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A new method for isotropic analysis of limited DOF parallel manipulators with terminal constraints

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SUMMARY

Based on the terminal constraints system (TCS) and reciprocal screw theory, a novel method is presented to determine the isotropic configurations of limited degree-of-freedom (DOF) parallel manipulators. From the available physical meaning of isotropy, the criteria to determine the isotropic configurations can be transformed to investigate whether the TCS acting on the moving platform works equally well in all directions. From the TCS study, the simplest form of constraints system matrix can be obtained. Then the constraint condition number is defined to measure the isotropy of spatial parallel manipulator based on the TCS. This method not only avoids solving the Jacobian matrix for some complex structural parallel manipulators but also points out the physical meaning of isotropy, which indicates that the TCS acting on the moving platform works equally well in all directions. Three examples are employed to illustrate this method.

KEYWORDS: Isotropic configurations; Limited DOF parallel manipulator; Terminal constraints system; Constraint condition number; Reciprocal screw theory.

Nomenclature

- $\hat{\$}$: The unit screw, which can be used to express the position and orientation of any vector.
- $\$^r$: The reciprocal screw, which is reciprocal to the screw system. The screws and reciprocal screws form the six bases of general spatial space.
- $\text{equiv.}(\$^r_A)_B$: The equivalent constraint screw transferred from reference point A to reference point B.
- $SF[\$^r]$: The simplest form of constraints system matrix.
- $\kappa_c(A)$: The constraint condition number, which is used to measure the isotropy of spatial parallel manipulator based on the TCS.

1. Introduction

With parallel manipulator near or at the isotropic configurations, the sensitivity of a manipulator in both velocity and torque errors is at a minimum, and the manipulator can be controlled equally well in all directions.[1–3] Many researchers have studied the creative mechanical design using the isotropic condition of the Jacobian matrix. However, the determination of all isotropic configurations is still a rather complex problem,[1] especially for some complex structural parallel manipulators.

Up to now, most researchers[2–9] have studied the isotropic configurations using the condition number of the Jacobian matrix, which is first used by Salisbury and Craig,[10] namely, the condition number reaches the minimum value of unity, and can also be expressed as the rows of the Jacobian matrix that must be mutually orthogonal and of equal Euclidean norms. This condition number was developed by Angeles[11] as a kinematic performance index of robotic mechanical systems.

Klein and Miklos[2] defined the positional isotropy, orientational isotropy, and spatial isotropy by the full row rank Jacobian matrix and its condition number, either kinematically nonredundant or redundant. Zanganeh and Angeles[4] used special structures of the forward and inverse Jacobian matrix to define a set of conditions under which a parallel manipulator can be rendered isotropic. Gogu[6–8,12] synthesized a serial of isotropic parallel manipulators via theory of linear transformations and evolutionary morphology based on distinguishing five types of Jacobian matrix. Baron and Bernier[13] defined the constraint manifold of isotropic designs, i.e., those having isotropic Jacobian matrices at their home position, through analyzing the applied constraints. Tsai and Huang[3] used a device called isotropic generator to avoid solving the Jacobian matrix and design 6-DOF isotropic parallel manipulator.

The aims of all these researchers were to design some parallel manipulators with ideal kinematic and dynamic performance. However, for some complex structural parallel manipulator, the general Jacobian matrix is difficult to solve. Therefore it is difficult to determine the isotropic configurations by using the condition number of Jacobian matrix.

In this paper, a novel method is presented to determine the isotropic configurations based on the theory of reciprocal screws by investigating the TCS of the whole mechanism. First, the reciprocal screw theory is used to obtain the reciprocal screw system of the parallel manipulator. On this basis, we can define and obtain the constraint condition number. Then the isotropic configurations can be determined

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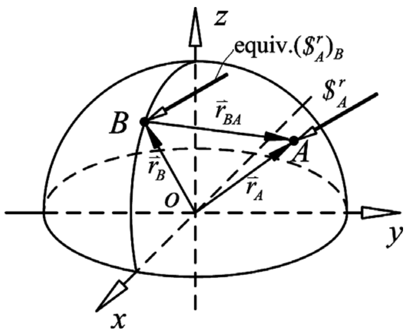

Fig. 1. Transformation of reciprocal screws.

through the concrete types of the constraint condition number. This method not only avoids solving the Jacobian matrix for some complex structural parallel manipulator but also points out the physical meaning of isotropy, which indicates that the TCS acting on the moving platform works equally well in all directions.

