J. Plasma Phys. (2020), *vol.* 86, 905860303

Published by Cambridge University Press
doi:10.1017/S002237782000046X

New kinetic cyclotron instability for electron beam in time-changing magnetic fields

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(Received 4 February 2020; revised 23 April 2020; accepted 24 April 2020)

This paper examines the velocity distribution function and cyclotron resonance conditions for a beam of electrons moving in a magnetic field which gradually changes with time. A spatial gradient of magnetic field is known to result in an unstable horseshoe distribution of electrons. The field gradient in time adds additional effects due to an induced electric field. The resultant anisotropic velocity distribution function, which we call a Luvdisk distribution, has some distinctive properties when compared to the horseshoe. Fitting the cyclotron resonance condition circle shows that the frequency of the resultant emission is under the local cyclotron frequency. While the spatial gradient results in the emission coming almost perpendicularly to the field, the direction of the radiation under a time-changing field has more variability. The Luvdisk distribution also arises when the magnetic field has a gradient both in space and time. The beam can be unstable if those gradients are added or subtracted from each other (if the gradients are of equal or different sign), which occurs even when the total change of magnetic field is negative. While the frequency of the emission is related to the final magnetic field value, its direction is indicative of the field's history which produced the instability.

Key words: astrophysical plasmas, intense particle beams, plasma instabilities

1. Introduction

The cyclotron instability in a beam of charged particles occurs when the beam's kinetic distribution comes into a resonance with the local cyclotron resonance condition circle. This is a collective instability driven by a gradient of the kinetic distribution function. The distribution function must be anisotropic to provide a substantial interaction of the cyclotron resonance condition with the high-gradient region of the distribution.

We are interested here in the cyclotron kinetic instability with perpendicular drive (i.e. driven by a gradient of the velocity distribution function perpendicular to the beam direction). Examples of anisotropic distribution functions responsible for such an instability include a ring distribution (Cairns *et al.* 2011), an even perpendicular spread

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(Gary *et al.* 1994) and a horseshoe distribution (Bingham & Cairns 2000; Vorgul *et al.* 2011). Such instabilities are important in astrophysical and laboratory plasma.

The horseshoe distribution is formed from an initial drifting Maxwellian when the beam is moving along a rising gradient of magnetic field. Energy and magnetic momentum conservation cause the beam in spreading in velocity space, perpendicular to the beam direction and curling backward (Bingham, Cairns & Kellett 2001; Vorgul, Cairns & Bingham 2005; Speirs *et al.* 2014). This distribution is responsible for periodic radio emission from many astrophysical bodies, including the Earth (auroral kilometric radiation, e.g. Ergun *et al.* (2000)) and other terrestrial planets (e.g. Lamy *et al.* 2018), stars (e.g. Kellett *et al.* 2007 and Trigilio *et al.* 2008) and brown dwarfs (e.g. Harding *et al.* 2013). The periodicity is due to the rotation of the source and to a high directivity of the emission. The radiation is detected when directly pointing towards the Earth, hence being seen as periodic pulsations in observations. The emission happens when the anisotropic distribution couples with the local cyclotron resonance condition. This means that the cyclotron resonance condition circle[1] intersects an extensive area of high gradient of the distribution function in velocity space. The wave of the resonant frequency will then grow, and radiation with distinct properties is produced.

For the horseshoe-distribution instability, the radiation has a narrow spectrum peaking just below the local electron cyclotron frequency; is emitted as an X-mode (corresponding to TE (transverse-electric) mode in laboratory experiments) and its direction is slightly backward with respect to the beam, but almost perpendicular to it (Speirs *et al.* 2014). The resultant radiation from an astrophysical body can have the direction changed by refraction while propagating through its magnetosphere, and the bandwidth of the spectrum can be affected by dispersion of the medium of propagation.

The magnetic field of many natural sources of cyclotron emission can be expected to be variable, due to fast motion or rotation, or due to physical processes at the field's source. Laboratory plasma can also have unstable magnetic field, or the field's changing can be introduced purposefully for specific effects. Cyclotron emission is a valuable non-invasive tool for assessing the local magnetic field both in the laboratory and in distant observations. In this paper, we explore the possibility of an electron cyclotron emission caused by a gradient of the magnetic field not in space, as for the horseshoe instability, but in time. We consider a magnetic field varying (gradually rising or falling) in time in a homogeneous medium, as well as in combination with the field's gradient in space.

