Robotica (1998) volume 16, pp. 181–192. Printed in the United Kingdom

Adaptive stabilization of uncertain nonholonomic mechanical systems

R. Colbaugh*, E. Barany† and K. Glass*

*Departments of *Mechanical Engineering and †Mathematical Sciences, New Mexico State University, Las Cruces, NM 88003 (USA)*

(Received in Final Form: July 18, 1997)

SUMMARY

This paper presents a new adaptive controller as a solution to the problem of stabilizing nonholonomic mechanical systems in the presence of incomplete information concerning the system dynamic model. The proposed control system consists of two subsystems: a slightly modified version of the kinematic stabilization strategy of M'Closkey and Murray[1] which generates a desired velocity trajectory for the nonholonomic system, and an adaptive control scheme which ensures that this velocity trajectory is accurately tracked. This approach is shown to provide arbitrarily accurate stabilization to any desired configuration and can be implemented with no knowledge of the system dynamic model. The efficacy of the proposed stabilization strategy is illustrated through extensive computer simulations with nonholonomic mechanical systems arising from explicit constraints on the system kinematics and from symmetries of the system dynamics.

KEYWORDS: Nonholonomic systems; Adaptive control; Dynamic model; Robot control; Lyapunov methods.

1. INTRODUCTION

There is great theoretical and practical interest in controlling mechanical systems in the presence of incomplete information concerning the system dynamic model. This challenging problem becomes even more difficult in the important case in which the system is subject to holonomic or nonholonomic (nonintegrable) constraints on its kinematics. Nonholonomically constrained mechanical systems, in particular, have attracted significant attention recently. Much of this interest is a consequence of the importance of such systems in applications. For example, nonholonomic constraints arise in systems with rolling contact, such as wheeled (and other) mobile robots and multifingered robotic hands, and in systems for which the dynamics admits a symmetry, such as space robots with angular momentum conservation. Observe that a wide variety of systems of practical importance are constrained in this way. Interest in studying nonholonomic systems is also motivated by their role as a class of "strongly" nonlinear systems for which traditional control methods are insufficient and new approaches must be developed. For instance, the investigation of these systems has led to recent progress in geometric control theory and geometric mechanics.

Most of the work reported to date on controlling nonholonomic mechanical systems has focused on the *kinematic control* problem, in which it is assumed that the system velocities are the control inputs and that the system dynamics can be represented using the system kinematic model. However, there are important reasons for formulating the nonholonomic system control problem at the *dynamic control* level, where the control inputs are those produced by the system actuators and the system model contains the mechanical system dynamics. For example, since this is the level at which control actually takes place in practice, designing controllers at this level can lead to significant improvements in performance and implementability and can help in the early identification and resolution of difficulties. It is interesting to consider the problem of controlling holonomically constrained robotic manipulators in this regard: the kinematic control problem in this case is trivial, but the dynamic control problem is nevertheless quite challenging. Another motivation for considering the dynamic control problem is that certain classes of nonholonomic systems are most naturally studied at this level, so that such an approach broadens the scope of potential applications. Recognizing the importance of addressing the nonholonomic system control problem at the dynamic control level, several researchers have considered this problem in recent years.[1–8] Significant progress has been made in understanding the fundamental characteristics of these systems, and several useful dynamic controllers have been presented. However, virtually all of the dynamic control strategies proposed to date have been developed under the assumption that there is little or no uncertainty associated with the system dynamic model.

This paper considers the dynamic control problem for uncertain nonholonomic mechanical systems, and proposes that a simple and effective approach to stabilizing these systems can be obtained by combining the kinematic stabilization strategy of M'Closkey and Murray[1] with the *performance-based adaptive control* methodology recently developed by the authors.[9,10] More specifically, a slightly modified version of the kinematic stabilizer given in reference 1 is used to generate a desired velocity trajectory for the system, and this velocity trajectory is tracked utilizing a performance-based adaptive controller. The resulting scheme is computationally efficient and easy to implement, and is

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

shown to provide arbitrarily accurate stabilization of the complete system without knowledge of the system dynamic model. The efficacy of the proposed approach is illustrated through extensive computer simulations with two classes of nonholonomic mechanical systems: those with explicit constraints on the system kinematics, such as occur in systems with rolling contact, and those with constraints that result from a symmetry of the system dynamics, such as are present in systems for which angular momentum is conserved.

2. PRELIMINARIES

The focus of this paper is the stabilization problem for uncertain nonholonomic mechanical systems. More specifically, we wish to develop a strategy for specifying the system control input, using only measurements of the system state and with no knowledge of the system dynamic model, so that the system evolves from its initial state to the desired final state. We are interested in nonholonomic systems arising from explicit kinematic constraints and from symmetries of the system dynamics. In this section we develop models for these two classes of systems and identify some of the useful structural features of these models.

Consider first the class of nonholonomic mechanical systems arising from the presence of explicit constraints on the system kinematics; these systems can be modeled as[2]

$$M(\mathbf{x})\mathbf{T} = H^*(\mathbf{x})\ddot{\mathbf{x}} + V^*_{cc}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}^*(\mathbf{x}) + A^T(\mathbf{x})\lambda \qquad (1a)$$

$$A(\mathbf{x})\dot{\mathbf{x}} = \mathbf{0} \qquad (1b)$$

where $\mathbf{x} \in \Re^n$ is the vector of system generalized coordinates, $\mathbf{T} \in \Re^p$ is the vector of actuator inputs, $M: \Re^n \to \Re^{n\times p}$ is bounded and of full rank, $H^*: \Re^n \to \Re^{n\times n}$ is the system inertia matrix, $V^*_{cc}: \Re^n \times \Re^n \to \Re^{n\times n}$ quantifies Coriolis and centripetal acceleration effects, $\mathbf{G}^*: \Re^n \to \Re^n$ arises from the system potential energy, $A: \Re^n \to \Re^{m\times n}$ is a bounded full rank matrix quantifying the nonholonomic constraints, $\lambda \in \Re^m$ is the vector of constraint multipliers, and all functions are assumed to be smooth. The mechanical system dynamics (1) possesses considerable structure. For example, for any set of generalized coordinates $\mathbf{x}$, the matrix H^* is symmetric and positive definite, the matrix V^*_{cc} depends linearly on $\dot{\mathbf{x}}$, and the matrices H^* and V^*_{cc} are related according to $\dot{H}^* = V^*_{cc} + V^{*T}_{cc}$. Additionally, we will assume in what follows that the inertia matrix H^* and potential energy gradient $\mathbf{G}^*$ are bounded functions with bounded first partial derivatives; these latter properties hold for virtually all mechanical systems of practical interest. In this case it is easy to show that the Coriolis/centripetal acceleration term can be written as $V^*_{cc}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} = D^*(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}]$, where $D^*: \Re^n \to \Re^{n\times n^2}$ is smooth and bounded and the notation $[\dot{\mathbf{x}}\dot{\mathbf{x}}] = [\dot{x}_1\dot{x}_1, \ldots, \dot{x}_1\dot{x}_n, \dot{x}_2\dot{x}_1, \ldots, \dot{x}_n\dot{x}_n]^T \in \Re^{n^2}$ is introduced.

The rows of A, say $\mathbf{a}_i \in \Re^{1\times n}$ for $i = 1, 2, \ldots, m$, are smooth covectors on the configuration space $\Re^n$ which quantify explicit kinematic constraints imposed on the system. These constraints could arise from rolling contact, for example. We will assume that these constraints cannot be integrated to yield constraints on the configuration coordinates $\mathbf{x}$; this assumption is made more precise below. It is well-known that the presence of these nonholonomic constraints complicates the control problem considerably. For instance, in this case the basic problem of stabilizing the system (1) to some goal configuration $\mathbf{x}_d$ cannot be solved using standard techniques.[11] This difficulty is only increased in the case of control in the presence of model uncertainty. One means of simplifying the problem of controlling these systems is to employ a reduction procedure to decrease the dimension of the dynamics (1). Observe that the assumption that A is full rank implies that the codistribution spanned by the rows $\mathbf{a}_i$ has dimension m. The annihilator of this codistribution is then an $r = n - m$ dimensional smooth distribution $\Delta = \text{span}[\mathbf{r}_1(\mathbf{x}), \mathbf{r}_2(\mathbf{x}), \ldots, \mathbf{r}_r(\mathbf{x})]$, where the $\mathbf{r}_i$ are smooth vector fields on the configuration space which satisfy $A\mathbf{r}_i = \mathbf{0}\ \forall\mathbf{x}$. Defining $R = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_r] \in \Re^{n\times r}$ permits this relationship to be expressed more concisely as $AR = 0$. As an example, let the matrix A be partitioned as $A = [A_1\ A_2]$, with $A_1 \in \Re^{m\times m}$ and $A_2 \in \Re^{m\times r}$ and where A_1 is nonsingular (this is always possible, possibly with a reordering of the configuration coordinates). Then R can be constructed as follows:

$$R = \begin{bmatrix} -A_1^{-1}A_2 \\ I_r \end{bmatrix} \qquad (2)$$

where I_r is the $r \times r$ identity matrix. Consider now the involutive closure of Δ, denoted Δ^* and defined as the smallest involutive distribution containing Δ. We will assume in what follows that Δ^* has constant rank n on the configuration space. In this case, Frobenius' theorem[12] shows that the constraints are nonintegrable and the system (1) is (completely) nonholonomic; thus there is no explicit constraint on the configuration space.

