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# PROPOSITIONAL LEARNING: FROM IGNORANCE TO KNOWLEDGE

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ABSTRACT

In this paper, I offer an account of propositional learning: namely, learning that *p*. I argue for what I call the "Three Transitions Thesis" or "TTT" according to which four states and three transitions between them characterize such learning. I later supplement the TTT to account for learning why *p*. In making my case, I discuss mathematical propositions such as Fermat's Last Theorem and the ABC Conjecture, and then generalize to other mathematical propositions and to non-mathematical propositions. I also discuss some interesting applications of the TTT, and reply to some noteworthy objections.

## 1. INTRODUCTION

Since at least Gettier (1963), epistemologists have devoted considerable attention to propositional knowledge: knowledge that *p* where *p* is some proposition.[1] Yet relatively little epistemological attention has been devoted to what we may call "propositional learning": learning that *p*.[2,3] This lack of attention proves curious, for one would think that anyone interested in knowledge would also be interested in learning inasmuch as the latter is to acquire knowledge or to come to know.[4] Educational psychologists, for their part, have offered a wide and variegated array of theories of learning, but none heretofore that focuses specifically on propositional learning.[5]

1 Also known as factive knowledge, it is standardly distinguished from procedural knowledge (knowing how to *A* where *A* is some activity or procedure) and objectual or acquaintance knowledge (knowing *x* where *x* is a person or thing). Whether epistemologists prior to Gettier (1963) were as interested in propositional knowledge as they are now is itself an interesting question. See on this matter Antognazza (2015), Dutant (2015), and Le Morvan (2017).

2 This disparity in attention is evidenced by the works devoted to these two topics. As of 5 July 2018, the *Philosopher's Index* listed 178 entries under the heading "propositional knowledge" but only one under the heading "propositional learning." The latter – namely, Claveau and Vergara Fernández (2015) – briefly discusses learning as the acquisition of knowledge, but is primarily focused on the epistemic contributions made by models, especially those in economics.

3 Note that the expression "propositional learning" may be construed as referring to someone's learning a proposition *p*, or to someone's learning that *p*. Someone's learning in the former case is not factive and so does not entail that *p* is true as it's possible to learn a proposition *p* that happens to be false; learning in the latter case is factive and so entails that *p*. Unless otherwise specified, by "propositional learning" I mean it in the sense of learning that *p*.

4 Cf. Audi (2011: 162) and Claveau and Vergara Fernández (2015: 406–7).

5 For a helpful overview of this array of theories, see Alexander *et al.* (2009) and Driscoll (2005).

I aim here to take a step in the direction of an epistemological theory of such learning. More specifically, I make a case for what I shall call the "Three Transitions Thesis" or "TTT" hereafter. According to the TTT, four states and three transitions between them characterize propositional learning.

These states and transitions, to be explained and clarified in the ensuing discussion, are the following. Someone *S* is in $\text{State}_1$ when *S* is preconceptually ignorant of a proposition *p*. *S* is in $\text{State}_2$ when *S* is postconceptually ignorant of *p*. *S* is in $\text{State}_3$ when *S* has knowledge of *p*. *S* is in $\text{State}_4$ when *S* has knowledge that *p*. The transitions in question are the $\text{State}_1 \rightarrow \text{State}_2$ transition, the $\text{State}_2 \rightarrow \text{State}_3$ transition, and the $\text{State}_3 \rightarrow \text{State}_4$ transition. Later, in discussing learning why *p*, I shall argue that it involves a fourth transition: namely, the $\text{State}_4 \rightarrow \text{State}_5$ transition, a transition from $\text{State}_4$ (where *S* has knowledge that *p*) to $\text{State}_5$ (where *S* has knowledge why *p*).

In making my case, I turn initially to mathematics, and in particular to two important mathematical propositions known as Fermat's Last Theorem (henceforth "FLT") and the ABC Conjecture (henceforth "ABC"). I turn to them because, in contrast with most other propositions, a helpful literature can be found on when they were discovered (and so when knowledge of them began) and when or if they were proven (and so when or if knowledge that they are true began).

As we shall see, the case of FLT provides a telling example of each of the transitions noted above, whereas the case of ABC provides a telling example of the first two. But fear not: understanding these examples does not require any expertise in mathematics. Note moreover that my findings generalize to other mathematical propositions, and indeed to non-mathematical propositions.

Before I continue, let me address a "So What?" question: Why should anyone care about the TTT? My answer is four-fold: (1) If, as epistemologists, we are interested in the nature of knowledge, we should also presumably be interested in the nature of learning as the acquisition of knowledge. (2) Insofar as we are interested in the nature of learning, we should also presumably be interested in the ways, if any, that it involves transitions from ignorance to knowledge. (3) Insofar as education is concerned with learning, epistemologists stand to make contributions to learning theory by contributing to the understanding of propositional learning. (4) As we will see, the TTT has a number of interesting applications. For these reasons, epistemologists and learning theorists (at least) should care about the TTT and whether it is true.

My case unfolds as follows. In Section 2, I clarify its main presuppositions. In Section 3, I discuss FLT and draw lessons from it concerning the TTT. In doing so, I distinguish between learning that *p* and learning why *p*, and supplement the TTT – with what I call the "TTT+" – to account for learning why *p*. In Section 4, I discuss ABC and show how my results generalize to other mathematical and non-mathematical propositions. In Section 5, I discuss some applications of the TTT and the TTT+ to a number of interesting philosophical problems. In Section 6, I address some noteworthy objections and offer replies. In Section 7, I conclude with some retrospective and prospective remarks.

## 2. MAIN PRESUPPOSITIONS

The TTT does not turn on any particular account of the nature of knowledge or of truth, and I leave open their natures except for assuming (*pace* skeptics) that we are capable (at least in some cases) of achieving knowledge and ascertaining truth.[6] Apart from presupposing they can be true and can be known, I also leave open the nature of propositions.[7]

I will focus on propositional learning and propositional knowledge, and henceforward I will only use the terms 'learning' and 'knowledge' to refer to them. I presuppose that to learn is to acquire knowledge. While I acknowledge that we often learn by being taught by others, I do not presuppose that all learning requires such teaching. While I acknowledge that learning often leads to a change in behavior, I do not presuppose that learning requires, or is nothing but, a change in behavior. While I acknowledge that learning is often purposeful and deliberate, I do not presuppose that all learning is so, leaving open the possibility that learning can occur accidentally.