The modelling is done by applying conservation laws to find a velocity distribution function of an electron beam resulting from change in magnetic field. Fitting a cyclotron resonance circle of a wave through its region of maximum gradient shows characteristic properties of the related emission.

2. Derivation of velocity distribution function

Similarly to the case of an electron beam moving into a stronger magnetic field, the resultant distribution function of the electrons can be found from an initial one by applying conservation laws (Bingham & Cairns 2000). The difference from the case when the beam's propagation conditions are stationary ($\boldsymbol{B} = \text{const.}$) comes from an additional electric field induced by the change in magnetic flux. The additional

[1]A circle described by the resonant wave's frequency being equal to the local cyclotron frequency with relativistic correction, Doppler-shifted by parallel propagation.

https://doi.org/10.1017/S002237782000046X Published online by Cambridge University Press

field then accelerates/decelerates the electrons and contributes to the final energy in conservation laws.

We consider the initial beam's distribution function being a drifting Maxwellian in velocity space, which in terms of parallel ($v_\parallel$) and perpendicular ($v_\perp$) velocities can be written as

$$f_0(v_\parallel, v_\perp) = A \cdot \exp\left(-\frac{m}{2k_B T}((v_\parallel - v_0)^2 + v_\perp^2)\right), \tag{2.1}$$

where v_0 is the drift velocity of the beam, m is the mass of an electron, T is an average temperature, and k_B is the Boltzman constant.

2.1. *Effect of the induced electric field*

Let the magnetic field change from the value B_0 to B ($B > B_0$) over the time Δt. Assume the vector field $\boldsymbol{B}$ is uniform and directed along the z-axis over the length where the time change is happening.

Changing magnetic flux induces an additional electric field, $\boldsymbol{E_B}$, according to

$$\oint_C \boldsymbol{E_B}\,\mathrm{d}\boldsymbol{l} = -\frac{\partial}{\partial t}\iint_S \boldsymbol{B}\boldsymbol{\cdot}\boldsymbol{n}\,\mathrm{d}S \approx -\frac{\partial}{\partial t} B\pi r_L^2, \tag{2.2}$$

where the normal to the surface, $\boldsymbol{n}$, is in the z-direction, $\boldsymbol{B} = B\boldsymbol{\cdot}\hat{z}$, r_L is the Larmor radius for the instantaneous B,

$$r_L = \frac{mv_\perp}{eB}, \tag{2.3}$$

with $v_\perp^2 = v_s^2 + v_\phi^2$ in cylindrical coordinates.

It is reasonable to assume E_B to be in the azimuthal direction for a uniform magnetic field along the z-axis, referring to the differential form of (2.2),

$$\nabla \times \boldsymbol{E_B} = -\frac{\partial \boldsymbol{B}}{\partial t}. \tag{2.4}$$

Then, the induced electric field gives an additional kinetic energy to the beam, $\Delta K = m(\Delta v_\phi)^2/2$. It can be calculated as work done by the electric force, $\boldsymbol{F_e} = -e\boldsymbol{E_B}$, over the electrons' path,

$$\Delta K = -e\int \boldsymbol{E_B}\,\mathrm{d}\boldsymbol{l}. \tag{2.5}$$

During the change of the magnetic field with time, the electrons move along a conical spiral with decreasing radius, which corresponds to the Larmor radius at an instantaneous local value of the magnetic field. The work done over this spiral path can then be calculated by summing up the work done over closed circular loops on that cylinder, and can be approximated by multiplying the work done over an average-length loop by a number of the loops over the time of the field changing, Δt. Then, combining equation (2.5) with (2.2), the change in kinetic energy is

$$\Delta K = -e \cdot N \cdot \oint \boldsymbol{E_B}\,\mathrm{d}\boldsymbol{l} = -e \cdot N \cdot \frac{\partial B}{\partial t} \cdot \pi \tilde{r_L^2}, \tag{2.6}$$

where $\tilde{r_L^2}$ is an average value of the square of the Larmor radius of the electrons along the conical spiral. The sign in final result of (2.6) is found using the right-hand rule

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for the directions of $\boldsymbol{E_B}$ and the azimuthal velocity and means that the induced field is slowing down the electrons, with no effect on the electrons close to the beam's axis and the larger effect for the electrons with larger transverse velocity.

