the system of equation (2).*

Theorem proof is similar to [10] for the network with multiple positive and negative customers.

THEOREM 2: *Each successive approximation $P_q(\vec{k},t),\ q \geq 1$, is represented in the form of a convergent power series*

$$P_q(\vec{k},t) = \sum_{l=0}^{\infty} d_{ql}^{+-}(\vec{k})t^l, \tag{6}$$

whose coefficients satisfy the recurrence relations:

$$\begin{aligned} d_{q+1l}^{+-}(\vec{k}) &= \frac{-\Lambda(\vec{k})^l}{l!}\left\{P(k,0) + \sum_{u=0}^{l-1}(-1)^{u+1}u!/\Lambda(\vec{k})^{u+1}D_{qu}^{+-}(\vec{k})\right\}, l \geq 0, d_{q0}^{+-}(k) \\ &= P(\vec{k},0), d_{0l}^{+}(k) = P(\vec{k},0)\delta_{l0}, D_{ql}^{+-}(\vec{k}) = \sum_{i,j=1}^{n}\sum_{s,c=1}^{r}\Phi_{icjs}^{+-}(\vec{k})d_{ql}^{+-}(\vec{k}+I_{ic}-I_{js}) \\ &\quad + \Phi_{icjs}^{++}(\vec{k})d_{ql}^{+-}(\vec{k}+I_{ic}+I_{js}) + \sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi_{icjsml}^{++}(\vec{k})d_{ql}^{+-}(\vec{k}+I_{ic}+I_{js}-I_{ml}). \end{aligned} \tag{7}$$

PROOF: We show that the coefficients of the power series (6) satisfy the recurrence relations (7). We substitute the successive approximations (6) in relation (4). Then, given that

$$e^{-\Lambda(\vec{k})t}\int_0^t e^{\Lambda(\vec{k})x}x^l dx = \left[\frac{1}{\Lambda(\vec{k})}\right]^{l+1} l! \sum_{j=l+1}^{\infty}\frac{[-\Lambda(\vec{k})]^l}{j!}, \quad l = 0,1,2,\ldots,$$

we obtain

$$\begin{aligned} \sum_{l=0}^{\infty} d_{ql}^{+-}(\vec{k})t^l = e^{-\Lambda(\vec{k})t}P(\vec{k},0) + \sum_{l=0}^{\infty}\sum_{i,j=1}^{n}\Bigg[&\sum_{c,s=1}^{r}\Phi_{icjs}^{+-}(\vec{k})\,d_{ql}^{+-}(\vec{k}+I_{ic}-I_{js}) \\ &+ \Phi_{icjs}^{++}(\vec{k})d_{ql}^{+-}(\vec{k}+I_{ic}+I_{js}) \\ &+ \sum_{i,j,m=1}^{n}\sum_{c,s,l=1}^{r}\Phi_{icjsml}^{++}(\vec{k})d_{ql}^{+-}(\vec{k}+I_{ic}+I_{js}-I_{ml})\Bigg]. \end{aligned}$$

Using (7), this series can be written in the form

$$\sum_{l=0}^{\infty} d_{ql}^{+-}(\vec{k})t^l = e^{-\Lambda(\vec{k})t}P(\vec{k},0) + \sum_{l=0}^{\infty} D_{ql}^{+-}(\vec{k})\left[\frac{1}{\Lambda(\vec{k})}\right]^{l+1} l! \sum_{u=l+1}^{\infty}\frac{[-\Lambda(\vec{k})]^u}{u!}t^u.$$

After interchanging summation indices and expanding $e^{-\Lambda(\vec{k})t}$ in a series in powers of t, we have

$$\sum_{l=0}^{\infty} d_{ql}^{+-}(\vec{k})t^l = e^{-\Lambda(\vec{k})t}P(\vec{k},0) + \sum_{l=0}^{\infty} D_{ql}^{+-}(\vec{k})\left[\frac{1}{\Lambda(\vec{k})}\right]^{l+1} l! \sum_{u=l+1}^{\infty}\frac{[-\Lambda(\vec{k})]^u}{u!}t^u.$$

If we equate the coefficients of t^l in expression (8), we obtain the relations (7) for the coefficients of the series (6).

