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Focusing of a ring ripple on a Gaussian electromagnetic beam in a magnetoplasma

SHIKHA MISRA and S.K. MISHRA

Ramanna Fellowship Program, Department of Education Building Lucknow University, Lucknow 226007, India
(msodha@rediffmail.com)

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Abstract. In this paper we present a theoretical investigation of the growth/propagation of a ring ripple, superposed on a Gaussian electromagnetic beam propagating along the direction of magnetic field in a magnetoplasma. The nature of propagation of the ripple is analysed in a paraxial-like approximation by radial expansion of the dielectric function, corresponding to the composite (Gaussian and ripple) electric field profile of the beam around the position of the maximum of the ripple. The two cases of collisional plasmas (with negligible thermal conduction) and collisionless plasmas (dominant ponderomotive nonlinearity) are considered. The effect of the magnetic field on the critical curves and focusing/defocusing of the ripple are studied and discussed.

1. Introduction

The nonlinear interaction of intense electromagnetic beams with plasmas has been the subject of many experimental and theoretical studies in recent years due to its relevance to laser-driven fusion [1–6] (i.e. inertial confinement fusion (ICF)), and some other exotic phenomena, e.g. ionospheric modification [7–13] and laser-based charged particle acceleration [14, 15]. Most of these studies are applicable only in the absence of a magnetic field, while in many situations of interest the magnetic field is significant enough to play an important role. It is well known that in the presence of an external static magnetic field the plasma displays an anisotropic character of the dielectric function [16–18], which leads to two distinct modes of wave propagation, namely the extraordinary and the ordinary modes. Several investigations have been devoted to plasmas embedded in an external magnetic field in the context of nonlinear processes [16–22]. Recently Sodha et al. [23] have developed a theory for self-focusing, considering both the conduction by the ionic and electronic species as well as collisions, in a fully ionized collisional magneto-plasma, while Sharma et al. [24] have analysed the three characteristic regimes for the propagation of an electromagnetic beam in a magnetized plasma. Further Sodha and Sharma [25] have applied the theory of self-focusing to the ionosphere, taking the Earth's magnetic field into account.

In many situations, laser or electromagnetic beam propagation through a non-linear medium may be perturbed by a plasma instability, characterized by a small-scale electron density perturbation resulting from irradiance fluctuations (small intensity spikes) in the plane, transverse to the direction of propagation. The perturbation grows at the cost of the main beam, is relevant to the physics of inertial confinement fusion. In the experiments on laser–plasma interaction the filamentary structures created in an underdense plasma undergo self-focusing [26, 27], which destroys the symmetry of energy deposition and triggers parametric instabilities that may lead to back and side scattering of the main beam. Thus, such instabilities are detrimental to ICF and other heating experiments.

There are two complementary approaches to the analysis of the growth/propagation of the instabilities. The first approach [28–51] considers an instability $E_1 \exp[i(k_\perp x + k_\parallel z)]$, superposed onto an intense electromagnetic laser beam $E_0 \exp[i(\omega t + kz)]$ and gets an expression for the spatial growth rate of the instability, i.e. $ik_\parallel$ in terms of relevant parameters, and looks for the condition when $k_\parallel$ is imaginary. However, this approach is limited to instability in one direction in the transverse plane and to idealized beams with uniform irradiance along the wavefront. The other approach is based on the direct [52,53] and indirect [54] evidence, suggesting that the filamentational instability in nonlinear media is caused by the occurrence of strong irradiance spikes, riding on an incident smooth-looking irradiance distribution in the plane, transverse to the direction of propagation. On the basis of the paraxial theory formulated by Akhmanov et al. [55] and developed by Sodha et al. [39, 56], the growth of a Gaussian ripple on a plane uniform beam [57–61] and of a ring ripple on a Gaussian electromagnetic beam [62–69] in a plasma have been investigated to a significant extent. An interesting critique of the two approaches has been made by Sodha and Sharma [70]. In a recent investigation, Misra and Mishra [71] have adopted a modified approach based on a paraxial-like approximation and analytically investigated the nature of the propagation of a ring ripple, superposed onto a Gaussian electromagnetic beam in a plasma. However, the behaviour of the ripple in a magnetized plasma has received relatively less attention [55,60,63] and in this context the present investigation aims at exploring the propagation of a ring ripple, riding on a Gaussian electromagnetic beam in a magnetoplasma.

The present work is based on the modified approach followed by Misra and Mishra [71] and represents the extension of the theory to plasmas, embedded in an external static magnetic field. The present investigation is inclusive of the following considerations.

(i) The radial field distribution profile of the electromagnetic beam has been taken as that of the composite electric field of the Gaussian beam and the ring ripple, and treated as such throughout the analysis. This assumption leads to the same dielectric function for both the ring ripple part as well as a Gaussian part of the electromagnetic beam and hence the same focusing factors (valid in the vicinity of the maximum of the ripple). This is in contrast to the earlier analyses [62–69], which separately consider the beam and the ripple, leading to incorrect results.

(ii) The diffraction term chosen in the present analysis is appropriate for the vicinity of the maximum of the irradiance of the ring ripple, occurring away from the beam axis ($r = 0$).

(iii) The r independent terms in the eikonal of the beam is taken into account, with the phase difference between the electric field vectors associated with the Gaussian part and the ring ripple part changing continuously.

(iv) All of the relevant parameters have been expanded in terms of the radial distance from the maximum of the ring ripple, which is away from the axis $r = 0$.

(v) The external static magnetic field B_0 in the homogeneous plasma has been taken to be along the direction of the propagation of the beam.

(vi) The plasma is electrically neutral everywhere.

(vii) Two cases of plasma nonlinearity have been considered, namely the ponderomotive nonlinearity (valid for collisionless plasmas) and the collisional nonlinearity, neglecting thermal conduction (which is valid, when $(\delta_c r_0^2/l^2) \gg 1$).

(viii) For computational convenience $\nu_e^2 \ll (\omega \mp \omega_c)^2$ has been assumed.

The authors have studied the critical curve for the ring ripple part, the dependence of the beam width parameter associated with the vicinity of the maximum of the ring ripple part on various factors in the presence of magnetic field for collisionless (with dominant ponderomotive nonlinearity) and collisional plasmas. To determine the dependence of the electron density on the irradiance of the beam (i.e. the electron temperature), the results of the kinetic theory [39] have been adopted. Furthermore, the effect of varying the magnitude of magnetic field on critical curves has also been explored. The numerical results and discussion concludes the paper.

