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ASYMPTOTICALLY OPTIMAL LOAD DISTRIBUTION FOR MULTIPATH STREAMING UNDER FEC

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Multipath streaming protocols have recently attracted much attention because they provide an effective means to provide high-quality streaming over the Internet. Most existing multipath streaming schemes also apply forward error correction (FEC) encoding in the stream so as to provide high-quality streaming of prestored or live media content. However, the problem of how to intelligently split the FEC-encoded stream among multiple available paths has not been fully addressed. Most previous work focused on protocol design or heuristic-based engineering approaches. Exact analysis turns out to be hard, as it involves heavy combinatorics computation. In this article, we develop an analytical model and use asymptotic analysis to address the problem of optimal load distribution. Using asymptotic approximations, we propose a closed-form formulation for the optimal load distribution problem. We then develop interesting properties of the optimal solution based on majorization, interchanging argument, and optimization techniques. These results are surprisingly simple yet insightful. We further demonstrate through simulation that our asymptotic solution works quite well in practice.

1. INTRODUCTION

Live streaming applications have become increasingly popular nowadays, driven by the widespread adoption of broadband networks. Such applications typically have stringent quality of service requirements, and the transport protocol must ensure that the streaming content is delivered in time, with minimal losses. Multipath streaming is one promising approach that leverages the availability of multiple paths between end hosts and exploits such path diversity to improve the streaming quality. Most of these multipath streaming schemes also apply forward error correction (FEC) encoding in the stream to guard against packet losses, so that the receiver can recover from packet losses without retransmission. With the joint benefits of multipath and FEC, these schemes can successfully provide high-quality streaming of prestored or live media content.

Exploiting multipath to improve the quality of streaming has attracted much attention. A broad overview of the general area can be found in Apostolopoulos and Trott [2]. Although multipath is believed to be beneficial, the problem of how to optimally split the load accross multiple lossy paths has not been well understood. Most previous work focuses on new protocol designs in which the benefits of multipath streaming are explored using heuristic policies and simulations; see, for example, Chu and Nahrstedt [7], Begen, Altunbasak, and Ergun [3], Nguyen and Zakhor [13], Abdouni, Cheng, Chow, Golubchik, Lee, and Lui [1], Wang, Wei, Guo, and Towsley [15], and Sharma, Kalyanaraman, Kar, Ramakrishnan, and Subramanian [14].

Analytical models for multipath streaming under FEC have been developed by Golubchik, Lui, Tung, Chow, Lee, Franceschinis, and Anglano [10], where the loss characteristics on a network path are characterized by a so-called Gilbert model (refer to Section 2) and it was illustrated that, compared with single-path streaming, multipath streaming can reduce the length of bursty losses and the correlation in consecutive packet losses, thus having the potential advantage of improving the quality of service delivery. Abdouni et al. [1] considered an optimization formulation for the load distribution problem in multipath streaming and showed that both the packet loss rate and the loss correlations are important when choosing an optimization objective. Chow, Golubchik, Lui, and Lee [6] further extended the previous studies to a functional Gilbert model where the loss characteristics can depend on the data rate. Most of these studies focused on exact distribution analysis (of the irrecoverable probability of an FEC group under a fixed a load splitting scheme), which can be derived through some iterative algorithms. However, as these algorithms involve heavy combinatorics computation, the task of finding the optimal load splitting scheme becomes almost prehibitive. Although optimal solutions were explored through numerical experiments and simulations, it is hard to give many engineering insights. Their solutions also require fairly accurate knowledge of the path loss model, which is difficult to obtain in practice.

In this article, we tackle the optimal load distribution problem for multipath streaming under FEC through an asymptotic analysis. We study the same model as in Golubchik et al. [10] but rely on an asymptotic approximation to get around the combinatorics computation challenges. Such asymptotic approximation then enables

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us to study the optimal load distribution problem through a closed-form optimization formulation. We further derive useful properties of the optimal load splitting solutions under different scenarios. Specifically, we show the following:

- Given a set of homogeneous paths, equal splitting over all paths achieves the lowest irrecoverable error if the probability of each path being lossy is lower than the maximum tolerable fraction of losses for a FEC group to avoid an irrecoverable error;
- Given a set of heterogeneous paths, it is asymptotically optimal to split the traffic such that the load on each path is inversely proportional to its loss rate.

These surprisingly simple results can offer valuable insights for the protocol design. To the best of our knowledge, this is the first work that provides closed-form optimal solutions for multipath load splitting, as the previous work all involve heavy combinatorics computation. We further demonstrate through simulation that our asymptotic solution works quite well in practice, even for small group sizes.

