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H_∞ stabilisation of networked control systems with time delays and packet losses

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This paper addresses the H_∞ state feedback stabilisation problem for networked control systems (NCSs) in the presence of time delays and packet losses. By introducing the concept of an effective sensor packet, the NCS is transformed into a new discrete-time switched model, where the parameters have a clear physical meaning and can be easily determined. In this framework, we derive the stability conditions of the closed-loop system in the H_∞ sense, and also provide the corresponding H_∞ stabilising controller design method. Finally, we give simulation and experimental results to demonstrate the effectiveness of the proposed approaches.

1. Introduction

Networked control systems (NCSs) are a class of spatially distributed control systems in which communication networks are employed for the connections between spatially distributed system components, such as sensors, actuators and controllers. The advantages of reduced system wiring, simple installation, increased system flexibility and resource sharing offered by NCSs mean they have received a lot of attention within the control community in recent years. Issues such as time delays (Nilsson *et al.* 1998; Hu and Zhu 2003; Shi and Yu 2009; Peng *et al.* 2009), packet losses (Zhang *et al.* 2001; Xion and Lam 2007), signal quantisation (Tian *et al.* 2007; Rasool and Nguang 2010) and multi-channels (Hu and Yan 2008; Li, J. *et al.* 2011) have been investigated, with many important results reported in the literature. NCSs have also found applications in the fields of remote medical treatment, intelligent vehicle systems, robotics, manufacture processing, and so on.

Time delay in NCSs is a major cause for system performance deterioration and potential system instability. In the literature, time delays have been modelled using

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various formulations, such as constant delay (Zhang *et al.* 2001), independent random delay (Nilsson *et al.* 1998) and random delay governed by a Markov chain (Zhang *et al.* 2005). Another important issue for NCSs is the packet loss phenomenon. Typically, the literature containts two approaches to describing packet losses: the first assumes that the packet losses follow certain probability distributions such as a Markov chain; the second is the deterministic approach, which specifies the packet losses in a time average sense, or places a bound on the number of consecutive packet losses. Note that the packets in NCSs usually suffer both time delays and packet losses simultaneously during network transmission. All this means that the analysis and synthesis of NCSs with time delays and packet losses is a challenging and practically important problem. In the literature, some important methodologies, such as stochastic control (Nilsson *et al.* 1998; Hu and Zhu 2003), predictive control (Liu *et al.* 2006), state feedback control (Xion and Lam 2007; Li *et al.* 2009), H_∞ control (Gao and Wang 2003; Yue *et al.* 2005; Wang 2007) and Fuzzy control (Dong *et al.* 2010), have been proposed to compensate for time delays and/or packet losses – for details on this topic, see the aforementioned references and the references they contain. It is worth noting that although the H_∞ stabilisation problem for NCSs with time delays and/or packet losses has been investigated for several years, there are still some interesting problems in need of further research. For example, as illustrated in our earlier work Li, H. *et al.* (2011), it is easy to reduce the conservativeness of an NCS model, as well as the corresponding method, by thoroughly investigating the effects and physical constraints of the network on the NCS. However, as far as we are aware, the H_∞ stabilisation problem for NCSs with time delays and/or packet losses including a comprehensive investigation of the physical constraints of the network on the NCS model still remains challenging, and this was the motivation for the present study.

In tackling this problem in the current paper, we propose a new NCS model, where the parameters have a clear physical meaning and can be easily determined. We derive the stability conditions in the H_∞ sense, and develop the corresponding H_∞ stabilising controller design technique. It is worth noting that it is easy to implement the proposed method for various applications since it is quite general and the parameters have a clear physical meaning. We also give simulation and experimental results to demonstrate the effectiveness and applicability of the proposed method.

1.1. *Organisation of the paper*

We formulate the stabilisation problem for NCSs with time delays and packet losses in Section 2 and give the main results in Section 3. We give simulation and experimental results in Section 4 to show the effectiveness of the results we have produced. Finally, we present our conclusions in Section 5.