In order to elucidate the transitions that mark propositional learning, I focus on ignorance and knowledge in relation to propositions, and on the salience of two distinctions: one between ignorance **of** a proposition $p$ and ignorance **that** $p$, and another between knowledge **of** $p$ and knowledge **that** $p$. I presuppose that knowledge and ignorance are complements (in ways to be exemplified and clarified below) such that knowledge entails non-ignorance and ignorance entails non-knowledge.[8]

Finally, a terminological matter. By "C$p$" I mean the conceptual capacity that I will presume is requisite for knowledge of a proposition $p$. Take for instance the following proposition:

$p_1$: Triangles are polygons.

To have knowledge of $p_1$ requires having a repertoire of concepts including the concepts of *triangle* and *polygon*. It also requires the ability to use such concepts to refer and to predicate. I presuppose that having this repertoire and this ability is required to have the conceptual capacity or C$p_1$ requisite for knowledge of $p_1$.[9] I presuppose that (i) for each known proposition $p$ there is a C$p$ associated with it, (ii) knowledge of $p$ requires having this C$p$, (iii) failure to have this C$p$ suffices for being ignorant of $p$, and (iv) failure to deploy this C$p$ (even if one has it) suffices for being ignorant of $p$.[10]

## 3. FLT

We turn now to FLT, a theorem fairly easy to grasp, but notoriously hard to prove.[11] We may state it as follows:

6 I have articulated and defended an account of knowledge in Le Morvan (2016), but I do not presuppose it here.

7 For a discussion of the epistemology and ontology of propositions, see Le Morvan (2015).

8 For a defense of the complementarity of knowledge and ignorance, see Le Morvan (2010, 2011, 2012, 2013). Note that readers who think that ignorance is not the complement of knowledge may substitute "non-knowledge" for "ignorance" in the ensuing discussion.

9 Note that having the C$p$ for $p$ is a necessary condition for knowledge of $p$ (or so I presume). I do not claim that it is a sufficient condition for such knowledge. What the necessary and sufficient conditions are for knowledge of $p$ is a topic too large to address in this paper.

10 What I mean by these presuppositions will become clearer as I discuss examples below. An interesting question may be raised here concerning the extent to which a C$p$ is acquired and its relation to other innate and acquired cognitive capacities. I will have to leave the addressing of this question to another occasion.

11 Prior to its proof which we will discuss in a moment, FLT was often characterized as the greatest mystery in mathematics. See Singh (2012). Stillwell (2015: 220) takes FLT to be one of four paradigmatic

*FLT*: No three positive integers *a*, *b*, and *c* can satisfy the equation

$$a^n + b^n = c^n \text{ for any whole number } n \text{ greater than 2.}$$

First formulated by Pierre de Fermat in 1637, FLT was finally proven by Andrew Wiles, in a proof he formally published in 1995.[12] For more than 350 years, prior unsuccessful attempts to prove it spurred not only the development of algebraic number theory in the 19th century, but also proof of the Modularity Theorem in the 20th century.[13]

To see how FLT helps shed light on propositional learning, consider it first in connection with four distinct individuals relative to $C_{FLT}$, the conceptual capacity requisite for knowledge of FLT.

To start, take someone who, we will suppose, did not have $C_{FLT}$: (say) Attila the Hun. Lacking $C_{FLT}$, he was thus ignorant not just that FLT is true, but of the theorem itself.

Consider now someone who, we will suppose, did have $C_{FLT}$, but who, for whatever reason, had not deployed it so as to have knowledge of FLT. Take (say) Descartes in 1636. Suppose for the sake of argument that, given his impressive mathematical training, knowledge, and abilities, Descartes had $C_{FLT}$ but had not deployed it in 1636 so as to have knowledge of FLT.[14] If so, he, like Attila, was thus ignorant not just that FLT is true, but of the theorem itself.[15]

We may note, however, a salient difference between Attila's and Descartes's ignorance of the theorem. The former (we suppose) lacked $C_{FLT}$, while the latter (we suppose) had $C_{FLT}$ but had never deployed it. Thus, Attila's ignorance of FLT was deeper than Descartes's. We may call "preconceptual ignorance of a proposition *p*" an ignorance resulting from a lack of a C*p*, and "postconceptual ignorance of a proposition *p*" ignorance resulting from not deploying a C*p*. Thus, on the suppositions we are making, Attila was preconceptually ignorant of FLT, whereas Descartes was postconceptually ignorant of it. This in turn entails that, whereas Attila had never undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition (i.e., from preconceptual to postconceptual ignorance) relative to FLT, Descartes by contrast did so at some point in his life.[16]

We turn now to Pierre de Fermat. In formulating FLT in 1637, de Fermat was neither preconceptually ignorant of it like Attila, nor postconceptually ignorant of it like Descartes. He not only had but deployed $C_{FLT}$ in formulating FLT, and his so doing shows that he had arrived at knowledge of the theorem itself.

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examples of theorems that are *historically deep* "in the sense that it took a long time to uncover them, and many other theorems had to be uncovered first."

12 Wiles (1995) and Taylor and Wiles (1995).

13 See Singh (2012). Formerly known as the Taniyama–Shimura–Weil Conjecture (among other names) prior to its being proven and thus recognized as a theorem, the Modularity Theorem connects topology and number theory.

14 FLT is an intellectual descendant of the Pythagorean Theorem, and given that he would have been intimately familiar with the latter, it's quite plausible to suppose that Descartes would have had $C_{FLT}$ even if FLT had not occurred to him until after de Fermat had formulated it.

15 See Le Morvan (2015) for a defense of the view that knowledge of *p* is required for any propositional attitude concerning *p* such as believing, doubting, hoping, knowing, considering, or entertaining that *p* is true. Being ignorant of FLT, Attila and Descartes thus could not have propositional attitudes concerning it.

16 Note that I do not claim that Descartes's undergoing this transition required his having been aware of his undergoing it. In fact, it seems that it is typically the case that those who undergo the $\text{State}_1 \rightarrow \text{State}_2$ transition do so without conscious awareness of it.