The rest of the article is organized as follows. In Section 2 we describe an analytical model to represent the multipath streaming system under FEC. An asymptotic analysis is then presented in Section 3, which enables a closed-form approximation to the irrecoverable error of a FEC frame under a given splitting scheme. In Section 4 the optimal load splitting problem is studied using the closed-form approximation. These analytical results are further validated through a simulation study in Section 5. Concluding remarks are presented in Section 6.

2. MODEL

Consider a streaming application that generates constant bit rate (CBR) stream data at rate λ and needs to transmit the data in realtime from sender s to receiver d. The streaming application will use FEC with group size N and redundancy r ($r > 0$) for the transmission. We assume a simple FEC scheme as follows: For every group of N data packets, we generate $N' = N(1 + r)$ packets. We refer to these N' packets as a FEC group. The encoding scheme is such that if the number of losses within a FEC group is less than or equal to Nr, then we can reconstruct the original N data packets with that FEC group.

There are K potential overlay paths between nodes s and d. We assume, as in Golubchik et al. [10], that the network loss rate on each path is dominated by some on and off background traffic on its bottleneck link, and we use a Gilbert model to model the loss process. Such a model characterizes the potential correlations between consecutive packet losses on a network path and is known to be more accurate than the independent loss model, as packet losses tend to occur in a burst due to buffer overflow. See, for example Cidon, Khamisy, and Sidi [8], and Bolot, Fosse-Parisis, and Towsley [5].

Under the Gilbert model, the packet loss process along path k is a two-state continuous-time Markov process $\{I_k(t)\}$, with $I_k(t) \in \{0, 1\}$. A packet transmitted at time t is considered lost if the state of path k is $I_k(t) = 1$; and otherwise it is considered successfully delivered. The infinitesimal generator for this Gilbert model of path k is

$$\mathbf{Q}_k = \begin{bmatrix} -\mu_0^{(k)} & \mu_0^{(k)} \\ \mu_1^{(k)} & -\mu_1^{(k)} \end{bmatrix}.$$

Let π_k denote the stationary probability that path k is in the lossy state. From the balance equation, it is easily checked that $\pi_k := \mu_0^{(k)}/(\mu_0^{(k)} + \mu_1^{(k)})$.

Let $P_{j,j'}^{(k)}(t)$ be the probability that path k is in state j' at time t, given that it was in state j at time 0; that is, $P_{j,j'}^{(k)}(t) = P\{I_k(t) = j' | I_k(0) = j\}$. The transition probabilities $P_{j,j'}^{(k)}(t)$ can be derived in closed form (Kulkarni [11]) as follows:

$$P_{j,j'}^{(k)}(t) = \begin{cases} (1-\pi_k)(1-e^{-\mu_\Sigma^{(k)} t}), & j=1, j'=0 \\ \pi_k(1-e^{-\mu_\Sigma^{(k)} t}), & j=0, j'=1 \\ \pi_k + (1-\pi_k)e^{-\mu_\Sigma^{(k)} t}, & j=1, j'=1 \\ (1-\pi_k) + \pi_k e^{-\mu_\Sigma^{(k)} t}, & j=0, j'=0 \end{cases} \quad \text{for all } t \geq 0,$$

where $\mu_\Sigma^{(k)} = \mu_0^{(k)} + \mu_1^{(k)}$.

Consider a simple multipath routing strategy under load splitting vector $\mathbf{x} = [x_k]$, $\sum_k x_k = 1$ as follows. For every N packets, exactly Nx_k packets[1] will use path k and they are evenly spaced throughout the time window N/λ with interarrival times $\tau_k = 1/\lambda(1+r)x_k$. This can be achieved by a proportional round-robin scheme based on the weight vector $\mathbf{x}$.

Let $X_n^{(k)} = 0$ (resp. $= 1$) if the nth transmitted packet on path k is lost (resp. successfully delivered) for $x_k > 0$. Then the packet delivery process $\{X_n^{(k)}\}$ on path k forms a discrete-time Markov chain, with transition matrix

$$\mathbf{P}_k = \begin{bmatrix} 1-\alpha_k & \alpha_k \\ \beta_k & 1-\beta_k \end{bmatrix},$$

where

$$\alpha_k = \pi_k(1-e^{-\mu_\Sigma^{(k)}\tau_k}) \quad \text{and} \quad \beta_k = (1-\pi_k)(1-e^{-\mu_\Sigma^{(k)}\tau_k}). \tag{1}$$