1.2. *Notation*

Throughout this paper, we write $\Re^n$ to denote the n dimensional Euclidean space and $P > 0$ ($\geqslant 0$) to mean that P is real symmetric and positive definite (semidefinite). We attach the superscript ^{T} to matrices to denote matrix transposition, and write I to denote

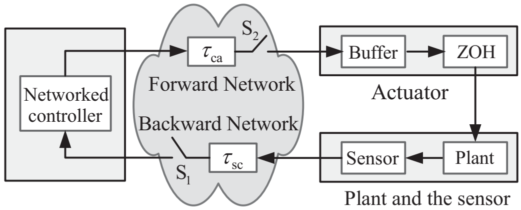

Fig. 1. The structure of the NCSs being studied

the identity matrix with appropriate dimensions. We write Z^+ to denote the set of non-negative integers. Finallly, we write $*$ in symmetric block matrices as an ellipsis for the terms introduced by symmetry.

2. Problem formulation

Consider the NCS shown in Figure 1, where the sensor and the actuator are time driven and the controller is event driven. The plant is described by the following discrete-time system:

$$\begin{aligned} x(k+1) &= Fx(k) + G_1u(k) + G_2w(k) \\ z(k) &= Cx(k) + D_1u(k) + D_2w(k) \end{aligned} \tag{1}$$

where:

— $x(k) \in \Re^n$ is the plant state;
— $u(k) \in \Re^m$ is the control input;
— $w(k) \in \Re^p$ is the exogenous disturbance input with $\{w(k)\} \in l_2[0, \infty)$;
— $z(k) \in \Re^q$ is the controlled output;
— F, G_1, G_2, C, D_1 and D_2 are known matrices with appropriate dimensions.

The networked controller takes the following form:

$$u = Lx(k) \tag{2}$$

where $L \in \Re^{m\times n}$ is the controller gain, which is to be designed. In NCS, a timestamp is added to the sensor packet and then passed to the corresponding control packet. The buffer will compare the time-stamp of the received control packet with the one stored in it, and it will be updated only when the received control signal is more recent than the existing one. Through such a mechanism, the ZOH (Zero-Order Hold) can always use the most recent control signal to control the plant.

In NCSs, networks exist in both of the channels from the sensor to the controller and from the controller to the actuator. It is well known that, for two spatially distributed network nodes, it is easier to obtain the round-trip network characteristics than the point-to-point network characteristic. Therefore, from the viewpoint of practical implementation, it is desirable to develop NCS approaches based on round-trip network characteristics. To this end, we introduce the following definitions and models to capture the nature of round-trip time delays and packet losses.

Definition 2.1. The round-trip time (RTT) delay is defined as

$$\tau = \tau_{sc} + \tau_{ca},$$

where τ_{sc} is the delay from the sensor to the controller and τ_{ca} is the delay from the controller to the actuator. A natural assumption for τ is that

$$\check{\tau} \leqslant \tau \leqslant \hat{\tau},$$

where $\check{\tau}$ and $\hat{\tau}$ are the lower and upper bounds of τ, respectively. We will use the notation $\lceil \hat{\tau}/h \rceil$, where h is the sampling period and $\lceil \cdot \rceil$ is the ceiling operator. We can then infer that $\lceil \tau/h \rceil$ takes values in a finite set

$$\Omega_1 = \{\lceil \check{\tau}/h \rceil, \lceil \check{\tau}/h \rceil + 1, \cdots, \lceil \hat{\tau}/h \rceil\}.$$

Definition 2.2. A sensor packet is said to have encountered a round-trip packet loss if the sensor packet is lost in the sensor-to-controller network or the corresponding control packet is lost in the controller-to-actuator network. We write η to denote the number of consecutive round-trip packet losses. A natural assumption for η is that $\eta \leqslant \hat{\eta}$, where $\hat{\eta}$ is the given upper bound for consecutive round-trip packet losses. It is clear that, η takes values in a finite set $\Omega_2 = \{0, 1, 2, \cdots, \hat{\eta}\}$.

The objective of the current paper is to design a networked controller (2) such that the closed-loop NCS with time delays and packet losses is asymptotically stable and the H_∞ performance constraint is also satisfied.

3. Main results

In this section, we address the H_∞ stabilisation problem for NCSs with a networked controller (2), and give complete results for system modelling, analysis and controller synthesis.