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But did he *also* know that it is true? Although he apparently claimed to have a proof too large to fit in the margin of his copy of the *Arithmetica* (an Ancient Greek mathematical text written by Diophantus in the 3rd century AD), it seems highly unlikely that he had a valid one, as Wiles's subsequent proof relied on more than 350 years' worth of later developments in mathematical theory.[17]

In any event, suppose for the sake of argument that de Fermat did not know **that** FLT is true despite his knowledge **of** it. As his case illustrates, knowledge **of** FLT does not suffice for knowledge **that** it is true. While knowledge **that** FLT is true presumably entails knowledge **of** it, knowledge **of** it does not entail knowledge **that** it is true. Knowledge **of** FLT is a necessary but not sufficient condition for knowledge **that** it is true, and knowledge **that** it is true is a sufficient but not necessary condition for knowledge **of** FLT.

In arriving at knowledge **of** FLT while still lacking knowledge that it is true, Pierre de Fermat was the first to undergo the $\text{State}_2 \rightarrow \text{State}_3$ transition relative to FLT (i.e., from postconceptual ignorance of it to knowledge of it). He was followed in this transition by those others who acquired knowledge of it.[18]

Wiles, upon proving it, was the first to know **that** it is true and thus to undergo the $\text{State}_3 \rightarrow \text{State}_4$ transition relative to FLT (i.e., from knowledge of it to knowledge that it is true).[19] He too was followed in this transition insofar as others acquired knowledge that FLT is true. Inasmuch as learning that FLT is true is to acquire knowledge that it is true, such learning thus requires undergoing the three transitions specified by the TTT.

I have so far used four different individuals to demonstrate the TTT and the distinctions between knowledge **of** a mathematical proposition and knowledge **that** it is true, and between ignorance **of** a mathematical proposition and ignorance **that** it is true. The TTT and these distinctions, however, can be demonstrated not just inter-personally but also intra-personally via a single individual.

Take Andrew Wiles once more, and consider four different points in his life: at 2 years old, at 9 years old, at 10 years old, and at 41 years old having proven the theorem.

We may reasonably suppose that Wiles at 2 years old was too young to have knowledge of FLT; at this age he lacked $C_{FLT}$ and was preconceptually ignorant of FLT.

Suppose that by 9 years old Wiles had acquired enough mathematical training to have $C_{FLT}$, but was ignorant of FLT having not yet heard or thought of it. He was thus at this

17 For discussion, see for instance Singh (2012), Edwards (2000), and Aczel (2007). Insofar as theorems are proven mathematical propositions, the expression "Fermat's Last Theorem" can misleadingly suggest that de Fermat himself had proven it which is unlikely to have been the case.

18 He also must have undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition at some earlier point in his life.

19 Notice that Wiles presumably would not have been in any position to prove and later know **that** it is true without knowledge **of** FLT itself and without having undergone the $\text{State}_1 \rightarrow \text{State}_2$ and $\text{State}_2 \rightarrow \text{State}_3$ transitions at some earlier points in his life. More generally, knowledge **of** a mathematical proposition is a precondition for proving and knowing **that** it is true. I presuppose that knowledge **that** a non-axiomatic mathematical proposition $p$ is true requires a proof of $p$, but that knowledge **of** $p$ does not require such proof. For instance, you can have knowledge of FLT (a non-axiomatic mathematical proposition) without knowing that it is true, but knowing that it is true requires a proof that it is true. In the case of an axiomatic mathematical proposition, I presuppose that one can know that it is true in virtue of its self-evidence. Furthermore, I assume that, if someone $S$ proves that $p$, then $S$ knows that $p$; I leave open whether $S$ can know that $p$ is true if $S$ has not proven that $p$ but it has been proven by someone else. For example, while Andrew Wiles presumably knows that FLT is true having proven it, I leave open whether others who have not proven it themselves can know that it is true (say, on the basis of reliable testimony).

stage in his life postconceptually ignorant of it, and had undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition relative to FLT.

Wiles has himself stated that he learned of FLT as a 10-year-old.[20] Fascinated by the existence of an unproven mathematical proposition so easy to state that he, a ten-year-old, could understand it, he resolved to be the first to prove it. At this stage in his life, Wiles was no longer preconceptually or postconceptually ignorant of FLT, for he had knowledge **of** it and had undergone the $\text{State}_2 \rightarrow \text{State}_3$ transition. But despite his knowledge **of** it, he did not know **that** it is true and he thus had not yet undergone the $\text{State}_3 \rightarrow \text{State}_4$ transition relative to FLT.

At age 41, having proven it, Wiles finally learned (came to know) **that** FLT is true. And with this cognitive achievement, he became the first to undergo the $\text{State}_3 \rightarrow \text{State}_4$ transition relative to it.

Notice that, having proven FLT, Wiles can quite plausibly be said to have learned not only that FLT is true but also *why* it is true, his proof being a demonstration why. Notice also that his having learned why FLT is true entails his having learned that it is true.

Contrast this situation with another. Consider Xiles, someone incapable of understanding Wiles's proof, let alone proving it. Xiles has obviously not learned why FLT is true. Suppose however that Wiles – who let's assume is a highly reliable source of truth – tells Xiles that FLT is true and Xiles believes it to be true on this basis. Has Xiles learned that FLT is true even though he does not know why it is true? If the answer is yes, then even though learning why FLT is true entails learning that it is true, the latter does not entail the former.

This suggests that, although learning that $p$ involves the three transitions specified by the TTT, learning why $p$ requires a fourth transition, namely the $\text{State}_4 \rightarrow \text{State}_5$ transition whereby $S$ goes from knowledge that $p$ (i.e., $\text{State}_4$) to knowledge why $p$ (i.e., $\text{State}_5$). In proving FLT, Wiles was the first to undergo this fourth transition. Note that even if we suppose that Wiles underwent both the $\text{State}_3 \rightarrow \text{State}_4$ and $\text{State}_4 \rightarrow \text{State}_5$ transitions simultaneously in proving FLT, the two transitions are logically distinct insofar as the former can occur without the latter (as shown for instance by the example of Xiles relative to FLT). Accordingly, let's call "TTT+" the thesis that learning why is delineated by the three transitions of the TTT with the addition of the fourth transition delineated above.