Let $S_n^{(k)} := \sum_{l=1}^n X_l^{(k)}$ be the number of successes in n transmissions on path k and let $S_0^{(k)} = 0$. Then the total number of successes in n transmissions over all paths under load splitting vector $\mathbf{x}$ is $S_n(\mathbf{x}) = \sum_{k=1}^K S_{nx_k}^{(k)}$. For a FEC group of N packets, if $S_{N(1+r)}(\mathbf{x})$ is smaller than N, then it is not possible to recover all N original data packets within the FEC group. The *probability of an irrecoverable error* within a FEC group is thus defined as

$$L_N(\mathbf{x}) := P(S_{N(1+r)}(\mathbf{x}) < N),$$

which we also refer to as the *group loss rate*. The percentage of data that cannot be recovered within a FEC group is typically called the *information loss rate*. Since the

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optimal decision that minimizes the information loss rate is quite similar to that which minimizes the group loss rate, it is sufficient to focus on minimizing the group loss rate L_N.

As presented in Golubchik et al. [10], the exact distribution of $S_n(\mathbf{x})$ can be derived using a complex two-dimensional recursive procedure with heavy combinatorics computation. This makes the task of finding the optimal load splitting scheme almost prehibitive. Although optimal solutions can be explored through numerical experiments and simulations, it is hard to give many engineering insights.

3. ASYMPTOTIC ANALYSIS

We next present an asymptotic analysis on $S_n(\mathbf{x})$, which is more mathematically tractable and helps provide useful guidelines in searching for an optimal design that achieves the least information loss.

Consider a fixed load splitting scheme defined by $\mathbf{x} = [x_k]$, $\sum_k x_k = 1$. The following asymptotic result for total successful transmissions $S_n^{(k)}$ on path k (with $x_k > 0$) is immediate from the well-known central limit theorem for Markov chains (see Cox and Miller [9, p. 138]).

LEMMA 1: *Consider Markov chain $\{X_n^{(k)}\}_{n\geq 1}$ with transition matrix $\mathbf{P}_k$ and cumulative sum $S_n^{(k)} = \sum_{l=1}^n X_l^{(k)}$. Then*

$$\sqrt{n}\left(\frac{S_n^{(k)}}{n} - m_k\right) \xrightarrow{d} \sigma_k(x_k)N(0,1) \quad \textit{as } n \longrightarrow \infty,$$

where

$$m_k = \frac{\alpha_k}{\alpha_k + \beta_k} \quad \textit{and} \quad \sigma_k^2(x_k) = \frac{\alpha_k\beta_k(2-\alpha_k-\beta_k)}{(\alpha_k+\beta_k)^3} \tag{2}$$

and $\xrightarrow{d}$ denotes convergence in distribution.

Plug Eq. (1) into Eq. (2); we see that

$$m_k = 1 - \pi_k \quad \text{and} \quad \sigma_k^2(x_k) = \pi_k(1-\pi_k)\frac{1+e^{-\mu_\Sigma^{(k)}\tau_k}}{1-e^{-\mu_\Sigma^{(k)}\tau_k}}, \tag{3}$$

where $\sigma_k^2(x_k)$ is a function of x_k due to the dependence of τ_k on x_k.

Lemma 1 states that the number of successful transmissions $S_n^{(k)}$ on path k (with $x_k > 0$) is asymptotically normal with mean nm_k and variance $n\sigma_k^2(x_k)$. We next show that the total successful transmissions on all paths $S_n(\mathbf{x}) = \sum_{k:x_k>0} S_{nx_k}^{(k)}$ is also asymptotically normal. The result might appear to be immediate from Lemma 1. However, since convergence in distribution is not additive in general (see, e.g., Billingsley [4]), a rigorous proof is needed. See the Appendix for the detailed proof.

THEOREM 2:

$$\sqrt{n}\left(\frac{S_n(\mathbf{x})}{n} - m(\mathbf{x})\right) \xrightarrow{d} \sigma(\mathbf{x})N(0,1) \quad \textit{as } n \longrightarrow \infty,$$

where

$$m(\mathbf{x}) := \sum_{k:x_k>0} x_k m_k \quad \textit{and} \quad \sigma^2(\mathbf{x}) := \sum_{k:x_k>0} x_k \sigma_k^2(x_k).$$

Note from Eq. (3) that $\sigma_k^2(x_k)$ is only defined for $x_k > 0$. We extend its definition to the case $x_k = 0$ and define $\sigma_k^2(0) = \pi_k(1 - \pi_k)$. Hence, we can rewrite $m(\mathbf{x})$ and $\sigma^2(\mathbf{x})$ as

$$m(\mathbf{x}) = \sum_{k=1}^{K} x_k m_k \quad \text{and} \quad \sigma^2(\mathbf{x}) = \sum_{k=1}^{K} x_k \sigma_k^2(x_k).$$

Theorem 2 shows that $S_n(\mathbf{x})$ is asymptotically normal with

$$E[S_n(\mathbf{x})] \sim nm(\mathbf{x}),$$
$$Var(S_n(\mathbf{x})) \sim n\sigma^2(\mathbf{x}),$$

where the notation $f_n \sim g_n$ denotes $\lim_{n\to\infty}(f_n/g_n) = c$ for $0 < c < \infty$.