2. Propagation of the ring ripple on a Gaussian electromagnetic beam

Propagation

Consider the propagation of a Gaussian electromagnetic beam with a small coaxial perturbation (the ring ripple), in a magnetoplasma along the direction of the magnetic field (i.e. z-axis). The complex amplitude of the effective electric field $E_\pm$ (associated with one of the two modes of the propagation) of the Gaussian electromagnetic beam with a coaxial ring ripple can be expressed as

$$E_\pm = (E_x \pm iE_y) = A_\pm \exp i\left(\omega t - \int_0^z k_\pm(z)\,dz\right), \tag{1a}$$

where

$$A_\pm(z=0) = E_{00\pm}\exp\left(-\frac{r^2}{2r_0^2}\right) + E_{10\pm}\left(\frac{r^2}{r_1^2} - \delta\right)^{n/2}\exp\left(-\frac{r^2}{2r_1^2}\right)\exp(i\phi_p), \tag{1b}$$

A refers to the complex amplitude of the electromagnetic beam, E_{00} and E_{10} correspond to the initial amplitude of the Gaussian beam (with initial beam width r_0) and the ring ripple (with initial beam width r_1), respectively, $k(z)$ is the wave number defined as $k_\pm(z) = (\omega/c)\sqrt{\varepsilon_{0\pm}(z)}$, $\varepsilon_{0\pm}(z)$ is the dielectric function, at $r = r_{\max}$, the value of r corresponding to the maximum of the electric field of the ring ripple part of the electromagnetic beam (far from the axis $r = 0$), n and δ are positive numbers, characterizing the position of the ring ripple on the wave front of the electromagnetic beam, ϕ_p is the initial phase difference between the electric field vectors of the Gaussian and the ring ripple parts of the beam, ω is the wave frequency and the signs + and − correspond to extraordinary and ordinary modes of propagation of the electromagnetic beam, respectively. The first term on the right-hand side of (1b) corresponds to the Gaussian part, while the second term represents the radial distribution of the coaxial perturbation in the form of the ring ripple, having its maximum at $r = r_{\max} = r_1\sqrt{(n+\delta)}$.

The effective electric field vector $E_\pm$ satisfies the wave equation (in cylindrical coordinate system) [39, 56],

$$\frac{\partial^2 E_\pm}{\partial z^2} + \delta_{m\pm}\left(\frac{\partial^2 E_\pm}{\partial r^2} + \frac{1}{r}\frac{\partial E_\pm}{\partial r}\right) + \varepsilon_\pm(r,z)\frac{\omega^2}{c^2}E_\pm = 0, \tag{2}$$

where $\varepsilon_\pm(r,z)$ is the effective dielectric function of the plasma and c is the speed of light in free space.

On substituting $E_\pm$ from (1a) and neglecting $(\partial^2 A_\pm/\partial z^2)$ (assuming $A_\pm(r,z)$ to be a slowly varying function of z) the wave equation for electromagnetic beam propagation along the direction of the magnetic field, namely along the z-axis, reduces to

$$2ik_\pm\frac{\partial A_\pm}{\partial z} + iA_\pm\frac{\partial k_\pm}{\partial z} = \delta_{m\pm}\left(\frac{\partial^2 A_\pm}{\partial r^2} + \frac{1}{r}\frac{\partial A_\pm}{\partial r}\right) + \frac{\omega^2}{c^2}(\varepsilon_\pm - \varepsilon_{0\pm})A_\pm, \tag{3}$$

where

$$\delta_{m\pm} = \frac{1}{2}\left(1 + \frac{\varepsilon_{0\pm}(0)}{\varepsilon_{0zz}}\right),$$

$$\varepsilon_{0zz} = 1 - \frac{\omega_{\rm pe}^2}{\omega^2}\left(1 + \frac{\nu_{\rm e}^2}{\omega^2}\right)^{-1}$$

is the dielectric tensor component along the z direction, $\omega_{\rm pe}$ is the plasma frequency unaffected by the electromagnetic beam and $\nu_{\rm e}$ is the collision frequency of electrons with heavier species.

The complex amplitude $A_\pm(r,z)$ of the electric field $E_\pm$ may be expressed as,

$$A_\pm(r,z) = A_{0\pm}(r,z)\exp(-ik_\pm(z)S_\pm(r,z)), \tag{4}$$

where $S_\pm(r,z)$ is termed as the eikonal, associated with the electromagnetic beam.

Substitution for $A_\pm(r,z)$ from (4) in (3) and separation of the real and imaginary parts, yield

$$\frac{2S_\pm}{k_\pm}\frac{\partial k_\pm}{\partial z} + 2\frac{\partial S_\pm}{\partial z} + \delta_{m\pm}\left(\frac{\partial S_\pm}{\partial r}\right)^2 = \frac{\delta_{m\pm}}{k_\pm^2 A_{0\pm}}\left(\frac{\partial^2 A_{0\pm}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{0\pm}}{\partial r}\right) + \frac{\omega^2}{k_\pm^2 c^2}(\varepsilon_\pm - \varepsilon_{0\pm}) \tag{5a}$$

and

$$\frac{A_{0\pm}^2}{k_\pm}\frac{\partial k_\pm}{\partial z} + \frac{\partial A_{0\pm}^2}{\partial z} + \delta_{m\pm}A_{0\pm}^2\left(\frac{\partial^2 S_\pm}{\partial r^2} + \frac{1}{r}\frac{\partial S_\pm}{\partial r}\right) + \delta_{m\pm}\frac{\partial A_{0\pm}^2}{\partial r}\frac{\partial S_\pm}{\partial r} = 0. \tag{5b}$$

In view of the interest of the current study (the ring ripple part, far from the axis $r = 0$), one can use a paraxial-like approximation, which is valid around $r = r_{\max}$, the position of the maximum irradiance of the ring ripple; this is analogous to the usual paraxial approach. One can thus express (5a) and (5b) in terms of z and a new variable $\chi_\pm$ as

$$\frac{2S_\pm}{k_\pm}\frac{\partial k_\pm}{\partial z} + 2\frac{\partial S_\pm}{\partial z} + \frac{\delta_{m\pm}}{r_1^2 f_\pm^2}\frac{(\lambda+\chi_\pm^2)}{\chi_\pm^2}\left(\frac{\partial S_\pm}{\partial \chi_\pm}\right)^2$$
$$= \frac{\delta_{m\pm}}{k_\pm^2 A_{0\pm} r_1^2 f_\pm^2}\left[\frac{\lambda}{\chi_\pm^2}\left(\frac{\partial^2 A_{0\pm}}{\partial \chi_\pm^2} - \frac{1}{\chi_\pm}\frac{\partial A_{0\pm}}{\partial \chi_\pm}\right) + \left(\frac{\partial^2 A_{0\pm}}{\partial \chi_\pm^2} + \frac{1}{\chi_\pm}\frac{\partial A_{0\pm}}{\partial \chi_\pm}\right)\right]$$
$$+ \frac{\omega^2}{k_\pm^2 c^2}(\varepsilon_\pm - \varepsilon_{0\pm}) \tag{6a}$$