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To find the radius of convergence of the power series (6), we can use the Cauchy–Hadamard formula $1/R(\vec{k}) = \lim_{l\to\infty} \sqrt[l]{|d_{ql}^{+-}(\vec{k})|}$. Similarly, as in [10], it can be shown that the radius of convergence of the series (10) is equal to $+\infty$. ■

4. FINDING EXPECTED REVENUES

Let us introduce vector I_{ic} of dimension $n \times r$, consisting of zeros, with the exception of the component with a number $r(i-1)+c$, which is equal to 1, I_{00} – $n \times r$ zero-vector, and $v_i(\vec{k},t)$ – be the expected revenue obtained by the i-th QS in time t, if at the initial time instant the network is in the state $\vec{k}$; $u(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}$ – the Heaviside function. The following transitions of the Markov random process to the state $(\vec{k},\, t)$ during the time Δt are possible:

In case (1), the revenue of the system S_i in this case will be $r_i(\vec{k})\Delta t + v_i(\vec{k} - I_{js}, t)$; if $i = j,\ s = c$ then the revenue of the system S_i will be $r_{0ic}(\vec{k} - I_{ic}) + v_i(\vec{k} - I_{ic}, t)$, where $r_{0ic}(\vec{k} - I_{ic})$, the revenue of the i-th system from the given transition.

In case (2), the revenue of the system S_i in this case will be $r_i(\vec{k})\Delta t + v_i(\vec{k} + I_{js}, t)$, if $i = j,\ s = c$ then the revenue of the system S_i will be $-R_{ic0}(\vec{k} + I_{ic}) + v_i(\vec{k} + I_{ic}, t)$, while $R_{ic0}(\vec{k} + I_{ic})$ the revenue of the i-th system from the given transition.

In case (3), the revenue of the system S_i in this case will be $r_i(\vec{k})\Delta t + v_i(\vec{k} + I_{ml} - I_{dh}, t)$; if $m = j,\ l = s,\ i = d,\ c = h$, then the revenue of the system S_i will be $-r_{jsic}(\vec{k} + I_{js} - I_{ic}) + v_i(\vec{k} + I_{js} - I_{ic}, t)$; if $k = i,\ l = c,\ j = d,\ s = h$, then the revenue of the system S_i will be $r_{icjs}(\vec{k} - I_{js} + I_{ic}) + v_i(\vec{k} - I_{js} + I_{ic}, t)$;

In case (4), the revenue of the system S_i in this case will be $r_i(\vec{k})\Delta t + v_i(\vec{k} + I_{ml} + I_{dh}, t)$; if $m = j, l = s, i = d, c = h$, then the revenue of the system S_i will be $r_{icjs}(\vec{k} + I_{js} + I_{ic}) + v_i(\vec{k} + I_{js} + I_{ic}, t)$;

In case (5), the revenue of the system S_i in this case will be $r_i(\vec{k} + I_{ml} + I_{dh} - I_{\alpha\beta}, t)\Delta t + v_i(\vec{k} + I_{ml} + I_{dh} - I_{\alpha\beta}, t)$; if $m = i,\ l = c$, the revenue of the system S_i will be $-r_{dh\alpha}(\vec{k} + I_{ic} + I_{dh} - I_{\alpha\beta}, t) + v_i(\vec{k} + I_{ic} + I_{dh} - I_{\alpha\beta}, t)$, if $\alpha = i, \beta = c$, then the revenue of the system S_i in this case will be $-r_{dh\alpha}(\vec{k} + I_{ml} + I_{dh} - I_{ic}, t) + v_i(\vec{k} + I_{ml} + I_{dh} - I_{ic}, t)$, otherwise $r_{dh\alpha}(\vec{k} + I_{ml} + I_{ic} - I_{\alpha\beta}, t) + v_i(\vec{k} + I_{ml} + I_{ic} - I_{\alpha\beta}, t)$,

In case (6), the revenue of the system S_i in this case will be $r_i(\vec{k})\Delta t + v_i(\vec{k}, t)$.