We can now approximate the irrecoverable error L_N for a large group size N. Recall that for a group of N data packets, the FEC scheme under redundancy r generates a total number of $N(1 + r)$ data packets and an irrecoverable error occurs when the total number of successfully transmitted packets is less than N. The following corollary is immediate.

COROLLARY 3: *For a large group size N, we can approximate the irrecoverable error $L_N(\mathbf{x})$ by*

$$L_N(\mathbf{x}) \approx \Phi\left(\frac{a - m(\mathbf{x})}{\sigma(\mathbf{x})} \frac{\sqrt{N}}{\sqrt{a}}\right). \tag{4}$$

Furthermore, if $a - m(\mathbf{x}) < 0$, then $L_N(\mathbf{x}) \to 0$ as $N \to \infty$ and

$$\ln(L_N(\mathbf{x})) \sim -N\frac{(a - m(\mathbf{x}))^2}{2a \cdot \sigma^2(\mathbf{x})}. \tag{5}$$

On the other hand, if $a - m(\mathbf{x}) > 0$, then $L_N(\mathbf{x}) \to 1$ as $N \to \infty$ and

$$\ln(1 - L_N(\mathbf{x})) \sim -N\frac{(a - m(\mathbf{x}))^2}{2a\sigma^2(\mathbf{x})}. \tag{6}$$

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PROOF: Denote $N' = N(1+r)$ and $a = 1/(1+r)$; then

$$\begin{aligned} L_N(\mathbf{x}) &= P(S_{N'}(\mathbf{x}) \le N) \\ &= P\left(\frac{S_{N'}(\mathbf{x})}{N'} \le a\right) \\ &= P\left(\sqrt{N'}\left(\frac{S_{N'}(\mathbf{x})}{N'} - m(\mathbf{x})\right) \le \sqrt{N'}(a - m(\mathbf{x}))\right) \\ &\approx \Phi\left(\frac{a - m(\mathbf{x})}{\sigma(\mathbf{x})}\frac{\sqrt{N}}{\sqrt{a}}\right), \end{aligned}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal random variable.

Based on the above approximation, we see that the irrecoverable error $L_N(\mathbf{x})$ converges to zero as N gets large if $a - m(\mathbf{x}) < 0$ and it goes to 1 if $a - m(\mathbf{x}) > 0$. Expressions (5) and (6) are based on the fact that for large $y(> 0)$, $\Phi^c(y) \sim e^{-y^2/2}$. ■

4. OPTIMAL LOAD SPLITTING

Consider a FEC scheme with fixed group size N and redundancy r. Based on Eq. (4), we see that in order to find the optimal load splitting scheme $\mathbf{x}$ that minimizes the irrecoverable error $L_N(\mathbf{x})$, it suffices to solve

$$\max_{\mathbf{x}:\sum_k x_k = 1} \frac{m(\mathbf{x}) - a}{\sqrt{a}\sigma(\mathbf{x})} = \max_{\mathbf{x}:\sum_k x_k = 1} \frac{m(\mathbf{x}) - a}{\sqrt{v(\mathbf{x})}}, \tag{7}$$

where $m(\mathbf{x}) = \sum_{k=1}^K x_k m_k$ and $v(\mathbf{x}) = a\sigma^2(\mathbf{x}) = \sum_{k=1}^K a x_k \sigma_k^2(x_k)$.

For most streaming applications, the data rate (constant bit rate) λ tends to be large; hence, the interarrival times τ_k are typically small. Using the approximation $e^{-\mu_\Sigma^{(k)}\tau_k} \approx 1 - \mu_\Sigma^{(k)}\tau_k$ in Eq. (3), we can then approximate $\sigma_k^2(x_k)$ as

$$\sigma_k^2(x_k) = b_k\left(\frac{\rho_k}{a}x_k - 1\right),$$

where $b_k = \pi_k(1-\pi_k)$ and $\rho_k = 2\lambda/\mu_\Sigma^{(k)}$. Consequently,

$$v(\mathbf{x}) = \sum_{k=1}^K a x_k \sigma_k^2(x_k) = \sum_{k=1}^K b_k x_k(\rho_k x_k - a).$$

We will focus on solving Eq. (7) under the above expression of $v(\mathbf{x})$. It can be easily shown that the optimal solution to Eq. (7) is unique and can be obtained by solving the equations from Karush–Kuhn–Tucker (KKT) conditions.