and

$$\frac{A_{0\pm}^2}{k_\pm}\frac{\partial k_\pm}{\partial z} + \frac{\partial A_{0\pm}^2}{\partial z} + \frac{\delta_{m\pm}A_{0\pm}^2}{r_1^2 f_\pm^2}\left[\frac{\lambda}{\chi_\pm^2}\left(\frac{\partial^2 S_\pm}{\partial \chi_\pm^2} - \frac{1}{\chi_\pm}\frac{\partial S\pm}{\partial \chi_\pm}\right) + \left(\frac{\partial^2 S_\pm}{\partial \chi_\pm^2} + \frac{1}{\chi_\pm}\frac{\partial S_\pm}{\partial \chi_\pm}\right)\right]$$
$$+ \frac{\delta_{m\pm}}{r_1^2 f_\pm^2}\frac{(\lambda+\chi_\pm^2)}{\chi_\pm^2}\frac{\partial A_{0\pm}^2}{\partial \chi_\pm}\frac{\partial S_\pm}{\partial \chi_\pm} = 0, \tag{6b}$$

where $\chi_\pm^2 = [(r/r_1 f_\pm)^2 - \lambda]$ is a parameter introduced for convenience, $\lambda = (n+\delta)$, $r_1 f_\pm(z)$ is the width of the ring ripple and $r_{\max}^2 = \lambda r_1^2 f_\pm^2$ indicates the position of the maximum irradiance for the ring ripple; it is shown later that (6a) and (6b) lead to retention of the original profile of the beam during propagation in the paraxial-like approximation, i.e. when $\chi_\pm^2 \lll n$.

In the paraxial-like approximation the relevant parameters (i.e. the dielectric function $\varepsilon_\pm(r,z)$, eikonal and irradiance) may be expanded around the maximum of the ring ripple, i.e. around $\chi_\pm = 0$. Thus, the dielectric function $\varepsilon_\pm(\chi_\pm, z)$ around $\chi_\pm = 0$ can be expressed as

$$\varepsilon_\pm(\chi_\pm, z) = \varepsilon_{0\pm}(z) - \chi_\pm^2 \varepsilon_{2\pm}(z), \tag{7}$$

where $\varepsilon_{0\pm}(z)$ and $\varepsilon_{2\pm}(z)$ are the coefficients associated with $\chi_\pm^0$ and $\chi_\pm^2$ in the expansion of $\varepsilon_\pm(\chi_\pm, z)$ around $\chi_\pm = 0$. The expressions for these coefficients are derived in the following.

Substitution for $\varepsilon_\pm(\chi_\pm, z)$ from (7) in (6a) and (6b) leads to,

$$\frac{2S_\pm}{k_\pm}\frac{\partial k_\pm}{\partial z} + 2\frac{\partial S_\pm}{\partial z} + \frac{\delta_{m\pm}}{r_1^2 f_\pm^2}\frac{(\lambda+\chi_\pm^2)}{\chi_\pm^2}\left(\frac{\partial S_\pm}{\partial \chi_\pm}\right)^2$$
$$= \frac{\delta_{m\pm}}{k_\pm^2 A_{0\pm} r_1^2 f_\pm^2}\left[\frac{\lambda}{\chi_\pm^2}\left(\frac{\partial^2 A_{0\pm}}{\partial \chi_\pm^2} - \frac{1}{\chi_\pm}\frac{\partial A_{0\pm}}{\partial \chi_\pm}\right) + \left(\frac{\partial^2 A_{0\pm}}{\partial \chi_\pm^2} + \frac{1}{\chi_\pm}\frac{\partial A_{0\pm}}{\partial \chi_\pm}\right)\right]$$
$$- \chi_\pm^2 \frac{\omega^2}{k_\pm^2 c^2}\varepsilon_{2\pm} \tag{8a}$$

and

$$\frac{A_{0\pm}^2}{k_\pm}\frac{\partial k_\pm}{\partial z} + \frac{\partial A_{0\pm}^2}{\partial z} + \frac{\delta_{m\pm}A_{0\pm}^2}{r_1^2 f_\pm^2}\left[\frac{\lambda}{\chi_\pm^2}\left(\frac{\partial^2 S_\pm}{\partial \chi_\pm^2} - \frac{1}{\chi_\pm}\frac{\partial S_\pm}{\partial \chi_\pm}\right) + \left(\frac{\partial^2 S_\pm}{\partial \chi_\pm^2} + \frac{1}{\chi_\pm}\frac{\partial S_\pm}{\partial \chi_\pm}\right)\right]$$
$$+ \frac{\delta_{m\pm}}{r_1^2 f_\pm^2}\frac{(\lambda+\chi_\pm^2)}{\chi_\pm^2}\frac{\partial A_{0\pm}^2}{\partial \chi_\pm}\frac{\partial S_\pm}{\partial \chi_\pm} = 0. \tag{8b}$$

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In the paraxial-like approximation ($\chi_\pm^2 \ll n$), the solution of (8b) may be written as

$$A_{0\pm}^2 = \frac{E_{0\pm}^2}{f_\pm^2}\exp[-\mu(\lambda+\chi_\pm^2)] + \frac{E_{1\pm}^2}{f_\pm^2}(n+\chi_\pm^2)^n \exp[-(\lambda+\chi_\pm^2)] + \frac{E_{0\pm}E_{1\pm}}{f_\pm^2}(n+\chi_\pm^2)^{n/2}\exp\left[-\frac{1}{2}(1+\mu)(\lambda+\chi_\pm^2)\right]\cos\phi_{\rm p}, \tag{9a}$$

where

$$S_\pm(\chi_\pm, z) = \frac{\chi_\pm^2}{2}\beta_\pm(z) + \varphi_\pm(z), \tag{9b}$$

$$\beta_\pm(z) = \frac{r_1^2 f_\pm}{\delta_{m\pm}}\frac{df_\pm}{dz},$$

$$E_{0\pm}^2 = E_{00\pm}^2\left(\frac{k_\pm(0)}{k_\pm(z)}\right) = E_{00_\pm}^2\left(\frac{\varepsilon_{0\pm}(0)}{\varepsilon_{0\pm}(z)}\right)^{1/2},$$

$$E_{1\pm}^2 = E_{10\pm}^2\left(\frac{k_\pm(0)}{k_\pm(z)}\right) = E_{10\pm}^2\left(\frac{\varepsilon_{0\pm}(0)}{\varepsilon_{0\pm}(z)}\right)^{1/2},$$

$\mu = (r_1^2/r_0^2), \varphi_\pm(z)$ is a function of z and $f_\pm(z)$ is the beam width parameter.