3.1. *Modelling of NCSs*

By considering the effects of network induced delays and packet losses, the control input of the plant can be described by the equation

$$u(k) = Lx(k - \delta_k) \tag{3}$$

where δ_k denotes the step delay of the control signal. From the definitions of Ω_1 and Ω_2, it is easy to conclude that δ_k takes values in a finite set

$$\Omega_3 = \{\lceil \check{\tau}/h \rceil, \lceil \check{\tau}/h \rceil + 1, \cdots, \lceil \hat{\tau}/h \rceil + \hat{\eta}\}.$$

By substituting (3) into (1), the closed-loop NCS is given by

$$\begin{aligned} x(k+1) &= Fx(k) + G_1 Lx(k - \delta_k) + G_2 w(k) \\ z(k) &= Cx(k) + D_1 Lx(k - \delta_k) + D_2 w(k). \end{aligned} \tag{4}$$

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It is easy to infer from the fact that $\delta_k \in \Omega_3$ together with the definition of Ω_3 that at time step k, the control signal no older than

$$k - \lceil \hat{\tau}/h \rceil - \hat{\eta}$$

can be used to control the plant. In view of this, we introduce the augmented states

$$\tilde{x}(k) = [x_k^T \; x_{k-1}^T \cdots x_{k-\lceil \hat{\tau}/h \rceil - \hat{\eta}}^T]^T$$
$$\tilde{z}(k) = [z_k^T \; z_{k-1}^T \cdots z_{k-\lceil \hat{\tau}/h \rceil - \hat{\eta}}^T]^T$$

into (4). The closed-loop system (4) can then be described by the following discrete-time system:

$$\begin{aligned} \tilde{x}(k+1) &= \left(\tilde{F} + \tilde{G}_1 L \tilde{E}_{k-\delta_k}\right) \tilde{x}(k) + \tilde{G}_2 w(k) \\ \tilde{z}(k) &= \left(\tilde{C} + \tilde{D}_1 L \tilde{E}_{k-\delta_k}\right) \tilde{x}(k) + \tilde{D}_2 w(k) \end{aligned} \tag{5}$$

with

$$\begin{aligned} &\tilde{F} = \begin{bmatrix} F & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \quad \tilde{G}_1 = \begin{bmatrix} G_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \tilde{G}_2 = \begin{bmatrix} G_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ &\tilde{C} = \begin{bmatrix} C & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \quad \tilde{D}_1 = \begin{bmatrix} D_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \tilde{D}_2 = \begin{bmatrix} D_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ &\tilde{E}_{k-\delta_k} = \begin{bmatrix} 0 & \cdots & I & \cdots & 0 \end{bmatrix} \end{aligned} \tag{6}$$

where all elements of $\tilde{E}_{k-\delta_k}$ are zero except for the $(k - \delta_k + 1)$th block, which is the identity. It is worth noting that the closed-loop system in (5) is a discrete-time switched system, where $k - \delta_k$ in $\tilde{E}_{k-\delta_k}$ is a piecewise constant function called the switch signal, which takes values in a finite set Ω_3.

Since the parameter δ_k is incorporated into the system (4), the transition of δ_k from one mode to another will be involved in the evolution of the system. We are thus led to ask the following natural questions, which will be of great importance for the development of this paper:

— What is the domain of δ_k?

— Are there any physical constraints on the transition of δ_k?

It is worth noting that in NCSs, if a control signal $L\hat{x}(k - \delta_k)$ is available for the actuator at time step k and no new control signal arrives, the actuator has at least $L\hat{x}(k - \delta_k)$ to use at time step $k + 1$, which implies that

$$\delta_{k+1} = \delta_k + 1.$$

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Therefore, the parameter δ_k in an NCS can increase by at most 1 at each time step. On the other hand, δ_k can decrease by as many steps as possible under the constraint $\delta_k \in \Omega_1$. Based on this analysis, we can construct a switch matrix $\Upsilon = [\delta_{ij}]$ as follows:

$$\Upsilon = \left[\begin{array}{ccccc|ccccc} 1 & 1 & 0 & \cdots & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 1 & 0 & \cdots & \cdots & \cdots & \vdots \\ 1 & \cdots & \cdots & \cdots & 1 & 1 & 0 & \cdots & \cdots & \vdots \\ 1 & \cdots & \cdots & \cdots & 1 & 0 & 1 & 0 & \cdots & 0 \\ \hline \vdots & \ddots & \ddots & \ddots & \vdots & 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & 1 \\ \vdots & \ddots & \ddots & \ddots & 1 & \vdots & \ddots & \ddots & \ddots & 0 \\ 1 & \cdots & \cdots & \cdots & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 1 & \cdots & \cdots & \cdots & 1 & 0 & \cdots & \cdots & \cdots & 0 \end{array}\right] \tag{7}$$