The derivative of the magnetic field in (2.6) can be approximated by

$$\frac{\partial B}{\partial t} \approx \frac{\Delta B}{\Delta t} = \frac{(B - B_0)}{\Delta t} \tag{2.7}$$

if the magnetic field change can be considered linear with time. The number of loops, N, can be calculated by dividing the total time over which the change occurred by the average time it took the electrons to travel along one loop of an average radius $\tilde{r_L}$ (τ),

$$N = \frac{\Delta t}{\tau} = \frac{\Delta t}{\dfrac{2\pi \tilde{r_L}}{\frac{1}{2}\left(\sqrt{v_\parallel^2 + v_\perp^2} + \sqrt{v_{\parallel 0}^2 + v_{\perp 0}^2}\right)}} = \frac{\Delta t}{4\pi \tilde{r_L}}\left(\sqrt{v_\parallel^2 + v_\perp^2} + \sqrt{v_{\parallel 0}^2 + v_{\perp 0}^2}\right), \tag{2.8}$$

where the square roots are the velocities of the electrons at $t = 0$ and $t = \Delta t$, correspondingly.

The change of the magnetic field in time results then in the change in kinetic energy of electrons equal to (combining (2.6) and (2.8))

$$\begin{aligned} \Delta K &= e \cdot \frac{\Delta t}{4\pi \tilde{r_L}} \cdot \left(\sqrt{v_\parallel^2 + v_\perp^2} + \sqrt{v_{\parallel 0}^2 + v_{\perp 0}^2}\right) \cdot \frac{\Delta B}{\Delta t} \cdot \pi \tilde{r_L}^2 \\ &= \frac{e}{4}\left(\sqrt{v_\parallel^2 + v_\perp^2} + \sqrt{v_{\parallel 0}^2 + v_{\perp 0}^2}\right) \cdot \Delta B \cdot \frac{m\tilde{v_\perp}}{eB} \\ &= \frac{m}{2} \cdot \frac{\Delta B}{4}\left(\frac{v_\perp}{B} + \frac{v_{\perp 0}}{B_0}\right)\left(\sqrt{v_\parallel^2 + v_\perp^2} + \sqrt{v_{\parallel 0}^2 + v_{\perp 0}^2}\right), \end{aligned} \tag{2.9}$$

where at the last step the Larmor radius was substituted with $\tilde{r_L} = m/2(v_\perp/B + v_{\perp 0}/B_0)$.

2.2. *Magnetic momentum conservation*

In order to find the transformed distribution function of the electrons after the change of magnetic field with time, the initial transverse and parallel velocities in the initial distribution function, equation (2.1), need to be expressed in terms of the final velocities. Conservation laws can be used to find these correspondences.

It was shown in Qin & Davidson (2006) that, similarly to the case of a beam moving in a non-uniform but stationary magnetic field, the magnetic momentum of the electrons is adiabatically conserved for the case when magnetic field is changing with time. Since the magnetic momentum can be written as

$$\mu = -\frac{e}{2m}L = -\frac{e}{2m}mv_\perp \cdot \frac{mv_\perp}{B} = \frac{emv_\perp^2}{2B}, \tag{2.10}$$

the magnetic momentum conservation yields the correspondence between the initial and final transverse velocity as

$$v_{\perp 0} = v_\perp \cdot \frac{B_0}{B}. \tag{2.11}$$

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2.3. *Energy conservation*

The electrons' kinetic energy after the field's change is equal to the energy before the change plus ΔK,

$$\begin{aligned}\frac{m}{2}(v_{\parallel 0}^2+v_{\perp 0}^2)+\Delta K &= \frac{m}{2}(v_{\parallel}^2+v_{\perp}^2)\\ &= \frac{m}{2}\left(v_{\parallel}^2+v_{\perp}^2+\frac{\Delta B}{4}\left(\frac{v_{\perp}}{B}+\frac{v_{\perp 0}}{B_0}\right)\left(\sqrt{v_{\parallel}^2+v_{\perp}^2}+\sqrt{v_{\parallel 0}^2+v_{\perp 0}^2}\right)\right).\end{aligned} \tag{2.12}$$