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Observe from the optimization formulation (7) that there are interesting trade-offs between single path versus multipath load splitting. On the one hand, sending more traffic to the less lossy (higher m_k) path increases $m(\mathbf{x})$, encouraging the usage of the single (least lossy) path; on the other hand, as losses tend to happen in a burst (when packets are sent during the lossy period), having smaller x_k would result a smaller variance $\sigma_k^2(x_k)$, encouraging the usage of a multipath to disperse the burstiness of traffic to reduce losses.

4.1. Homogeneous Case

Suppose that all paths are homogeneous with $\pi_k \equiv \pi$, $\rho_k \equiv \rho$, and $b_k \equiv b$. We have the following theorem.

THEOREM 4: *Suppose the paths are homogeneous.*

(*i*) *If* $\pi < 1 - a$, *then the optimal solution to Eq.* (7) *is multipath routing with equal splitting.*

(*ii*) *If* $\pi > 1 - a$, *then the optimal solution to Eq.* (7) *is single-path routing with no splitting.*

PROOF: When the paths are homogeneneous, $m(\mathbf{x}) - a = 1 - a - \pi$ and it does not depend on $\mathbf{x}$. In order to solve Eq. (7), it suffices to minimize (resp. maximize) $\sigma^2(\mathbf{x})$ when $\pi < 1 - a$ (resp. $\pi > 1 - a$).

Note that $v(\mathbf{x})$ is symmetric and convex in all the arguments x_ks. Therefore, $\sigma^2(\mathbf{x})$ is a Schur convex function of $\mathbf{x}$ (see Marshall and Olkin [12]). The claimed results then follow immediately from properties of Schur convexity since

$$v(1, 0, \ldots, 0) \geq v(x_1, \ldots, x_K) \geq v\left(\frac{1}{K}, \ldots, \frac{1}{K}\right).$$

■

Note that $1 - a$ defines the *maximum tolerable fraction of losses* for a FEC group to avoid an irrecoverable error. Theorem 4 states that given a set of homogeneous paths, when the probability that a path is in lossy state (π) is smaller than the maximum tolerable fraction of losses ($1 - a$), multipath routing is beneficial. On the other hand, if all paths are very lossy, then one should avoid multipath and use single-path routing instead. In other words, in order to take advantage of multipath routing, the FEC scheme should increase its redundancy r so that the tolerable fraction of losses is larger the the fraction of time that the path is lossy.

4.2. Heterogeneous Case

We next consider the general case when the K paths are heterogeneous. Without loss of generality, assume the K paths are ordered such that $\pi_1 \leq \pi_2 \leq \cdots \leq \pi_K (<$

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$\frac{1}{2}$). To make the problem interesting, assume $\pi_1 < 1 - a$; that is, there is at least one path whose probability of being in the lossy state is lower than the maximum tolerable fraction of losses under FEC. Therefore, the optimal solution $\mathbf{x}^*$ should satisfy $m(\mathbf{x}^*) - a > 0^2$

In this case, we can show that the optimal load splitting should be monotone in the path quality, where a less lossy path should be assigned more load in routing.

THEOREM 5: *Suppose $\pi_1 < 1 - a$. The optimal solution to Eq.* (7) *must satisfy*

$$x_1^* \geq x_2^* \geq \cdots \geq x_K^*$$

that is, the optimal load splitting tends to assign more load to less lossy paths.

PROOF: Suppose, to the contrary, that the optimal solution $\mathbf{x}^*$ to Eq. (7) has $x_i^* < x_j^*$ for some $i < j$, where $\pi_i < \pi_j$.

Let $\tilde{x}_i = x_i^* + \epsilon$, $\tilde{x}_j = x_j^* - \epsilon$, and $\tilde{x}_k = x_k^*$ for $k \neq i, j$ for some small $\epsilon > 0$. Then $m(\tilde{\mathbf{x}}) - m(\mathbf{x}^*) = \epsilon(m_i - m_j) > 0$ and

$$v(\tilde{\mathbf{x}}) - v(\mathbf{x}^*) = 2\epsilon\rho\left[b_i\left(x_i^* - \frac{a}{2\rho}\right) - b_j\left(x_j^* - \frac{a}{2\rho}\right)\right] + \epsilon^2\rho(b_i + b_j) < 0$$

for ϵ sufficiently small, where the last inequality holds because $x_i^* < x_j^*$ and $b_i < b_j$.