2. New Method for Isotropic Design

2.1. Screws transformation

The position and orientation of any vector can be expressed by a screw, and a unit screw, $\hat{\$}$, is defined by a straight line with an associated pitch,[14]

$$\hat{\$} = [s\ ;\ s_o + hs]^T, \tag{1}$$

where s is a unit vector pointing in the direction of the screw axis, $s_o = r \times s$ defines the moment of the screw axis about the origin of a reference frame, r is the position vector of any point on the screw axis with respect to the reference frame, and h is the pitch of the screw.

If the pitch of a screw is equal to zero, the unit screw is reduced to

$$\hat{\$}_o = [s\ ;\ s_o]^T. \tag{2}$$

Here the screw (2) denotes the motion of revolute joint or pure force, whereas s is the axis of revolute joint or the action line of the denoted force.

If the pitch of a screw is infinite, the unit screw is reduced to

$$\hat{\$}_\infty = [0\ ;\ s]^T. \tag{3}$$

Here the screw (3) denotes the motion of prismatic joint or pure couple, where s is the axis of prismatic joint or the action line of the denoted couple.

In order to analyze the motion constrained by the constraint screws system, we should transform all the constraint screws to the target coordinate system with respect to the same reference point,[15,16] namely, we need do some equivalent transformations on the constraint screws system. Assume, a reciprocal screw acted on point A at a rigid body, which is show in Fig. 1, and has the form as given below:

$$\$^r_A = [s\ ;\ s_{Ao} + hs]^T, \tag{4}$$

where $s_{Ao} = r_A \times s$, denotes the moment of vector s with respect to point A.

When the constraint screw $\$^r_A$ is transferred from reference point A to reference point B, the equivalent screw, equiv. $(\$^r_A)_B$, of this constraint screw $\$^r_A$ can be obtained as follows:

$$\text{equiv.}(\$^r_A)_B = [s\ ;\ s_{Ao} - r_{BA} \times s + hs]^T, \tag{5}$$

where $r_{BA} = r_A - r_B$.

Two screws, $\$$ and $\$^r$, are said to be reciprocal if they satisfy the condition[17]

$$\$^T \$^r = 0, \tag{6}$$

where the transpose of a screw is defined as $\$^T = [S_4\ S_5\ S_6;\ S_1\ S_2\ S_3]^T$, such that

$$\$^T \$^r = S_4 S_{r1} + S_5 S_{r2} + S_6 S_{r3} + S_1 S_{r4} + S_2 S_{r5} + S_3 S_{r6}, \tag{7}$$

and where S_i denotes the ith coordinate of the screw $\$$, and S_{ri} denotes the ith coordinate of the reciprocal screw $\$^r$. The screws and reciprocal screws form the six bases of general spatial space. When the n linear independent screws are obtained, we could gain the other 6-n reciprocal screws by Eq. (6), and *vice-versa*.

2.2. New method

The concept of isotropy is often defined through the condition number of Jacobian matrix. And the parallel manipulator is referred to as isotropic configuration when the condition number reaches the minimum value of unity.[10]

The physical meaning of the above isotropy definition is that the TCS acted on the moving platform equally well in all directions. Therefore the criteria to determine the isotropic configurations can be transformed to investigate whether the TCS acting on the moving platform works equally well in all directions.

If the reciprocal screws applied on the moving platform do not meet at one point, we should transform them to one reference point, namely, the equivalent constraint screw system, which can be obtained by Eq. (5) of equivalent transformation, and then we could examine whether the equivalent constraint screw system is isotropic. If the equivalent constraint screw system is isotropic at one moment, then we call this the parallel manipulator isotropy.

Here we make a clear redefinition of isotropy based on the following discussed physical meaning:

1. If the moving platform can well proportionately translate in all directions under the equivalent TCS, then this configuration is called the positional isotropy (*PI*), and the relationship between the equivalent TCS and the isotropic translations can be expressed as shown in Fig. 2.
2. If the moving platform can well proportionately revolve about all directional axes under the equivalent TCS, then this configuration is called the oriented isotropy (*OI*), and the relationship between the equivalent TCS and the isotropic orientations can be expressed as shown in Fig. 3.
3. If the moving platform possesses a combined well-proportioned position and orientation, which can be

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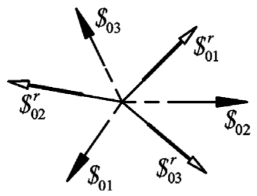

Fig. 2. Relationship between the equivalent TCS and isotropic translations.