Substituting $v_{\perp 0}^2$ from (2.11), (2.12) can be used to express $v_{\parallel 0}$ in terms of the velocity components after the field's change,

$$v_{\parallel 0}^2 = v_{\parallel}^2+v_{\perp}^2\left[1-\frac{B_0}{B}+\left(\frac{\Delta B}{4B}\left(1+\sqrt{\frac{B_0}{B}}\right)\right)^2\right]+\frac{\Delta B}{2B}v_{\perp}\left(1+\sqrt{\frac{B_0}{B}}\right)\sqrt{v_{\parallel}^2+v_{\perp}^2}. \tag{2.13}$$

2.4. *Transformed distribution function*

Substituting the initial velocities in terms of the final velocity components from (2.11) and (2.13) into the initial velocity distribution function in (2.1) the transformed distribution function is equal to

$$\begin{aligned} f(v_{\parallel}, v_{\perp}) \;=\; & A\cdot\exp\big(-(m/2k_BT)\big(\big(v_{\parallel}^2+v_{\perp}^2\big[1-B_0/B+(\Delta B/4B(1+\sqrt{B_0/B}))^2\big]\\ & +\Delta B/2Bv_{\perp}(1+\sqrt{B_0/B})\sqrt{v_{\parallel}^2+v_{\perp}^2}-v_0\big)^2+B_0/Bv_{\perp}^2\big)\big).\end{aligned} \tag{2.14}$$

The final value of the magnetic field B in (2.13) determines the final Larmor radius, while the change of the field, ΔB, comes from the time derivative of the field and is a factor in the term accounting for the additional energy from the induced electric field ΔK in (2.9). There are no requirements for the final B to be only a result of the change in time, and (2.13) can describe the distribution function as a result of propagation under a combination of gradients of magnetic field in space (by $B-B_0-\Delta B$) and in time (by ΔB), resulting in the final field being equal to B. It is easy to see that for $\Delta B\rightarrow 0$ this distribution function tends to the horseshoe distribution arising when a beam of electrons moves into a spatial gradient of magnetic field (Bingham & Cairns 2000; Vorgul *et al.* 2005),

$$f(v_{\parallel}, v_{\perp}) = A\cdot\exp\left(-\frac{m}{2kT}\left(\left(\sqrt{v_{\parallel}^2+(1-B_0/B)\cdot v_{\perp}^2}-v_0\right)^2+B_0/B\cdot v_{\perp}^2\right)\right). \tag{2.15}$$

Typical plots of the distribution are shown in figure 1.

This anisotropic distribution has regions with a positive gradient transverse to the beam direction, which suggests the possibility of it being unstable to a cyclotron instability. Since this distribution has instability properties distinct from those of a conventional horseshoe distribution (more of which will be addressed in § 4) and to reflect on its shape in velocity space, it was called a Luvdisk distribution.

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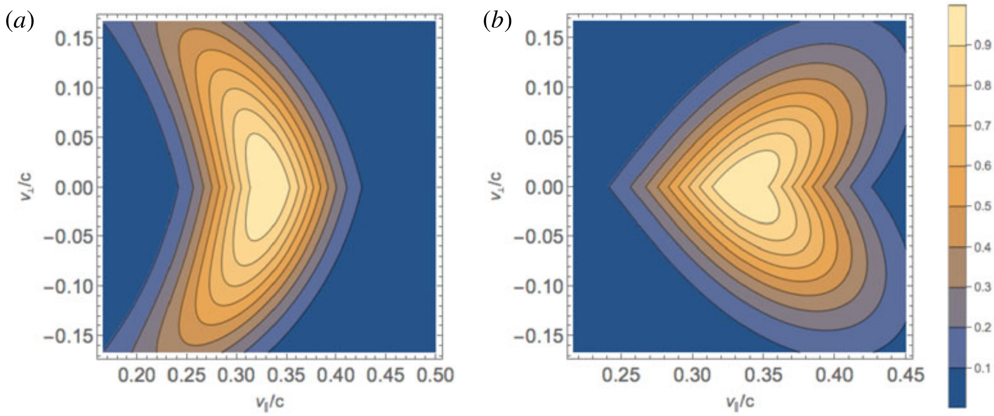