Now define a partition of $\mathbf{x}$ corresponding to the partition specified for A, so that $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T]^T$ with $\mathbf{x}_1 \in \Re^m$ and $\mathbf{x}_2 \in \Re^r$. Observe that the definition (2) and the constraint equation (1b) imply that the system velocities are determined by $\dot{\mathbf{x}}_2$ via $\dot{\mathbf{x}} = R(\mathbf{x})\dot{\mathbf{x}}_2$. This parameterization then permits (1) to be reformulated as

$$\dot{\mathbf{x}} = R(\mathbf{x})\mathbf{v} \qquad (3a)$$

$$\mathbf{F} = H(\mathbf{x})\dot{\mathbf{v}} + V_{cc}(\mathbf{x}, \mathbf{v})\mathbf{v} + \mathbf{G}(\mathbf{x}) \qquad (3b)$$

where $\mathbf{v} = \dot{\mathbf{x}}_2$, $\mathbf{F} = R^TM\mathbf{T}$, $H = R^TH^*R$, $V_{cc} = R^T(H^*\dot{R} + V^*_{cc}R)$, and $\mathbf{G} = R^T\mathbf{G}^*$. In what follows, it is assumed that $p \geq r$ and R^TM is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (3b) is fully actuated. Note that (3) consists of a "reduced" dynamic model (3b) together with a purely kinematic relationship (3a), and therefore provides a simpler description of the nonholonomic mechanical system than that given by (1). Moreover, as shown in the next lemma, the dynamics (3b) retains the mechanical system structure of the original system (1).

Lemma 1: The dynamic model terms H, $\mathbf{G}$ are bounded functions of $\mathbf{x}$ whose time derivatives $\dot{H}$, $\dot{\mathbf{G}}$ are also bounded in $\mathbf{x}$ and depend linearly on $\mathbf{v}$, the matrix H is symmetric and positive definite, and the matrices H and V_{cc} are related according to $\dot{H} = V_{cc} + V_{cc}^T$. Additionally, $V_{cc}\mathbf{v}$ satisfies $V_{cc}(\mathbf{x}, \mathbf{v})\mathbf{v} = D(\mathbf{x})[\mathbf{vv}]$ with D a bounded function with bounded first partial derivatives.

Proof: All of the properties can be established through direct calculation using the definitions of H, V_{cc}, D and $\mathbf{G}$ and the properties of H^*, V_{cc}^*, D^*, and $\mathbf{G}^*$.[13] ■

We now turn our attention to those nonholonomic mechanical systems which arise from the presence of a symmetry in the system dynamics. More specifically, consider the class of mechanical systems for which the system Lagrangian is G-invariant for some Lie group G (see, for example, reference 14 for a discussion of G-invariant Lagrangian systems), and suppose for concreteness that $G = SO(2)$ or multiple copies of $SO(2)$ (more general situations can be treated using techniques similar to those developed here, although there may be technical complications). By decomposing the configuration space into irreducible representations of $SO(2)$, it is always possible to choose (local) configuration coordinates $\mathbf{x}$ so that each component transforms as $x_i \to x_i + n_i\alpha$ for some integer n_i and $\alpha \in [0, 2\pi)$. The coordinates for which $n_j = 0$ (i.e., the invariant coordinates) are then local coordinates for $\Re^n/G$. We collect these together and write $\mathbf{x} = [\mathbf{x}_1^T \ \mathbf{x}_2^T]^T$, where $\mathbf{x}_2 \in \Re^r$ are the invariants and $\mathbf{x}_1 \in \Re^n$ transform nontrivially. Choosing coordinates in this way permits the G-invariant system Lagrangian to be written in the form $L(\mathbf{x}, \dot{\mathbf{x}}) = \dot{\mathbf{x}}^T H^*(\mathbf{x}_2)\dot{\mathbf{x}}/2 - U(\mathbf{x}_2)$ for some potential U and inertia matrix

$$H^* = \begin{bmatrix} J_1(\mathbf{x}_2) & Q(\mathbf{x}_2) \\ Q^T(\mathbf{x}_2) & J_2(\mathbf{x}_2) \end{bmatrix}$$

with submatrices J_1, J_2, Q which are independent of $\mathbf{x}_1$.

Let us restrict our attention to those systems for which the control input $\mathbf{T} \in \Re^p$ does not break the symmetry of the dynamics; no generality is lost with this assumption because, if this is not the case, then the $\mathbf{x}_1$ variables can be controlled directly and the (controlled) system is not nonholonomic. The fact that L is independent of $\mathbf{x}_1$ means that the Euler-Lagrange equations corresponding to the $\mathbf{x}_1$ coordinates have the character of a velocity constraint:

$$\frac{\partial L}{\partial \dot{\mathbf{x}}_1} = J_1(\mathbf{x}_2)\dot{\mathbf{x}}_1 + Q(\mathbf{x}_2)\dot{\mathbf{x}}_2 = \mathbf{l} \qquad (4)$$

where $\mathbf{l} \in \Re^m$ is constant. If the system starts from rest then $\mathbf{l} = 0$ and (4) can be used to parameterize the system velocities via $\dot{\mathbf{x}} = R(\mathbf{x}_2)\dot{\mathbf{x}}_2$, with R defined as

$$R = \begin{bmatrix} -J_1^{-1}Q \\ I_r \end{bmatrix} \qquad (5)$$

We again assume that the smallest involutive distribution containing the span of the columns of R has constant rank n, in which case Frobenius' theorem[12] shows that the constraints (4) are nonintegrable and the system is nonholonomic.

Now an analysis which exactly parallels the one given above for kinematic nonholonomic systems can be applied to reduce the original $2n$-dimensional symmetric mechanical system to a $2r$-dimensional mechanical system together with n kinematic equations:

$$\dot{\mathbf{x}} = R(\mathbf{x}_2)\mathbf{v} \qquad (6a)$$

$$\mathbf{F} = H(\mathbf{x}_2)\dot{\mathbf{v}} + V_{cc}(\mathbf{x}_2, \mathbf{v})\mathbf{v} + \mathbf{G}(\mathbf{x}_2) \qquad (6b)$$

where $\mathbf{F} = B(\mathbf{x}_2)\mathbf{T}$ for some matrix $B \in \Re^{r \times p}$ which depends only on $\mathbf{x}_2$ (because the inputs do not break the system symmetry). It is assumed that $p \geq r$ and B is full rank, so that any desired $\mathbf{F}$ can be realized through proper specification of $\mathbf{T}$ and the system (6b) is fully actuated. Note that (6b) is a $2r$ order differential equation which defines the evolution of the $2r$ states $(\mathbf{x}_2, \mathbf{v})$, and that the behavior of the remaining configuration coordinates $\mathbf{x}_1$ is completely determined by the kinematic relationship (6a). Moreover, an analysis similar to the one summarized in Lemma 1 can be used to show that the reduced system (6b) retains the mechanical system structure of the original system.

3. ADAPTIVE STABILIZATION SCHEME

We now consider the problem of stabilizing the uncertain nonholonomic mechanical systems discussed above. That is, we seek a strategy for specifying the control input $\mathbf{F}$, using no knowledge of the system dynamic model, which will cause the nonholonomic system (3) or (6) to converge to an arbitrarily small neighbourhood of the goal configuration $\mathbf{x}_d$. In what follows it will be assumed (without loss of generality) that $\mathbf{x}_d = \mathbf{0}$. We begin by briefly reviewing a strategy for stabilizing the *kinematic system* (3a) or (6a) which has recently been proposed by M'Closkey and Murray,[1] and then introduce an adaptive velocity tracking controller for mechanical systems of the form (3b) or (6b). We then show how these schemes can be combined to yield a stabilizer for the complete nonholonomic system.