Then, using the formula of total probability and taking the limit $\Delta t \to 0$, we obtain a system of DDE for the expected revenues of the network:

$$
\begin{aligned}
\frac{dv_i(\vec{k},t)}{dt} &= r_i(\vec{k}) - \left[\lambda^+ + \lambda^- + \sum_{j=1}^{n}\sum_{s=1}^{r} \mu_{js} u(k_{js})\right] v_i(\vec{k},t) \\
&\quad + \sum_{j=1}^{n}\sum_{s=1}^{r} \lambda^+ p_{0js}^+ u(k_{js}) v_i(\vec{k} - I_{js}, t)
\end{aligned}
$$

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$$
\begin{aligned}
&+\sum_{j=1}^{n}\sum_{s=1}^{r}\left(\lambda^{(1)}p_{0js}^{-}q_{js0}+\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}p_{js0}\right.\\
&\left.+\sum_{j=1}^{n}\sum_{s=1}^{r}\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}\sum_{m=1}^{n}\sum_{l=1}^{r}p_{jsml}^{-}(1-u(k_{ml}))\right)v_i(\vec{k}+I_{js},t)\\
&+\sum_{c=1}^{r}\left\{\lambda^{+}p_{0ic}^{+}u(k_{ic})v_i(\vec{k}-I_{ic},t)+\left(\lambda^{(1)}p_{0ic}^{-}q_{ic0}+\mu_{ic}\frac{k_{ic}+1}{\sum_{s^*=1}^{r}k_{is^*}+1}p_{ic0}\right.\right.\\
&\left.+\mu_{ic}\frac{k_{ic}+1}{\sum_{s^*=1}^{r}k_{is^*}+1}\sum_{m=1}^{n}\sum_{l=1}^{r}p_{icml}^{-}(1-u(k_{ml}))\right)v_i(\vec{k}+I_{ic},t)\\
&+\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{+}+\lambda^{(1)}p_{0ic}q_{icjs}\right)u(k_{js})v_i(\vec{k}+I_{ic}-I_{js},t)\\
&+\sum_{m,d=1}^{n}\sum_{l,h=1}^{r}\left(\mu_{ml}\frac{k_{ml}+1}{\sum_{l^*=1}^{r}k_{ml^*}+1}p_{mldh}^{+}+\lambda^{(1)}p_{0ml}q_{mldh}\right)\\
&\quad\times u(k_{dh})v_i(\vec{k}+I_{ml}-I_{dh},t)\\
&+\sum_{m,d=1}^{n}\sum_{l,h=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l^*=1}^{r}k_{ml}+1}p_{mldh}^{-}q_{dh0}v_i(\vec{k}+I_{ml}+I_{dh},t)\\
&+\sum_{j=1}^{n}\sum_{c,s=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{-}q_{ic0}v_i(\vec{k}+I_{ic}+I_{js},t)\\
&+\sum_{m,d,\alpha=1}^{n}\sum_{l,h,\beta=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mldh}^{-}q_{dh\alpha\beta}u(k_{dh})v_i(\vec{k}+I_{ml}+I_{dh}-I_{\alpha\beta},t)\\
&+\sum_{d,\alpha=1}^{n}\sum_{h,\beta=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{l=1}^{r}k_{il}+1}p_{icdh}^{-}q_{dh\alpha\beta}u(k_{dh})v_i(\vec{k}+I_{ic}+I_{dh}-I_{\alpha\beta},t)\\
&+\sum_{m,\alpha=1}^{n}\sum_{l,\beta=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mlic}^{-}q_{ic\alpha\beta}u(k_{dh})v_i(\vec{k}+I_{ml}+I_{ic}-I_{\alpha\beta},t)\\
&+\sum_{m,d=1}^{n}\sum_{l,h=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mldh}^{-}q_{dhic}u(k_{dh})v_i(\vec{k}+I_{ml}+I_{dh}-I_{ic},t)\\
&-\left[\lambda^{(1)}p_{0js}^{-}q_{js0}+\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}p_{js0}\right.\\
&\left.+\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}p_{jsml}^{-}(1-u(k_{ml}))\right]R_{ic0}(\vec{k}-I_{ic},t)\\
&+\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{c=1}^{r}\left[\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{+}+\lambda^{(1)}p_{0ic}q_{icjs}\right)u(k_{ic})r_{icjs}(\vec{k}-I_{ic}+I_{js},t)\right.\\
&-\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{c=1}^{r}\left[\left(\mu_{js}\frac{k_{js}+1}{\sum_{l^*=1}^{r}k_{jl^*}+1}p_{jsic}^{+}+\lambda^{(1)}p_{0js}q_{jsic}\right)u(k_{js})r_{icjs}(\vec{k}-I_{js}+I_{ic},t)\right.
\end{aligned}
$$