FIGURE 1. Typical plots of the velocity distribution function resulting from a magnetic field changing in time (Luvdisk distribution). (*a*) Magnetic field change with time $B/B_0 = 7.5$, (*b*) magnetic change $B/B_0 = 3.5$ combined with magnetic field gradient in space with $B/B_0 = 10$.

3. Fitting the cyclotron resonance condition circle

An anisotropic distribution function in velocity space can be unstable, resulting in a cyclotron emission of a wave whose frequency, ω, and propagation vector, $\boldsymbol{k} = (\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})$, satisfy the cyclotron resonant condition of the frequency of the emission being equal to the local cyclotron frequency ($\omega_c = qB/m$) Doppler-shifted by the parallel propagation and corrected by the relativistic factor γ,

$$\omega - k_{\parallel} v_{\parallel} = \frac{\omega_c}{\gamma} = \omega_c \sqrt{1 - \frac{v_{\parallel}^2 + v_{\perp}^2}{c^2}}. \tag{3.1}$$

The emission occurs when the cyclotron resonance ellipse in velocity space, as described by (3.1), comes through a substantial region of a high gradient ($\partial f/\partial v_{\perp} > 0$) in the anisotropic distribution function (Bingham & Cairns 2000; Vorgul *et al.* 2005; Cairns, Vorgul & Bingham 2008).

To explore the parameter space of the resultant emission, the cyclotron resonance ellipse should fit through the high gradient of the Luvdisk distribution described by (2.14). This resonance ellipse is extremely sensitive to the parameters of the wave, ω and $k_{\parallel}$, which means a sharp resonance with a narrow frequency spectrum and high directivity. In order for this ellipse to pass through the area of a high gradient of the distribution function, we derive constraints on the parameters to fit the velocity coordinates of the unstable region of the distribution.

Rewriting the resonance condition (3.1) in dimensionless notation as

$$\tilde{\omega} - \tilde{k_{\parallel}} \tilde{v_{\parallel}} = \sqrt{1 - \tilde{v_{\parallel}}^2 + \tilde{v_{\perp}}^2}, \tag{3.2}$$

where

$$\left.\begin{aligned} \tilde{\omega} &= \frac{\omega}{\omega_c}, \\ \tilde{k_{\parallel}} &= \frac{k_{\parallel} c}{\omega_c}, \\ \tilde{v_{\parallel,\perp}} = \frac{v_{\parallel,\perp}}{c}, &\quad \tilde{v_0} = \frac{v_0}{c}, \end{aligned}\right\} \tag{3.3}$$

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and looking for the resonant ellipse to come through two points of the distribution's highest gradient, measured in terms of the fractions of the beam's bulk velocity, v_0,

$$\tilde{v_\parallel} = \alpha \cdot v_0, \quad \tilde{v_\perp} = \beta \cdot v_0, \tag{3.4a,b}$$

the resultant frequency and parallel wavenumber of the wave can be found as

$$\left.\begin{aligned} \tilde{\omega} &= \frac{\alpha_1\sqrt{1-\alpha_2^2\tilde{v_0}^2} - \alpha_2\sqrt{1-(\alpha_1^2+\beta_1^2)\tilde{v_0}^2}}{\alpha_1-\alpha_2}, \\ \tilde{k_\parallel} &= \frac{\sqrt{1-\alpha_2^2\tilde{v_0}^2} - \sqrt{1-(\alpha_1^2+\beta_1^2)\tilde{v_0}^2}}{(\alpha_1-\alpha_2)\tilde{v_0}}. \end{aligned}\right\} \tag{3.5}$$

The indices 1 and 2 in (3.5) correspond to the coordinates of two points on the maximum gradient curve of the velocity distribution function, choosing the second point at its intersection with the $v_\parallel$-axis ($\beta_2 = 0$). They can be found by equating the second derivative of the distribution function to zero, $\partial^2 f/\partial v_\perp^2 = 0$, which we solve using Mathematica.

