Robotica (2009) volume 27, pp. 539–546. © 2008 Cambridge University Press
doi:10.1017/S026357470800492X Printed in the United Kingdom

An on-line task modification method for singularity avoidance of robot manipulators

Changwu Qiu†*, Qixin Cao‡ and Shouhong Miao§

†*School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai, P. R. China.*
‡*State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, P. R. China.*
§*School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai, P. R. China.*

(Received in Final Form: June 11, 2008. First published online: September 10, 2008)

SUMMARY
In this paper, we present an on-line task modification method (OTMM) to realize singularity avoidance for nonredundant and redundant manipulators at the velocity level. The method introduces a correction vector, constructed from the task velocity and the singular vector corresponding to the minimum singular value, into the task velocity. The performance is simply affected by the choice of the lower limit of the minimum singular value and a scalar adjusting function, which is monotone with respect to the minimum singular value. The method makes unnecessary avoiding the singularity point by off-line path planning for nonredundant or redundant manipulators, and the effort to check whether the singularity is escapable for redundant manipulators. The simulation results show the effectiveness of the OTMM for on-line singularity avoidance in manipulator motion control.

KEYWORDS: On-line task modification; Singularity avoidance; Nonredundant; Redundant; Manipulator.

1. Introduction

One of the important problems in the research of a robot manipulator's control is the inverse kinematics problem. At the same time, any methods of inverse kinematics resolution must properly address the singularity problem associated with nonredundant or redundant manipulators. At singular configurations, the inverse kinematics algorithms that rely on inverse Jacobian matrix (for nonredundant or redundant manipulators) may break down because of the Jacobian is rank deficiency. Understanding and treatment of singular configurations of a robot manipulator is important in the implementation of a controller for the manipulator.

The Jacobian matrix represents the linear mapping relation between the task velocity and the joint velocity space. A column vector of the Jacobian is the finite screw motion that the end-effector can fulfill with unit velocity of the corresponding joint while all other joints are fixed. When rank deficiency of the Jacobian exists, the task velocity vector may not be obtained from the linear combination of the column vectors of the Jacobian matrix. Therefore, with resolved motion rate control algorithms we have only two ways to avoid the singularities: One is to avoid the situation of the Jacobian rank deficiency, and the other is to make the task trajectory bypass the singularities to avoid singular configurations.

Much effort in research community has been paid on dealing with kinematic singularities in motion control of robot manipulators. The classical Singularity Robust Inverse (SRI)[1−5] introduced a regularization term into the Jacobian matrix to avoid the ill-conditioning of the Jacobian in the neighbor of the singularities. The SRI method is simple but sensitive in parameter selection. The works of Cheng *et al.*,[6] Aboaf *et al.*,[7] and Chiaverini *et al.*[8] differentiate the achievable components and degenerate components of the task velocity. The special control policy was utilized to treat the unachievable components while the exact inverse kinematic solution was used for the achievable motion components. This method makes the design of control algorithm complex.

Duleba *et al.*[9] developed a modified Jacobian method of transversally passing through singular configurations of *corank* 1 for nonredundant and redundant robotic manipulators. When the determinant of the Jacobian (nonredundant case) or the submatrix of maximum independent groups in the Jacobian (redundant case) was lower than a given threshold, the ill-conditioned row of the Jacobian was substituted with the differential of the determinant of the Jacobian (nonredundant case) or with the differential of one among the nonvanishing determinants (redundant case), and the corresponding task component was substituted with the negative determinant at that moment. The modified Jacobian method revised the task and the mapping relation between the joint velocity and the task velocity space simultaneously at singularities. This method depends strongly on the form of the forward kinematics relation to assure that the updated Jacobian is not rank deficiency again. A failed example for hyperbolic singularity is presented in ref. [9].

Mayorga *et al.*[10,11] presented a singularity avoidance approach for redundant manipulators based on establishing a local sufficient condition to insure the rank preservation of the Jacobian. The condition is set up based on the differential of the Jacobian. The normal form technique[12,13] expresses original kinematics around singularity in the

* Corresponding author. E-mail: qcw@sjtu.org

normal form. The part of the task path corresponding to singular configurations is moved from the task to the joint space, and trajectory planning is performed there. Far away from singularities the basic Newton algorithm is used to generate a trajectory. Finally, the trajectory parts are joined. Based on the analysis of the differential of the Jacobian, the types of singularity are differentiated. However, the normal form approach is high computation load in deriving the diffeomorphisms, and needs to switch on and off different algorithms to traversing singularity within motion control process.

