Robotica (2005) volume 23, pp. 785–788. © 2005 Cambridge University Press
doi:10.1017/S0263574705001761 Printed in the United Kingdom

Multi-Input Fuzzy control of an inverted pendulum using an armature controlled DC motor

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(Received in Final Form: February 14, 2005)

SUMMARY

This paper presents a design methodology to stabilize a class of multi-variant nonlinear system after a high disturbance occurs. It investigates application of Takagi-Sugeno type fuzzy controller (T-S-FC) to an inverted pendulum mechanism, actuated by an armature-controlled DC electrical motor.

Fuzzy controllers use heuristic information in developing design methodologies for control of non-linear dynamic systems. This approach eliminates the need for comprehensive knowledge and mathematical modeling of the system, and in cases of more complex systems, approximation and simplifications in order to achieve feasible mathematical model is not required.

The paper presents the stages of development of the Fuzzy Controller for an inverted pendulum by developing a two-input, Mamdani type system. It evaluates the performance of the system. Then a four-input T-S-FC type is developed. The research compares performances of each controller and presents the result of tests. A model for a DC motor is developed in this study, in order to measure the effect of time delays and response time caused by inherent properties of the physical system. The final part will demonstrate the complete operational system with the DC electrical motor included in the test system.

KEYWORDS: Fuzzy control; Inverted pendulum; Multi-variant system; Design methodology.

I. INTRODUCTION

The conventional approach in controlling the inverted pendulum system is to use a PID (Proportional, Integral, and Derivative) controller. In order to model the system the developer would have to know every technical detail about the system and be able to model it mathematically. Fuzzy Logic control (FLC) challenges this traditional approach by using educated guesses about the system to control it.[1] Passino states that differential equations are the language of conventional control (PID), while "rules" about how the system works is the language of fuzzy control.[2]

Fuzzy logic has found its way into the everyday life of people, since Lotfi Zedah first introduced fuzzy logic in 1962. In Japan, the use of fuzzy logic in household appliances is common. Fuzzy logic can be found in such common household products as video cameras, rice cookers and washing machines.[3] From the weight of the clothes, fuzzy logic would be able to determine how much water as well as the time needed to effectively wash the clothes. Japan developed one of the largest fuzzy logic projects, when they opened the Sendai Subway in 1987.[4] In this subway, trains are controlled by fuzzy logic.

Fuzzy Logic is a subset of traditional Boolean logic. Boolean logic states that something is either true or false, on or off, 0 or 1. Fuzzy logic extends this into saying that something is somewhat true, or not completely false. In fuzzy logic there is no clear definition as to what is exactly true or false. Fuzzy logic uses a degree of membership (DOM) to generalize the inputs and outputs of the system.[5] The DOM ranges from [0 1], where the degree of membership can lie anywhere in between.

The majority of Inverted pendulum systems developed using fuzzy logic, are developed using a two dimensional approach, where only the angle and angular velocity of the pendulum's arm are measured. The following research will show why this method is insufficient for the development of an inverted pendulum on a limited size track. To have an efficient fuzzy controller for an inverted pendulum, the system must also include inputs for the position of the cart that the pendulum is balanced upon and the velocity of the cart. Two-dimensional fuzzy controllers are very simple examples of fuzzy control research. Many of them will balance the inverted pendulum, but are not in control of the cart's position on the track. Adeel Nafis[6] proposed a two-dimensional fuzzy controller to balance the Inverted pendulum on a track. Tests showed that the controller would balance the pendulum but neglected to control the position of the cart and eventually the cart's position would exceed the length of the track. Another FLC was proposed by Passino;[2] again this cart had the same result as the previous FLC.

Control of the system requires that the cart holding the pendulum be moved by some mechanism. For simulation purposes an armature controlled, DC motor[7] was used.