$$= \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix}$$

where

$$\Upsilon_{11} \in \Re^{(\lceil \hat{\tau}/h \rceil - \lceil \check{\tau}/h \rceil + 1) \times (\lceil \hat{\tau}/h \rceil - \lceil \check{\tau}/h \rceil + 1)}$$
$$\Upsilon_{22} \in \Re^{\hat{\eta} \times \hat{\eta}},$$

and i and j take values in Ω_3. The switching matrix Υ says that the switching signal δ_{ij} transfers from mode i to mode j with feasibility δ_{ij}, where 0 and 1 stand for feasible and unfeasible transitions, respectively. Note that the switching matrix Υ is quite general since both time delays and packet losses are taken into account. In particular, the switching matrix Υ captures the physical meaning of the time delay and packet loss in a clear way, and makes the transition feasibility between different modes apparent.

3.2. *H_∞ performance analysis*

Before proceeding, we need to introduce the following definition, which will be useful for the development of our work.

Definition 3.1. Given a scalar $\gamma > 0$, the closed-loop system (5) is said to be asymptotically stable with an H_∞ performance γ if:

(1) The system with $w(k) \equiv 0$ is asymptotically stable.
(2) The controlled output $\tilde{z}(k)$ satisfies

$$\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2$$

for all non-zero

$$w(k) \in l_2[0, \infty)$$

under zero initial conditions.

With the above definition, the following theorem states and establishes the sufficient conditions for the existence of the network-based H_∞ controller for the system (5).

Theorem 3.2. Given a scalar $\gamma > 0$, the closed-loop NCS (5) is asymptotically stable with an H_∞ performance γ if there exist positive definite matrices P_i, with $i \in \Omega_3$, satisfying

$$\delta_{ij} \begin{bmatrix} -P_j & 0 & P_j(\tilde{F} + \tilde{G}_1 L \tilde{E}_i) & P_j \tilde{G}_2 \\ * & -I & \tilde{C} + \tilde{D}_1 L \tilde{E}_i & \tilde{D}_2 \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{8}$$

where $i, j \in \Omega_3$ and $\delta_{ij} \in \Upsilon$.

Proof. We first establish the asymptotic stability of the closed-loop system in (5) with $w(k) \equiv 0$. Consider the following state-dependent Lyapunov function:

$$V(\tilde{x}(k)) = \tilde{x}^T(k) P_{k-\delta_k} \tilde{x}(k) \tag{9}$$

where $P_{k-\delta_k}$ are matrices upon the switch state $k - \delta_k$ in the NCS model (5). Let $k - \delta_k$ at time steps k and $k+1$ be i and j, respectively, where $i, j \in \Omega_3$. The difference between the values of the Lyapunov function is given by:

$$\begin{aligned} \triangle V(\tilde{x}(k)) &= V(\tilde{x}(k+1)) - V(\tilde{x}(k)) \\ &= \tilde{x}^T(k+1) P_j \tilde{x}(k+1) - \tilde{x}^T(k) P_i \tilde{x}(k). \end{aligned} \tag{10}$$

Two cases arise:

(1) No new control signal arrives at the actuator at time step $k+1$.
In this case, i in (10) takes values in

$$\Omega_4 = \Omega_3 - \{\lceil \hat{\tau}/h \rceil + \hat{\eta}\}$$

and j in (10) is specified to be

$$j = i + 1.$$

Then, for the solution of the closed-loop system (5) with $w(k) \equiv 0$, we have

$$\begin{aligned} \triangle V(\tilde{x}(k)) &= \tilde{x}^T(k+1) P_{i+1} \tilde{x}(k+1) - \tilde{x}^T(k) P_i \tilde{x}(k) \\ &= \tilde{x}^T(k) \left(\left(\tilde{F} + \tilde{G} L \tilde{E}_i\right)^T P_{i+1} \left(\tilde{F} + \tilde{G} L \tilde{E}_i\right) - P_i \right) \tilde{x}(k) \end{aligned} \tag{11}$$

where $i \in \Omega_4$. It is clear that this case corresponds to the non-zero elements above the diagonal of Υ.

(2) The actuator has a new control signal available at time step $k+1$.