Bedrossian[14] presented a general methodology for the singularity analysis of redundant manipulators. The singular configurations are classified based on whether the robot can be reconfigured into a nonsingular posture by its self-motion. For the escapability that the redundant manipulator can reconfigure itself from a singular posture to a nonsingular posture via self-motion, Seng *et al.*[15] established the criteria for the classifications of escapable and inescapable singularities. Donelan[16] presented a deeper analysis about classifying singularities of robot manipulators with language of differential topology and singularity theory. Nakamura *et al.*[17] and Cheng *et al.*[18] presented the methods to avoid the escapable singularities utilizing the self-motion capacity of the redundant manipulators.

For redundant manipulators, self-motion capacity offers the manipulators the ability to avoid the escapable singularities. However, to judge the escapability of the singularities needs high computation load to analyze the differential of the Jacobian matrix. On the other hand, the methods to avoid rank deficiency of the Jacobian, in singularity avoidance methods[1–13] for nonredundant manipulators or for inescapable singularities of redundant manipulators, are indirectly or directly making a modification to the task path. Therefore, we can conclude that a unified task modification method can realize singularity avoidance for the singularities of the nonredundant manipulators or the escapable and inescapable singularities of the redundant manipulators. Marani *et al.*[19] proposed an on-line trajectory control scheme that used the manipulability measure as a distance criterion to avoid singularities for generic manipulators. The proposed approach introduced a correction vector constructed for the task velocity with the gradient of the manipulability surface. This method provides a general approach to treat the requirement of avoiding singularity.

Tan *et al.*[20] designed a hybrid motion controller to realize singularity-free tracking algorithms for robot manipulators. The robot workspace is partitioned into subspaces based on the singular configurations of the robot. Switching between continuous controllers is involved when the robot travels across the subspaces. In some works (Muszyński *et al.*,[12] K. Tchoń *et al.*,[13] Duleba *et al.*,[9] Cheng *et al.*,[6] Aboaf *et al.*,[7] and Chiaverini *et al.*[8]), a switching mechanism is also needed in motion control algorithm design for robot manipulator with singularity traversing consideration. The switching design makes the design of motion control algorithm complex.

In this paper, we propose an on-line task modification method (OTMM) to realize singularity avoidance for nonredundant and redundant manipulators by introducing a correction vector into the task velocity. The method is developed based on the observation of the geometry relation between the task velocity and the singular vector at singular configurations. The OTMM provides a general and intuitive geometry approach to realize singularity avoidance and does not differentiate between the type of manipulator (nonredundant or redundant) and the escapability of the singularities for redundant manipulators. The task velocity is filtered by modifying the manipulator's task directly in the vicinity of singularity, and then the design of resolved motion rate control algorithm need not consider the special treatment for singularity, such as a switching mechanism in the design of control algorithm. The escapable/inescapable singularities are dealt with the same way for redundant manipulator just like the one for nonredundant manipulator. Therefore, the resolved motion rate control system can be designed as a general one accompanied with a filter layer for its input.

The paper is organized as follows. Section 2 briefly analyzes the attributes of the singular configuration of the manipulator. Section 3 constructs the OTMM and analyzes its geometric principle. Section 4 shows the effectiveness of our method with simulation examples for the trajectory following task with a 2-link planar manipulator and a 4-link planar manipulator. The effects for different choices of the scalar adjusting function for the correction vector are presented. Finally, Section 5 presents the conclusions.