II. MULTI-INPUT FUZZY LOGIC CONTROLLERS

Fuzzy Logic Controllers can have more than one input. Two-input FLC's are easy to implement and receive great performance responses from simulations. Layne [1] modeled a fuzzy controller that had great performance balancing the pendulum but the cart's positioning was unstable, making it

Table I. Rule-base for the Inverted Pendulum.

X/X-dot	− 2	− 1	0	1	2
− 2	2	2	2	1	0
− 1	2	2	1	0	− 1
0	2	1	0	− 1	− 2
1	1	0	− 1	− 2	− 2
2	0	− 1	− 2	− 2	− 2

an impractical rule set for real life implementation. Two-input FLC's are the most commonly researched inverted pendulum systems.

The 2-input system received theta (angle θ) and theta-dot (angular velocity $\dot{\theta} = \omega$) as its inputs. The system uses 5 membership functions for each input, and another 5 for the outputs (Force). The system consists of 25 (that is 5 to power 2; 5^2) rules. Table I shows the rule base for the inverted pendulum system.

According to Table I a value of − 2 represents a negative large value for angle or angular velocity, and 2 represents a positive large angle/angular velocity. If there is a situation where the angle is 0 and the angular velocity is 1 then the rule − 1 will be fired.

Figure 1 shows a simulation that is run over a time period of 1 second. The pendulum has an initial angle of 0.2 radians (dashed line). When the simulation is run, the angle of pendulum balances quickly, in about 1 second, but the position of the cart is not controlled (continuous line) so the cart's position will eventually drift off into the end of the track, even though the pendulum's arm is balanced.

The benefit of adding two more inputs to the system to control the X-position of the cart and the velocity of the cart will greatly benefit the stability of the system. There is a cost for better stability; this is a greater computation time, and greater complexity in the model. The cost of adding more inputs increases exponentially with the number of inputs added. The above two-input system used five membership function for each input used; this resulted in a 25 (i.e. 5^2) rule base. By adding two more inputs to the system, the systems rule base would grow to 625 (i.e. 5^4) rules. Development time for a rule base this size can be very time consuming, both in development and in computational time. Bush proposed using an equation to calculate the rules, rather than taking the time to develop the rules individually.[8] The system was a 5^4 system with 17 output membership functions (OMF). The equation used was:

$$I + (J - 1) + (-K + 5) + (L + 5) \quad (1)$$

This equation results in values ranging between 1 and 17. This corresponds to the OMF that is to be used in the calculation of the output. The performance of the system using this approach is not consistent with that of the original simulation, given by the author of the above Equation 1.[8] The force given to the cart holding the pendulum was found not to be enough to balance the pendulum and the system failed within a small amount of time. It can be concluded that this system would be a good starting point for one to base a large rule set on, but the system would need some tweaking of the rules and membership functions to get to balance the system effectively.

The final FLC controller that was modeled for simulation was a Takagi-Sugeno type fuzzy controller. All the previous FLC's modeled were of Mamdani type. A Takagi-Sugeno type fuzzy controller[9−12] varies from the traditional Mamdani type controller by using linear or constant OMF's instead of triangular, trapezoidal, Gaussian or any other method the developer decided to use. The system uses 4-inputs with only 2 input membership functions for each. This resulted in a 2^4, 16 rule system. The linear output membership functions are calculate using the equation

$$y = c_0 + (c_1 * x_1) + (c_2 * x_2) + (c_3 * x_3) + (c_4 * x_4) \quad (2)$$

Where c_n is the parameters of the OMF, and x_n is the values of θ, θ-dot, X and X-dot respectively. The system modeled here uses fuzzy logic toolbox of Matlab.[9]

The control of all 4 parameters with only 2 membership functions causes the system to run very quickly. The down side to this quick response is that it takes more time for the system to stabilize when there are so few membership functions. The system will overshoot the targeted position and eventually come to rest. The settling time of this system takes more time than any other system.