Let's take stock. Ignorance **of** FLT is distinct from ignorance **that** FLT is true, and knowledge **of** FLT is distinct from knowledge **that** FLT is true. Ignorance **of** FLT comes in two main kinds: preconceptual and postconceptual, with the former being deeper than the latter. Relative to FLT, someone undergoes the $\text{State}_1 \rightarrow \text{State}_2$ transition when going from preconceptual to postconceptual ignorance of it, the $\text{State}_2 \rightarrow \text{State}_3$ transition when going from postconceptual ignorance of it to knowledge of it, and the $\text{State}_3 \rightarrow \text{State}_4$ transition when going from knowledge of it to knowledge that it is true.[21] Learning that FLT is true requires undergoing all three transitions specified by TTT. Someone undergoes the $\text{State}_4 \rightarrow \text{State}_5$ transition relative to FLT when going from knowledge that FLT is true to knowledge why it is true. Learning why FLT is true requires undergoing all four transitions specified by TTT+.

20 http://www.pbs.org/wgbh/nova/physics/andrew-wiles-fermat.html.

21 Notice that, relative to at least some mathematical proposition, any of us may be in a state of ignorance or knowledge akin to Attila's, or Descartes's, or de Fermat's, or Wiles's relative to FLT.

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## 4. BEYOND FLT TO ABC AND OTHER PROPOSITIONS

We have so far discussed FLT in making the case for the TTT and TTT+, but the lessons we have drawn from it apply to other propositions, mathematical and non-mathematical alike.

As a mathematical example, consider ABC.[22] Also known as the Oesterlé-Masser conjecture, ABC was proposed, but not proven, first by Masser in 1985 and then by Oesterlé in 1988.[23] It may be stated as follows:

*ABC:* For every infinitesimal $\varepsilon > 0$, there exist only finitely many triples of positive coprime integers $a$, $b$, $c$ such that $a + b = c$ and such that $c > d^{1+\varepsilon}$, where $d$ is the product of the distinct prime factors of the product $abc$.

In August 2012, Shinichi Mochizuki claimed to have proven it in a series of four papers on the topic of what he labelled "inter-universal Teichmüller theory."[24] He has subsequently revised his work on the theory, and we await verification by other mathematicians of the validity of his attempted proof which many mathematicians find extremely difficult to understand. Given its complexity, this attempted verification has been quite slow-going.

Whether Mochizuki's attempted proof is ever verified, notice that we can distinguish between preconceptual and postconceptual ignorance of ABC. Anyone in the former state of ignorance lacks $C_{ABC}$ and has not undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition relative to ABC. Anyone in the latter state of ignorance has $C_{ABC}$ and so has undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition relative to ABC but not yet the $\text{State}_2 \rightarrow \text{State}_3$ transition. If Masser was the first to have knowledge **of** ABC without knowledge **that** it is true, he was the first to undergo the $\text{State}_2 \rightarrow \text{State}_3$ transition relative to it. If Mochizuki's proof of ABC is valid and he is the first to have knowledge **that** ABC is true and knowledge **why** it is true, he is thus the first to undergo the $\text{State}_3 \rightarrow \text{State}_4$ transition and $\text{State}_4 \rightarrow \text{State}_5$ transition relative to it. Mochizuki's learning that ABC is true – insofar as it is his acquisition of knowledge that it is true – requires his having undergone the three transitions specified by the TTT, and his learning why it is true requires his having undergone the four transitions specified by the TTT+.[25]

While mathematical propositions such as FLT and ABC prove particularly useful in demonstrating the transitions requisite for propositional learning, the lessons drawn here concerning it extend to non-mathematical propositions.

Take for instance the following true non-mathematical proposition:

$p_2$: The *Ognissanti Madonna* was painted by Giotto.

Now consider an art history student Sarah who, newly enamored of medieval art, contemplates the *Ognissanti Madonna* for the first time. Suppose that she has already learned enough concerning art history to have undergone the $\text{State}_1 \rightarrow \text{State}_2$ transition relative

22 ABC has recently been characterized as one of the greatest unsolved problems in mathematics, the solution of which could change the face of number theory. See Castelvecchi (2015).

23 See Masser (1985) and Oesterlé (1988).

24 See Mochizuki (2017a, b, c, d).

25 These points concerning propositional learning generalize to other mathematical propositions as well.

to $p_2$ and therefore has $Cp_2$. She is postconceptually ignorant of $p_2$ until she wonders whether the painting before her is a Giotto, whereupon $p_2$ occurs to her for the first time and she undergoes the State$_2$→State$_3$ transition relative to $p_2$ (thereby going from postconceptual ignorance of $p_2$ to knowledge of it). Desirous of knowing whether it is a work of Giotto's, she asks a world-renowned Giotto expert who confirms that the painting is indeed by Giotto. Supposing that Sarah now knows that $p_2$ is true, she has thereby undergone the State$_3$→State$_4$ transition (from knowledge of $p_2$ to knowledge that it is true).

The example above, unlike the FLT and ABC examples, is obviously hypothetical. It nonetheless helps to show that the three transitions specified by the TTT are hardly unique to mathematical propositions. *Any* case of propositional learning, or so it seems at least, requires undergoing these three transitions.

Returning to the example of Sarah, suppose she acquires expertise in art history that allows her to identify for herself distinguishing features of a Giotto work and so learns why $p_2$. If so, she has undergone the State$_4$→State$_5$ transition relative to $p_2$ (from knowledge that $p_2$ to knowledge why $p_2$). This too shows that the latter transition is hardly unique to mathematical propositions.[26]

## 5. APPLICATIONS

While the TTT and TTT+ should be of interest to epistemologists and learning theorists in and of itself, a thesis is even more interesting inasmuch as it is fruitful in terms of its applications. Happily the TTT and TTT+ bear such fruit. In what follows, I adumbrate some of their applications in relation to a number of interesting philosophical problems.

### *5.1 Reichenbach's distinction and the Disconnection Problem*

Reichenbach (1938) famously distinguished between two epistemological contexts: the context of discovery and the context of justification. The former concerns how (i.e., the methods or processes by which) hypotheses or claims are generated (or learned or discovered), whereas the latter concerns how they are justified (or, more strongly, known to be true). Reichenbach argued that epistemologists and philosophers of science should concentrate on the context of justification, and leave to sociologists and psychologists the context of discovery.