4. Analysis of the emission features

Electron cyclotron instabilities result in radiation at a frequency close to a local electron cyclotron one, with the horseshoe-type instability known to be emitted at a frequency slightly below the cyclotron one. Equation (3.5) suggests that the frequency of radiation is below the cyclotron frequency (i.e. $\tilde{\omega} < 1$) when $\alpha_1^2 + \beta_1^2 > \alpha_2^2$, i.e. when the transformed distribution curls along an ellipse with a vertical axis being larger than a horizontal one, but it is above the cyclotron frequency otherwise. The former case also yields a negative propagation constant $v_\parallel$ (meaning the radiation being emitted at some angle backward with respect to the beam's direction), while the latter corresponds to a positive parallel propagation constant, with the wave emitted rather forward.

4.1. *Emission in a spatially uniform field changing with time*

If a beam was passing through a region without a spatial gradient of the field, and its unstable distribution was formed due to the effect of a change of the local magnetic field with time, the distributions curls towards lower velocities. A typical resulting Luvdisk distribution is shown in figure 2(*a*). For comparison, figure 2(*b*) shows the initial Maxwellian distribution, and figure 2(*c*) shows a horseshoe distribution formed by a gradient of the magnetic field in space (with the same initial and final values as for the time-changing field correspondent to figure 2*a*).

The horseshoe curls around an ellipse with the vertical axis being longer than the horizontal one, resulting in the radiation being emitted at a frequency just below the electron cyclotron frequency at the direction close to perpendicular to the beam one but slightly backward (at approximately 4°). The Luvdisk distribution curls inside the horseshoe ellipse, and its frequency of radiation, suggested by the fitted ellipse, ranges approximately from $0.9 \cdot \omega_c$ to ω_c. The corresponding wavenumber, according to (3.5), suggests the direction of radiation can be found by the tangent of the propagation

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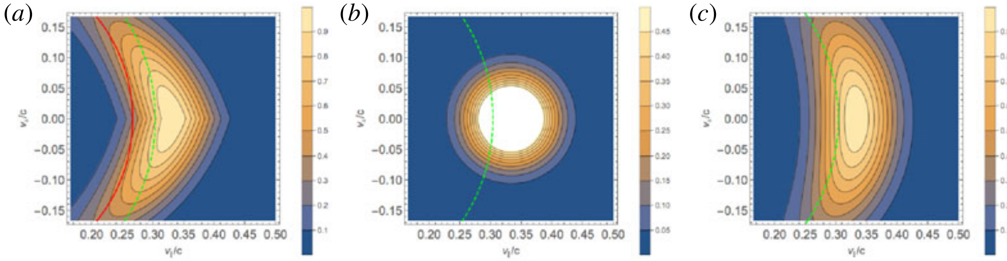

FIGURE 2. (*a*) Typical plots of the Luvdisk distribution velocity distribution function resulting from the magnetic field changing in time (magnetic field change with time $B/B_0 = 7.5$, $v_0 = c/3$), with the cyclotron resonance ellipse fitted through its highest gradient in the transverse direction (solid red line) and a resonance ellipse corresponding to a horseshoe distribution (dashed green line, as in *c*); (*b*) initial beam's Maxwellian distribution; (*c*) horseshoe distribution as a result of a spatial gradient of a stationary magnetic field (with $B/B_0 = 7.5$, $v_0 = c/3$) with a cyclotron resonant condition ellipse fit (dashed green line).

vector's angle with respect to the perpendicular direction, $\tan\phi = k_{\parallel}/\omega/c$ (where ω/c is a magnitude of the propagation vector) as

$$\tan\phi = \frac{1}{\tilde{v_0}\alpha_2} - \frac{\sqrt{1-\tilde{v_0}^2\alpha_2^2}}{\tilde{v_0}^2\sqrt{1-\tilde{v_0}^2\beta_1^2}}. \tag{4.1}$$