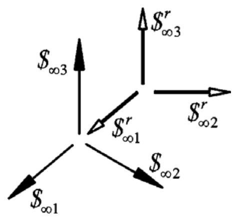

Fig. 3. Relationship between the equivalent TCS and isotropic orientations.

expressed as ***r**-PI* & ***k**-OI* isotropic system, where ***r*** denotes the number of translational DOFs, and ***k*** denotes the number of rotational DOFs, then this configuration is called the combined isotropy (*CI*).

In this section, a novel method is presented to determine the isotropic configurations based on the physical meaning of isotropy and the theory of reciprocal screws by investigating the TCS of the whole mechanism. Comparing with the works[6,18] of the former scholars, the distinct merit of the method addressed here is that there is no need to construct the general Jacobian matrix, which is a complex process for some complex structural parallel manipulator. Therefore, it is a direct method to determine the isotropic configurations by investigating the TCS of the whole mechanism.

This new method can be presented in the following two steps:

Step 1: Investigate the TCS.

Each limb provides certain reciprocal screws on the moving platform, and these reciprocal screws form the TCS of the manipulator. These reciprocal screws can be easily gained through studying the types of kinematic screws and the algebra operation of the reciprocal product.[17]

As the elementary matrix transformation does not change the rank of matrix, we can obtain the simplest form of constraints system matrix ($\$^r$) after the TCS gain. And then the analysis of the TCS can be transformed to investigate the simplest form (*SF*) of constraints system matrix, *A*, which can be expressed as given in Eq. (8),

$$A_{m\times n} = SF[\$^r]. \tag{8}$$

Step 2: Define and check the constraint condition number.

With the gained simplest form of constraints system matrix, we can define the constraint condition number, $\kappa_c(A)$, to measure the isotropy of spatial parallel manipulator based

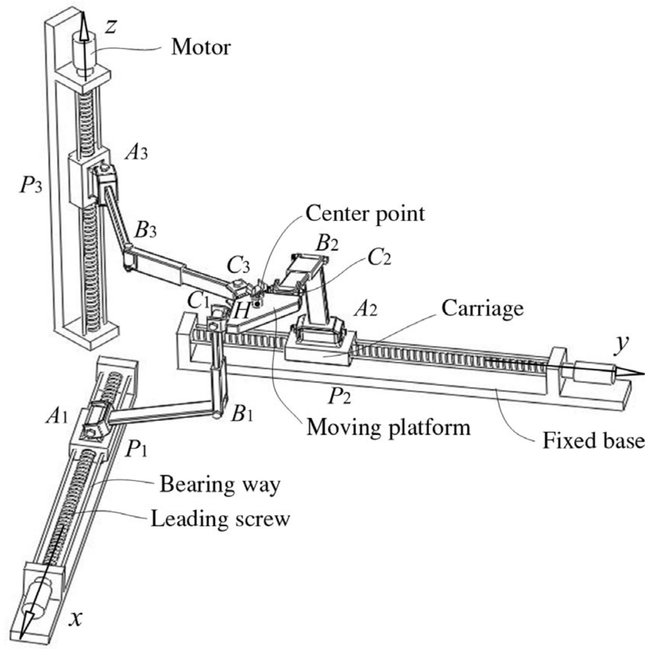

Fig. 4. The architecture of a spatial parallel manipulator.

on the TCS, similarly,

$$\kappa_c(A) = \begin{cases} norm(A).norm(A^{-1}) & m = n, unsingular \\ norm(A).norm(A^{+}) & m \neq n \end{cases} \tag{9}$$

where A^{-1} is the inverse matrix and $A^{+} = (A^T A)^{-1} A^T$ is the pseudo-inverse of *A*, and when the constraint condition number is equal to one, this configuration is called isotropy.

However, some researchers[19] pointed that the matrix involved in the condition number calculation is not homogeneous in terms of units for a robot having both translation and orientation DOFs. Hence, in this research the problem of constraint condition number needs further research.

3. Application and Discussion

3.1. A fully isotropic parallel manipulator

In this section, we take a fully isotropic parallel manipulator configuration, which is proposed by Gogu[7] based on the theory of linear transformations, as an example to illustrate this new method. Here we carry out modeling and analysis of this fully isotropic configuration, as shown in Fig. 4. This manipulator consists of three identical limbs, which are made up of one prismatic joint (P) and three revolute joints (R), and are denoted by PRRR kinematic chains; and this parallel manipulator has three translational DOFs at ordinary positions.