2. Singularity Analysis

For the linear attribute of the first-order kinematics mapping, the majority of efforts have been focused on finding the solution of the inverse kinematics at the velocity level. The kinematics of manipulators is frequently represented as

$$x = f(q), \tag{1}$$

$$\dot{x} = J(q) \times \dot{q}, \tag{2}$$

where $x \in R^n$ represents the task in Cartesian space, $f(q) \in R^n$ is a vector function expressing the forward kinematics relation, $q \in R^m (m \geq n)$ represents the joint variables in configuration space, and $J(q) = ¶f/¶q = [J_1 \quad L \quad J_m] \hat{I} R^{n\times m}$ is the end-effector Jacobian matrix which is consisted of column vectors for joint twists with respect to Cartesian space. If rank $(J(q)) = n$, the joint velocity can be resolved as

$$\dot{q} = J^{\#}(q)\dot{x} \tag{3}$$

where $J^{\#}(q) = J^{-1}(q) \hat{I} R^{n\times n}$ for $m = n$ and $J^{\#}(q) = J^T(q)(J(q)J^T(q))^{-1} \hat{I} R^{m\times n}$ for $m > n$. When rank$(J(q)) < n$, there is no definition for the inverse or pseudoinverse of the Jacobian at such configurations. These configurations are named singular configurations. At singular configurations, the joint velocity may break or chatter with resolved motion rate control algorithms based on Jacobian inverse or pseudoinverse, which is not practically feasible in manipulators' control system and is also dangerous for the robot's structure. Therefore, singularity avoidance measures

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

must be considered in designing motion control algorithm of manipulators.

The most powerful tool to investigate the singularity of the robot manipulators (nonredundant or redundant) is Singular Value Decomposition (SVD).[21] The SVD has a reputation for being numerically expensive to compute. The determinant of the manipulator's Jacobian matrix gives illegible information about the absolute proximity to singularities, since the minimum singular value is the only reliable measure of the absolute proximity to singularities.[21,22] Decomposition of the Jacobian matrix has the form

$$J = U\mathring{a}V^T \tag{4}$$

where $U \in R^{n\times n}$ and $V \in R^{m\times m}$ are orthogonal matrices, $\sum \in R^{n\times m}$ contains singular values on its main diagonal and has the form $[\mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)|0_{n\times(m-n)}]$, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$. The column vectors of U and V are respectively the left and right singular vectors of the matrix J. When the manipulator reached a singularity configuration, there are $\mathrm{rank}(J(q)) = z < n$, $\mathrm{corank}(J(q)) = n - z$, and $\sigma_j = 0$, $j = \{n - z + 1, \ldots, n\}$. Then, we can partition the matrix U as $U = [U_z, U_{n-z}]$ and get equation $\mathrm{rang}(J) = \mathrm{rang}(U_z)$. The subspace $\mathrm{rang}(U_z)$ and $\mathrm{rang}(U_{n-z})$ can be named respectively as achievable and unachievable subspace for the task velocity space. At singular configurations, we cannot find a group of feasible joint velocities for the task velocity with nonzero projective components in the subspace $\mathrm{rang}(U_{n-z})$.

We have pointed out in Section 1 that there are only two ways to avoid singularity: One is to avoid the situation of the Jacobian rank deficiency, and the other is to make the task trajectory bypass the singularities to avoid singular configurations. For the singularity of nonredundant manipulator or inescapable singularity of redundant manipulator, avoiding rank deficiency of the Jacobian means changing the linear mapping relation between the joint velocity and the task velocity space. Taking the SRI[1–5] method as example, the physics meaning of the SRI can be expressed as

$$\min(\dot{x} - J(q)\dot{q})^T(\dot{x} - J(q)\dot{q}) + \alpha^2 \times \dot{q}^T\dot{q}, \tag{5}$$

where $\alpha \geq 0$ is a damping factor for the joint velocity. The analytical resolution of (5) can be expressed as

$$\dot{q} = J^T(JJ^T + \alpha \times I)^{-1}\dot{x}. \tag{6}$$

When the algorithm (5) or (6) is applied in the vicinity of singularity, there will be a trade-off between the tracking accuracy and the joint velocity norm. In fact, the SRI method avoids singularity by modifying indirectly the task velocity with (5) or (6). According to the matrix formula $(C + D)^{-1} = C^{-1}(I - (C^{-1} + D^{-1})^{-1}C^{-1})\pm$, where $C \in R^{n\times n}$ and $D \in R^{n\times n}$, we can represent (6) as