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$$+\lambda^{+}p_{0ic}^{+}u(k_{ic})r_{0ic}(\vec{k}-I_{ic},t)+\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{-}q_{js0}v_i(\vec{k}+I_{ic}+I_{js},t)$$
$$+\sum_{d,\alpha=1}^{n}\sum_{h,\beta=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{l=1}^{r}k_{il}+1}p_{icdh}^{-}q_{dh\alpha\beta}u(k_{dh})r_{id\alpha}(\vec{k}+I_{ic}+I_{dh}-I_{\alpha\beta},t)$$
$$+\sum_{m,\alpha=1}^{n}\sum_{l,\beta=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mlic}^{-}q_{ic\alpha\beta}u(k_{dh})r_{mi\alpha}(\vec{k}+I_{ml}+I_{ic}-I_{\alpha\beta},t)$$
$$+\sum_{m,d=1}^{n}\sum_{l,h=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mldh}^{-}q_{dhic}u(k_{dh})r_{mdi}(\vec{k}+I_{ml}+I_{dh}-I_{ic},t). \quad \textbf{(8)}$$

Let $\vec{V}^T(\vec{k},t)=(v_1(\vec{k},t),v_2(\vec{k},t),\dots,v_n(\vec{k},t))$. Then (8) is represented in the form

$$\frac{d\vec{V}(\vec{k},t)}{dt}=-\Delta^{(\partial)}(\vec{k})\vec{V}(\vec{k},t)+\sum_{i,j,m=0}^{n}\sum_{c,s,l=1}^{n}\Theta_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}-I_{js},t)$$
$$+\Phi_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}+I_{js},t)+\Phi_{icjsml}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}+I_{js}-I_{ml},t)+\vec{E}(\vec{k}), \quad \textbf{(9)}$$

where

$$\vec{E}^T(\vec{k})=(E_1(\vec{k}),E_2(\vec{k}),\dots,E_n(\vec{k})),E_i(\vec{k})=r_i(\vec{k})+\lambda^{+}p_{0ic}^{+}u(k_{ic})r_{0ic}(\vec{k}-I_{ic})$$
$$-\left[\lambda^{(1)}p_{0js}^{-}q_{js0}+\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}p_{js0}\right.$$
$$\left.+\mu_{js}\frac{k_{js}+1}{\sum_{s^*=1}^{r}k_{js^*}+1}p_{jsml}^{-}(1-u(k_{ml}))\right]$$
$$\times R_{ic0}(\vec{k}-I_{ic},t)+\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{c=1}^{r}\left[\left(\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{+}+\lambda^{(1)}p_{0ic}q_{icjs}\right)\right.$$
$$\times u(k_{ic})r_{icjs}(\vec{k}-I_{ic}+I_{js},t)$$
$$-\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{c=1}^{r}\left[\left(\mu_{js}\frac{k_{js}+1}{\sum_{l^*=1}^{r}k_{jl^*}+1}p_{jsic}^{+}+\lambda^{(1)}p_{0js}q_{jsic}\right)\right.$$
$$\times u(k_{js})r_{icjs}(\vec{k}-I_{js}+I_{ic},t)+\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r}k_{il^*}+1}p_{icjs}^{-}q_{js0}v_i(\vec{k}+I_{ic}+I_{js},t)$$
$$+\sum_{d,\alpha=1}^{n}\sum_{h,\beta=1}^{r}\mu_{ic}\frac{k_{ic}+1}{\sum_{l=1}^{r}k_{il}+1}p_{icdh}^{-}q_{dh\alpha\beta}u(k_{dh})r_{id\alpha}(\vec{k}+I_{ic}+I_{dh}-I_{\alpha\beta},t)$$
$$+\sum_{m,\alpha=1}^{n}\sum_{l,\beta=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mlic}^{-}q_{ic\alpha\beta}u(k_{dh})r_{mi\alpha}(\vec{k}+I_{ml}+I_{ic}-I_{\alpha\beta},t)$$
$$+\sum_{m,d=1}^{n}\sum_{l,h=1}^{r}\mu_{ml}\frac{k_{ml}+1}{\sum_{l=1}^{r}k_{ml}+1}p_{mldh}^{-}q_{dhic}u(k_{dh})r_{mdi}(\vec{k}+I_{ml}+I_{dh}-I_{ic},t).$$