There is clearly a grain of wisdom in Reichenbach's distinction, for even if a hypothesis is generated non-rationally, it does not follow that a good case cannot be made for it. Take the well-known example of Kekulé's dream, in which he "saw" atoms dancing around,

26 One very important difference between this example and the previous mathematical examples of learning why $p$ is that learning why $p$ in the latter cases presumably involves acquiring the ability to give, or at least comprehend a proof of, a mathematical proposition, whereas this seems far too stringent a standard in most non-mathematical cases. In the case of Sarah relative to $p_2$, her learning why the *Ognissanti Madonna* is a Giotto presumably involves (something like) her acquiring the ability to recognize distinguishing features of this work in virtue of which it counts as a Giotto. Learning why $p$ in many cases of non-mathematical propositions may call for the ability to adduce causal considerations, in others evidential considerations. Giving a full account of learning why $p$ is a project too large to undertake here, and I deliberately leave open what the proper account of it should be.

then forming themselves into strings, moving about in a snake-like fashion, culminating in an image of a snake eating its own tail. Kekulé's dream purportedly led him to the insight that the benzene molecule has a ring-like structure. Even though this idea came to him in a dream and so non-rationally, there are still good reasons for the thesis, for the benzene molecule has indeed been shown to have a ring-like structure.

While it is clearly true that non-rational factors may lead to the generation of hypotheses, and that psychology and sociology may provide valuable insights thereon, an unfortunate legacy of adherence to Reichenbach's distinction is what we may call the *Disconnection Problem*. As DeNicola (2017) has recently argued, the "traditional adherence to Reichenbach's boundary between discovery and justification has had the negative consequences of disconnecting the theory of knowledge from the process of learning, and of divorcing normative epistemology from educational practice" (2017: 203).[27] In addressing this problem, DeNicola does *not* advocate that "epistemology should become or replace cognitive psychology," and he acknowledges that "the two fields have properly different aims and methods" (2017: 204). His point rather is that epistemically relevant issues arise within the process of learning or coming to know, that "any adequate epistemology should be inclusive of them and attend to them" (2017: 204). Arguing that "it is illuminating to regard knowledge as an epistemic achievement, as success in the effort to learn," DeNicola contends that virtue epistemology "allows us to reconnect process with product, learning to knowing, and education to epistemology" (2017: 116–17). In this way, virtue epistemology "serves to bridge Reichenbach's two contexts" (2017: 117).

Whether or not DeNicola is right about the merits of virtue epistemology, he provides a persuasive case that the Disconnection Problem deserves attention, and resolution if possible. Worth noting therefore is that TTT and TTT+ provide another way of bridging Reichenbach's two contexts. They do so by giving an account of learning in terms of transitions from states of ignorance to states of knowledge, and thereby help resolve the Disconnection Problem by reconnecting the theory of knowledge and the process of learning. This is not to say that the TTT and TTT+ lead us by themselves to a "logic of discovery"; rather, they demonstrate that epistemological reflection can have a bearing on understanding learning (and thereby potentially on educational practice), that epistemologically relevant issues arise in the process of learning, and that theorizing about learning need not be left solely to psychologists, sociologists, and educational theorists.

## *5.2 Teaching and learning*

An important and ongoing debate in the philosophy of education concerns whether teaching implies learning. Dewey famously and influentially argued that it does.[28] According, however, to what has become known as the *Standard Thesis* in the philosophy of education, teaching does not imply learning, but rather the intention to bring about learning by

27 As mentioned earlier, it's telling how little attention has been paid by epistemologists to propositional learning despite their more than ample attention to propositional knowledge.

28 Dewey (1933: 35–6) famously compared teaching to the selling of commodities: "No one can sell unless someone buys. We should ridicule a merchant who said that he had sold a great many goods although no one had bought any. But perhaps there are teachers who think they have done a good day's teaching irrespective of what people have learned. There is the same exact equation between teaching and learning that there is between selling and buying."

reasonable methods under certain restrictions of manner.[29] While the TTT and TTT+ do not allow us to conclusively resolve this debate, they do provide new grounds for preferring the Standard Thesis over the Deweyan position with regard at least to propositional learning.

Take the latter position in relation to such learning. If teaching implies learning, and learning is acquiring knowledge, then a teacher *T* teaching a student *S* implies that *S* acquires knowledge why *p*, or knowledge that *p*, or knowledge of *p*. Suppose though that *T* assists *S* (through instruction) in making the $\text{State}_1 \rightarrow \text{State}_2$ transition relative to *p* such that *S* has thereby acquired C*p* although *S* has not yet deployed it so as to have knowledge of *p* (or that *p* or why *p*). Since *S* has not yet acquired knowledge of *p* (or that *p* or why *p*), *S* has not yet learned relative to *p* insofar as learning is the acquisition of knowledge. Yet, arguably, *T* has still taught *S* in assisting *S* in making the $\text{State}_1 \rightarrow \text{State}_2$ transition relative to *p*.

Accordingly, even if, in many cases, teaching results in student learning and learning is to acquire knowledge, not all cases of teaching are also cases where a student acquires some form of knowledge. *Contra* Dewey, teaching does not imply learning, for sometimes teaching involves imparting or facilitating not learning itself but rather a *precondition* for learning.

This point seems to square better with the Standard Thesis than the Deweyan position, for intending to bring about learning presumably involves intending to impart or facilitate a precondition for learning such as a student's $\text{State}_1 \rightarrow \text{State}_2$ transition relative to a proposition *p*.[30] And to the extent that the Deweyan position is modified to incorporate this point, it would, so it seems, collapse into a version of the Standard Thesis, and thereby fail to offer a genuine alternative to it.

## *5.3 Models and learning*

Claveau and Vergara Fernández (2015) provide an important account of how models make significant epistemic contributions in facilitating or stimulating learning which they categorize in terms of the acquisition of knowledge that *p*.[31] They focus on economic models in making their case.[32] However compelling the case they offer, a noteworthy limitation of their account is that, with its conception of learning as the acquisition of knowledge and of knowledge solely in terms of knowledge that *p*, it does not adequately address other kinds of epistemic contributions that models might make in terms of the acquisition of knowledge of *p*, or of knowledge why *p*, or indeed of the transition from pre-conceptual to post-conceptual ignorance of *p*.[33]

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29 See Noddings (2016: 49).