For further algebraic analysis, it is convenient to expand the $A_{0\pm}^2$ as a polynomial in $\chi_\pm^2$; thus,

$$A_{0\pm}^2 = g_{0\pm} + g_{2\pm}\chi_\pm^2 + g_{4\pm}\chi_\pm^4 + g_{6\pm}\chi_\pm^6, \tag{10}$$

where

$$g_{0\pm} = \frac{E_{0\pm}^2}{f_\pm^2}(e^{-\mu\lambda} + p_\pm^2 n^n e^{-\lambda} + 2p_\pm n^{n/2} e^{-(\mu+1)\lambda/2}\cos\phi_{\rm p}), \tag{11a}$$

$$g_{2\pm} = -\frac{E_{0\pm}^2}{f_\pm^2}(\mu e^{-\mu\lambda} + p_\pm \mu n^{n/2} e^{-(\mu+1)\lambda/2}\cos\phi_{\rm p}), \tag{11b}$$

$$g_{4\pm} = \frac{E_{0\pm}^2}{f_\pm^2}\left(\frac{\mu^2}{2}e^{-\mu\lambda} - \frac{1}{2n}p_\pm^2 n^n e^{-\lambda} + p_\pm n^{n/2} e^{-(\mu+1)\lambda/2}\cos\phi_{\rm p}\left(\frac{\mu^2}{4} - \frac{1}{2n}\right)\right), \tag{11c}$$

$$g_{6\pm} = \frac{E_{0\pm}^2}{f_\pm^2}\left(-\frac{\mu^3}{6}e^{-\mu\lambda} + \frac{1}{3n^2}p_\pm^2 n^n e^{-\lambda} + p_\pm n^{n/2} e^{-(\mu+1)\lambda/2}\cos\phi_{\rm p}\left(\frac{\mu}{4n} + \frac{1}{3n^2} - \frac{\mu^3}{24}\right)\right) \tag{11d}$$

and $p_\pm = (E_{1\pm}/E_{0\pm})$.

On substituting $A_{0\pm}^2$ and $S_\pm$ from (10) and (9b) into (8a), and equating the coefficients of $\chi_\pm^0$ and $\chi_\pm^2$ on both sides of the resulting equation, one obtains

$$\frac{g_{0\pm}^2 f_\pm}{\delta_{m\pm}}\left(\varepsilon_{0\pm}\frac{d^2 f_\pm}{d\xi^2} + \frac{1}{2}\frac{d\varepsilon_{0\pm}}{d\xi}\frac{df_\pm}{d\xi}\right) + 2g_{2\pm}g_{0\pm}\varepsilon_{0\pm}\left[\frac{\Phi_\pm}{\varepsilon_{0\pm}}\frac{d\varepsilon_{0\pm}}{d\xi} + \left(2\frac{d\Phi_\pm}{d\xi} - \frac{\lambda}{\delta_{m\pm}}\left(\frac{df_\pm}{d\xi}\right)^2\right)\right] = \frac{\delta_{m\pm}}{f_\pm^2}(4g_{0\pm}(2g_{4\pm} + 3\lambda g_{6\pm}) + g_{2\pm}^2) - \rho_0^2 g_{0\pm}^2 \varepsilon_{2\pm} \tag{12a}$$

and

$$\varepsilon_{0\pm}g_{0\pm}^2\left[\frac{\Phi_\pm}{\varepsilon_{0\pm}}\frac{d\varepsilon_{0\pm}}{d\xi}+\left(2\frac{d\Phi_\pm}{d\xi}-\frac{\lambda}{\delta_{m\pm}}\left(\frac{df_\pm}{d\xi}\right)^2\right)\right]$$
$$=\frac{\delta_{m\pm}}{f_\pm^2}(2g_{0\pm}(g_{2\pm}+2\lambda g_{4\pm})-\lambda g_{2\pm}^2), \tag{12b}$$

where $\xi=(c/r_1^2\omega)z$ is the dimensionless distance of propagation, $\rho_0=(r_1\omega/c)$ is the dimensionless initial width of the ring ripple and $\Phi_\pm=(\omega/c)\varphi_\pm$ is the dimensionless phase function associated with the eikonal.

The parameter $\Phi_\pm$ can be eliminated between (12a) and (12b); thus,

$$\frac{g_{0\pm}^2f_\pm}{\delta_{m\pm}}\left(\frac{1}{2}\frac{d\varepsilon_{0\pm}}{d\xi}\frac{df_\pm}{d\xi}+\varepsilon_{0\pm}\frac{d^2f_\pm}{d\xi^2}\right)+\frac{2\delta_{m\pm}}{f_\pm^2}\frac{g_{2\pm}}{g_{0\pm}}(2g_{0\pm}(g_{2\pm}+2\lambda g_{4\pm})-\lambda g_{2\pm}^2)$$
$$=\frac{\delta_{m\pm}}{f_\pm^2}(4g_{0\pm}(2g_{4\pm}+3\lambda g_{6\pm})+g_{2\pm}^2)-\rho_0^2g_{0\pm}^2\varepsilon_{2\pm}. \tag{13}$$

The dependence of the beam width parameter $f_\pm$ on the dimensionless distance of propagation ξ can be obtained by the numerical integration of (13) after substituting suitable expressions for $\varepsilon_{0\pm}(z)$ and $\varepsilon_{2\pm}(z)$, with the initial boundary conditions $f_\pm=1$ and $df_\pm/d\xi=0$ at $\xi=0$; furthermore, the variation of $\Phi_\pm$ with distance of propagation ξ may be obtained by integrating (12a) and (12b) simultaneously with an additional boundary condition $\Phi_\pm=0$ at $\xi=0$.

Dielectric function

Following Sodha et al. [39, 56], the effective dielectric function of the plasma can be expressed as

$$\varepsilon_\pm(r,z)=1-\Omega^2\left[\frac{(1\mp\omega_c/\omega)}{(\nu_e/\omega)^2+(1\mp\omega_c/\omega)^2}\right]\left(\frac{N_{0e\pm}}{N_0}\right), \tag{14}$$

where $\Omega=(\omega_{pe}/\omega)$, $\omega_{pe}=(4\pi N_0e^2/m)^{1/2}$ is the electron plasma frequency, ω_c is the electron cyclotron frequency, N_0 is the undisturbed electron density of the plasma, N_{0e} is the electron density of the plasma in the presence of the electromagnetic field, m is the mass of the electron and e is the electronic charge.