In order to obtain the TCS of this parallel manipulator, we need to analyze the reciprocal screws of each limb applied to the moving platform.[17] As each limb is identical, we can take one limb of all the limbs as an example to study and create an absolute coordinates system *o-xyz* as shown in Fig. 5. The center of the moving platform is *H* and we assume that the length of the leading screw is *L*.

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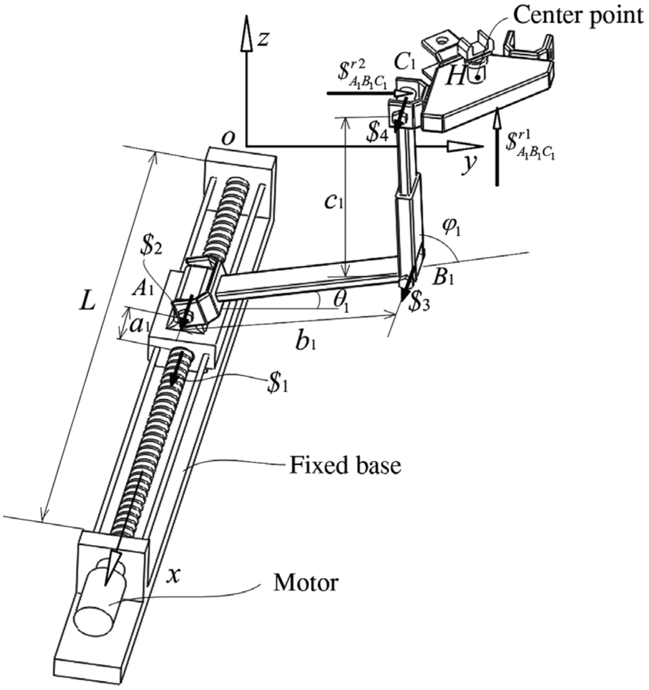

Fig. 5. Screw analysis of selected limb.

The screws of the selected limb can be obtained, and with the coordinate system, the Plücker coordinates of each joint are

$$\left.\begin{array}{l} \$_1 = [0 \quad 0 \quad 0\,;\,1 \quad 0 \quad 0]^T, \\ \$_2 = [S^2\,;\,S_0^2]^T, \\ \$_3 = [S^3\,;\,S_0^3]^T, \\ \$_4 = [S^4\,;\,S_0^4]^T, \end{array}\right\} \tag{10}$$

where $S^2 = S^3 = S^4 = (1 \;\; 0 \;\; 0)$, $S_0^2 = \mathrm{r}_{A_1} \times S_0^2 = (0 \; a_1 \; 0)$,

$$S_0^3 = \mathrm{r}_{B_1} \times S_0^3 = (0 \;\; a_1 + b_1 s\theta_1 \quad -b_1 c\theta), \text{ and}$$

$$S_0^4 = \mathrm{r}_{C_1} \times S_0^4 = (0 \;\; a_1 + b_1 s\theta_1 + c_1 s(\theta_1 + \varphi_1) \\ -b_1 c\theta_1 - c_1 c(\theta_1 + \varphi_1)).$$

Here a_1, b_1, and c_1 are the link lengths of the selected limb, θ_1 is the angle from y-axis positive direction to link vector A_1B_1, and φ_1 is the angle from link vector A_1B_1 to link vector B_1C_1.

Therefore, the kinematic screws of the selected limb can be expressed as

$$\$_{A_1B_1C_1} = [\$_1 \; \$_2 \; \$_3 \; \$_4]^T. \tag{11}$$

The reciprocal screws of the kinematic chain $A_1B_1C_1$ can be obtained using the algebra operation of reciprocal product.[14,17]

$$\left.\begin{array}{l} \$^{r1}_{A_1B_1C_1} = [0 \;\; 0 \;\; 0\,;\,0 \;\; 0 \;\; 1]^T, \\ \$^{r2}_{A_1B_1C_1} = [0 \;\; 0 \;\; 0\,;\,0 \;\; 1 \;\; 0]^T \,. \end{array}\right\} \tag{12}$$

According to the physical meaning of the reciprocal screws, the reciprocal screws $\$^{r1}_{A_1B_1C_1}$ and $\$^{r2}_{A_1B_1C_1}$ denote pure moment of couples, which are shown in Fig. 5. And the movement constrained by $\$^{r1}_{A_1B_1C_1}$ and $\$^{r2}_{A_1B_1C_1}$ are $[0 \;\; 0 \;\; 1\,;\,0 \;\; 0 \;\; 0]^T$ and $[0 \;\; 1 \;\; 0\,;\,0 \;\; 0 \;\; 0]^T$, respectively. Therefore, the manipulator shall not execute the rotation around the axis $(0 \;\; 0 \;\; 1\,)$ and $(0 \;\; 1 \;\; 0\,)$.