30 Interestingly, in relation to a proposition *p*, one can demarcate kinds of teaching relative to the propositional learning transitions ($\text{State}_1 \rightarrow \text{State}_2$, $\text{State}_2 \rightarrow \text{State}_3$, $\text{State}_3 \rightarrow \text{State}_4$, $\text{State}_4 \rightarrow \text{State}_5$) students make as a result of this teaching.

31 As they put it: "For the purpose of this paper, we propose to take learning to be the process of 'coming to know' ... and to rely on the traditional account of knowledge as true justified belief" (Claveau and Vergara Fernández 2015: 406–7). Though cognizant of Gettier-type challenges to this account, they argue that it suffices for the purposes of exploring the epistemic contributions of economic models.

32 In particular, they focus on Diamond–Mortensen–Pissarides (or DMP) model of the labor market.

33 In fairness, Claveau and Vergara Fernández do discuss learning what they call "model propositions" (propositions about a model) and "real-world propositions" (propositions about the world outside of

An interesting application of the TTT and the TTT+ is how they allow us to accept Claveau and Vergara Fernández's important insight about the epistemic contributions of models, but to overcome the limitation noted above by extending and generalizing this insight beyond knowledge that *p*, and indeed beyond economic models.

As an example, take a skillful chemistry teacher and the following proposition:

$p_3$: The benzene molecule is composed of six carbon atoms joined in a ring with one hydrogen atom attached to each.

Suppose this teacher adroitly uses plastic manipulatives to fashion with her students a physical model of the ring-like ($C_6H_6$) molecular structure of benzene. Imagine that, in doing so, she skillfully helps them transition from preconceptual ignorance to postconceptual ignorance of $p_3$, and then from postconceptual ignorance to knowledge of $p_3$, and then from knowledge of $p_3$ to knowledge that $p_3$, and finally to knowledge why $p_3$.[34] Other such examples abound. Claveau and Vergara Fernández are quite right that models make significant contributions to learning; accepting the TTT and the TTT+ allows us to see that these contributions extend well beyond economic models and knowledge that *p*.

## 5.4 *Some new problems for epistemology*

As noted at the outset of this paper, since at least Gettier (1963), epistemologists have devoted considerable attention to propositional knowledge: knowledge that *p* where *p* is some proposition. Our discussion of the TTT and TTT+, however, brings to light new problems worth epistemological investigation.

We have seen, for instance, that knowledge that *p* requires knowledge of *p* (although not vice-versa). But while considerable attention has been devoted to the problem of providing an illuminating account of the former, little epistemological attention has been devoted to the latter.[35] If knowledge that *p* requires knowledge of *p*, then properly understanding the former requires understanding the latter, and so more attention (than it has hitherto received by epistemologists) is worth devoting to understanding knowledge of *p*.[36]

We have also seen that knowledge of *p* requires a transition from preconceptual ignorance to postconceptual knowledge of *p*, but little epistemological attention has been devoted to the problems of providing illuminating accounts of either of these states.[37] Worthwhile research projects for epistemology lie therein.[38]

the model), but their doing so elides the distinction between learning as acquisition of knowledge of some proposition *p* with learning as acquisition of knowledge that *p* is true. See especially Claveau and Vergara Fernández (2015: 409–13).

34 To be sure, while logically distinct, some of these transitions may occur simultaneously as students learn.

35 See Le Morvan (2015) for an attempt to do so.

36 If, moreover, any propositional attitude that *p* (such as belief that *p*, or doubt that *p*, or desire that *p*, or hope that *p*) requires knowledge of *p* – for being ignorant of *p* precludes having any propositional attitude that *p* – this is all the more reason why knowledge of *p* is worthy more attention than it has hitherto received.

37 See Le Morvan (2015).

38 A worthwhile research project also lies in further exploring the relationship between knowledge that *p* and knowledge why *p*; while the latter presumably entails the former, on a particularly stringent form

Even with respect to knowledge that $p$, recognition of the TTT and TTT+ reorients us from a *synchronic* preoccupation with such knowledge and draws our attention to its *diachronic* dimension. That is, instead of focusing solely, as epistemologists have traditionally been wont to do, on the conditions for propositional knowledge *at a point in time*, the TTT and TTT+ call our attention to how we transition to such knowledge.

Take the condition we may call *cognitive limbo* wherein, relative to a proposition $p$, one has knowledge of $p$ but not yet knowledge that $p$. We presumably currently find ourselves in this situation with respect to *ABC* as we await verification of Mochizuki's putative proof; we have knowledge of *ABC* without knowing that *ABC* is true.

Cognitive limbo is hardly unique to mathematical propositions. Picture the following scenario. Sarah finds a lump on her breast. Her doctor, judging the lump to be suspicious, advises her to have a mammogram. She does so. It turns out to be inconclusive, so her doctor advises her to have a biopsy taken of the lump. She then anxiously awaits its result. Consider in this context the following proposition:

$p_4$: The lump on Sarah's breast is benign.

Suppose that $p_4$ is true, but Sarah does not know this before the result of the biopsy arrives. Suppose as well that, as she anxiously awaits this result, she has knowledge of $p_4$. She thus finds herself in cognitive limbo.

The example above is but one of many that can be given of this condition. Take for instance the following propositions:

$p_5$: In 2020, the USA elects a female president.
$p_6$: In 2030, worldwide sales of electric cars surpass sales of cars with internal combustion engines.
$p_7$: In 2040, the Earth's average temperature is more than 2 degrees Celsius higher than the 2010 level.
$p_8$: In 2050, humans colonize Mars.

Suppose for the sake of argument that, although not the case now, in the future we will learn that these propositions are true on whatever may be the correct account of such knowledge. Though we have not yet learned that they are true, our being able to entertain them implies that we now have knowledge of them. We are thus in cognitive limbo relative to them.

More generally, notice how much of our lives we spend in cognitive limbo – believing or dreading or hoping or anticipating or expecting or considering that something or other is (or will be) true. Being in this condition can be quite unpleasant. Think of Sarah's anxiety in waiting to learn the results of her biopsy, or the dread of a financially-strapped factory worker waiting to learn that her plant will lay her off. Cognitive limbo, however, is not invariably unpleasant. Think of the giddy anticipation of a child waiting to learn that he has received a new bike for his birthday, or a reader's rapture from being engrossed in the suspense of a novel while waiting to learn that things will work out in the end for her favorite characters.