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In this case, i in (10) takes values in Ω_3 and j in (10) takes values in Ω_1. Then, for the solution of the closed-loop system (5) with $w(k) \equiv 0$, we have

$$\begin{aligned} \triangle V(\tilde{x}(k)) &= \tilde{x}^T(k+1)P_{i+1}\tilde{x}(j) - \tilde{x}^T(k)P_i\tilde{x}(k) \\ &= \tilde{x}^T(k)\left(\left(\tilde{F}+\tilde{G}L\tilde{E}_i\right)^T P_j\left(\tilde{F}+\tilde{G}L\tilde{E}_i\right) - P_i\right)\tilde{x}(k) \end{aligned} \tag{12}$$

where $i \in \Omega_3$ and $j \in \Omega_1$. It is clear that this case corresponds to the non-zero elements in and below the diagonal of Υ.

Summarising, we can conclude that if

$$\delta_{ij}\left(\left(\tilde{F}+\tilde{G}_1L\tilde{E}_i\right)^T P_j\left(\tilde{F}+\tilde{G}_1L\tilde{E}_i\right) - P_i\right) < 0 \tag{13}$$

for $i, j \in \Omega_3$ and $\delta_{ij} \in \Upsilon$, then we have

$$\lim_{k\to\infty} V(\tilde{x}(k)) = 0$$
$$\lim_{k\to\infty} \tilde{x}(k) = 0,$$

which imply that NCS (5) is asymptotically stable.

On the other hand, using the Schur complement, (8) is equivalent to

$$\delta_{ij}\left(\Pi_i{}^T\tilde{P}_j\Pi_i - \hat{P}_i\right) < 0 \tag{14}$$

where

$$\begin{aligned} \Pi_i &= \begin{bmatrix} \tilde{F}+\tilde{G}_1L\tilde{E}_i & \tilde{G}_2 \\ \tilde{C}+\tilde{D}_1L\tilde{E}_i & \tilde{D}_2 \end{bmatrix} \\ \tilde{P}_j &= \begin{bmatrix} P_j & 0 \\ 0 & I \end{bmatrix} \\ \hat{P}_i &= \begin{bmatrix} P_i & 0 \\ 0 & \gamma^2 I \end{bmatrix}. \end{aligned} \tag{15}$$

It is clear that the conditions (13) are implied by (14). Therefore, if the conditions (8) hold, the closed-loop system in (5) is asymptotically stable.

We will now establish the H_∞ performance for the NCS (5). To this end, we assume zero initial conditions, and consider the following index:

$$\begin{aligned} J_\infty &= \sum_{k=0}^{\infty}[\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k)w(k)] \\ &\leqslant \sum_{k=0}^{\infty}[\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k)w(k)] + V(\tilde{x}(\infty)) - V(\tilde{x}(0)) \\ &= \sum_{k=0}^{\infty}[\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)] \\ &= \sum_{k=0}^{\infty}\delta_{ij}[\xi(k)^T(\Pi_i{}^T\tilde{P}_j\Pi_i - \hat{P}_i)\xi(k)] \end{aligned} \tag{16}$$

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where

$$\xi(k) = [\tilde{x}(k)^T, w(k)^T]^T.$$

Using similar arguments to those used earlier, we can conclude that (8) guarantees (14), so we have

$$\delta_{ij}\xi(k)^T(\Pi_i{}^T\tilde{P}_j\Pi_i - \hat{P}_i)\xi(k) < 0$$

for all non-zero $\xi(k)$, which means $J_\infty < 0$. Therefore, we can conclude from (16) that for all non-zero $w(k) \in l_2[0, \infty)$, we have

$$\|\tilde{z}(k)\|_2 < \gamma\|w(k)\|_2,$$

which completes the proof. □

It is worth noting that the physical meaning of δ_{ij} in the stability condition is to guarantee that the NCS is not only stable in a special mode, but also remains stable when it switches from one mode to another.

3.3. *H_∞ controller design*

We are now ready to address the H_∞ controller design problem. Note that the sufficient conditions (8) are non-linear in the state feedback gain matrix L. In order to circumvent the synthesis problem, the following theorem proposes equivalent conditions to (8).

Theorem 3.3. Given a scalar $\gamma > 0$, the closed-loop NCS (5) is asymptotically stable with an H_∞ performance γ if there exist $P_i > 0$ and $Q_j > 0$ satisfying

$$\delta_{ij}\begin{bmatrix} -Q_j & 0 & \tilde{F} + \tilde{G}_1 L\tilde{E}_i & \tilde{G}_2 \\ * & -I & \tilde{C} + \tilde{D}_1 L\tilde{E}_i & \tilde{D}_2 \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{17}$$

and

$$P_j Q_j = I \tag{18}$$

where $i, j \in \Omega_3$ and $\delta_{ij} \in \Upsilon$.