Since $m(\mathbf{x}^*) - a > 0$, we then have $(m(\tilde{\mathbf{x}}) - a)/\sqrt{v(\tilde{\mathbf{x}})} > (m(\mathbf{x}^*) - a)/\sqrt{v(\mathbf{x}^*)}$, which contradicts the assumption that $\mathbf{x}^*$ is optimal to Eq. (7). ■

The next theorem establishes the asymptotic behavior for the high redundancy case.

THEOREM 6: *As the redundancy r becomes large, the optimal load splitting approaches*

$$x_k^* = \frac{c}{\pi_k}, \qquad k = 1, \ldots, K,$$

where $c^{-1} = \sum_{k=1}^{K}(1/\pi_k)$; that is, as the FEC redundancy becomes large, the optimal load x_k on path k becomes inversely proportional to the path loss rate π_k.

PROOF: We solve Eq. (7) by dropping K (since it does not affect the optimal solution) and using the Lagrangian

$$J(\mathbf{x}) = \frac{m(\mathbf{x}) - a}{\sqrt{v(\mathbf{x})}} + \theta\sum_{k=1}^{K} x_k.$$

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Setting $\partial J/\partial x_k = 0$, we have

$$-m_k + \frac{m(\mathbf{x}) - a}{v(\mathbf{x})} b_k \left(\rho x_k - \frac{a}{2}\right) = \theta\sqrt{v(\mathbf{x})}. \tag{8}$$

Having a weighted sum of Eq. (8) with weights x_k, we then have

$$\theta = \left[-a + \frac{m(\mathbf{x}) - a}{v(\mathbf{x})} \left(\sum_k b_k x_k\right) \frac{a}{2}\right] \left(\sqrt{v(\mathbf{x})}\right)^{-1}.$$

As redundancy r becomes larger (i.e., $a \to 0$), we have $\theta \to 0$, and Eq. (8) becomes

$$-m_k + \frac{m(\mathbf{x})}{v(\mathbf{x})} b_k \rho x_k = 0.$$

Thus,

$$x_k = \frac{m_k}{b_k \rho} \frac{v(\mathbf{x})}{m(\mathbf{x})} = \frac{1}{\pi_k \rho} \frac{v(\mathbf{x})}{m(\mathbf{x})}.$$

It is easily checked that, by setting $x_k = c/\pi_k$ for all k, we have $v(\mathbf{x})/m(\mathbf{x}) = c\rho$. Since $\sum_k x_k = 1$, we have $c^{-1} = \sum_{k=1}^{K}(1/\pi_k)$. ■

4.3. Independent Case

We next illustrate that without the Gilbert (on and off) model, if one assumes that the packet delivery process on path k is i.i.d. Bernoulli with fixed loss probability π_k at all times (as assumed in Sharma et al. [14]), then the benefit of multipath routing no longer exists.

Without loss of generality, we assume that the loss probabilities on all paths $\pi_k < 1/2$. Based on the standard central limit theorem, we have a result similar to Lemma 1 with $m_k = 1 - \pi_k$ and $\sigma_k^2(x_k) = \pi_k(1 - \pi_k) = b_k$. Note that the larger m_k is, the smaller the variance σ_k^2. The following result is then immediate with a simple interchange argument.

THEOREM 7: *If the packet delivery process $\{X_n^{(k)}\}_{n\geq 1}$ on each path is i.i.d. Bernoulli with loss probability π_k ($< 1/2$), then multipath splitting is not preferred and the policy that uses the single best path (with the least loss rate π_k) achieves the least irrecoverable error.*

Theorem 7 shows that under i.i.d. Bernoulli transmissions, there is no variance reduction on each path by dispersing the traffic over a multipath, and multipath routing has no advantage over single-path routing. This further illustrates the need for dependent models to study the benefit of multipath routing. Since packet losses tend to occur in a burst due to buffer overflow, one needs to account for the dependence structure in consecutive packet losses.

5. MODEL VALIDATION

In this section we present a simulation study to evaluate the performance of the solutions obtained through analytical models. We consider a two-path streaming case under FEC in which we use brute-force search to find the optimal splitting in different scenarios. Our simulation results show that the optimal splitting solution obtained by the brute-force search agrees amazingly well with that obtained using our analytical model, even under relatively small group sizes.