$$\dot{q} = J^T(JJ^T)^{-1}(\dot{x} - \dot{x}_\alpha), \tag{7}$$

where $\dot{x}_\alpha = ((JJ^T)^{-1} + \frac{1}{\alpha}I)^{-1}(JJ^T)^{-1}\dot{x}\frac{1}{2}$ is a corrected vector for the task velocity. The damping parameter α plays a key role in the SRI method. It needs a bigger α to get a feasible solution but a smaller one to decrease the tracking error. In order to smooth the damping action within the bounding area of singularity point, Liu *et al.*[5] adopted an adaptive method to define the damping parameter α as a function of the minimum singular value. However, their method is still sensitive to the choice of α and the bounding limit of singularity for that the damping parameter α is acting directly in the joint space and the velocity direction variety of end-effector is not considered. Kirćanski *et al.*[23] and O'Neil *et al.*[24] had proved that the controller based on pseudoinverse method will cause instability in the vicinity of singularity. Therefore, direct modification of the mapping relation from joint space to task space is not preferred.

3. On-Line Task Modification Method

For nonredundant manipulators, singularity-free motion can be achieved with off-line path planning. However, it requires *a priori* knowledge of the singular configurations of the manipulator and applies severe restrictions to the manipulator's workspace. For a redundant manipulator, self-motion can realize avoidance of the escapable singularities but this cannot be achieved for inescapable singularities.[14,15] Therefore, an on-line general singularity avoidance method for nonredundant and redundant manipulators will be a better method. On the other hand, we must remove the drawback of the singularity avoidance methods that revised directly the Jacobian mapping relation between the joint velocity and task velocity space. Finally, we hope that the designed singularity avoidance method can remove the burden of checking whether the singular configurations for redundant manipulators can be avoided by self-motion.

At a singularity configuration, to avoid Jacobian deficiency means changing the mapping relation between the task velocity and the joint velocity space. It will inevitably cause some perturbation to the task trajectory. Analysis about the SRI in the former section can prove this point. Therefore, a basic idea is to circumscribe singularities by directly modifying the task trajectory and make the input for the resolved motion rate control kernel be out of the singularity problem. The singular direction vectors u_s, $\{s = 1, \ldots, n - z\}$, which are the column vectors of the submatrix U_{n-z}, span the instantaneous unachievable task velocity space at singularity configurations. Assuming $m = 2$, $n = 2$, and $z = 1$, Fig. 1 shows the geometry relation between

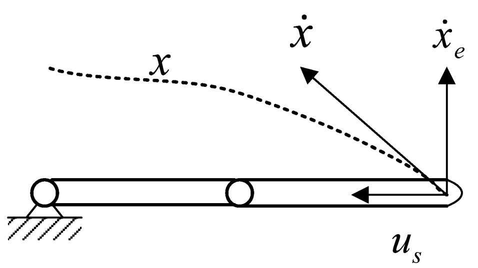

Fig. 1. The geometry relation between the viable end-effector velocity, the arbitrary end-effector velocity, and the left singular vector corresponding to the singular value $\sigma_{\min} = 0$.

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

the viable end-effector velocity, the arbitrary end-effector velocity, and the left singular vector corresponding to the singular value $\sigma_{\min} = 0$ for a 2-link planar manipulator. In Fig. 1, we can find only $\pm\dot{x}_e$ which can be realized by the manipulator instantaneously. For a nonredundant or redundant manipulator, the achievable components of task velocity can be represented as $\dot{x}_a = \dot{x} - (\dot{x}^T u_s) \times u_s$ when $\mathrm{corank}(J(q)) = 1$.