3.1 Stabilization of kinematic system

Consider the problem of stabilizing the kinematic system

$$\dot{\mathbf{x}} = R(\mathbf{x})\mathbf{v} \qquad (7)$$

to the origin $\mathbf{x} = \mathbf{0}$ using the velocities $\mathbf{v}$ as control inputs. It is well known that such stabilization cannot be achieved through the use of continuous static feedback.[11] Moreover, while it is possible to stabilize (7) using smooth time-varying feedback, such controllers will necessarily exhibit slower than exponential rates of convergence;[1] this, in turn, has limited the applicability of these strategies. Recognizing these difficulties, M'Closkey and Murray have recently proposed a class of nonsmooth feedback controllers which are smooth everywhere except at the origin and which achieve (a form of) exponential convergence. In this section we briefly review this approach to stabilizing kinematic systems.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

The kinematic controllers proposed in reference 1 are developed using tools from homogeneous systems theory; thus we give a few definitions from this field before stating the main result we require from reference 1. A dilation on $\Re^n$ is defined by assigning n positive rationals $r_1(=1) \leq r_2 \leq \cdots \leq r_n$, a positive scalar λ, and a map $\Delta_\lambda: \Re^n \to \Re^n$ as follows:

$$\Delta_\lambda \mathbf{x} = [\lambda^{r_1} x_1, \ldots, \lambda^{r_n} x_n]^T$$

A function $f: \Re^n \to \Re$ is homogeneous of degree $l \geq 0$ with respect to Δ_λ if $f(\Delta_\lambda \mathbf{x}) = \lambda^l f(\mathbf{x})$. A vector field $\mathbf{X}$ on $\Re^n$ is homogeneous of degree $m \leq r_n$ with respect to Δ_λ if $\mathbf{X}(f)$ is homogeneous of degree $l - m$ whenever f is homogeneous of degree l. A map $\rho: \Re^n \to \Re$ is called a homogeneous norm with respect to the dilation Δ_λ if it is positive definite and homogeneous of degree one. Finally, consider a nonautonomous differential equation with an isolated equilibrium at the origin, and let $\mathbf{x}(t, \mathbf{x}_0, t_0)$ denote the solution of the equation passing through $\mathbf{x}_0$ at time t_0. The origin is locally ρ-exponentially stable if there exists a homogeneous norm ρ and two positive constants k_1 and k_2 such that locally

$$\rho(\mathbf{x}(t, \mathbf{x}_0, t_0)) \leq k_1 \rho(\mathbf{x}_0) e^{-k_2(t-t_0)}$$

It is in this sense that the kinematic controllers proposed in reference 1 are exponentially stable.

We now present the main result we wish to use from reference 1. Suppose that a smooth time-periodic feedback $\mathbf{v}_d^*(t, \mathbf{x})$ is known which stabilizes the kinematic system (7). We note that several such controllers have been proposed by various authors (see, for instance, reference 1 for a review of some of this work), so that this assumption is not restrictive. Additionally, suppose that the vector fields $\mathbf{r}_i(\mathbf{x})$ which define the kinematic system (7) are homogeneous degree one with respect to some dilation Δ_λ. Again we note that this assumption is not restrictive, because if the system (7) is not of this type it can be approximated by such a system, and it is shown in reference 1 that the terms neglected in this approximation do not cause any difficulties, at least locally. In this case, we have the following result.

Lemma 2 (M'Closkey/Murray)**:** Under the mild technical conditions described in reference 1 there exists a homogeneous degree one function $\rho^*(t, \mathbf{x})$ and a map $\gamma: \Re^n \to \Re^n$ such that the modified feedback $\mathbf{v}_d^{**}(t, \mathbf{x}) = \rho^*(t, \mathbf{x}) \mathbf{v}_d^*(t, \gamma(\mathbf{x}))$ ρ-exponentially stabilizes the kinematic system (7) and is smooth everywhere except at the origin. Moreover, in this case there exists a Lyapunov function $V_1(t, \mathbf{x})$ for the system which satisfies the bounds

$$c_1 \rho^2(\mathbf{x}) \leq V_1 \leq c_2 \rho^2(\mathbf{x})$$
$$\dot{V}_1 \leq -c_3 \rho^2(\mathbf{x})$$

for homogeneous norm ρ and positive constants c_i, where $\dot{V}_1$ denotes the derivative of V_1 along the closed-loop system $\dot{\mathbf{x}} = R(\mathbf{x}) \mathbf{v}_d^{**}$.

Proof: The proof is given in reference 1. ∎

In order to illustrate some of the ideas summarized above, we consider the problem of stabilizing the following simple kinematic system:

$$\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 \\ \dot{x}_3 &= x_2 v_1 \end{aligned} \tag{8}$$

This is an example of a so-called "chained form" system, and although it is quite simple it is worth mentioning that all two-input nonholonomic systems with three states are locally feedback equivalent to this form. It is shown in reference 15 that the smooth time-varying controller

$$\begin{aligned} v_1^* &= -x_1 + x_3 \cos t \\ v_2^* &= -x_2 + x_3^2 \sin t \end{aligned} \tag{9}$$

asymptotically stabilizes the system (8). This controller can be modified to yield a ρ-exponentially stabilizing controller as follows:

$$\begin{aligned} v_1^{**} &= -x_1 + \frac{x_3}{\rho(\mathbf{x})} \cos t \\ v_2^{**} &= -x_2 + \frac{x_3^2}{\rho^3(\mathbf{x})} \sin t \end{aligned} \tag{10}$$

where the homogeneous norm is defined as $\rho(\mathbf{x}) = (x_1^4 + x_2^4 + x_3^2)^{1/4}$. It is easily verified that both the control law (10) and the system vector fields in (8) are homogeneous degree one with respect to the dilation $\Delta_\lambda \mathbf{x} = [\lambda x_1, \lambda x_2, \lambda^2 x_3]^T$. Later in the paper we illustrate the performance obtainable when using the kinematic stabilizer (10) as part of a complete control system.

3.2 Adaptive velocity tracking

Observe that the complete nonholonomic mechanical system model (3) (or (6)) consists of two cascaded dynamical systems. As a consequence, the system velocity $\mathbf{v}$ cannot be commanded directly, as is assumed in the design of controllers at the kinematic control level, and instead must be realized as the output of the mechanical system dynamics (3b) (or (6b)) through proper specification of the control input $\mathbf{F}$. Additionally, recall that it is desirable to obtain accurate control of mechanical systems despite incomplete information regarding the system dynamic model. Thus, in order for the kinematic stabilization scheme summarized in the previous section to be practically useful, we require a strategy for causing the system (3b) (or (6b)) to track a desired trajectory for the system velocity $\mathbf{v}$ in the presence of model uncertainty.

Toward this end, let us rewrite the "reduced system" portion of the mechanical system model (3) (or (6)) using the relation $V_{cc}\mathbf{v} = D[\mathbf{vv}]$:

$$\mathbf{F} = H(\mathbf{x})\dot{\mathbf{v}} + D(\mathbf{x})[\mathbf{vv}] + \mathbf{G}(\mathbf{x}) \tag{11}$$

We now present a velocity tracking controller for this system which is implementable without information regarding the system dynamic model. Let $\mathbf{v}_d(t)$ denote

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

the desired trajectory for the system velocity $\mathbf{v}$, and consider the following adaptive control scheme:

$$\begin{aligned}\mathbf{F} &= A(t)\dot{\mathbf{v}}_d + B(t)[\mathbf{v}\mathbf{v}_d] + \mathbf{f}(t) + k\mathbf{e} \\ \dot{\mathbf{f}} &= -\sigma\mathbf{f} + \beta\mathbf{e} \\ \dot{A} &= -\sigma A + \beta\mathbf{e}\dot{\mathbf{v}}_d^T \\ \dot{B} &= -\sigma B + \beta\mathbf{e}[\mathbf{v}\mathbf{v}_d]^T\end{aligned} \tag{12}$$

where $\mathbf{e} = \mathbf{v}_d - \mathbf{v}$ is the tracking error, $\mathbf{f}(t) \in \Re^r$, $A(t) \in \Re^{r\times r}$, $B(t) \in \Re^{r\times r^2}$ are adaptive elements, k is a positive constant, and σ, β are adaptation gains.

The stability properties of the proposed adaptive velocity tracking controller (12) are summarized in the following lemma.

Lemma 3: The adaptive controller (12) ensures that $\mathbf{e}$, $\mathbf{f}$, A, and B are globally uniformly bounded and that the velocity tracking error $\mathbf{e}$ converges exponentially to a compact set which can be made arbitrarily small.

Proof: Applying the control law (12) to the mechanical system dynamics (11) yields the closed-loop system

$$H\dot{\mathbf{e}} + V_{cc}\mathbf{e} + k\mathbf{e} + \mathbf{\Phi} + \Phi_A\dot{\mathbf{v}}_d + \Phi_B[\mathbf{v}\mathbf{v}_d] = 0 \tag{13}$$

where $\mathbf{\Phi} = \mathbf{f} - \mathbf{G}$, $\Phi_A = A - H$, and $\Phi_B = B - D$. Consider the Lyapunov function candidate

$$V_2 = \tfrac{1}{2}\mathbf{e}^T H\mathbf{e} + \frac{1}{2\beta}(\mathbf{\Phi}^T\mathbf{\Phi} + \mathrm{tr}[\Phi_A\Phi_A^T + \Phi_B\Phi_B^T]) \tag{14}$$

and note that V_2 is a positive definite and proper function of $\mathbf{e}$, $\mathbf{\Phi}$, Φ_A, and Φ_B. Computing the derivative of (14) along (13) and simplifying using an analysis similar to the one utilized in reference 9 (Theorem 1) yields the following upper bound for $\dot{V}_2$:

$$\dot{V}_2 \le -\frac{k}{2}\|\mathbf{e}\|^2 - \frac{\sigma}{2\beta}(\|\mathbf{\Phi}\|^2 + \|\Phi_A\|_F^2 + \|\Phi_B\|_F^2) + \frac{\eta_1}{\beta} \tag{15}$$

where η_1 is a positive scalar constant which does not increase as β is increased.