Proof. From (18) we have

$$Q_j = P_j^{-1}. \tag{19}$$

Substituting (19) into (17), we have

$$\delta_{ij}\begin{bmatrix} -P_j^{-1} & 0 & \tilde{F} + \tilde{G}_1 L\tilde{E}_i & \tilde{G}_2 \\ * & -I & \tilde{C} + \tilde{D}_1 L\tilde{E}_i & \tilde{D}_2 \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{20}$$

Performing congruence transformations on (20), by diag$\{P_j, I, I, I\}$, we get (8). Then, according to Theorem 3.2, we can conclude that if the conditions (17) and (18) hold, NCS (5) is asymptotically stable with an H_∞ performance γ. □

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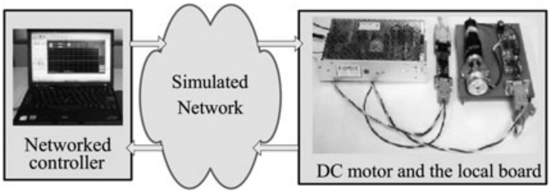

Fig. 2. The actual networked DC motor system setup

Note that the conditions stated in Theorem 3.3 do not form a convex set due to the matrix equality constraints in (18). In the literature, there are several approaches to solving such non-convex problems, amongst which the *cone complementarity linearisation* (CCL) approach is the most commonly used since it is simple and very efficient for numerical implementation (Ghaoui *et al.* 1997; Zhang *et al.* 2005). For these reasons, we used the CCL approach for the current paper to calculate L from Theorem 3.3. Note that the CCL approach is quite standard, and it is easy to adapt the way we used it in Li *et al.* (2009) to solve the controller design problem in the current paper, but to save space and avoid repetition, we will omit the details of the CCL-based controller design procedure here.

4. Illustrative examples

In this section, we will illustrate the applicability and effectiveness of the proposed approaches. To this end, we constructed a real-world networked DC motor system and subjected it to a comprehensive study to develop both simulation and experimental results.

The system setup is shown in Figure 2, where the experimental apparatus consists of a PC controller, a local board and a DC motor with sensors. The PC controller is used to implement the networked controller. The local board is on the plant side and used for two functions:

(1) to convert the control signal read from the buffer into a pulse width-modulation (PWM) signal, and then send the PWM signal to drive the DC motor;
(2) to encapsulate the plant state and its timestamp into a packet and send it to the PC controller via the network.

Both the local board and the sensor are time-driven, and they are synchronised using the same clock signal. We set the sampling period to 0.3s and let

$$x = [i_a,\ \omega]^T$$

where i_a and ω are the armature winding current and the rotor angular speed, respectively. The DC motor can be described by the equations

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0.0046 \\ 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 2.2685 \\ 7.6794 \end{bmatrix} u(k) \\ & \quad \begin{bmatrix} 1.4721 \\ -570.6426 \end{bmatrix} w(k) \qquad (21) \\ z(k+1) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + 0.1u(k). \end{aligned}$$

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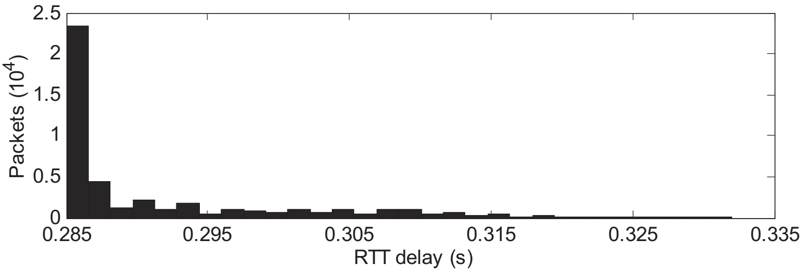

Fig. 3. Histograms for the RTT delays of the network under consideration

The Internet connection between Tsinghua University, Beijing, China and Stanford University, CA, USA was investigated and the RTT delay and packet loss information measured for a 24-hour period (00:00-24:00). 45000 packets were sent and only 129 packets lost, with the maximum consecutive packet loss being 1. The histogram of RTT delays for the 44871 received packets is shown in Figure 3, from which it can be seen that the lower and the upper bounds of τ_k are $\check{\tau} = 0.285s$ and $\hat{\tau} = 0.332s$, respectively. This further demonstrates that the delay and packet loss models formulated in this paper are natural and realistic.