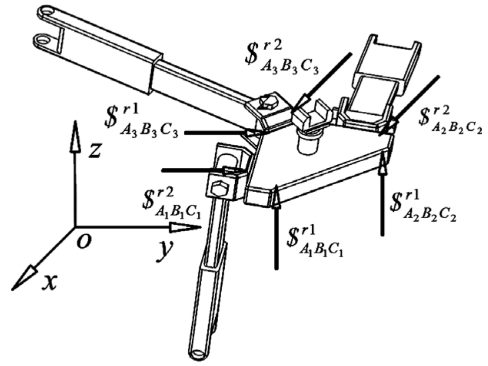

Fig. 6. The TCS.

Similarly, we can obtain the reciprocal screws applied to the moving platform of the other two limbs,

$$\left.\begin{array}{l} \$^{r1}_{A_2B_2C_2} = [0 \quad 0 \quad 0\,;\,0 \quad 0 \quad 1]^T, \\ \$^{r2}_{A_2B_2C_2} = [0 \quad 0 \quad 0\,;\,1 \quad 0 \quad 0]^T, \\ \$^{r1}_{A_3B_3C_3} = [0 \quad 0 \quad 0\,;\,0 \quad 1 \quad 0]^T, \\ \$^{r2}_{A_3B_3C_3} = [0 \quad 0 \quad 0\,;\,1 \quad 0 \quad 0]^T. \end{array}\right\} \tag{13}$$

The reciprocal screws applied to the moving platform are six pure moment of couples, which form the TCS. However, they are in linear correlation, as shown in Fig. 6.

Therefore, the terminal constraints screw system can be written as follows:

$$\$^r = \left[\$^{r1}_{A_1B_1C_1} \;\; \$^{r2}_{A_1B_1C_1} \;\; \$^{r1}_{A_2B_2C_2} \;\; \$^{r2}_{A_2B_2C_2} \;\; \$^{r1}_{A_3B_3C_3} \;\; \$^{r2}_{A_3B_3C_3}\right]^T. \tag{14}$$

At ordinary position, the rank of the terminal constraints screw system, $\$^r$, is

$$\mathrm{Rank}(\$^r) = 3 \text{ (constrain three rotational DOFs)}. \tag{15}$$

So according to refs. [16] and [20], the DOF of the manipulator at ordinary position is

$$F = 6 - \mathrm{Rank}(\$^r) = 6 - 3 = 3, \tag{16}$$

namely, this parallel manipulator possesses three translational DOFs.

From studying the constraints screw system in Eq. (14), we can easily obtain the simplest form of constraints system matrix, A, while the moving platform is at ordinary position,

$$A = SF(\$^r) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{17}$$

According to the redefinition of isotropy, we found that the constraint condition number, κ_c, is always equal to one,

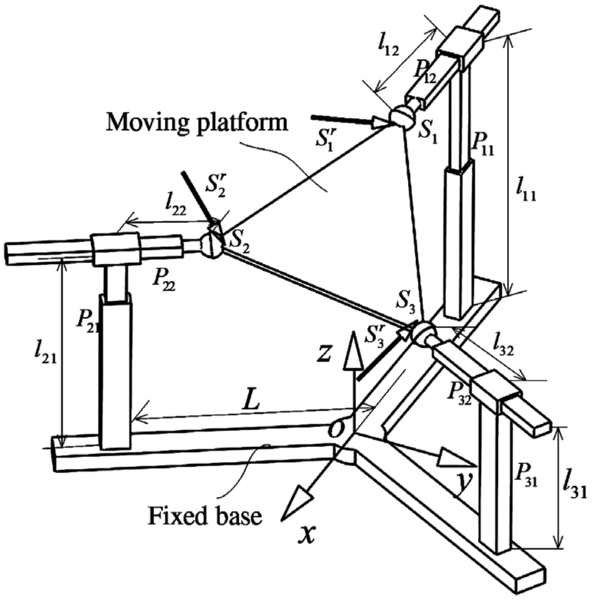

Fig. 7. The 3-$\underline{P}_V P_H S$ parallel manipulator and the applied constraint system.

and can be expressed as

$$\kappa_c(A) = 1. \tag{18}$$

Namely, the moving platform of this parallel manipulator can well proportionately translate in all directions under the TCS, and this parallel manipulator is positional isotropy.