Examination of (14) and (15) reveals that there exist positive scalar constants λ_i, independent of β, such that V_2 and $\dot{V}_2$ can be bounded as

$$\lambda_1\|\mathbf{e}\|^2 + \frac{\lambda_2}{\beta}\|\mathbf{\Psi}\|^2 \le V_2 \le \lambda_3\|\mathbf{e}\|^2 + \frac{\lambda_4}{\beta}\|\mathbf{\Psi}\|^2$$

$$\dot{V}_2 \le -\lambda_5\|\mathbf{e}\|^2 - \frac{\lambda_6}{\beta}\|\mathbf{\Psi}\|^2 + \frac{\eta_1}{\beta}$$

where $\mathbf{\Psi} = [\|\mathbf{\Phi}\|\ \|\Phi_A\|_F\ \|\Phi_B\|_F]^T$. The ultimate boundedness result presented in reference 9 (Lemma 2) now applies and permits the conclusion that all signals are uniformly bounded and that $\mathbf{e}$, $\mathbf{\Psi}$ converge exponentially to the closed balls B_{r_1}, B_{r_2}, respectively, where $r_1^2 = \delta\eta_1/\lambda_1\beta$ and $r_2^2 = \delta\eta_1/\lambda_2$ with $\delta = \max(\lambda_3/\lambda_5, \lambda_4/\lambda_6)$. Observe that the radius of the ball to which $\mathbf{e}$ is guaranteed to converge can be decreased as desired simply by increasing β. ■

A few observations can be made concerning the adaptive control strategy (12). First note that the control law is simple and requires no information concerning the mechanical system dynamics. Thus the proposed scheme provides a computationally efficient, modular, and readily implementable approach to velocity tracking for mechanical systems. Note also that the controller ensures global exponential convergence of the tracking error to an arbitrarily small neighborhood of zero; thus both the transient and steady-state performance of the strategy will be good. Finally, observe that increasing the adaptation gain β does not ordinarily lead to large control action. To see this, observe from the ultimate bound on $\mathbf{\Psi}$ that increasing β does not increase the size of the discrepancy between adaptive terms and model terms, and therefore does not cause unwarranted growth in the control law terms.

3.3 Stabilization of complete system

In this section we show that the kinematic stabilization strategy summarized in Lemma 2 and the velocity tracking controller proposed in (12) can be combined, in slightly modified form, to yield a scheme for providing arbitrarily accurate stabilization of the complete nonholonomic mechanical system (3) or (6). Our basic approach is as follows: we first "smooth" the input $\mathbf{v}_d^{**}$ defined in Lemma 2 in such a way that this stabilizing input can be naturally combined with the velocity tracking controller (12), and we then show that the resulting control law possesses the desired stability and convergence properties using a Lyapunov stability result for stability with respect to sets.[16] This process is made more precise in the following theorem:

Theorem 1: Suppose that a kinematic control input $\mathbf{v}_d$ is constructed in such a way that it is differentiable and satisfies the following two conditions:

a. The input $\mathbf{v}_d$ is such that $\mathbf{v}_d = \mathbf{v}_d^{**}$ whenever $\mathbf{x}$ is outside the set $B_\varepsilon = \{\mathbf{x} \mid \rho(\mathbf{x}) \le \varepsilon\}$, where ρ is the homogeneous norm associated with the stabilizing feedback $\mathbf{v}_d^{**}$ and ε is a (small) user-specified constant.

b. The input $\mathbf{v}_d$ guarantees that $\dot{V}_1 \le 0$ whenever $\mathbf{x} \in B_\varepsilon$, where V_1 is the Lyapunov function defined in Lemma 2.

Then, if k is chosen large enough, this definition for $\mathbf{v}_d$ can be used with the adaptive tracking controller (12) to provide arbitrarily accurate stabilization of the complete nonholonomic mechanical system (3) or (6).

Proof: Applying the proposed control law to the nonholonomic mechanical system (3) or (6) yields the closed-loop system

$$\dot{\mathbf{x}} = R\mathbf{v}_d - R\mathbf{e} \tag{16a}$$

$$H\dot{\mathbf{e}} + V_{cc}\mathbf{e} + k\mathbf{e} + \mathbf{\Phi} + \Phi_A\dot{\mathbf{v}}_d + \Phi_B[\mathbf{v}\mathbf{v}_d] = 0 \tag{16b}$$

where all terms are defined as before. Consider the Lyapunov function candidate $V_3 = V_1 + V_2$. Note that V_3 is a positive definite and proper function of $\mathbf{x}$, $\mathbf{e}$, $\mathbf{\Phi}$, Φ_A, and Φ_B; indeed, the following bounds on V_3 can easily be derived:

$$\lambda_1\rho^2(\mathbf{x}) + \lambda_2\|\mathbf{e}\|^2 + \frac{\lambda_3}{\beta}\|\mathbf{\Psi}\|^2 \le V_3 \le \lambda_4\rho^2(\mathbf{x}) + \lambda_5\|\mathbf{e}\|^2 + \frac{\lambda_6}{\beta}\|\mathbf{\Psi}\|^2 \tag{17}$$

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

where the λ_i are positive constants which are independent of β and ε. Computing the derivative of V_3 along (16) and simplifying using Lemmas 2 and 3 yields the following upper bound for $\dot{V}_3$:

$$\dot{V}_3 \le \dot{V}_1 - \frac{\partial V_1}{\partial \mathbf{x}} R\mathbf{e} - \frac{k}{2}\|\mathbf{e}\|^2 - \frac{\sigma}{2\beta}(\|\mathbf{\Phi}\|^2 + \|\mathbf{\Phi}_A\|_F^2 + \|\mathbf{\Phi}_B\|_F^2) + \frac{\eta_1}{\beta} \quad (18)$$

where $\dot{V}_1 = \partial V_1/\partial t + (\partial V_1/\partial \mathbf{x})R\mathbf{v}_d$ and all other terms are defined as before.

It is shown in reference 1 that $\|(\partial V_1/\partial \mathbf{x})R\mathbf{e}\| \le c_4\rho(\mathbf{x})\|\mathbf{e}\|$ with c_4 a positive constant. If k is chosen large enough then this bound together with some routine manipulation permits the following bounds on $\dot{V}_3$ to be obtained:

$$\dot{V}_3 \le \begin{cases} -\lambda_8\|\mathbf{e}\|^2 - \dfrac{\lambda_9}{\beta}\|\mathbf{\Psi}\|^2 + \dfrac{\eta_1}{\beta} + \dfrac{\eta_2\varepsilon^2}{k} & \mathbf{x} \in B_\varepsilon \\ -\lambda_7\rho^2(\mathbf{x}) - \lambda_8\|\mathbf{e}\|^2 - \dfrac{\lambda_9}{\beta}\|\mathbf{\Psi}\|^2 + \dfrac{\eta_1}{\beta} & \text{otherwise} \end{cases} \quad (19)$$

where η_2 and the λ_i are positive constants which are independent of β and ε. The bounds given in (17) and (19) imply that, if ε^2 is chosen to be inversely proportional to β, V_3 is uniformly ultimately bounded with ultimate bound B given by $B = \eta_3/\beta$, where η_3 is a positive constant which does not increase as β increases.

Define $\mathbf{z} = [\mathbf{x}^T\ \mathbf{e}^T\ \mathbf{\Psi}^T]^T$ and let $|\mathbf{z}|_{\mathscr{A}}$ denote the distance between $\mathbf{z}$ and a (nonempty) set $\mathscr{A}$ (see reference 16 for a more careful definition of this notion of distance to a set). Consider the set $\mathscr{A} = \{\mathbf{x}, \mathbf{e}, \mathbf{\Psi} \mid V_3 \le \eta_3/\beta\}$, and observe from the above calculations that $\mathbf{x} \in B_\varepsilon$ implies $\mathbf{x} \in \mathscr{A}$. In this case, the preceding analysis shows that $\mathscr{A}$ is compact, invariant, and such that

$$\alpha_1(|\mathbf{z}|_{\mathscr{A}}) \le V_3 - V_3^* \le \alpha_2(|\mathbf{z}|_{\mathscr{A}})$$

$$\dot{V}_3 \le -\alpha_3(|\mathbf{z}|_{\mathscr{A}})$$

for proper class $\mathscr{K}$ functions α_1, α_2 and class $\mathscr{K}$ function α_3, where $V_3^* = \min(V_3, \eta_3/\beta)$. The Lyapunov stability theorem for stability with respect to sets given by Lin and Sontag in reference 16 can now be applied and permits the conclusion that $\mathbf{z}$ converges to $\mathscr{A}$. This, in turn, implies that the nonholonomic mechanical system (3) or (6) can be stabilized to any desired degree of accuracy. To see this, note from (17) and the definition of the set $\mathscr{A}$ that the dimensions of $\mathscr{A}$ in the $\mathbf{x}$ and $\mathbf{e}$ directions can be reduced as desired by increasing β (and correspondingly decreasing ε). ∎