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$$\Phi_{icjs}^{(\partial)}(\vec{k}) = \mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r} k_{ic}+1} p_{icjs}^{-}\delta_{0m}\delta_{0l},$$

$$\Phi_{icjsml}^{(\partial)}(\vec{k}) = \mu_{ic}\frac{k_{ic}+1}{\sum_{l=1}^{r} k_{il}+1} p_{icjs}^{-} q_{jsml} u(k_{js}), \Delta^{(\partial)}(\vec{k}) = \lambda^{+} + \lambda^{-} + \sum_{j=1}^{n}\sum_{s=1}^{r} \mu_{ic} u(k_{ic}),$$

$$\Theta_{icjs}^{(\partial)}(\vec{k}) = \delta_{0j}\delta_{0s}\left(\lambda^{(1)} p_{0ic}^{-} q_{ic0} + \mu_{ic}\frac{k_{ic}+1}{\sum_{s^*=1}^{r} k_{is^*}+1} p_{ic0} \right.$$
$$\left. + \mu_{ic}\frac{k_{ic}+1}{\sum_{s^*=1}^{r} k_{is^*}+1}\sum_{m=1}^{n}\sum_{l=1}^{r} p_{icml}^{-}(1-u(k_{ml}))\right)$$
$$+ \delta_{0i}\delta_{0c}\lambda^{+} p_{0js}^{+} u(k_{js}) + \left(\mu_{ic}\frac{k_{ic}+1}{\sum_{l^*=1}^{r} k_{il^*}+1} p_{icjs}^{+} + \lambda^{(1)} p_{0ic} q_{icjs}\right) u(k_{js}).$$

The solution of the system (9) has the form:

$$\vec{V}(\vec{k},t) = e^{-\Delta^{(\partial)}(\vec{k})t}\left(\vec{V}(\vec{k},0) + \int_0^t e^{\Delta^{(\partial)}(\vec{k})x}\left\{\sum_{i,j.m=0}^{n}\sum_{c,s,l=1}^{n}\Theta_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}-I_{js},x)\right.\right.$$
$$\left.\left. +\Phi_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}+I_{js},x) + \Phi_{icjsml}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}+I_{js}-I_{ml},x)\right\} dx\right)$$
$$+ \frac{\vec{E}(k)}{\Delta^{(\partial)}(\vec{k})}\left[1 - e^{-\Delta^{(\partial)}(\vec{k})t}\right]. \tag{10}$$

Let $\vec{V}_q(\vec{k},t)$ – be the approximation of $\vec{V}(\vec{k},t)$ at the q-th iteration, $\vec{V}_{q+1}(k,t)$ – the solution of (9) obtained by successive approximations. Then it follows from (3):

$$\vec{V}_{q+1}(\vec{k},t) = e^{-\Delta^{(\partial)}(\vec{k})t}\left(\vec{V}(\vec{k},0) + \int_0^t e^{\Delta^{(\partial)}(\vec{k})x}\left\{\sum_{i,j.m=0}^{n}\sum_{c,s,l=1}^{n}\Theta_{icjs}^{(\partial)}(\vec{k})\vec{V}_q(\vec{k}+I_{ic}-I_{js},x)\right.\right.$$
$$\left.\left. +\Phi_{icjs}^{(\partial)}(\vec{k})\vec{V}_q(\vec{k}+I_{ic}+I_{js},x) + \Phi_{icjsml}^{(\partial)}(\vec{k})\vec{V}_q(\vec{k}+I_{ic}+I_{js}-I_{ml},x)\right\} dx\right)$$
$$+ \frac{\vec{E}(k)}{\Delta^{(\partial)}(\vec{k})}\left[1 - e^{-\Delta^{(\partial)}(\vec{k})t}\right]. \tag{11}$$