Compared with the general isotropic determination, we could find that the constraint condition number of *SF* and the condition numbers of Jacobian matrix are all equal to one for the above-mentioned parallel manipulator. However, these represent different information. The Jacobian matrix[21] expresses the one-to-one correspondence between the actuated joint space and the operational space of the moving platform, while the *SF* denotes the TCS of the moving platform and explains the isotropic determination from the perspective isotropic determination of constraint force.

3.2. *Isotropic determination for 3-[PP]S type of parallel manipulator*

A 3-[PP]S type of parallel mechanism, such as 3-$\underline{P}_V P_H S$ parallel manipulator (the subscripts V and H indicating that the direction of the P joint is vertical or horizontal, and the joint symbol with underline means the joint is active), is employed to illustrate the isotropic determinations, which has been described by Liu and Bonev[22] for articulated tool head, as shown in Fig. 7. With three identical and symmetrically configured kinematic limbs, this parallel manipulator possesses two rotational DOFs and one translational DOF. Here, we create an absolute coordinate system *o*-*xyz* and set the parameters of this parallel manipulator as shown in Fig. 7.

With the absolute coordinate system, we could obtain the Plücker coordinates of each joint in the first kinematic limb, as follows:

$$\left.\begin{array}{l} \$_{11} = [0 \quad 0 \quad 0\,;\, 0 \quad 0 \quad 1]^T, \\ \$_{12} = [0 \quad 0 \quad 0\,;\, 1 \quad 0 \quad 0]^T, \\ \$_{13} = [1 \quad 0 \quad 0\,;\, 0 \quad l_{11} \quad 0]^T, \\ \$_{14} = [0 \quad 1 \quad 0\,;\, -l_{11} \quad 0 \quad L - l_{12}]^T, \\ \$_{15} = [0 \quad 0 \quad 1\,;\, 0 \quad L - l_{12} \quad 0]^T. \end{array}\right\} \tag{19}$$

The reciprocal screws applied on the moving platform of the first kinematic limb can be obtained by using the algebra operation of the reciprocal product,[14,17]

$$\$_1^r = [0 \;\; 1 \;\; 0\,;\, -l_{11} \;\; 0 \;\; l_{12} - L]^T. \tag{20}$$

Similarly, we can obtain the reciprocal screws applied on the moving platform of the other two kinematic limbs,

$$\left.\begin{array}{l} \$_2^r = [\sqrt{3} \;\; 1 \;\; 0\,;\, -l_{21} \;\; \sqrt{3}l_{21} \;\; 2(L - l_{22})]^T, \\ \$_3^r = [-\sqrt{3} \;\; 1 \;\; 0\,;\, -l_{31} \;\; -\sqrt{3}l_{31} \;\; 2(L - l_{32})]^T. \end{array}\right\} \tag{21}$$

Therefore, the terminal constraints screw system can be written as follows:

$$\$^r = \{\$_1^r, \$_2^r, \$_3^r\}. \tag{22}$$

And the *SF* of this parallel manipulator is obtained,

$$SF = \begin{bmatrix} 0 & 1 & -l_{11} & 0 & l_{12} - L \\ \sqrt{3} & 1 & -l_{21} & \sqrt{3}l_{21} & 2(L - l_{22}) \\ -\sqrt{3} & 1 & -l_{31} & -\sqrt{3}l_{31} & 2(L - l_{32}) \end{bmatrix}. \tag{23}$$

In this section, the 2-norm is employed to perform the analysis of the constraint condition number. In order to facilitate the calculation, we assume that all the parameters in *SF* are nondimensional parameters and set the length, *L*, is one parameter. As the three kinematic limbs are symmetrically configured relative to the fixed base, the spatial and contour atlases of constraint condition number can be obtained with the appointed inputs, $l_{11} = 1, l_{21} = 1.5$, $l_{31} = 1$, and the set values of some variables, as illustrated in Figs. 8 and 9 when $l_{12} = 1$.

From these spatial and contour atlases of constraint condition number we can obtain the useful isotropic information about this 3-[PP]S type of parallel manipulator and find the isotropic configurations. And according to the definition of the constraint condition number, the manipulator will possess the same performance capability in a certain area with the same value of the constraint condition number.

3.3. *Isotropic determination for a 4-DOF parallel manipulator*

Here, we take a 4-DOF parallel manipulator[23] as a further example, which possesses 4 DOFs, as shown in ref. [23] Fig. 1, to illustrate the isotropic determination. The spatial parallel manipulator possesses four kinematic chains with two parallel sideways.