Several observations can be made regarding the adaptive stabilization strategy described in Theorem 1. First note that the controller is simple and requires no information concerning the mechanical system dynamics. Thus the proposed scheme provides a computationally efficient, modular, and readily implementable approach to stabilizing nonholonomic mechanical systems. Note also that the controller ensures global convergence of the system state to an arbitrarily small neighborhood of the origin, and achieves this convergence with well-behaved transient behavior. Finally, observe that, given the kinematic stabilizer $\mathbf{v}_d^{**}$ proposed in reference 1, the smoothed input $\mathbf{v}_d$ is not difficult to obtain. For example, since the input $\mathbf{v}_d^{**}$ is derived from a smooth stabilizing input $\mathbf{v}_d^*$ (see Lemma 2), simply "blending" these two inputs together often provides a suitable controller. Additionally, we mention that in general the construction of the smoothed input $\mathbf{v}_d$ is simplified by the fact that this input need only guarantee that $\dot{V}_1 = 0$ (rather than $\dot{V}_1 < 0$) in the region where it is different from $\mathbf{v}_d^{**}$. Thus, for instance, in all of the simulations presented in this paper $\mathbf{v}_d$ is constructed by smoothly connecting $\mathbf{v}_d^{**}$ (the input in the region outside B_ε) with the *zero* input inside B_ε.

4. SIMULATION RESULTS

The effectiveness of the proposed approach for stabilizing uncertain nonholonomic mechanical systems is now examined through computer simulations with four such systems: a two wheel mobile robot, a three wheel mobile robot, a "hopping" robot, and a "free-flying" space robot. The mobile robots are representative of the class of nonholonomic systems which result from explicit constraints on the system kinematics, while the hopping robot and free-flying space robot are examples of systems with nonholonomic constraints arising from the presence of symmetry in the system dynamics.

The first system considered in this simulation study is the simple two wheel mobile robot (with front castor for balance) shown in Figure 1 and described in reference 5. The dynamic model (1) has the following form for this

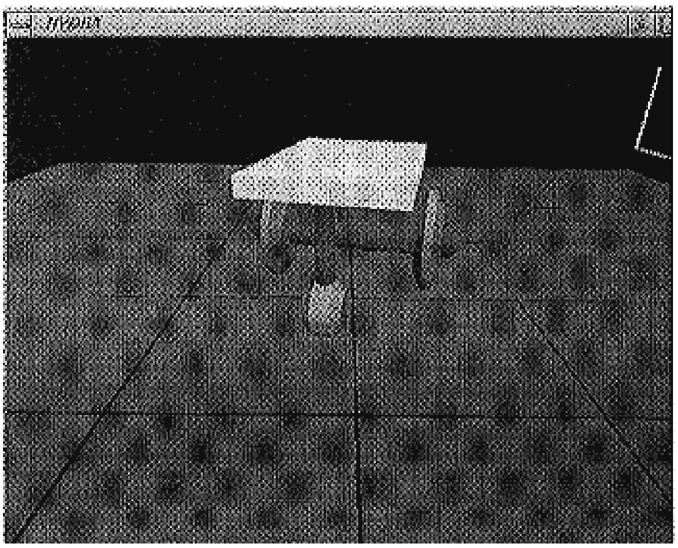

Fig. 5. Illustration of hopping robot.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

system:

$$\begin{aligned} m\ddot{x} &= \lambda \cos\theta - p_1(T_1 + T_2)\sin\theta + d_1(t) \\ m\ddot{y} &= \lambda \sin\theta + p_1(T_1 + T_2)\cos\theta + d_2(t) \\ J\ddot{\theta} &= p_2(T_1 - T_2) \\ 0 &= \dot{x}\cos\theta + \dot{y}\sin\theta \end{aligned} \tag{20}$$

where x, y, θ are the position and orientation coordinates of the (axle of the) mobile robot, m, J are the system inertial parameters, p_1, p_2 are kinematic parameters, λ is the constraint multiplier, and T_1, T_2 are the torques provided at the wheels. The terms d_1, d_2 are external disturbances and are included to permit the robustness of the proposed stabilization scheme to be examined.

We now apply the adaptive control strategy summarized in Theorem 1 to the problem of stabilizing the mobile robot to the goal $x = y = \theta = 0$. In implementing the controller, we assume that no information is available regarding the system dynamic model. Observe that the kinematic model for this system is (locally) feedback equivalent to the chained form (8). Thus the kinematic stabilizer (10) can be used to generate the desired velocity for the system (subject to smoothing around the origin), and this velocity trajectory can be tracked with the adaptive scheme (12). The controller is applied to the mathematical model of the mobile robot through computer simulation with a sampling period of two milliseconds. All integrations required by the controller are implemented using a simple trapezoidal integration rule with a time-step of two milliseconds. Additional details concerning the simulation strategy for this study can be found in reference 17. The system model parameters are defined as $m = J = 10$ and $p_1 = p_2 = 1$. The external disturbances d_1 and d_2 are modeled as the sum of a constant (bias) force with magnitude equal to one half the maximum undisturbed control force and a zero-mean Gaussian signal with standard deviation of one tenth the maximum undisturbed control force. The controller parameters k and ε are set as $k = 10$ and $\varepsilon = 0.001$, the adaptive gains $\mathbf{f}$, A, and B are set to zero initially, and the adaptation parameters are set as follows: $\sigma = 0.1$ and $\beta = 10$. It is noted that no attempt was made to "tune" the gains to obtain the best possible performance. The control strategy was tested using a wide range of initial conditions; sample results are given in Figures 2a and 2b, and indicate that accurate stabilization of the system is achieved.

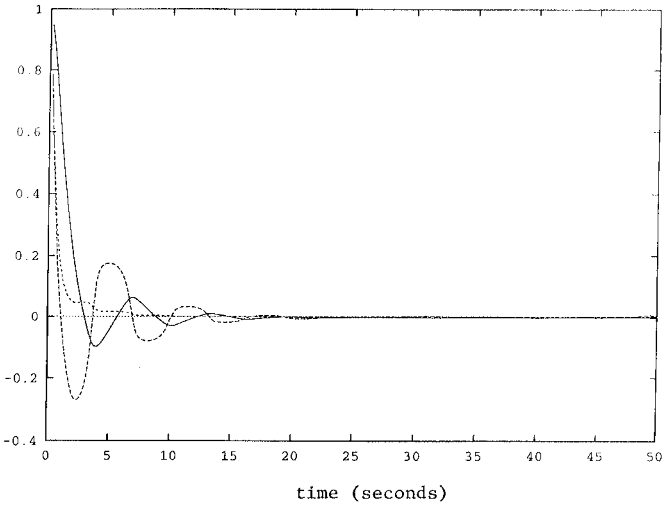

Fig. 2b. Response of x_1 (solid), x_2 (dashed), and x_3 (dotted) coordinates of two wheel mobile robot for one sample initial condition.

Consider next the more complicated three wheel mobile robot shown in Figure 3 and described in reference 17. For this system the dynamic model (1) has the following form:

$$\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ l\sin\phi\cos\phi & 0 \\ 0 & 1 \end{bmatrix}\mathbf{T} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & (I_r + I_f) & I_f \\ 0 & 0 & I_f & I_f \end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix} -\sin\theta & -\sin(\theta+\phi) \\ \cos\theta & \cos(\theta+\phi) \\ 0 & l\cos\phi \\ 0 & 0 \end{bmatrix}\lambda \tag{21}$$

$$\begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\sin(\theta+\phi) & \cos(\theta+\phi) & l\cos\phi & 0 \end{bmatrix}\dot{\mathbf{x}} = 0$$

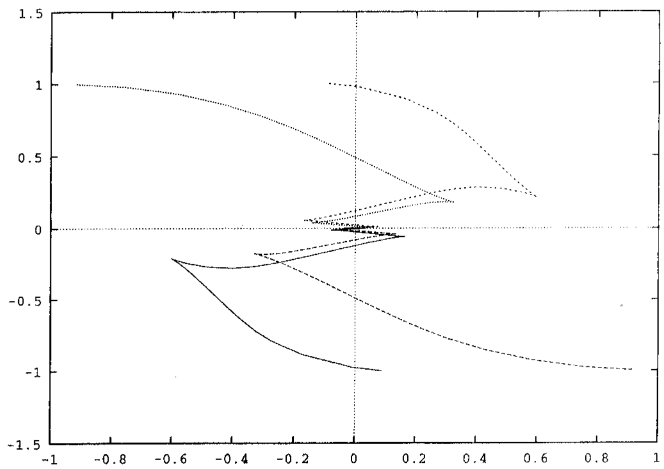

Fig. 2a. Response of (x_1, x_3) coordinates of two wheel mobile robot for four sample initial conditions.