Following a paraxial like approximation (i.e. $\chi_\pm^2\ll 1$) one can expand the dielectric function $\varepsilon_\pm(\chi_\pm,z)$ in axial and radial parts around the maximum of the ring ripple ($\chi_\pm=0$). Thus, one obtains from (7),

$$\varepsilon_{0\pm}(z)=\varepsilon_\pm(\chi_\pm,z)_{\chi_\pm=0}, \tag{15a}$$

and

$$\varepsilon_{2\pm}(z)=-\left(\frac{\partial\varepsilon_\pm(\chi_\pm,z)}{\partial\chi_\pm^2}\right)_{\chi_\pm=0}. \tag{15b}$$

Ponderomotive nonlinearity

In a collisionless magnetoplasma the redistribution of the electron density takes place on account of the ponderomotive force on the electrons and modifies the dielectric function of the plasma. The magnitude of the ponderomotive force is proportional to the gradient of the irradiance. Such a nonlinearity sets in a period of the order (r_0/c_s), where r_0 is the width of the beam and c_s is the ion sound speed. Hence, for a collisionless plasma (with $\nu_{\mathrm{e}} \approx 0$) at moderate fields (when the quiver speed of the electron is much smaller than the speed of light in vacuum), the modified electron density function $N_{0e\pm}$ is given by [39, 56],

$$N_{0e\pm} = N_0 \exp\left[-\gamma_{1\pm}\beta E_\pm E_\pm^*\right], \tag{16}$$

where

$$\gamma_{1\pm} = \frac{(1 \mp \omega_{\mathrm{c}}/2\omega)}{(1 \mp \omega_{\mathrm{c}}/\omega)^2},$$

$$\beta = (e^2/16k_{\mathrm{B}}T_0\omega^2 m),$$

k_{B} is the Boltzmann constant and T_0 is the temperature of the atoms/ions.

Substituting $N_{0\mathrm{e}\pm}$ from (16) in (14) and using (15a), (15b) and (10), one can easily obtain $\varepsilon_{0\pm}(z)$ and $\varepsilon_{2\pm}(z)$ as

$$\varepsilon_{0\pm}(z) = 1 - \frac{\Omega^2}{(1 \mp \omega_{\mathrm{c}}/\omega)} \exp(-\gamma_{1\pm}\beta g_{0\pm}) \tag{17a}$$

and

$$\varepsilon_{2\pm}(z) = -\frac{\Omega^2}{(1 \mp \omega_{\mathrm{c}}/\omega)} \gamma_{1\pm}\beta g_{2\pm} \exp(-\gamma_{1\pm}\beta g_{0\pm}). \tag{17b}$$

Collisional nonlinearity

In collisional plasmas, the radial distribution of the temperature of constituent particles (electrons and ions) takes place in the transverse plane on account of the non-uniform radial dependence of irradiance of the beam; this non-uniformity in temperature creates the pressure gradients of the electron and ion gases. In the steady state with plasma neutrality these pressure gradients are balanced by the space charge field, and lead to a redistribution of the electron density and, hence, the modified dielectric function. The collisional non-linearity sets in a period $1/\delta_{\mathrm{c}}\nu_{\mathrm{e}}$, where δ_{c} is the fractional loss of excess energy by an electron in a collision with heavier species (ions and neutral) and ν_{e} is the electron collision frequency. For a collisional magnetoplasma the modified electron density function $N_{0\mathrm{e}\pm}$ is thus given by [39, 56],

$$N_{0\mathrm{e}\pm} = N_0(1 + \gamma_{2\pm}\alpha E_\pm E_\pm^*)^{(s/2)-1}, \tag{18}$$

where

$$\alpha = (e^2/3k_{\mathrm{B}}T_0\omega^2 m\delta_{\mathrm{c}}),$$

$$\gamma_{2\pm} = \left[\frac{\omega^2}{\nu_{\mathrm{e}}^2 + (\omega \mp \omega_{\mathrm{c}})^2} + \frac{\omega^2}{\nu_{\mathrm{e}}^2 + (\omega \pm \omega_{\mathrm{c}})^2}\right]$$

and the collision frequency ν_{e} is proportional to the sth power of the random electron speed. For electron–ion collision dominant plasma $s = -3$ and for electron–neutral collision dominant plasma $s = 1$. Equation (18) is based on the fact that in the magnetoplasma the heating of the electrons takes place on account of both the modes of propagation and thermal conduction does not play a significant part in the energy balance of electrons (which is justified [39] when $(\delta_{\mathrm{c}}r_0^2/l^2) \gg 1$). It is also assumed that the heavier particles are abundant enough to provide a heat sink at almost constant temperature for energy loss by the electrons.

Substituting $N_{0e\pm}$ from (18) in (14) and using (15a), (15b) and (10), one can obtain $\varepsilon_{0\pm}(z)$ and $\varepsilon_{2\pm}(z)$ as

$$\varepsilon_{0\pm}(z) = 1 - \Omega^2 \left[\frac{(1 \mp \omega_c/\omega)}{(\nu_e/\omega)^2 + (1 \mp \omega_c/\omega)^2} \right] (1 + \gamma_{2\pm}\alpha g_{0\pm})^{(s-2)/2} \tag{19a}$$

and

$$\varepsilon_2(z) = -\Omega^2 \gamma_{2\pm}[(2-s)/2]\alpha g_{2\pm} \left[\frac{(1 \mp \omega_c/\omega)}{(\nu_e/\omega)^2 + (1 \mp \omega_c/\omega)^2} \right] (1 + \gamma_{2\pm}\alpha g_{0\pm})^{(s-4)/2}. \tag{19b}$$

Critical condition for focusing: critical curves

Using the expression for $\varepsilon_{2\pm}(z)$ for any specific non-linearity, (13) ensures vanishing of $d^2 f_\pm/d\xi^2$ for a value of $p_\pm$, for a given $E_{0\pm}^2$, corresponding to the dimensionless ripple beam width $\rho_0 f_\pm$, when

$$\rho_0^2 g_{0\pm}^2 \varepsilon_{2\pm} = \frac{1}{f_\pm^2}(4g_{0\pm}(2g_{4\pm} + 3\lambda g_{6\pm}) + g_{2\pm}^2), \tag{20a}$$

which at $\xi = 0$ reduces to

$$\rho_0^2 \varepsilon_{2\pm}(0) = \left[\frac{1}{g_{0\pm}^2}(4g_{0\pm}(2g_{4\pm} + 3\lambda g_{6\pm}) + g_{2\pm}^2) \right]_{\xi=0}. \tag{20b}$$

On substitution for the coefficients $g_{0\pm}$, $g_{2\pm}$, $g_{4\pm}$ and $g_{6\pm}$ from (12a), one obtains (20b), which represents the critical power curve plotted as ρ_0 versus $p_\pm$ and separates the self-focusing region from the rest. For the points lying above the critical power curve the beam undergoes oscillatory convergence (self-focusing) while for the points below this curve the beam executes oscillatory divergence or steady divergence. The points on the curve lead to self-trapped mode propagation of the ring ripple. Thus, by using (20a), one can obtain the critical power curve for any specific mode for both kinds of nonlinearities.