Through the analysis of the reciprocal screws of each kinematic chains, the TCS[23] could be obtained as in Eq. (24),

$$\$^r = \left\{\$_{P_1B_1}^r, \quad \$_{P_1B_1}^r, \quad \$_{P_1B_1}^r, \quad \$_{P_1B_1}^r\right\}, \tag{24}$$

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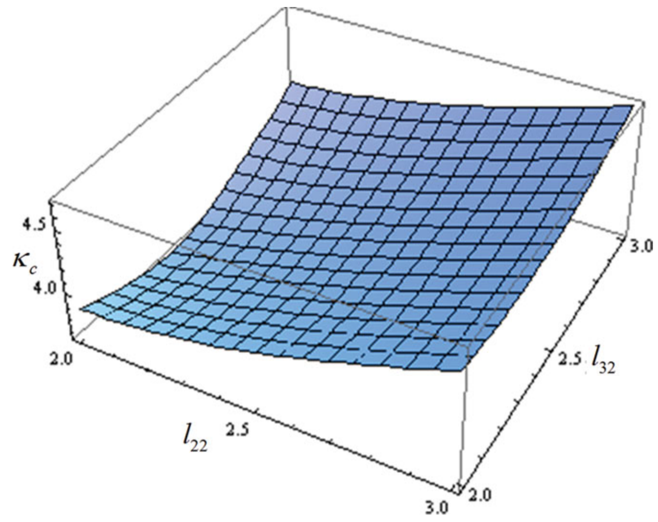

Fig. 8. The spatial atlas of constraint condition number when $l_{12} = 1$.

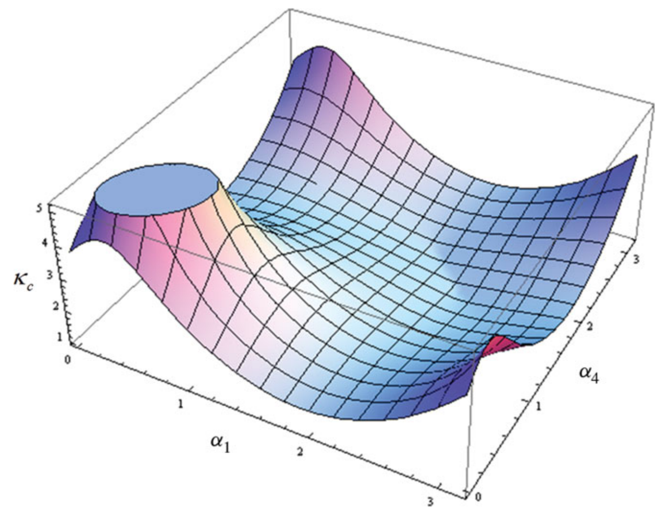

Fig. 10. The spatial atlas of constraint condition number when $\alpha_1 = 18^\circ$ and $\alpha_4 = 36^\circ$.

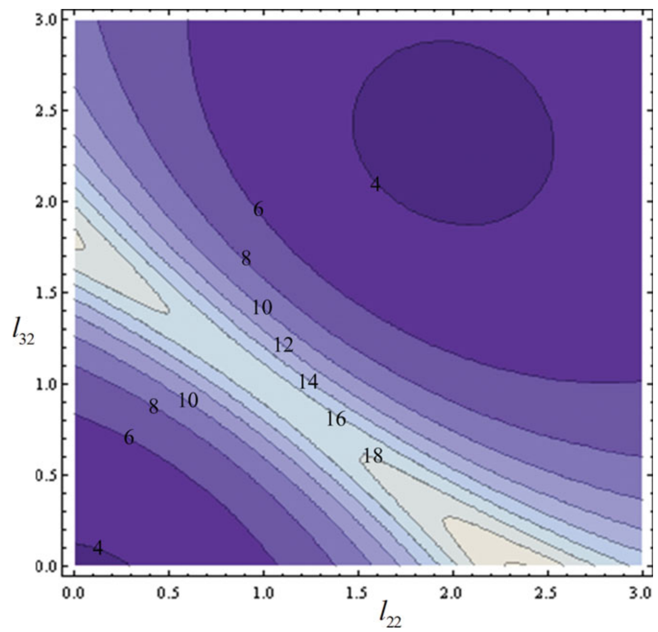

Fig. 9. The contour atlas of constraint condition number when $l_{12} = 1$.