Our simulation topology consists of access routers and overlay nodes, as shown in Figure 1. Between the sender and the receiver, there are multiple overlay paths, one through each relay node. The bandwidth and delay for the access links are chosen to reflect the common settings found in residential broadband links, which have asymmetric upload and download capacity. Background traffic is introduced on the core links between the core routers. To emulate the bursty loss behavior in the Internet, the background traffic for each link is composed of five FTP sessions and UDP On/Off traffic. Note that we do not directly set the path loss rate; we control it by introducing the background traffic and measure the resulting path loss rate. When the UDP traffic is active, it sends UDP packets at 10 Mbp with packet size of 1400 bytes. The queue length of the routers is 50. The sender generates CBR live traffic of 768 kbps with 500 B packets.

Each simulation run consists of 30 s of a warm-up time (for the background traffic to ramp up) and 10,000 s of streaming. Table 1 shows the default parameters used in the simulation.

Figure 2 presents a sample set of results obtained in a two-path case. It suffices to note that the loss rates on Path 1 and Path 2 are 5.29% and 8.26%, respectively. Thus, based on Theorem 6, we can calculate the asymptotic optimal splitting as ($x_1 = 61\%$, $x_2 = 39\%$) as redundancy becomes large.

In these simulations, we fix the FEC group size to 20 and gradually increase the FEC redundancy from 0% to 60%. For each redundancy value, we vary the load on

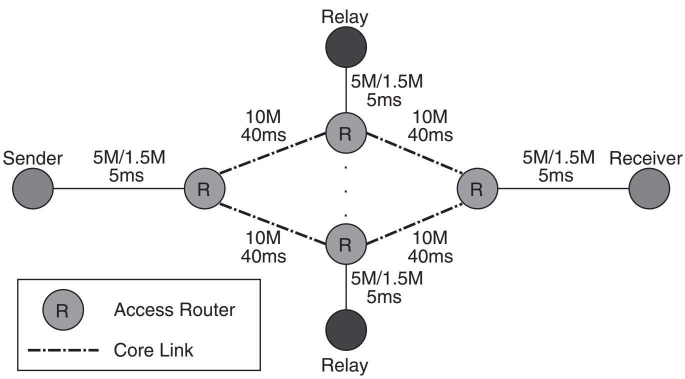

FIGURE 1. Topology for performance evaluation.

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Table 1. Default EMS Settings

Parameter	Value
Loss requirement, L_{req}	5%
Delay constraint	500 ms
Adapt window, T_s	30 s
OLS step size, Δ_L	3%
FEC redundancy adapt step size, Δ_R	5%
FEC redundancy adapt threshold, α and β	0.5
FEC redundancy adapt threshold, γ	0.75
FEC redundancy init value	10%
FEC group size init value	10
FEC group size adapt threshold, b	3

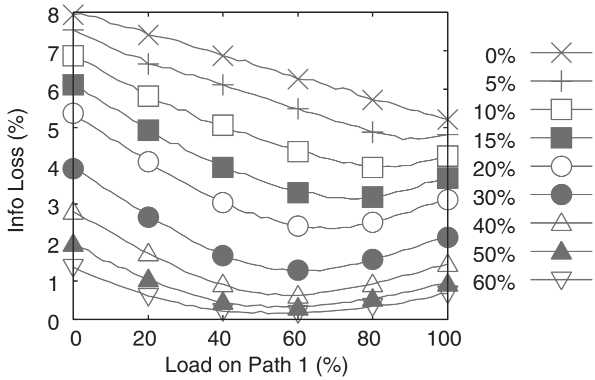

Figure 2. Optimal splitting with two paths.

Path 1 (x_1) from 0% to 100%, with an increment of 3%. The load on Path 2 is always $x_2 = 1 - x_1$. The results of such brute-force search are shown in Figure 2, which plots how the information loss rates changes as we vary x_1. We can see that as the FEC redundancy becomes large, the optimal splitting vector is indeed approaching the asymptotic optimal one provided by Theorem 6.

We have also validated the model using three-path cases and a subset of the results are summarized in Table 2 (in which the FEC group size and redundancy are 20% and 40%, respectively). In all cases, we have observed that our asymptotic analysis matches the simulation results very well.

Table 2. Optimal Splitting with Three Paths

	Path 1	Path 2	Path 3
Loss rate	3.69%	6.89%	9.97%
Optimal load (asymptotic analysis)	52.4%	28.1%	19.4%
Optimal load (brute-force search)	51%	24%	25%

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From both analytical and simulation results, we can see that the optimal load splitting depends on both path quality and the FEC redundancy. In general, we can make the following remarks:

- With large FEC redundancy, the optimal load splitting is close to the asymptotic optimal one, with the load on a path inversely proportional to its loss rate (Theorem 6).
- With small FEC redundancy, the optimal load splitting is biased toward those less lossy paths (Theorem 5).