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of knowledge internalism, the former also entails the latter. Such a stringent view strikes me as setting the bar for knowledge that $p$ implausibly high, but I shall not argue for that judgment here.

Despite its ubiquity in our lives, the condition of cognitive limbo has heretofore not been identified and conceptualized within a broader epistemic framework, and no philosophical account of it has yet been offered in the literature.[39] The conceptual resources afforded by the TTT allow us to do so: we are in cognitive limbo when we have not made the $\text{State}_3 \rightarrow \text{State}_4$ transition (from knowledge of *p* to knowledge that *p*) relative to some proposition *p*.[40]

## 6. FIVE OBJECTIONS AND REPLIES

As with any philosophical theory, a number of objections can be lodged against the position defended here, and space does not permit answering them all. I will, however, address five noteworthy objections.

### 6.1 *An objection from the non-factivity of learning*

You take learning that *p* to be the acquisition of knowledge that *p*. But propositional learning is not factive, for it is possible to learn that *p* where *p* is false. Therefore, learning cannot be the acquisition of knowledge. For instance, as Nola and Irzik (2005) point out:

> From Ancient times until the seventeenth century Europeans learned *that* the Sun orbits the Earth, since that was the prevailing belief over that period. In addition the educated learned *why* this occurred, where the explanation could not possibly be anything like the one we now adopt. And they learned it just as effectively as we now learn the opposite, viz., *that*, or *why*, the Earth orbits the Sun. It does not follow that when one learns *that* … one has learned a truth; one can just as well learn what is false. Learning that p is independent whether p is true or false. (Nola and Irzik 2005: 33)

*Reply.* I find Nola's and Irzik's putative counter-example unpersuasive. Suppose someone rejected the widely held epistemological thesis that knowledge is factive on the basis of claiming people once knew that the Earth is flat. As Nola and Irzik themselves acknowledge:

> Of course we can *claim to know* that p when p is false. But *claiming to know* is quite different from knowing; it is just another way of talking of belief. By not noting this difference people often claim that we can know what is false and say 'we once knew that the Earth is flat'. But the correct response is to say: 'well, people then *thought* that they knew, or *claimed* to know, but they did not *really know* that the Earth is flat. The word 'know', it is often said, is a success word. Just as 'winning' is a success word because it carries the implication that you came first (you cannot win and yet come second or third), so 'knowing' is a success word in that it latches onto the truth and not the false. (Nola and Irzik 2005: 70)

A response can be given to Nola's and Irzik's putative counter-example to the factivity of learning that ironically parallels their own response to the putative counter-example they

39 I suspect this is the case because of the traditional epistemological preoccupation with knowledge that *p at some point in time*.

40 I have focused here on a kind of cognitive limbo wherein one has knowledge of some proposition *p* without yet having knowledge that *p*. Interestingly, in light of the TTT+, one can also distinguish another kind of cognitive limbo wherein one has knowledge that *p* without knowledge why *p*.

envisage to the factivity of knowledge. To wit: even if people once *claimed* to have learned or *thought* they had learned that the Sun orbits the Earth, they were in fact mistaken. I see no good reason to think that 'learn' is any less a success or achievement verb than is 'know'.[41]

### 6.2 *An objection from learning as the mere acquisition of belief*

Why not suppose that learning that *p* is simply the acquisition of the belief that *p*? Would this not be a simpler account of such learning?[42]

*Reply*. Yes, it would be simpler, but it would also be simpler to give an account of knowledge merely as belief than in terms (say) of justified true belief (and whatever other condition might be necessary). Such an account of knowledge is widely (and I think rightly) regarded by epistemologists as incomplete. Similarly, though it might be simpler to account for learning that *p* merely in terms of the acquisition of belief that *p*, such an account comes at a steep intuitive cost in its apparent incompleteness.[43]

### 6.3 *An objection from the acquiring of knowledge without learning*

You take learning to be the acquisition of knowledge. But could one not acquire knowledge that *p* without learning that *p*? Suppose a device could be implanted in our brains that would give us immediate knowledge that *p* – say, that Milan is north of Rome – without our having to learn that *p*. Would this not show that knowledge that *p* can be acquired without learning that *p*? Moreover, we acquire knowledge by perception and/or by rational intuition. Are we to suppose that each time people acquire an instance of knowledge that *p* by these means they are learning that *p*?

*Reply*. Consider the following two conditionals:

(i) If someone *S* learns that *p*, then *S* acquires knowledge that *p*.
(ii) If someone *S* acquires knowledge that *p*, then *S* learns that *p*.

The thesis that to learn that *p* is to acquire knowledge that *p* entails (i) but is consistent with rejecting (ii). Thus, even if the objector is right that one can acquire knowledge that *p* without learning that *p*, it does not follow that it is false that learning that *p* is to acquire knowledge that *p*.

### 6.4 *An objection from lack of details*

You characterize propositional learning in terms of transitions to knowledge of *p* and to knowledge that *p* without providing an account of what these forms of knowledge

41 Interestingly, Hazlett (2010) argues at considerable depth against the epistemologically orthodox view that propositional knowledge is factive. Turri (2011) provides a forceful rebuttal thereto. The kind of counter-arguments Turri advances could, it seems to me, be adapted to rebut arguments purporting to show that learning is not factive.

42 This is in effect the view of Nola and Irzik (2005).

43 Also, common usage favors taking learning to be the acquisition of knowledge – for instance, the *OED*'s definition 1 of 'learn' is to acquire knowledge – and all other things being equal, it's preferable for an account to square with common usage than to be at odds with it.

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amount to. But would not different accounts of such knowledge (e.g., internalist vs. externalist, foundationalist vs. coherentist) yield quite different accounts of when learning occurs? You have not addressed such differences.

*Reply*. The objector is right that different accounts of knowledge may indeed yield quite different accounts of when learning occurs, and that I have not addressed such differences. My aim though has been to give an account of propositional learning that abstracts away from these differences, and that is compatible with a number of theories of knowledge. Consider an analogy from computer science: sometimes it can be very useful to abstract away from the very complicated details of the implementation of a program in order to achieve a "big picture" of the overall program without getting lost in the details. My aim has been to provide a "big picture" account of propositional learning even if the details of what the acquisition of knowledge amounts to are left deliberately unaddressed.