3. Scheme of computation

For a numerical appreciation of the results, the critical curves and the dependence of the beam width parameter $f_\pm$ (in the vicinity of the maximum of the ring ripple) on ξ for a chosen set of parameters and different kinds of nonlinearities have been computed by considering the propagation of the extraordinary mode of the electromagnetic beam in a magnetoplasma.

The critical curves for the propagation of the ring ripple, between $p_{0\pm}(= E_{10\pm}/E_{00\pm})$ that is, $p_\pm(\xi = 0)$ and the initial dimensionless width of the ring ripple ρ_0, have been plotted with the help of (20b), by using appropriate expressions for $\varepsilon_{0\pm}$ and $\varepsilon_{2\pm}$, corresponding to the different modes for both kinds of nonlinearities and chosen sets of parameters $E_{0\pm}^2$, δ, ϕ_p, μ, n, ω_c, s and Ω. The critical curve for the collisional magnetoplasma corresponds to the approximation $\nu_e^2 \ll (\omega \mp \omega_c)^2$. Furthermore, computations have been performed to investigate the variation of the beam width parameter $f_\pm$ associated with the propagation of the ring ripple on the dimensionless distance of propagation ξ in the plasma. Starting with a combination of parameters $\beta E_{00\pm}^2$ (or $\alpha E_{00\pm}^2$), $p_{0\pm}$, ρ_0 and Ω, one can obtain the solution for the beam width parameter $f_\pm$ and $\Phi_\pm$ by simultaneous numerical integration of (12a)

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and (12b) using the parameters $\varepsilon_{0\pm}$ and $\varepsilon_{2\pm}$ under appropriate boundary conditions, namely $f_\pm = 1$, $df_\pm/d\xi = 0$ and $\Phi_\pm = 0$ at $\xi = 0$.

4. Numerical results and discussion

The present analysis investigates the propagation characteristics of a coaxial ring ripple superimposed onto a Gaussian electromagnetic beam, propagating through a homogeneous plasma, along the direction of the static magnetic field B_0 (z-axis); the electric field profile of the propagating beam is assumed to be composed of the radial electric field distributions of the Gaussian beam as well as that of the ring ripple. A paraxial-like approach has been adopted to analyse the characteristics of the propagation. The nature of the propagation of the ring ripple is characterized by the dielectric function (tensor in the presence of a magnetic field) which becomes modified by the effective electric field of the electromagnetic beam (i.e. the composite field of both the Gaussian and ring ripple parts). In view of the interest of the present study (the ring ripple), all of the characteristic parameters (i.e. dielectric function $\varepsilon_\pm(r, z)$, eikonal $S_\pm$ and irradiance) have been expanded around the maximum of the electric field of the ring ripple part (i.e. at $r^2 = \lambda r_1^2 f_\pm^2$) and evaluated on the basis of a paraxial-like approach. Such a modification is a significant departure from earlier studies [62–69] in which the Gaussian beam and the ring ripple have been treated as separate entities having a separate dielectric function for each part. This approach was based on the assumption that the dielectric function of the main beam is modified only by its own field, while that for the ripple is modified by the resultant electric field on account of the ring ripple as well as that of the main beam. This simplification leads to erroneous conditions for the growth or decay of the ring ripple. In the current study the modified diffraction term is appropriate for the region around the maximum of the ring ripple, far from the axis $r = 0$; it also depends on other parameters such as irradiance, δ, ϕ_p, μ, n and $p_{0\pm}$, in contrast to the earlier investigations [62–69].

The parameter $\Phi_\pm$ does not occur in (13) for $f_\pm$ and hence at first look it seems that the parameter $\Phi_\pm$ is not relevant to self-focusing of the ring ripple. However, a more careful look reveals that the second term on the left-hand side of (13) is a consequence of the inclusion of $\Phi_\pm$ in the analysis; this was ignored in earlier studies. It may be emphasized that (13) is valid only in the vicinity of the maximum of the irradiance of the ring ripple and is not applicable in the vicinity of $r = 0$, where the maximum irradiance of the Gaussian part of the beam occurs.

The results of the present analysis can be appreciated through numerical computation of the critical power curves and the dependence of the beam width parameter $f_\pm$ on the dimensionless distance of propagation ξ for a chosen set of parameters $E_{0\pm}^2$, δ, ϕ_p, μ, n, ω_c and $p_{0\pm}$, for collisionless (dominant ponderomotive nonlinearity) and collisional magnetoplasmas. The figures herein correspond to the extraordinary mode (+) of the propagation of the electromagnetic beam; however, the corresponding curves for ordinary mode (−) may be obtained in the same fashion by replacing ω_c with $(-\omega_\mathrm{c})$.

The critical curve for the ring ripple characterizes its propagation characteristics and defines a specific region for self-focusing in the ρ_0 (dimensionless initial width) $-p_{0\pm}$ (ratio of the initial amplitude of the ripple and the Gaussian beam) space. A beam corresponding to a point $(\rho_0, p_\pm)$ in the $\rho_0 - p_{0\pm}$ space above the critical curve executes oscillatory (or self-)focusing, while for points lying below the critical

