CORRECTIONS FOR HYDROSTATIC ATMOSPHERIC MODELS : RADII AND EFFECTIVE TEMPERATURES OF WOLF RAYET STARS

C. De Loore[1], P. Hellings[1], H.J.G.L.M. Lamers[2]
1 Astrophysical Institute, Pleinlaan 2 B-1050 Brussels
2 Space Research Laboratory, Beneluxlaan 21 NL-3527 Utrecht

Abstract.

With the assumption of planparallel hydrostatic atmospheres, used generally for the computation of evolutionary models, the radii of WR stars are seriously underestimated. The true atmospheres may be very extended, due to the effect of the stellar wind. Instead of these hydrostatic atmospheres we consider dynamical atmospheres adopting a velocity law. The equation of the optical depth is integrated outwards using the equation of continuity.

The "hydrostatic" radii are to be multiplied with a factor 2 to 8, and the effective temperatures with a factor 0.8 to 0.35 when Wolf Rayet characteristics for the wind are considered, and WR mass loss rates are used. With these corrections the effective temperatures of the theoretical models, which are helium burning Roche lobe overflow remnants, range between 30 000 K and 50 000 K. Effective temperatures calculated in the hydrostatic hypothesis can be as high as 150 000 K for helium burning RLOF-remnants with WR mass loss rates.

Formulae.

To compute the optical depth of the wind we proceed in the following way: the equation of continuity and the adopted velocity law are inserted in the equation of optical depth.

Optical depth : $d\tau = -\kappa\rho dr$ (1)

Continuity : $\dot{M} = -4\pi\rho r^2 v(r)$ (2)

Velocity law : $v(r) = v_{inf} (1 - R_s/r)^{\beta}$ (3)

With v_{inf} the final wind velocity and R_s the hydrostatic effective radius of the star. These equations may be combined to obtain:

C. W. H. de Loore and A. J. Willis (eds.), Wolf-Rayet Stars: Observations, Physics, Evolution, 53–56.

$$d\tau = \frac{\kappa \dot{M}}{4\pi r^2 v(r)} = \frac{\kappa \dot{M}}{4\pi r^2 v_{inf} (1 - R_s/r)^{\beta}} \tag{4}$$

Because of the low densities in the outer parts of the wind we only consider electron scattering as an absorption source. Hence we use $\kappa = \sigma_e = 0.22\ (1 + X)$ where X is the hydrogen abundance by weight. Clearly we obtain $\sigma_e = 0.22$ for Wolf Rayet stars. We can integrate equation (4) from 0 to τ for the left hand side and from ∞ to r for the right hand side. The solutions are:

$$\tau(r) = 1.5942\ 10^8 \frac{\dot{M}}{R_s\ v_{inf}\ (1 - \beta)} \left\{ \frac{1}{(1 - R_s/r)^{\beta - 1}} - 1 \right\} \tag{5}$$

when $\beta \neq 1$, and

$$\tau(r) = 1.5942\ 10^8 \frac{\dot{M}}{R_s\ v_{inf}} \ln\ (1 - R_s/r) \tag{6}$$

when $\beta = 1$.

In these expressions $\dot{M}$ is expressed in solar masses per year, all distances in solar radii and velocities is km/sec. For a given τ the corresponding r can be found. We consider the radius for which $\tau = 2/3$ as the new radius of the star. Taking $\tau = 1$ gives results differing less than 10% from the $\tau = 2/3$ case.

Information on the velocity structure in WR-winds can be found in Hartmann et al.(1977), Hartmann (1978) and Rumpl (1980). The change in the optical depth due to the second acceleration at about ten stellar radii is less than 0.01 so that we neglected this peculiarity. We used rather slow velocity laws (β = 2, 3 and 4) and wind velocities ranging from 500 to 2000 km/sec for v_{inf}. In our evolutionary code the radius of the star is output and the mass loss rate is calculated with

$$\dot{M} = -\ N\ L/c^2 \tag{7}$$

Values for N between 500 and 1000 give mass loss rates between $1\ 10^{-5}$ and $5\ 10^{-5}\ M_o$/yr for our helium burning Roche lobe remnants. These rates are comparable to those of WR stars. Once the new radius is found we compute the new effective temperature using rT^2 is constant. This assumes a constant luminosity throughout the wind. If the luminosity decreases outwards the corrections for the effective temperatures become more important.