As an initial approximation, we take the stationary distribution $\vec{V}_0(\vec{k},t) = \vec{V}(\vec{k}) = \lim_{t\to\infty}\vec{V}(\vec{k},t)$, which satisfies the relation

$$\Delta^{(\partial)}(\vec{k})\vec{V}(\vec{k}) = \sum_{i,j=0}^{n}\sum_{c,s=1}^{n}\Theta_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}-I_{js}) + \Phi_{icjs}^{(\partial)}(\vec{k})\vec{V}(\vec{k}+I_{ic}+I_{js})$$
$$+ \Phi_{icjsml}^{(\partial)}(\vec{k})\vec{V}_q(\vec{k}+I_{ic}+I_{js}-I_{ml}) + \vec{E}(\vec{k}). \tag{12}$$

The following theorems are valid for successive approximations.

THEOREM 3: *Sequential approximations $\vec{V}_q(\vec{k},t)$, $q = 0,1,2,\ldots,$ converge for $t\to\infty$ to a stationary solution of the system of equation* (2), *and the sequence constructed according to*

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scheme (4), *for any zeroth approximation bounded by* $\{\vec{V}_q(\vec{k},t)\}$, $q = 0, 1, 2, \ldots$, *converges for* $q \to \infty$ *to a unique solution of the system of equation* (2).

Theorem 4: *Each successive approximation* $\vec{V}_q(\vec{k},t)$, $q \geq 1$ *is represented in the form of a convergent power series:*

$$\vec{V}_q(\vec{k},t) = \sum_{l=0}^{\infty} \vec{g}_{ql}^{(\partial)}(\vec{k}) t^l, \tag{13}$$

whose coefficients satisfy the recurrence relations:

$$\begin{aligned}
&\vec{g}_{q+1l}^{(\partial)}(\vec{k}) = \frac{-\Delta^{(\partial)}(\vec{k})^l}{l!} \left\{ \vec{V}(\vec{k},0) - \frac{\vec{E}(\vec{k})}{\Delta^{(\partial)}(\vec{k})} + \sum_{u=0}^{l-1} \frac{(-1)^{u+1} u!}{\Delta^{(\partial)}(\vec{k})^{u+1}} \vec{G}_{q+1l}^{(\partial)}(\vec{k}) \right\}, \\
&l \geq 0 \vec{g}_{q0}^{(\partial)}(\vec{k}) = \vec{V}(\vec{k},0), \vec{g}_{0l}^{(\partial)}(\vec{k}) = \vec{V}(\vec{k},0)\delta_{l0}, \\
&\vec{G}_{ql}^{(\partial)}(\vec{k}) = \sum_{i,j,m=1}^{n} \sum_{c,s,l=1}^{r} \Big[\Theta_{icjs}^{(\partial)}(\vec{k}) \vec{g}_{ql}^{(\partial)}(\vec{k} + I_{ic} - I_{js}) \\
&\qquad + \Phi_{icjs}^{(\partial)}(\vec{k}) \vec{g}_{ql}^{(\partial)}(\vec{k} + I_{ic} + I_{js}) + \Phi_{icjsml}^{(\partial)}(\vec{k}) \vec{g}_{ql}^{(\partial)}(\vec{k} + I_{ic} + I_{js} - I_{ml}) \Big].
\end{aligned} \tag{14}$$

Theorems 3 and 4 are proofed similarly as in [18] for the network with batch removal of positive customers.

5. MODEL EXAMPLE

Let $n = 5$ and $r = 3$ be types of positive customers and signal. Let the probabilities of arriving of positive customers and signals to the i-th system of types c be equal respectively $p_{011}^{+} = 0,15$; $p_{012}^{+} = 0,1$; $p_{013}^{+} = 0,05$; $p_{021}^{+} = 0,05$; $p_{022}^{+} = 0,03$; $p_{022}^{+} = 0,02$; $p_{031}^{+} = p_{041}^{+} = p_{051}^{+} = 0,1$; $p_{032}^{+} = p_{042}^{+} = p_{052}^{+} = 0,06$; $p_{033}^{+} = p_{043}^{+} = p_{053}^{+} = 0,04$; $p_{0i1}^{-} = 0,1, p_{0i2}^{-} = 0,06, p_{0i3}^{-} = 0,04$, $i = \overline{1,5}$; and $\sum_{i=1}^{5}\sum_{c=1}^{3} p_{0ic}^{+} = 1$; $\sum_{i=1}^{5}\sum_{c=1}^{3} p_{0ic}^{-} = 1$. Let also the intensities of incoming streams of positive customers and signals are equal respectively $\lambda^{+} = 100$ and $\lambda^{(\partial)} = 90$.