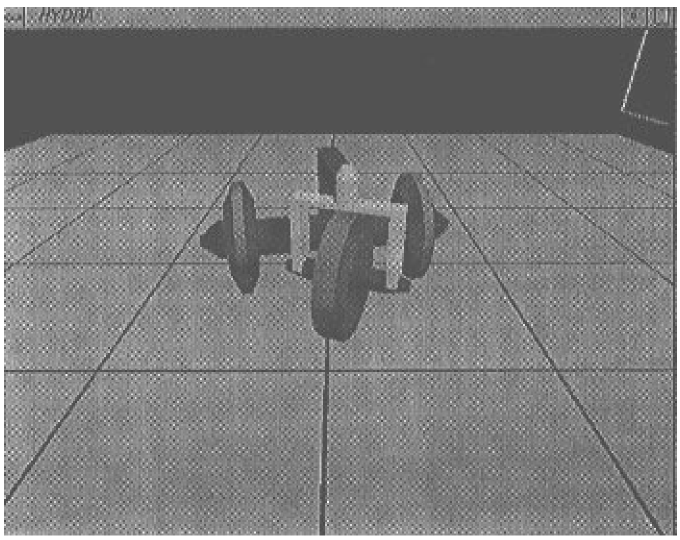

Fig. 3. Illustration of three wheel mobile robot.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

where (x, y) is the rear axle position, θ and ϕ are the body angle and steering angle, respectively, $\mathbf{x} = [x\ y\ \theta\ \phi]^T \in \Re^4$ is the system configuration vector, I_r, I_f, m are the system inertial parameters, l is the wheel base length, $\mathbf{T} \in \Re^2$ is the control input, and $\lambda \in \Re^2$ is the vector of constraint multipliers.

We now apply the adaptive control strategy summarized in Theorem 1 to the problem of stabilizing this mobile robot to the goal $x = y = \theta = \phi = 0$; in implementing the controller, we assume that no information is available regarding the system dynamic model. Observe that the kinematic model for this system is (locally) feedback equivalent to the following chained form:

$$\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 \\ \dot{x}_3 &= x_2 v_1 \\ \dot{x}_4 &= x_3 v_1 \end{aligned} \tag{22}$$

Thus a generalization of the kinematic stabilizer (10) can be used to generate the desired velocity for the system (subject to smoothing around the origin), and this velocity trajectory can be tracked with the adaptive scheme (12). The controller is applied to the mathematical model of the mobile robot through computer simulation with a sampling period of two milliseconds. Additional details concerning the simulation strategy for this study can be found in reference 17. The system model parameters are defined as $m = I_f = I_r = 20$ and $l = 0.5$. The controller parameters and adaptive terms in the control law are set to the values used in the previous simulation, despite the fact that the two mobile robots have quite different properties. This choice for the controller terms is made to demonstrate that these gains need not be "tuned" for a particular system to obtain good performance. The control strategy was tested using a wide range of initial conditions; sample results are given in Figure 4, and indicate that accurate stabilization of the system is achieved.

As indicated above, the performance of the proposed stabilization scheme was also examined through computer simulations with systems whose nonholonomic nature is a consequence of a symmetry of the system dynamics. Consider first a simple model of a "hopping robot". In this study, the system is modeled as a rigid robot "body" with inertia J pinned to the ground at its center of mass, and a two degree-of-freedom robot "leg" composed of a revolute joint and a prismatic joint connected in series (see Figure 5). The leg has total mass m, assumed for simplicity to be a point mass which slides along the leg under the action of the prismatic joint (see Figure 5). The robot leg has two actuators, one at each joint, while the joint which connects the robot body to the ground is unactuated. Note that pinning the body of the robot in this way permits the body to rotate freely but prevents translation. Thus the nonholonomic constraint arising from angular momentum conservation is retained, while the holonomic translational constraints are replaced with holonomic pinned constraints; observe that this simplifies the subsequent analysis but removes none of the essential structure of the model of a hopping robot in the free flight stage of motion. Let θ_1 denote the angle of the body and (θ_2, r) be polar coordinates for the mass of the leg. It is easily verified that in this case the system model is of the form (6), where the mechanical system dynamics (6b) is standard and the kinematic map (6a) can be obtained from the nonholonomic constraint corresponding to angular momentum conservation:

$$J\dot{\theta}_1 + mr^2\dot{\theta}_2 = 0$$

where it has been assumed that the system is initially at rest.

We now apply the adaptive control strategy summarized in Theorem 1 to the problem of stabilizing the hopping robot to the goal $\theta_1 = \theta_2 = 0°$, $r = 200$ cm in the presence of dynamic model uncertainty (note that $r = 0$ is a singularity of the system, so we have chosen a nonzero

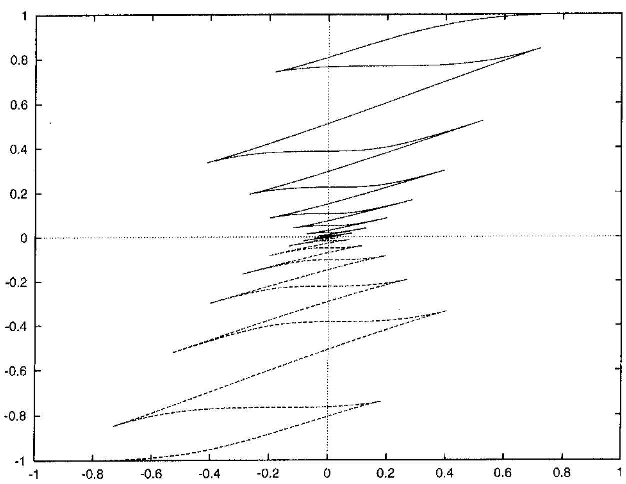

Fig. 4. Response of (x_1, x_4) coordinates of three wheel mobile robot for two sample initial conditions.

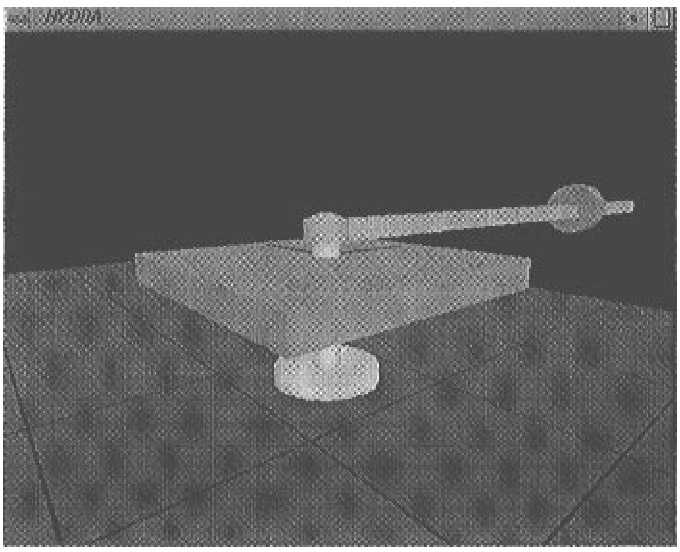

Fig. 5. Illustration of hopping robot.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

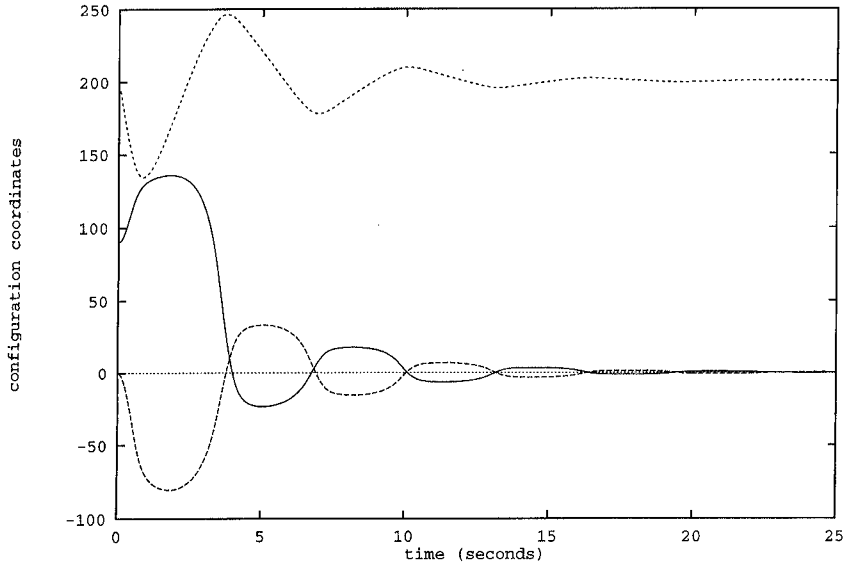

Fig. 6a. Response of θ_1 (solid), θ_2 (dashed), and r (dotted) coordinates of hopping robot for first initial condition.

goal for r). Observe that the kinematic model for this system is (locally) feedback equivalent to the chained form (8). Thus the kinematic stabilizer (10) can be used to generate the desired velocity for the system (subject to smoothing around the origin), and this velocity trajectory can be tracked with the adaptive scheme (12). The controller is applied to the mathematical model of the hopping robot through computer simulation with a sampling period of two milliseconds. Additional details concerning the simulation strategy for this study can be found in reference 17. The system model parameters are chosen as $J=10$ and $m=5$. The adaptive tracking scheme (12) is implemented exactly as described in the preceding simulation; thus, obviously, no attempt was made to "tune" the gains in this simulation. The control strategy was tested using a wide range of initial conditions; sample results are given in Figures 6a through 6c and indicate that accurate stabilization of the system is achieved.