We will use these guidelines in designing the algorithms searching for the optimal load distribution in the general settings.

6. CONCLUDING REMARKS

In this article, we study the optimal load distribution problem for multipath streaming under FEC for emerging real-time live streaming applications with tight loss requirements. Through modeling and asymptotic analysis, we have developed a closed-form approximation for finding optimal load splitting solutions and derived properties of the optimal solutions with useful engineering insights. The effectiveness of our analytical results is further confirmed by simulation.

In addition to the optimal load distribution, our asymptotic framework can also be used to explore the optimal decision on the FEC group size and redundancy. This is part of our ongoing research, as choice of these parameters also directly affects the overall service delay, which is another important attribute in the quality of service delivery of streaming applications. Our preliminary investigation indicates that an analytical framework can also be helpful in addressing both delay and information loss constraints, but the analysis will be much more involved. We, therefore, defer it to future research.

Notes

1. To be technically correct, it should be $\lfloor Nx_k \rfloor$ packets instead since we can only take integer values. This, however, will not matter much since we are interested in asymptotic analysis for large N.

2. If, instead, $\pi_k \geq 1 - a$ for all k, then $m(\mathbf{x}) - a < 0$ under all load splitting policy $\mathbf{x}$. Note from Corollary 3 that the irrecoverable error $L_N(\mathbf{x})$ becomes close to zero for large N if $m(\mathbf{x}) > a$, and $L_N(\mathbf{x})$ converges to 1 if $m(\mathbf{x}) < a$. We should therefore choose the redundance r high enough such that $a = c1/(1 + r)$ is at least smaller than m_1 so as to ensure that the irrecoverable rate is low.

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APPENDIX

Proof of Theorem 2.

To prove Theorem 2, first we need the following lemma, which is also known as the continuity theorem.

LEMMA A.1 (Continuity Theorem. See Billingsley [4]): *Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables with characterisitc function $\varphi_{X_n}(s) = E[e^{isX_n}]$ for $n \geq 1$. Let $F_n(\cdot)$ be the cumulative distribution function (cdf) of X_n, $n \geq 1$, and F be the cdf of (a possibly defective random variable) X. Then*

$$\lim_{n\to\infty} F_n(t) = F(t)$$

at all continuity points of F if and only if

$$\lim_{n\to\infty} \varphi_{X_n}(s) = \varphi_X(s) \quad \textit{for all } s > 0.$$

PROOF OF THEOREM 2: Define $\bar{Z}_n^{(k)} = \sqrt{n}(S_n^{(k)}/n^{-m_k})$. Based on Lemma 1, we know that $\bar{Z}_n^{(k)}$ converges in distribution to a Normal random variable $Z_*^{(k)}$ with mean 0 and variance $\sigma_k^2(x_k)$.

Apply Lemma A.1, we then have

$$\lim_{n\to\infty} \varphi_{\bar{Z}_n^{(k)}}(s) = \varphi_{Z_*^{(k)}}(s) = e^{-\frac{1}{2}\sigma_k^2(x_k)s^2} \qquad \text{for all } s > 0.$$

Define $\bar{Z}_n(\mathbf{x}) = \sqrt{n}[S_n(\mathbf{x})/n - m(\mathbf{x})]$, where $m(\mathbf{x}) = \sum_{k:x_k>0} x_k m_k$. It is easily checked that

$$\bar{Z}_n(\mathbf{x}) = \sum_{k:x_k>0} \sqrt{x_k}\bar{Z}_{nx_k}^{(k)}.$$

Hence, for all $s > 0$,

$$\begin{aligned}
\lim_{n\to\infty} \varphi_{\bar{Z}_n(\mathbf{x})}(s) &= \lim_{n\to\infty} e^{is\sum_{k:x_k>0}\sqrt{x_k}\bar{Z}_{nx_k}^{(k)}} \\
&= \lim_{n\to\infty} \prod_{k:x_k>0} \phi_{\bar{Z}_{nx_k}^{(k)}}(s\sqrt{x_k}) \\
&= \prod_{k:x_k>0} e^{-\frac{1}{2}\sigma_k^2(x_k)s^2 x_k} = e^{-\frac{1}{2}\sigma^2(\mathbf{x})s^2}. \qquad \textbf{(A.1)}
\end{aligned}$$

Note that the right-hand side of (A.1) is the moment generating function of a normal random variable with mean 0 and variance $\sigma^2(\mathbf{x})$. Based on Lemma A.1, we can then claim that $\bar{Z}_n(\mathbf{x})$ converges in distribution to a normal random variable with mean 0 and variance $\sigma^2(\mathbf{x})$. ∎