Applications

Table 1 gives the factors T_{eff} has to be multiplied with for $\dot{M} = 3\ 10^{-5}\ M_o$/yr and $R_s = 10\ R_o$. Table 2 gives the same for R_s=1 R_o

https://doi.org/10.1017/S0074180900028679 Published online by Cambridge University Press

v_{inf} =	500	1000	2000
β = 2	0.64	0.76	0.86
3	0.59	0.70	0.78
4	0.56	0.65	0.73

Table 1 : multiplication factors for $\dot{M}$=3 10^{-5} and 10 R_o

v_{inf} =	500	1000	2000
β = 2	0.26	0.35	0.47
3	0.25	0.34	0.45
4	0.25	0.33	0.43

Table 2 : multiplication factors for $\dot{M}$=3 10^{-5} and 1 R_o

For a constant mass loss rate the corrections become more important when the velocity law is slower, the final velocity is smaller and the hydrostatic radius of the star is smaller. This can be understood since in these cases the density must be higher to explain the same mass loss rate.

In the application to the evolution code we have taken helium burning Roche lobe overflow remnants. The stars considered had initial masses between 60 and 40 solar masses. During their core hydrogen burning phase mass loss by stellar wind was taken into account (form.(7)) but no overshoot. All systems were computed through their Roche lobe overflow phase according early case B of mass transfer, i.e. mass transfer during shell hydrogen burning. The results are displayed in figure 1 : crosses give the positions in the HR-diagram of some non-corrected WR-stars and of one non corrected evolutionary track. At right the corrected models and track.

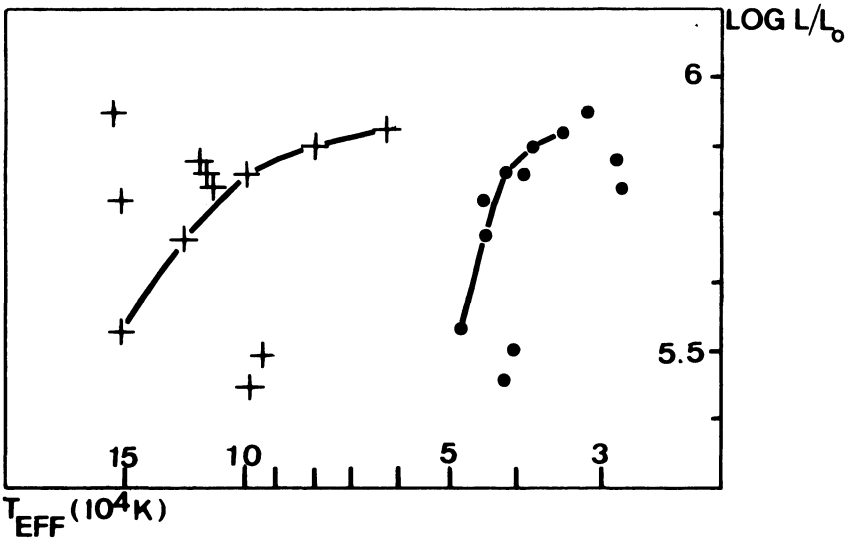

fig 1 : Hydrostatically computed models and evolutionary track of mass loosing helium burning RLOF-remnants (crosses) and their corrections (dots) in the HR-diagram.

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As can be seen, the non-corrected models have effective temperatures between 50 000 and 150 000 K, whereas their corrected models are situated between 30 000 and 50 000 K, which agrees better with observations. All the corrections in Fig 1 were calculated with = 3 and v_{inf} = 1000 km/s.

REFERENCES

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DISCUSSION

Underhill: How can you justify basing your calculations on arbitrary velocity laws of the type you have chosen, when it is known that stars are natural systems that behave in a manner consistent with the conservation laws of mass, momentum and energy, and that an acceptable solution to the wind ewuation derived from conservation laws does not have the form you have adopted ?

de Loore: Since our purpose was not to determine the exact velocity profile, we have used velocity laws generally used in the calculations of line profiles in extended atmospheres (see. Castor, et al., 1979, Astrophys.J.Suppl., 39, 481; Hamman, 1981, Astron.Astrophys. 93, 353). For technical reasons we are not able to include a detailed computation of extended atmospheres in our binary evolution program.

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