Figure 2 is the result of the simulation. The pendulum is started with an initial disturbance of 0.2 radians. As shown, the fuzzy controller overcompensates for this initial

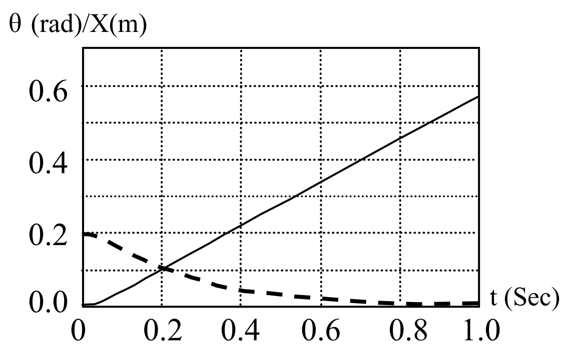

Fig. 1. Angle (θ/rad – dashed) and Position (X/m – continuous) of Pendulum V. S. time t (Sec).

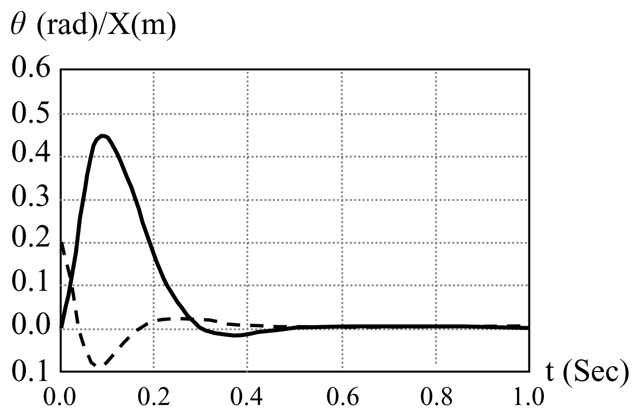

Fig. 2. Angle (θ/rad – dashed) and Position (X/m – continuous) of Pendulum V. S. time t (Sec).

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disturbance and sends the pendulum's angle (dashed line) in an opposite direction in an attempt to balance it, this is the overshoot. It takes approximately 5 seconds for the pendulums arm to balance.

III. ARMATURE-CONTROLLED DC MOTOR

The DC motor model chosen for the simulation is an armature-controlled DC motor. The motor is modeled using the motors' transfer function [7].

$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{(R_a + L_a s)(Js + b) + (K_b K_m)} \tag{3}$$

The parameters of the motor are as follows:

K_m	Motor constant	[(N*m)/A]
K_b	Motor constant	[(N*m)/A]
J	Rotor Inertia	[(N*m*s^2)/rad]
R	Electric Resistance	[Ω]
L_a	Electric Inductance	[H]
b	damping ratio	[N*m*s]

In this simulation it was assumed that $K_m = K_b$.

The transfer function of this DC motor yields angular velocity (ω) as the motor shaft output. In the simulation, ω was easily converted into the force on the cart. The motor responded well, reaching its maximum force exerted on the cart in less than 0.5 seconds.

IV. SIMULATION RESULTS

The simulation consists of four main components, the Fuzzy controller, DC Motor, the cart and the inverted pendulum, Figure 3. The cart passes the fuzzy controller 4 parameters $\theta, \dot{\theta}, X, and\ \dot{X}$. Based on these 4 parameters the fuzzy controller outputs a voltage to the motor. The motor in turn calculates the force that will be exerted on the cart. The system then calculates the new values for parameters $\theta, \dot{\theta}, X, and\ \dot{X}$ and the cycle will be repeated.

The fuzzy controller used in the simulation, with the DC motor included, is a 2^4 FLC as described above. The system runs identical to the 2^4 system only the settling time for the simulation, with the motor included, is larger. Figure 4 shows the results of the simulation ran using the same fuzzy controller as of [9] with the DC motor included in the simulation.