Let the rates of customers of type c in the network systems are equal:

$\mu_{11} = 50; \mu_{12} = 30; \mu_{13} = 20; \mu_{21} = 50; \mu_{22} = 40; \mu_{23} = 20; \mu_{31} = 50; \mu_{32} = 40; \mu_{33} = 20; \mu_{51} = 50; \mu_{52} = 30; \mu_{41} = 50; \mu_{42} = 40; \mu_{43} = 20, \mu_{53} = 20$. Suppose also that the probabilities of the transitions of positive customers and signals between the QS are equal

$$\begin{aligned}
&p_{1111}^{+} = 0,01, p_{1112}^{+} = 0,012, p_{1113}^{+} = 0,011, p_{1211}^{+} = 0,01, p_{1212}^{+} = 0,012, p_{1213}^{+} = 0,011, \\
&p_{1311}^{+} = 0,01, p_{1312}^{+} = 0,012, p_{1313}^{+} = 0,012, p_{1111}^{-} = 0,01, p_{1112}^{-} = 0,012, p_{1121}^{+} = 0,03; \\
&p_{1122}^{+} = 0,036; p_{1123}^{+} = 0,033; p_{1113}^{-} = 0,011, p_{1211}^{-} = 0,01, p_{1212}^{-} = 0,012, p_{1213}^{-} = 0,011, \\
&p_{1311}^{-} = 0,01, p_{1312}^{-} = 0,012, p_{1313}^{-} = 0,012, p_{1221}^{+} = 0,03; p_{1222}^{+} = 0,036; p_{1223}^{+} = 0,033; \\
&p_{1321}^{+} = 0,03; p_{1322}^{+} = 0,036; p_{1323}^{+} = 0,036; p_{1121}^{-} = 0,01, p_{1122}^{-} = 0,012, p_{1123}^{-} = 0,011, \\
&p_{1221}^{-} = 0,01, p_{1222}^{-} = 0,012, p_{1223}^{-} = 0,011, p_{1321}^{-} = 0,01, p_{1322}^{-} = 0,012, p_{1323}^{-} = 0,012, \\
&p_{12i1}^{+} = 0,005, p_{12i2}^{+} = 0,006, p_{12i3}^{+} = 0,0055, p_{13i1}^{+} = 0,005, p_{13i2}^{+} = 0,006, p_{13i3}^{+} = 0,006, \\
&p_{11i1}^{-} = 0,005, p_{11i2}^{-} = 0,006, p_{11i3}^{-} = 0,0055, p_{12i1}^{-} = 0,005, p_{12i2}^{-} = 0,006, p_{12i3}^{-} = 0,0055,
\end{aligned}$$