The direction ranges from nearly perpendicular backward propagation to approximately 30° forward propagation for a beam with bulk velocity equal to $v_0 = c/3$. The backward-propagation angle is limited by the second term in (4.1), and with $\tilde{v_0} < 1$ and the maximum-gradient-fit cyclotron resonance ellipse semi-axis constants α_2 and β_1 being less than 1 but close to it, this angle is small. When the first term in (4.1) dominates, meaning a forward propagation, the angle is limited by the causality light cone corresponding to the position of the maximum traverse gradient of the distribution function at the $v_{\parallel}$ axis, which is determined by the beam's velocity and the distribution spread after the field's change. For a non-relativistic beam it does not exceed 30° forward from the perpendicular to the beam's direction.

4.2. *Effects of a space gradient of magnetic field combined with its change in time*

When the electron beam is moving in a non-uniform magnetic field (e.g. towards a magnetic pole of an astrophysical body) and the field's change in time occurs, different scenarios are possible. When the spatial gradient dominates, the transformed distribution in velocity space is still a horseshoe distribution, with the corresponding properties of the cyclotron instability. If the change in time dominates over the spatial gradient, the Luvdisk distribution is formed with the emission properties as described in the above subsection. If, however, the field is rising both in space and in time by a comparable quantity, the perpendicular spread of the distribution function happens without it curling towards lower or higher parallel velocities. Then, no positive gradient of the distribution in the transverse direction is produced, and there is no cyclotron maser instability with perpendicular drive.

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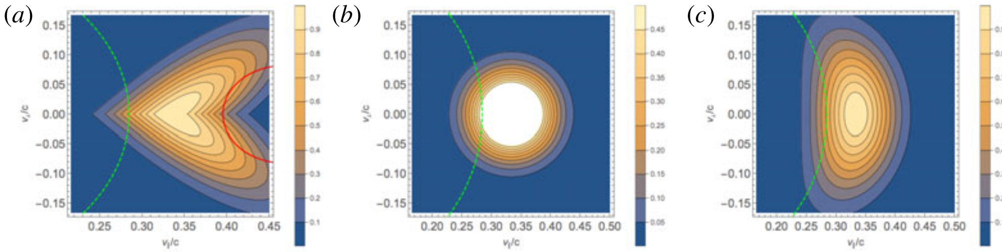

FIGURE 3. (*a*) Typical plots of the velocity Luvdisk distribution function resulting from the magnetic field space gradient $B_{final}/B_{initial} = 10$ combined with the field changing downwards with time ($B_{initial}/B_{final} = 2.5$), with the total change $B/B_0 = 7.5$, $v_0 = c/3$. The solid red line shows the cyclotron resonance ellipse fitted through the highest gradient in the transverse direction, and the dashed green line shows the resonance ellipse corresponding to a horseshoe distribution (as in *c*); (*b*) initial beam's Maxwellian distribution; (*c*) horseshoe distribution as a result of a spatial gradient of a stationary magnetic field (with $B/B_0 = 7.5$, $v_0 = c/3$) with a cyclotron resonant condition ellipse fit (dashed green line).

The case when the space gradient of the magnetic field is positive along the beam's path while the field's change in time is negative, or the other way round, is significantly different. If the resultant change of magnetic field is still positive (final value at the end point is larger than the initial value at the starting point of the beam's motion), the resultant velocity distribution is a mirrored Luvdisk distribution, curling towards higher parallel velocities, as shown in figure 3(*a*). To fit a cyclotron resonance ellipse through maximum positive gradient in transverse velocity direction, the ellipse should be to the right of the beam, as shown by the solid red curve. The fit ellipse parameters suggest the possible emission due to the cyclotron instability happening at a frequency up to 25 % larger than the local electron cyclotron frequency ($\omega/\omega_c = 1.128$ for the parameters shown in figure 3*a*). The radiation is expected to be emitted forward with respect to the beam's propagation, forming an angle with respect to the perpendicular to the beam direction which can reach up to approximately 45° (being equal to 30° for the parameters used in figure 3*a*). Figure 3(*b*,*c*) shows the initial Maxwellian distribution and the horseshoe distribution for the same initial/final values of magnetic field and other parameters as in figure 3(*a*) for comparison.