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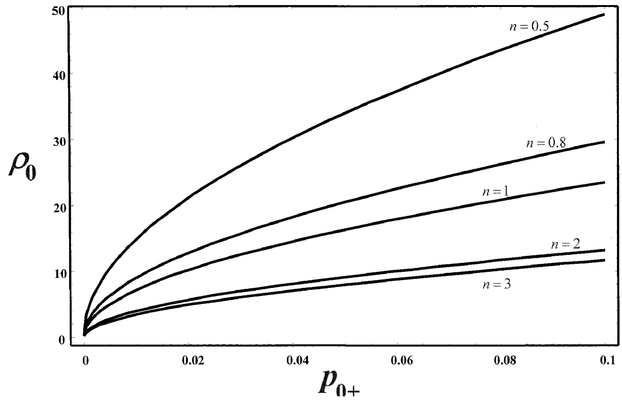
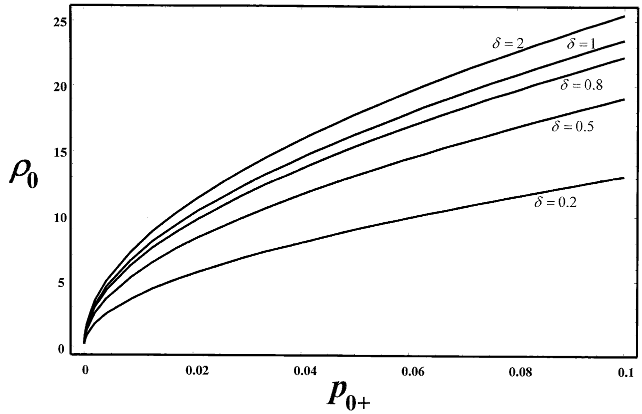
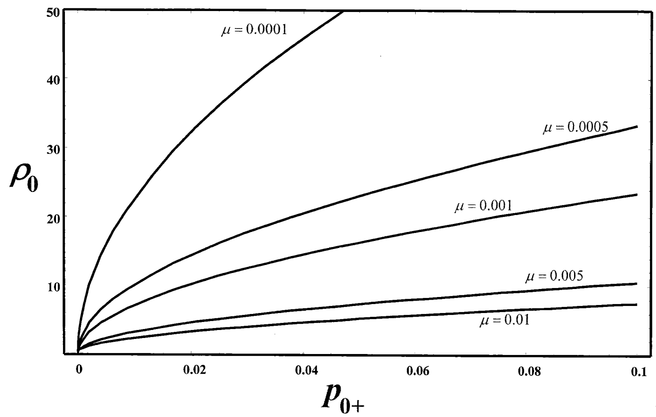


(a)



(b)



(c)

Figure 1. Critical curves for the ring ripple (the dependence of the initial beam width $\rho_0 (= r_1\omega/c)$ on $p_{0+} (= E_{10+}/E_{00+})$), corresponding to ponderomotive nonlinearity for the extraordinary mode of propagation of the beam in a magnetoplasma, for the standard set of parameters $\Omega^2 = 0.8$, $(\omega_c/\omega) = 0.2$, $\mu = 0.001$, $\delta = 1$, $n = 1$, $\phi_p = \pi/3$ and $\beta E_{00+}^2 = 1$; (a)–(e) refer to the effect on the critical width by varying the parameter n, δ, μ, ϕ_p and βE_{00+}^2, respectively, keeping the rest of the constants the same (the magnitude of the varying parameter is indicated above the curve).

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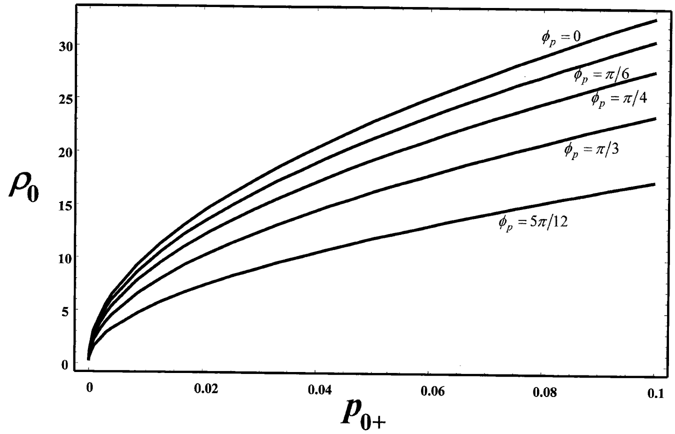
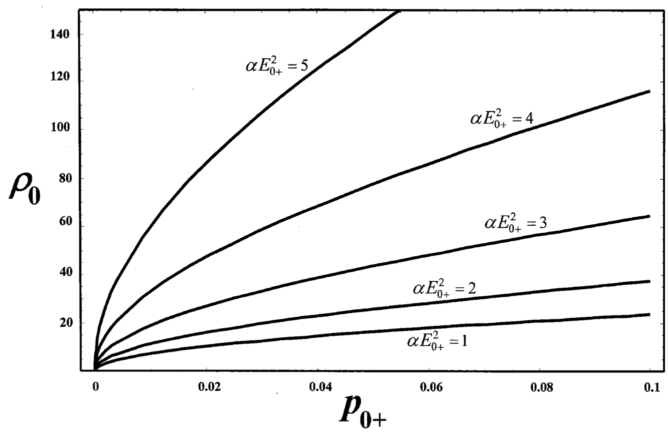


(d)



(e)

Figure 1. Continued.

curve it displays oscillatory divergence or steady divergence. The set of Fig. 1 illustrates the dependence of the initial dimensionless amplitude of the electric field p_{0+} (associated with the ring ripple) on the dimensionless initial ring ripple width $\rho_0 (= r_1\omega/c)$ for self-trapping corresponding to the collisionless plasma with dominant ponderomotive nonlinearity for the extraordinary mode of propagation of the electromagnetic beam. The figures correspond to a standard set of parameters $\beta E_{00+}^2 = 1.0$, $\delta = 1.0$, $\phi_{\mathrm{p}} = \pi/3$, $\mu = 0.001$, $n = 1.0$, $\Omega^2 = 0.8$, $(\omega_{\mathrm{c}}/\omega) = 0.2$ and the effect of the variation of each individual parameter (keeping the rest of them the same) on the critical curves of the ring ripple, has been shown in different figures; the nature of the curves is justified by the saturating character of the nonlinearities.

Figure 1(a) illustrates the effect of variation of the parameter n on the critical curve of the ring ripple and predicts that the self-focusing region increases with the increasing value of n. This can be understood in terms of the nonlinearity containing term $\varepsilon_{2+}(z)$ which falls sharply with increasing p_{0+} for lower values of n. The ring ripple exhibits weaker interaction with the field of the Gaussian part