$p^-_{13i1} = 0,005, p^-_{13i2} = 0,006, p^-_{13i3} = 0,006, i = \overline{3,5}, p_{110} = 0,05; p_{120} = 0,03; p_{120} = 0,02;$
$p^+_{2311} = 0,01, p^+_{2312} = 0,012, p^+_{2313} = 0,012, p^+_{2111} = 0,01, p^+_{2112} = 0,012, p^+_{2113} = 0,011,$
$p^+_{2211} = 0,01, p^+_{2212} = 0,012, p^+_{2213} = 0,011, p^+_{2311} = 0,01, p^+_{2321} = 0,02, p^+_{2322} = 0,024,$
$p^+_{2323} = 0,024, p^-_{2321} = 0,012, p^-_{2322} = 0,012, p^-_{2323} = 0,01, p^+_{2121} = 0,02, p^+_{2122} = 0,024,$
$p^+_{2123} = 0,022 p^+_{2222} = 0,024, p^+_{2223} = 0,022, p^+_{2321} = 0,02, p^-_{2121} = 0,01, p^-_{2122} = 0,012,$
$p^-_{2123} = 0,011, p^-_{2221} = 0,01, p^+_{11i1} = p^+_{11i3} = 0,005, p^+_{11i2} = 0,006, p^-_{2222} = 0,012,$
$p^-_{2223} = 0,011, p^-_{2321} = 0,01, p^-_{22i1} = p^+_{22i1} = 0,005, p^-_{22i2} = p^+_{22i2} = 0,006,$
$p^-_{22i3} = p^+_{22i2} = 0,0055, p^-_{23i1} = p^+_{23i1} = 0,005, p^-_{23i2} = p^+_{23i2} = 0,006, p^+_{j1i1} = p^-_{j1i1} = 0,01,$
$p^-_{23i1} = p^+_{23i1} = 0,005, p^-_{23i2} = p^+_{23i2} = 0,006, p^+_{j1i2} = p^-_{j1i2} = 0,012, p^+_{j1i3} = p^-_{j1i3} = 0,011,$
$p^+_{i3j2} = p^-_{i3j2} = 0,012, p^+_{i3j3} = p^-_{i3j3} 0,012, i,j = \overline{3,5,} p_{i10} = 0,1; p_{i20} = 0,07; p_{i30} = 0,03,$
$i = \overline{2,5}. q_{icjs} = 0,02, i \neq j, i,j > 0, q_{icis} = 0,02; q_{ic0} = 1/30.$

Let $r_i(\bar{k}) = 2, r_{0ic}(\vec{k}) = 2, R_{ic0} = 4, r_{icjs}(\vec{k}) = 4, r_{ij\alpha} = 3.$

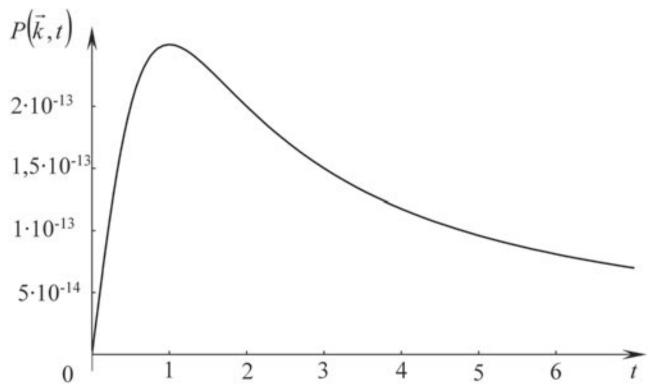

FIGURE **1.** State probability $\vec{k}$ at [0; 7].

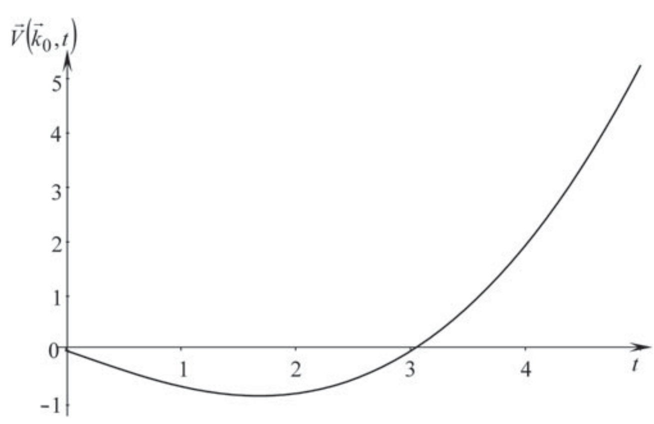

FIGURE **2.** Revenues of the second QS at [0; 5].

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Find the state probability $\vec{k} = (1,1,1,2,2,2,3,3,3,4,4,4,5,5,5)$ and expected revenues of the second QS, if the state $\vec{k}_0 = (1,1,1,1,1,1,2,2,2,3,3,3,2,2,2)$ under the initial condition $v_2(\vec{k}_0, 0) = 0$. Solving the problem using the programming language C # on the interval $[0,7]$ with $\varepsilon = 10^{-6}$, we obtain the dependence presented in Figures 1 and 2.

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