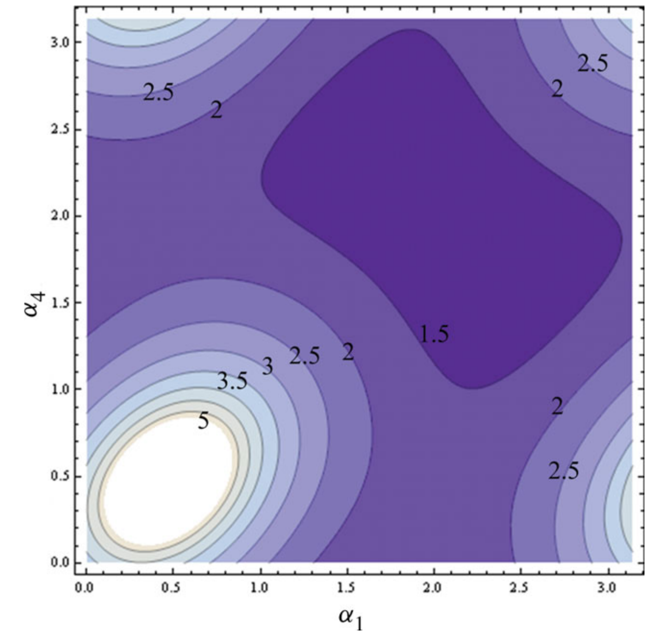

Fig. 11. The contour atlas of constraint condition number when $\alpha_1 = 18^\circ$, and $\alpha_4 = 36^\circ$.

where

$$\begin{aligned}
\$^r_{P_1B_1} &= (0 \quad 0 \quad 0\,;\ \cos\alpha_1 \quad \sin\alpha_1 \quad 0)\\
\$^r_{P_1B_1} &= (0 \quad 0 \quad 0\,;\ \cos\alpha_2 \quad \sin\alpha_2 \quad 0)\\
\$^r_{P_1B_1} &= (0 \quad 0 \quad 0\,;\ \cos\alpha_3 \quad \sin\alpha_3 \quad 0)\\
\$^r_{P_1B_1} &= (0 \quad 0 \quad 0\,;\ \cos\alpha_4 \quad \sin\alpha_4 \quad 0)
\end{aligned}$$

and α_i $(i = 1, 2, 3, 4)$ denotes the angle from x-axis to line OB_{1P}, see Figure in ref. [23], Section 5.3.

Then we can obtain the *SF* of this parallel manipulator, as

$$A = SF(\$^r) = \begin{bmatrix} \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 & \sin\alpha_4 \\ \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 & \cos\alpha_4 \end{bmatrix}. \quad (25)$$

We use the two-norm to perform the analysis of the constraint condition number. The spatial and contour atlases of constraint condition number can be obtained when the values of two angles, as illustrated in Figs. 10 and 11, are set: $\alpha_1 = 18^\circ$ and $\alpha_4 = 36^\circ$.

From these spatial and contour atlases of constraint condition number, certain area with the same value of the constraint condition number could be determined. In this area, the parallel manipulator will possess the same performance capability. However, as the parameters become greater than three, the spatial atlas could not be expressed in three dimensions until set the values of some parameters.

3.4. Discussion

The isotropic results and the atlases of the constraint condition number provide further support for the proposed new method that we can make an analysis on the isotropic condition of spatial parallel manipulator based on the TCS and reciprocal screw system. As we can avoid solving the Jacobian matrix through the analysis of the reciprocal screws, especially for some complex structural parallel manipulators, the calculation process of constraint condition number becomes more simplified. In this paper we use the iterative search strategy to find the isotropic configurations of the given spatial parallel manipulators and obtain the atlases of constraint condition number, as the analytic form of constraint condition number is still complex for some complex structural parallel manipulators.

4. Conclusion

A novel method is presented to determine the isotropic configuration of spatial parallel manipulator in this paper. The major work and conclusions of this paper are drawn as follows:

1. The isotropic configuration of spatial parallel manipulator can be obtained through analyzing the TCS. Based on this viewpoint, the novel method can avoid solving the general Jacobian matrix, and according to the reciprocal screw theory, the physical meaning of isotropy is pointed out, which indicates that the TCS acting on the moving platform works equally well in all directions.
2. The constraint condition number is defined to measure the isotropy of spatial parallel manipulator based on the TCS. From the obtained spatial and contour atlases of constraint condition number, we can obtain the useful isotropic information about the parallel manipulators and find the isotropic configurations area.

Acknowledgments

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