Finally, we turn our attention to another symmetric mechanical system: a simple model of a "free-flying" space robot (see Figure 7). The system is modeled as a rigid "vehicle" with inertia J pinned to the ground at its center of mass, and a two link planar manipulator with link lengths $l_1 = l_2 = l$ and link masses $m_1 = m_2 = m$, assumed for simplicity to be concentrated at the distal ends of the two links. The manipulator has two actuators, one at each joint, while the vehicle's pinned connection to the ground is unactuated. Note that pinning the vehicle in this way permits the body to rotate freely but prevents translation. Thus, just as was the case with the hopping robot above, the nonholonomic constraint arising from angular momentum conservation is retained and this simplification removes none of the essential structure of the system. Let ϕ denote the angle of the vehicle and (θ_1, θ_2) be coordinates for the manipulator. It is easily verified that in this case the system model is of the form (6), where the mechanical system dynamics (6b)

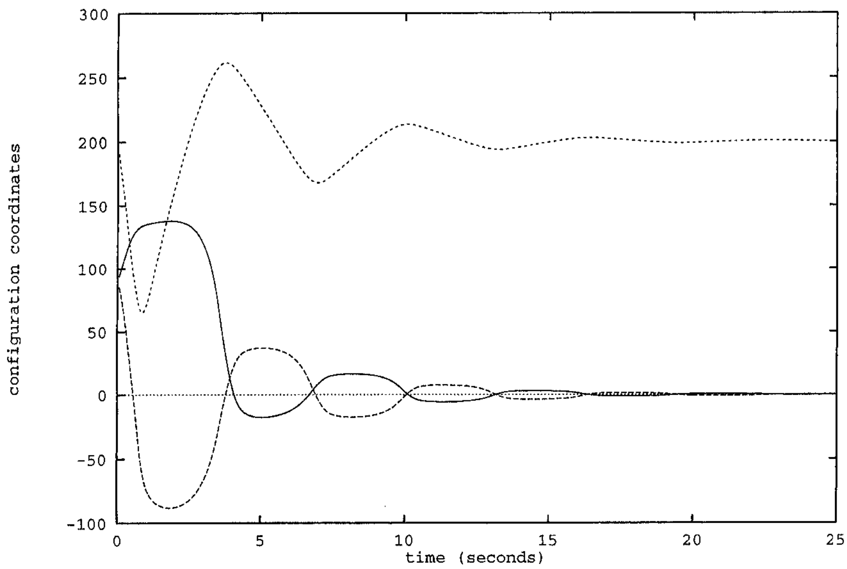

Fig. 6b. Response of θ_1 (solid), θ_2 (dashed), and r (dotted) coordinates of hopping robot for second initial condition.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

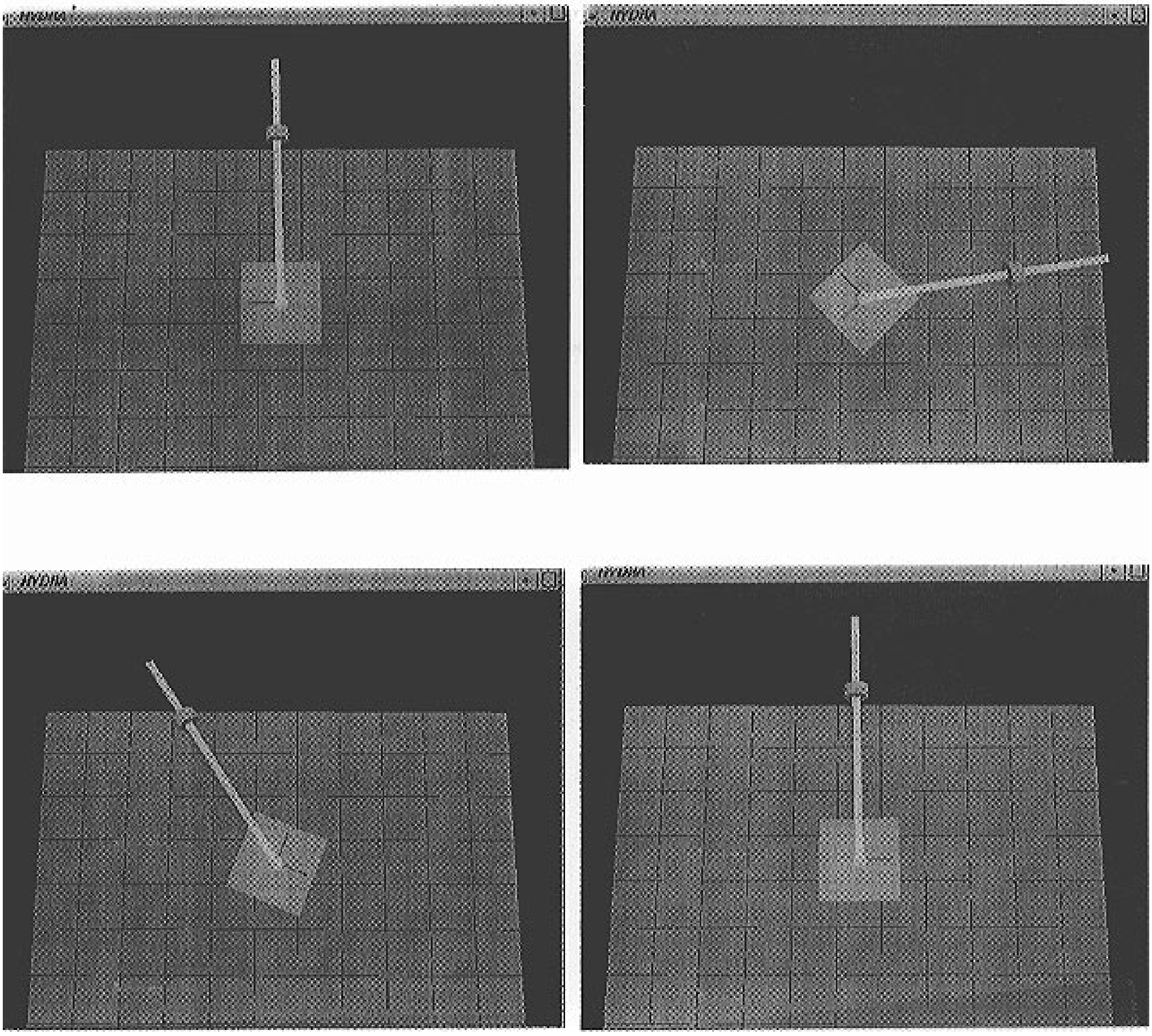

Fig. 6c. Sample of configurations in hopping robot simulation.

is standard and the kinematic map (6a) can be obtained from the nonholonomic constraint corresponding to angular momentum conservation:

$$p_1\dot{\phi} + p_2\dot{\theta}_1 + p_3\dot{\theta}_2 + p_4 \cos\theta_2(2\dot{\phi} + 2\dot{\theta}_1 + \dot{\theta}_2) = 0$$

where the p_i are system parameters obtained through routine manipulation and it has been assumed that the system is initially at rest.

We now apply the adaptive control strategy summarized in Theorem 1 to the problem of stabilizing the space robot to the goal $\phi = \theta_1 = 0°$, $\theta_2 = 75°$ in the presence of dynamic model uncertainty (note that $\theta_2 = 0$ is a singularity of the system, so we have chosen a nonzero goal for θ_2). Observe that the kinematic model for this system is (locally) feedback equivalent to the chained form (8). Thus the kinematic stabilizer (10) can be used to generate the desired velocity for the system (subject to smoothing around the origin), and this velocity trajectory can be tracked with the adaptive scheme (12). The controller is applied to the mathematical model of the space robot through computer simulation with a sampling period of two milliseconds. Additional details concerning the simulation strategy for this study can be found in reference 17. The system model parameters are chosen as follows: $J = 20$, $m = 5$, and $l = 1$. The adaptive tracking scheme (12) is implemented exactly as described in the preceding

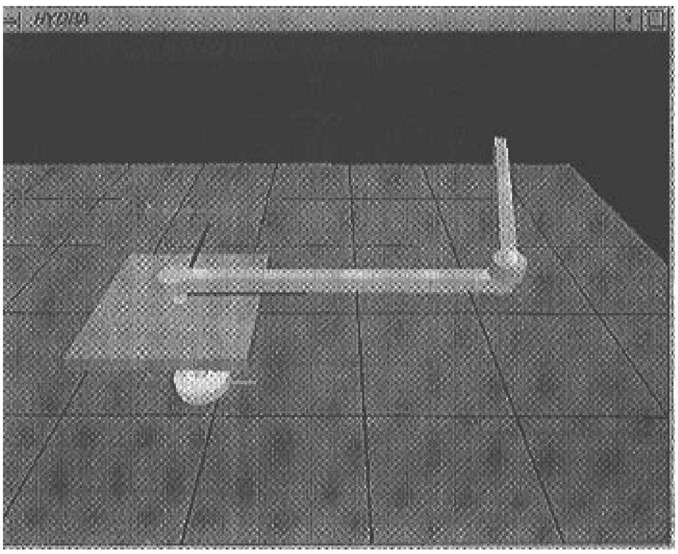

Fig. 7. Illustration of free-flying space robot.