From Fig. 1, we can find that the left singular vector corresponding to the singular value $\sigma_{\min} = 0$ provides important information to construct the achievable task velocity. The column vector of U corresponding to the minimum singular vector represents the degenerated direction of the task velocity when the manipulator moves near to the singularity. In order to make the end-effector to bypass the singularities and avoid the failure of the inverse or pseudoinverse of the Jacobian matrix, the column vector of U corresponding to the minimum singular vector can be used to revise the task velocity and hold back the movement of the end-effector to the singularity point. A general on-line singularity avoidance method for nonredundant or redundant manipulators with $\mathrm{corank}(J(q)) = 1$ is formulated as

$$\dot{x}_m = \dot{x} - k \times p(\sigma_{\min}) \times (u_S^T \dot{x}) \times u_S, k = \begin{cases} 0 & \sigma_{\min} > \sigma_S \\ 1 & \sigma_{\min} \le \sigma_S \end{cases}, \tag{8}$$

where $\dot{x}_m \hat{I} R^n$ is the modified task velocity, $\sigma_{\min} \in R$ is the minimum singular value of the matrix J, $\sigma_S \in R$ is the low limit of the minimum singular value of the matrix J, $p(\sigma_{\min})\hat{I}[0, 1]$ is a monotone function, where $p(\sigma_{\min}) = 1$ when $\sigma_{\min} = 0$ and $p(\sigma_{\min}) = 0$ when $\sigma_{\min} = \sigma_S$. The $p(\sigma_{\min})$ is designed to adjust the extent of suppressing the components of the task velocity along the column vector corresponding to the minimum singular value, which represents the degenerated direction of the range space of the Jacobian. The choices of the function $p(\sigma_{\min})$ can be chosen by trial-and-error. We will give some simulation results in Section 4 for the effect of different choices of the function $p(\sigma_{\min})$.

Method (8) can be utilized to realize on-line singularity avoidance and was named OTMM. The OTMM realizes singularity avoidance by directly modifying the task velocity and avoids the instability drawback caused by modifying the Jacobian mapping relation in joint space. There is no need to design switching mechanism between the normal resolved motion rate control algorithm, which is based on the Jacobian inverse or pseudoinverse, and the special singularity avoidance control algorithm in the vicinity of singularities.

The OTMM avoids the singularity of the task trajectory by directly modifying the task velocity vector, so it is effective not only for nonredundant manipulators but also for redundant manipulators. The choice of $p(\sigma_{\min})$ and the low limit of the minimum singular value σ_S are the exclusive factors for the performance of singularity avoidance control algorithm and the trajectory tracking accuracy. Simulation results in Section 4 will validate the effectiveness of the OTMM.

4. Simulation Results

In order to demonstrate the effectiveness of the OTMM in resolved motion rate control of manipulators, several simulation tests have been performed. For nonredundant manipulators, a 2-link planar manipulator is considered. The kinematic model of the 2-link planar manipulator is $[x, y]^T = [l_1\cos(q_1) + l_2\cos(q_1 + q_2); l_1\sin(q_1) + l_2\sin(q_1 + q_2)]^T$. As an example of a redundant manipulator, a 4-link planar manipulator is selected and its kinematic model is $[x, y]^T = [\sum_{i=1}^4 l_i\cos(\sum_{j=1}^i q_j); \sum_{i=1}^4 l_i\sin(\sum_{j=1}^i q_j)]^T$. We use Matlab to implement the control algorithm and perform the numerical simulations of the robot motion.

In order to compare the performance of different methods, we implement respectively the resolved motion rate control algorithm without singularity avoidance consideration, the SRI algorithm proposed by Liu *et al.*,[5] and the OTMM for a 2-link manipulator with the same mission, which tracks a circle with radius 0.6 m and center (1.2, 0) in clockwise direction. The end-effector moves with even velocity to finish the mission in 10 s. The parameters of the manipulator and its initial states are $[l_1, l_2] = [1, 0.8]$ m and $[q_1, q_2] = [0.927, -2.5]$ rad. The initial joint velocity of the manipulator is $[\dot{q}_1, \dot{q}_2] = [0, 0]$ rad/s and the sample time interval is 20 ms. The simulation results are showen in Figs. 2–6 respectively for that mission. Figures 4–6 present the simulation results with OTMM for different choices of the function $p(\sigma_{\min})$.

Figure 2 shows the simulation results of the resolved motion rate control algorithm without singularity avoidance consideration. In this simulation test, the basic Newton algorithm is used to compute the joint velocity. The configuration with joint angle [0, 0] is a singular configuration for the 2-link planar manipulator. In Fig. 2(c), we can find that there is joint velocity chattering and break when the manipulator passes a singularity. The initial jump of the joint velocity shown in Fig. 2(c) does not impact the inspection of the effectiveness of the algorithm. The end-effector tracking error history showen in Fig. 2(d) reveales the same phenomena caused by singularity with the basic Newton algorithm. This situation must be avoided for safety consideration of robot structure and stability of control system.