An anisotropic unstable distribution can be formed even when the field at the final point is smaller than at the point where the beam started travelling, if this final field is a result of a negative space gradient (e.g. when the beam travels outward from a magnetic pole) and a positive (upward) change of the field in time, even when the total field's change over that path is still downwards. Figure 4(*a*) shows a typical example of the resultant anisotropic distribution, with the fitted cyclotron resonance condition fitted through the maximum transverse gradient of the distribution shown by the solid red line. It supports an instability resulting in emitted radiation which is similar to that of the horseshoe-distribution instability. It is emitted nearly perpendicularly to the beam but slightly backward at a frequency just below the electron cyclotron frequency at the final location. The horseshoe, however, is not formed in the case when the same initial and final values of the field are a result of only a spatial gradient, as shown in figure 4(*c*) for the same initial Maxwellian beam, shown in figure 4(*b*).

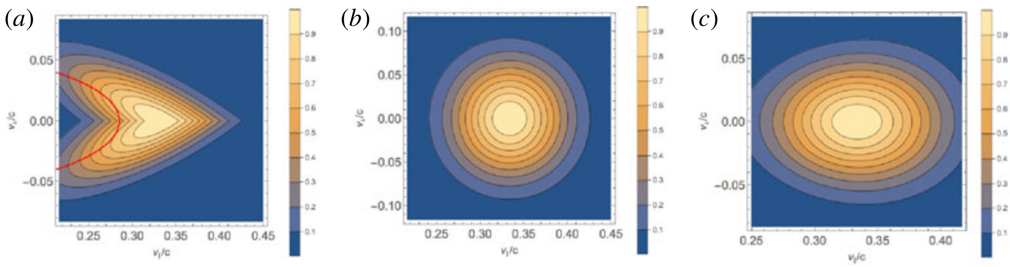

FIGURE 4. (*a*) Typical plots of the velocity Luvdisk distribution function resulting from the magnetic field space gradient $B_{final}/B_{initial} = -2$ combined with the field changing upwards with time ($B_{final}/B_{initial} = 2.5$), with the total change $B/B_0 = 0.5$, $v_0 = c/3$. The solid red line shows the cyclotron resonance ellipse fitted through the highest gradient in the transverse direction, and the dashed green line shows the resonance ellipse corresponding to a horseshoe distribution (as in *c*); (*b*) initial beam's Maxwellian distribution; (*c*) the distribution as a result of a spatial gradient of a stationary magnetic field (with $B/B_0 = 0.5$, $v_0 = c/3$) with a cyclotron resonant condition ellipse fit (dashed green line).

5. Conclusions

The change of a magnetic field with time can result in an anisotropic Luvdisk distribution of an electron beam in velocity space which can be unstable to a maser cyclotron instability. The properties of this instability are distinct from those associated with the horseshoe distribution, which is formed due to a gradient of magnetic field in space. When happening on its own, the time change of the field can result a cyclotron emission even if the field is homogeneous. The direction of the emission can vary from slightly backward to up to 30° forward with respect to the beam's direction. When a spatial gradient of the field is accompanied by its time change, the emission can happen even when the final value of the field is lower than the initial one, which would not be possible if only a spatial gradient was present. When the total change of the field from spatial and temporal gradients is upward, while the spatial and temporal gradients are in opposite directions, a frequency above the local cyclotron frequency can be expected, and the emission is radiated forward.

The analysis based on the distribution function does not allow us to explore the growth rate and nonlinear effects of the instability saturation, for which kinetic equations should be solved. The growth rate, however, could be expected to be higher when the spread of the velocity distribution function along the cyclotron resonance condition circle is larger, meaning higher electrons population in the positive perpendicular gradient ($\partial f/\partial v_\perp > 0$) areas (while purely perpendicular spread of the velocity distribution function produces only a negative gradient in perpendicular direction, which does not drive the instability). for larger perpendicular spread of the distribution function. The distinct features of the instability can be useful in analysing electron cyclotron emission from astrophysical objects or laboratory/tokamak plasmas, helping to retrieve the magnetic field evolution by the change in direction and frequency of the radiation.

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