Applying the proposed method to the above NCS, we obtain the following network controller:

$$u = [-0.095\ 0.011]\, x. \tag{22}$$

It is worth noting that the networked DC motor system is designed to drive the DC motor to a pre-set angle, so we introduce the reference input r into the networked controller and rewrite (22) as

$$u = [-0.095\ 0.011]\, x + 0.095r \tag{23}$$

4.1. *Numerical simulation results*

For the numerical simulation, the networked dc motor system was simulated using MATLAB with a fully controlled environment. The disturbance input $w(k)$ was given by

$$w(k) = \begin{cases} 0.05 & 0 \leqslant k \leqslant 10 \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

With the initial condition $[0,\ 0]^T$ and reference input $r = 40$ degrees, typical simulation results for the networked DC motor system are given in Figure 4. These show that the position of the networked DC motor converges to the reference input, which demonstrates that the resulting NCS is asymptotical stable.

4.2. *Experimental results*

With the same initial state, reference input and control parameters as used for the numerical simulation, typical experimental results for the networked DC motor system

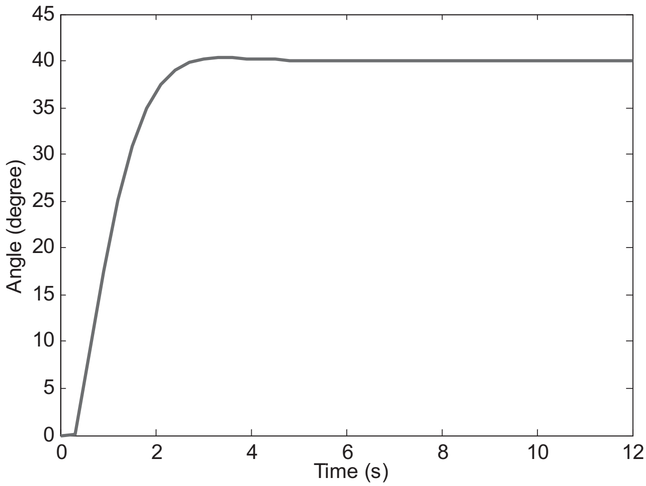

Fig. 4. Simulation result of networked system

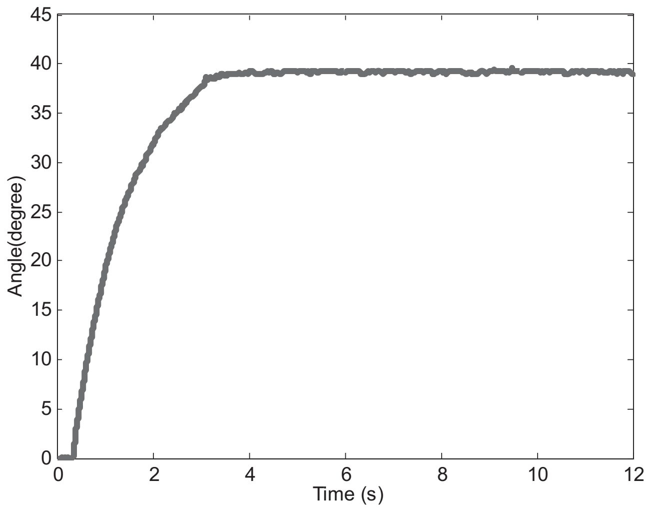

Fig. 5. Simulation result of networked system

are given in Figure 5 and are consistent with the simulation results shown in Figure 4, with the exception of a small steady-state error in the experimental result. Note that this steady-state error does not come as a surprise since there are inevitable modelling errors, and, most importantly, there are inevitable non-linearities such as a dead zone and fabrication errors in the experimental apparatus.

5. Conclusions

In this paper, we have investigated the H_∞ stabilisation problem for a class of NCSs with time delays and packet losses. In doing this, we constructed a new discrete-time switched NCS model for the resulting NCSs. In this framework, we derived the stability

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conditions for the closed-loop NCSs and proposed an iterative algorithm for designing the corresponding H_∞ stabilising controller. The simulation and experimental results demonstrate the effectiveness of the proposed approach.

In future work, we will extend our results to the output feedback case, where full state measurement is not available. We will also introduce integral control to the networked controller to address the steady-state error problem for NCSs.

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