### *6.5 An objection from dogmatism*

Your whole account of propositional learning quite dogmatically assumes that we are capable of acquiring knowledge in at least some cases. Nothing in what you argue addresses skeptical challenges, a central project of epistemology.

*Reply*. The objector is right and I plead guilty as charged. I have not argued against skeptical challenges here, nor have I defended the thesis that we are capable of acquiring knowledge. Nevertheless, let me say that while addressing skeptical challenges is an important epistemological project, it need not be considered the *only* worthwhile epistemological project. Providing an account of knowledge, or justification, or, in this case, propositional learning, can still be valuable even if the account does not answer skeptics. Why, after all, should answering skeptics be all that matters epistemologically?

## 7. CONCLUSION

For too long epistemologists have devoted too little attention to the topic of learning. In this paper, my aim has been to delineate an account of propositional learning in terms of transitions from ignorance to knowledge, a task begun but not ended here. I invite others to bring their insights to bear on this undeservedly neglected topic. It's a field of epistemological inquiry worth much further plowing, as the TTT and TTT+ may yield a bountiful harvest of applications, some of which I have adumbrated here.[44]

## REFERENCES

**Aczel, A.** 2007. *Fermat's Last Theorem*. New York, NY: Basic Books.

**Alexander, P. A., Schallert, D. L. and Reynolds, R. E.** 2009. 'What Is Learning Anyway? A Topographical Perspective Considered.' *Educational Psychologist*, 44: 176–92.

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**Antognazza, M. R.** 2015. 'The Benefit to Philosophy of the Study of its History.' *British Journal for the History of Philosophy*, 23: 161–84.
**Audi, R.** 2011. *Epistemology: A Contemporary Introduction to the Theory of Knowledge*, 3rd edition. New York, NY: Routledge.
**Castelvecchi, D.** 2015. 'The Impenetrable Proof.' *Nature*, 526: 178–81.
**Claveau, F. and Vergara Fernández, M.** 2015. 'Epistemic Contributions of Models: Conditions for Propositional Learning.' *Perspectives on Science*, 23: 405–23.
**Dewey, J.** 1933. *How We Think*. Chicago, IL: Henry Regnery.
**DeNicola, D.** 2017. *Understanding Ignorance*. Cambridge, MA: MIT Press.
**Driscoll, M. P.** 2005. *Psychology of Learning for Instruction*. New York, NY: Pearson Education.
**Dutant, J.** 2015. 'The Legend of the Justified True Belief Analysis.' *Philosophical Perspectives*, 29: 95–145.
**Edwards, H.** 2000. *Fermat's Last Theorem: A Genetic Introduction to Algebraic Number Theory*. New York, NY: Springer-Verlag.
**Gettier, E.** 1963. 'Is Justified True Belief Knowledge?' *Analysis*, 23: 121–3.
**Hazlett, A.** 2010. 'The Myth of Factive Verbs.' *Philosophy and Phenomenological Research*, 80: 497–522.
**Le Morvan, P.** 2010. 'Knowledge, Ignorance, and True Belief.' *Theoria*, 76: 309–18.
—— 2011. 'On Ignorance: A Reply to Peels.' *Philosophia*, 39: 335–44.
—— 2012. 'On Ignorance: A Vindication of the Standard View.' *Philosophia*, 40: 379–93.
—— 2013. 'Why the Standard Conception of Ignorance Prevails.' *Philosophia*, 41: 239–56.
—— 2015. 'On the Ignorance, Knowledge, and Nature of Propositions.' *Synthese*, 192: 3647–62.
—— 2016. 'Knowledge and Security.' *Philosophy*, 91: 411–30.
—— 2017. 'Knowledge Before Gettier.' *British Journal for the History of Philosophy*, 25: 1216–38.
**Masser, D. W.** 1985. 'Open Problems.' In W. W. L. Chen (ed.), *Proceedings of the Symposium on Analytic Number Theory*. London: Imperial College.
**Mochizuki, S.** 2017a. 'Inter-universal Teichmüller Theory I: Construction of Hodge Theaters.' http://www.kurims.kyoto-u.ac.jp/~motizuki/Inter-universal%20Teichmuller%20Theory%20I.pdf.
—— 2017b. 'Inter-universal Teichmüller Theory II: Hodge-Arakelov-theoretic Evaluation' http://www.kurims.kyoto-u.ac.jp/~motizuki/Inter-universal%20Teichmuller%20Theory%20II.pdf.
—— 2017c. 'Inter-universal Teichmüller Theory III: Canonical Splittings of the Log-theta-lattice.' http://www.kurims.kyoto-u.ac.jp/~motizuki/Inter-universal%20Teichmuller%20Theory%20III.pdf.
—— 2017d. 'Inter-universal Teichmüller Theory IV: Log-volume Computations and Set-theoretic Foundations.' http://www.kurims.kyoto-u.ac.jp/~motizuki/Inter-universal%20Teichmuller%20Theory%20IV.pdf.
**Noddings, N.** 2016. *Philosophy of Education*, 4th edition. Boulder, CO: Westview Press.
**Nola, R. and Irzik, G.** 2005. *Philosophy, Science, Education, and Culture*. Dordrecht: Springer.
**Oesterlé, J.** 1988. 'Nouvelles approches du 'théorème' de Fermat.' *Astérisque, Séminaire Bourbaki*, 694: 165–86.
**Reichenbach, H.** 1938. *Experience and Prediction*. Chicago, IL: University of Chicago Press.
**Singh, S.** 2012. *Fermat's Last Theorem*. New York, NY: HarperCollins Publishers.
**Stillwell, J.** 2015. 'What Does 'Depth' Mean in Mathematics?' *Philosophia Mathematica*, 23: 215–32.
**Taylor, R. and Wiles, A.** 1995. 'Ring Theoretic Properties of Certain Hecke Algebras.' *Annals of Mathematics*, 141: 553–72.
**Turri, J.** 2011. 'Mythology of the Factive.' *Logos & Episteme*, II: 143–52.
**Wiles, A.** 1995. 'Modular Elliptic Curves and Fermat's Last Theorem.' *Annals of Mathematics*, 141: 443–551.

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