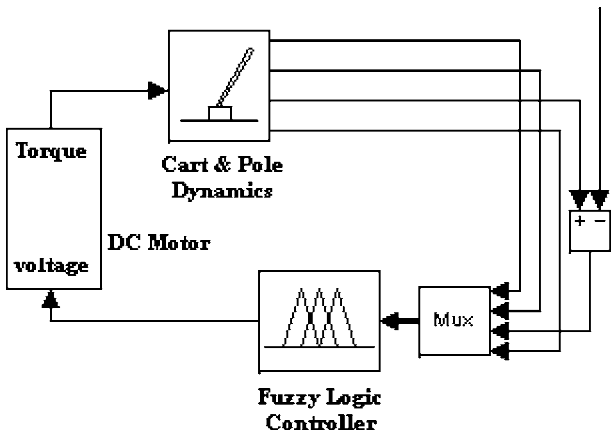

Fig. 3. Block diagram of Fuzzy Control Inverted Pendulum.

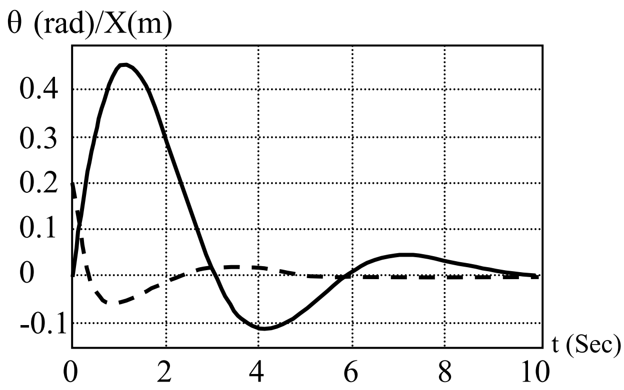

Fig. 4. Angle (θ/rad – dashed) and Position (X/m – continuous) of Pendulum V. S. time t (Sec).

The DC motor has a delay, where it takes the motor a given time to reach a maximum force. This in turn causes the simulation to take longer to reach a steady state. The following parameters are used in the simulation of the motor:

K_m/ K_b	$= 50 \times 10^{-3}$ Nm/A	L	= 0.5 H
J	$= 1 \times 10^{-3}$ Nms^2/rad	b	= 0.1 Nms
R	= 1.00 Ω	r (radius)	= 0.03 m

Figure 4 shows that it takes approximately 8 seconds for the pendulum's angle to become steady, and even longer for the cart's position to stabilize. The difference in the response time of this system can be found in the motor. The motor has a time constant which delays the motor's response time to an inputted voltage. A typical armature controlled DC motor has a time constant around 100 ms. The shorter the time constant of the motor, the quicker the system will respond.

The simulation shows that the system responds well even with a motor attached to the system. The cost of implementing a motor into the simulation is response time for the pendulum to stabilize. Simulations done without the addition of the DC motor can not be considered for real life implementation because the motor is needed to investigate the response time that the system will observe in real life.

V. CONCLUSION

Developing Fuzzy controllers can be a time consuming procedure. There is no absolute answer when it comes to developing fuzzy logic controllers. When developing a FLC, the developer must consider whether precision will be sacrificed for performance and simplicity. The 5^2 system developed was very simple and computed quickly. The drawback of this system was that precision was compromised. The 2^4 system was also very simple and ran quickly but the performance of the system was not good. The settling time for the pendulum and cart could be quicker. The 5^4 system was very complex and performance was slow, but if tuned correctly, a system of this size would be very precise. Unfortunately time did not allow for this system to be tuned correctly during the course of this research.

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A real life implementation of the system would require a high performance DC motor. Simulation results showed that the system would work for this type of motor. Having a smaller time constant in the DC motor would result in a shorter response time of the system. The FLC would need to be fine tuned for other types of motors.

With the DC motor implemented in the simulation, the system did not react as well to high disturbances as it did when the motor was neglected in the simulation. This means that the system will react well to small disturbances and be able to recover from them quickly. In order for this system to handle large disturbances a motor with high performance dynamics need to be used that has a very small time constant.

Acknowledgment

This research was supported by St. Francis Xavier University Council Research grants, 91574-UCR-1337 and 91102-UCR-1275.

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