https://doi.org/10.1017/S0263574798000514 Published online by Cambridge University Press

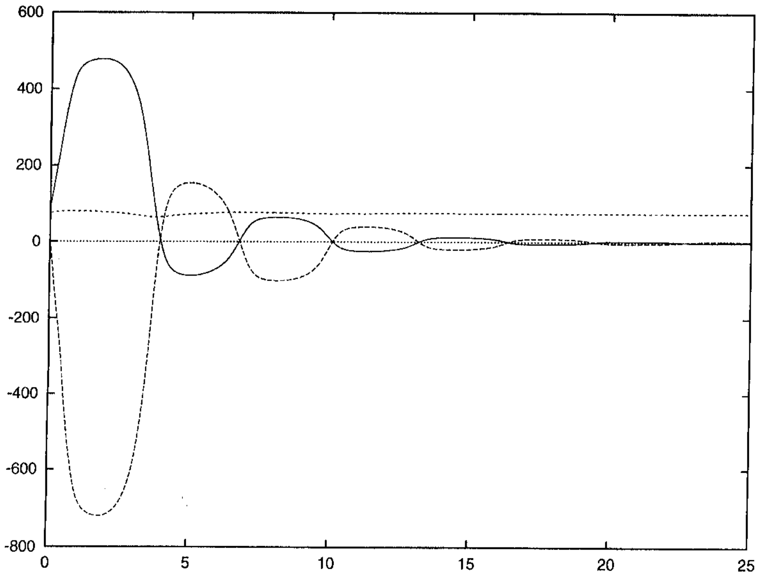

Fig. 8a. Response of ϕ (solid), θ_1 (dashed), and θ_2 (dotted) coordinates of free-flying space robot for first initial condition.

simulation; thus, obviously, no attempt was made to "tune" the gains for this simulation. The control strategy was tested using a wide range of initial conditions; sample results are given in Figures 8a and 8b and indicate that accurate stabilization of the system is achieved.

5. CONCLUSIONS

This paper considers the problem of stabilizing nonholonomic mechanical systems in the presence of incomplete information concerning the system dynamic model. It is shown that a simple and effective solution to this problem can be obtained by combining the kinematic stabilization strategy of M'Closkey and Murray[1] with adaptive control methods. The resulting control scheme is computationally efficient and easy to implement, and provides arbitrarily accurate stabilization without knowledge of the system dynamic model.

The performance of the proposed stabilization strategy is illustrated through computer simulations with nonholonomic systems arising from both explicit kinematic constraints and symmetries of the system dynamics. This simulation study indicates that the control scheme possesses significant advantages compared with other controllers for nonholonomic systems. For example, the model independence of the strategy provides enhanced simplicity, modularity, and robustness relative to other schemes, and facilitates convenient implementation with a wide range of nonholonomic systems. Additionally, the performance of the proposed controller was found to be superior to other stabilizers in terms of convergence rate and overall transient behavior. These latter qualities are to be expected in view of the structure of the control system, which combines an exponentially convergent

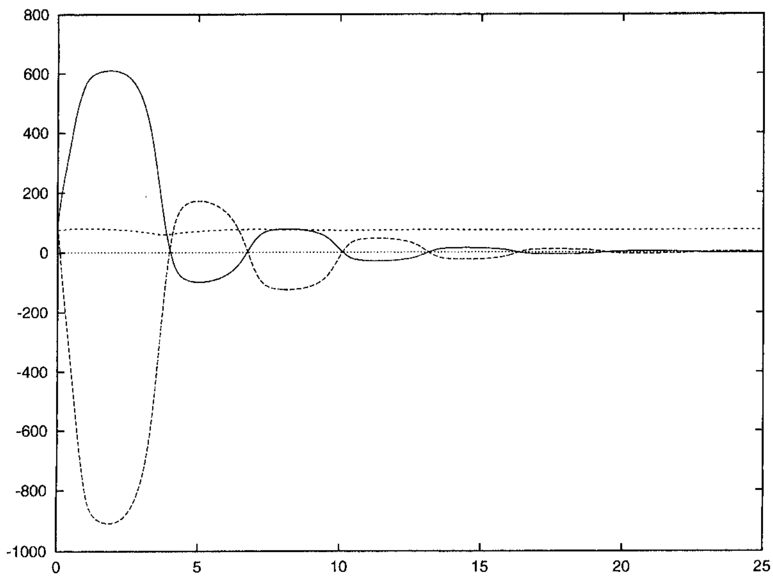

Fig. 8b. Response of ϕ (solid), θ_1 (dashed), and θ_2 (dotted) coordinates of free-flying space robot for second initial condition.

kinematic stabilizer with a high performance adaptive tracking strategy. It is believed that this combination of implementability, robustness, and good transient performance makes the controller well suited for many robotics and automation applications.

Acknowledgments

The research described in this paper was supported in part through contracts with the Army Research Office and the Department of Energy (WERC).

References

1. R. M'Closkey and R. Murray, "Exponential Stabilization of Driftless Nonlinear Control Systems Using Homogeneous Feedback" *Technical Report CIT/CDS 95-012* California Institute of Technology (May, 1995).
2. A. Bloch, M. Reyhanoglu and N. McClamroch, "Control and Stabilization of Nonholonomic Dynamic Systems" *IEEE Transactions on Automatic Control* **37,** No. 11, 1746–1757 (1992).
3. G. Walsh and L. Bushnell, "Stabilization of Multiple Input Chained Form Control Systems" *Proc. 32nd IEEE Conference on Decision and Control,* San Antonio, TX, (December, 1993) pp. 959–964.
4. N. Sarkar, X. Yun and V. Kumar, "Control of Mechanical Systems with Rolling Constraints: Application to Dynamic Control of Mobile Robots" *Int. J. Robotics Research* **13,** No. 1, 55–69 (1994).
5. C. Su and Y. Stepanenko, "Robust Motion/Force Control of Mechanical Systems with Classical Nonholonomic Constraints" *IEEE Transactions on Automatic Control,* **39,** No. 3, 609–614 (1994).
6. R. M'Closkey and R. Murray, "Extending Exponential Stabilizers for Nonholonomic Systems from Kinematic Controllers to Dynamic Controllers" *Proc. IFAC Symposium on Robot Control,* Capri, Italy (September, 1994) pp. 243–248.
7. B. Maschke and A. van der Schaft, "A Hamiltonian Approach to Stabilization of Nonholonomic Mechanical Systems" *Proc. 33rd IEEE Conference on Decision and Control,* Orlando, FL (December, 1994) pp. 2950–2954.
8. R. Murray, "Nonlinear Control of Mechanical Systems: A Lagrangian Perspective" *Proc. IFAC Symposium on Nonlinear Control System Design,* Lake Tahoe, CA (June, 1995) (preprint).
9. R. Colbaugh, H. Seraji and K. Glass, "Adaptive Compliant Motion Control for Dexterous Manipulators" *Int. J. Robotics Research* **14,** No. 3, 270–280 (1995).
10. R. Colbaugh, K. Glass and E. Barany, "Adaptive Regulation of Manipulators Using Only Position Measurements" *Int. J. Robotics Research* **16,** No. 5. (in press) (1997).
11. R. Brockett, "Asymptotic Stability and Feedback Stabilization" *Differential Geometric Control Theory* (Birkhauser, Boston, 1983).
12. H. Nijmeijer and A. van der Schaft, *Nonlinear Dynamical Control Systems* (Springer-Verlag, New York, 1990).
13. R. Colbaugh, E. Barany and K. Glass, "Adaptive Control of Nonholonomic Mechanical Systems" *Proc. 35th IEEE Conference on Decision and Control,* Kobe, Japan (December, 1996) pp. 1428–1434.
14. J. Marsden and T. Ratiu, *Introduction to Mechanics and Symmetry* (Springer-Verlag, New York, 1994).
15. A. Teel, R. Murray and G. Walsh, "Nonholonomic Control Systems: From Steering to Stabilization With Sinusoids" *Proc. 31st IEEE Conference on Decision and Control,* Tucson, AZ (December, 1992) pp. 1603–1609.
16. Y. Lin and E. Sontag, "On Control-Lyapunov Functions Under Input Constraints" *Proc. 33rd IEEE Conference on Decision and Control,* Orlando, FL (December, 1994) pp. 640–645.
17. R. Colbaugh, "Control of Uncertain Nonholonomic Mechanical Systems: Case Studies" *Robotics Laboratory Report* (New Mexico State University, November, 1995).