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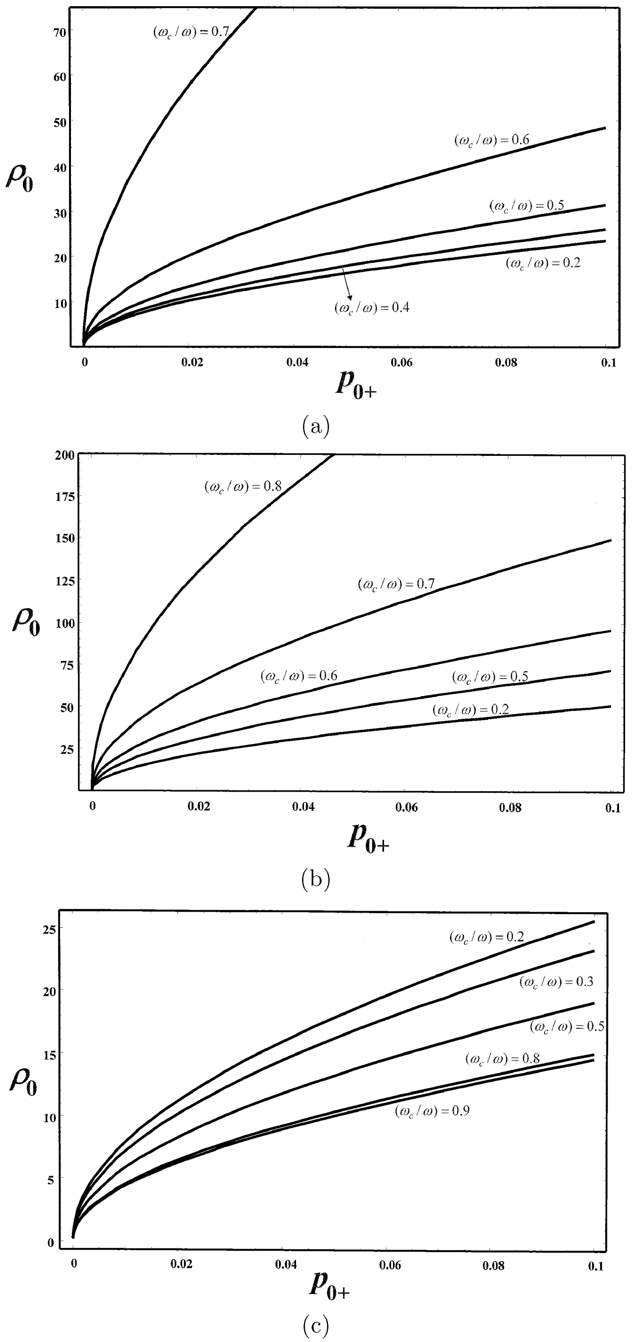

Figure 2. The effect of the variation of the static magnetic field (i.e. ω_c/ω) on the critical curves for the extraordinary mode of propagation of the beam in a magnetoplasma, for the parameters $\Omega^2 = 0.8$, $\mu = 0.001$, $\delta = 1$, $n = 1$, $\phi_p = \pi/3$ and βE^2_{00+} (or αE^2_{00+}) $= 1$; (a) refers to collisionless magnetoplasma (with dominant ponderomotive nonlinearity) while (b) and (c) correspond to collisional nonlinearity with $s = -3$ and $s = 1$, respectively (the magnitude of the varying parameter is indicated above the curve).

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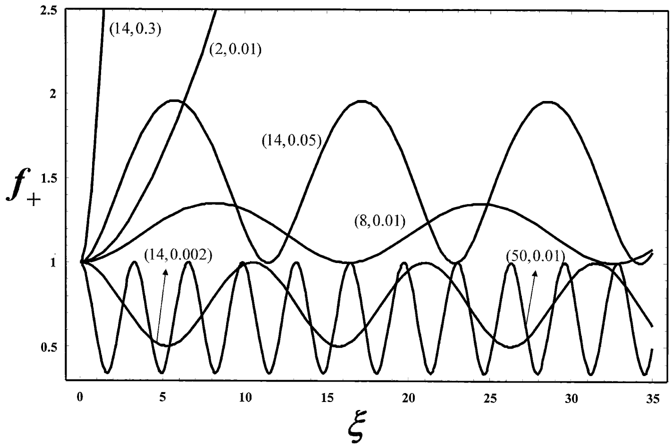

Figure 3. Variation of the dimensionless extraordinary beam width parameter f_+ on the dimensionless distance of propagation ξ, in a collisionless magnetoplasma with dominant ponderomotive nonlinearity, for the parameters $\Omega^2 = 0.8$, $(\omega_c/\omega) = 0.2$, $\mu = 0.001$, $\delta = 1$, $n = 1$, $\phi_p = \pi/3$ and $\beta E^2_{00+} = 2.0$; the curves refer to a chosen set of parameters (ρ_0, p_+) as indicated over the curves.

of the electromagnetic beam as the position of the ring ripple moves away from the axis $r = 0$ (i.e. δ increases); thus, the region of self-focusing also decreases (this nature has been displayed in Fig. 1(b)). The larger the parameter μ, the ratio of the initial width of the ring ripple part to that of the Gaussian part, i.e. $(r_1/r_0)^2$, the larger is the region for self-focusing as expressed in Fig. 1(c). Furthermore, Fig. 1(d) suggests that the focusing occurs at lower initial ring ripple width ρ_0 with increase in the initial phase difference ϕ_p between the electric fields of the Gaussian beam and that of the ring ripple; this behaviour is characteristic of saturating nonlinearity and high values of axial irradiance (for the ring ripple). Similarly one can explain the nature of the curves of Fig. 1(e) which indicates larger critical width of the ring ripple for higher irradiance of the Gaussian part of the electromagnetic beam. The computations (not given here) have also been made for collisional magnetoplasma and the critical curves were found to follow similar trends.

The set of Fig. 2 describes the effect of the variation of the applied external magnetic field on the critical power curve of the ring ripple for both kinds of nonlinearities. From Fig. 2(a) it is seen that for a collisionless plasma the lower critical curves (ρ_0 versus p_{0+}) correspond to lower values of (ω_c/ω) with consequent enhancement of the region for self-focusing; this can be understood in terms of saturating nonlinearity, which implies that the main parameter responsible for self-focusing is not $\varepsilon(EE^*)$ but $[d\varepsilon_\pm/d(EE^*)]$. Furthermore, Figs 2(b) and (c) correspond to the critical curves for collisional nonlinearity with electron–ion collision dominant plasma ($s = -3$) and for electron–neutral collision dominant plasma ($s = 1$), respectively; the critical curves display opposite trends of the dependence on (ω_c/ω). This is because, as a result of electron heating, larger redistribution of electrons takes place for $s = -3$ than that for the case $s = 1$.

Figure 3 expresses the dependence of the beam width parameter f_+ on the dimensionless distance of propagation ξ for a collisionless magnetoplasma with dominant ponderomotive nonlinearity. Figure 3 describes the characteristic propagation of the ring ripple on a Gaussian beam in the three regions for chosen points (ρ_0, p_{0+}) corresponding to the self-focusing, oscillatory divergence and steady divergence; the relevant parameters are $\beta E_{00+}^2 = 2.0$, $\delta = 1.0$, $\phi_{\mathrm{p}} = \pi/3$, $\mu = 0.001$, $n = 1.0$, $\Omega^2 = 0.8$ and $(\omega_{\mathrm{c}}/\omega) = 0.2$. The curves are in conformance with the above discussion regarding critical curves.

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