Figure 3 shows the simulation results for the SRI algorithm proposed by Liu *et al.*[5] Because there are no distinct differences in robot motion and joint history, the figures about the motion process and joint history are omitted. We choose the same low limit of singular value with $\sigma_S = 0.05$ for the SRI method and the OTMM in this and the next simulation tests. The other parameters are $w_0 = \det(J)|_{\sigma_{\min}=0.05}$ and $\alpha = 0.003$ for the SRI algorithm.[5] Though, we have obeyed the SRI algorithm[5] to implement the manipulator control, there is joint velocity break and chattering phenomena shown in Fig. 3. The results in Fig. 3 indicate that the SRI method is sensitive to the parameters chosen and the robot structure.

Figures 4–6 show the simulation results with the proposed OTMM in tracking a circular path which is same to the former simulation tests. The scalar adjusting funtions $p(\sigma_{\min})$ are chosen as $p(\sigma_{\min}) = 1 - \sigma_{\min}/\sigma_S$, $p(\sigma_{\min}) = 1 - \sqrt{\sigma_{\min}/\sigma_S}$, and $p(\sigma_{\min}) = 1 - \sqrt[3]{\sigma_{\min}/\sigma_S}$ respectively. The initial

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

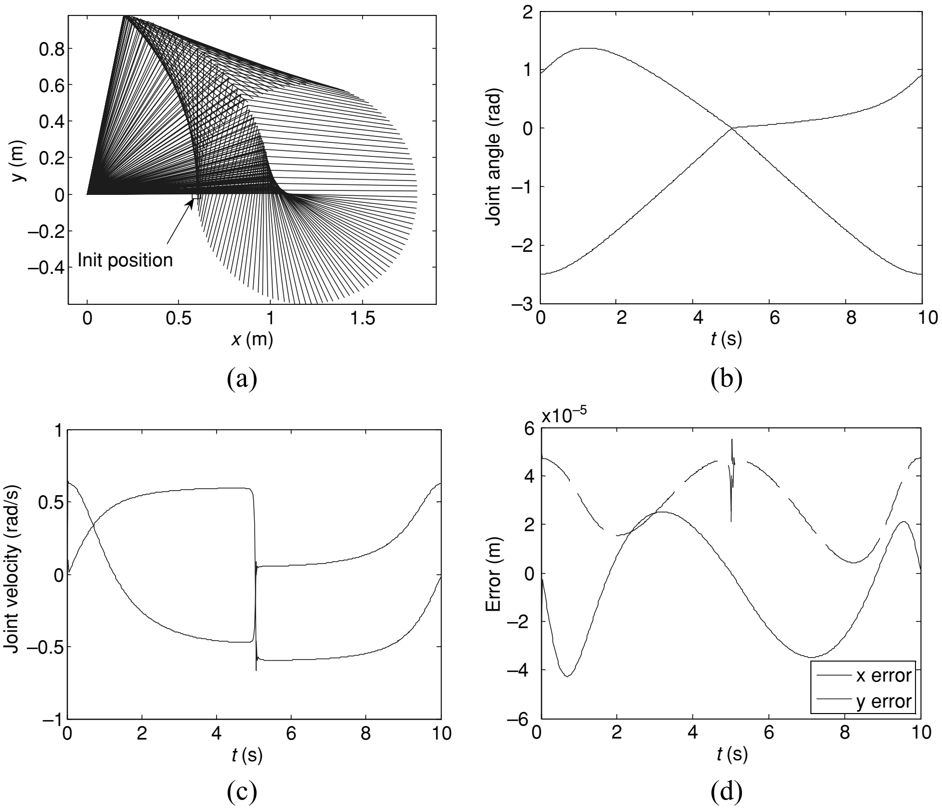

Fig. 2. A 2-link planar manipulator follows a circular trajectory without singularity avoidance consideration.

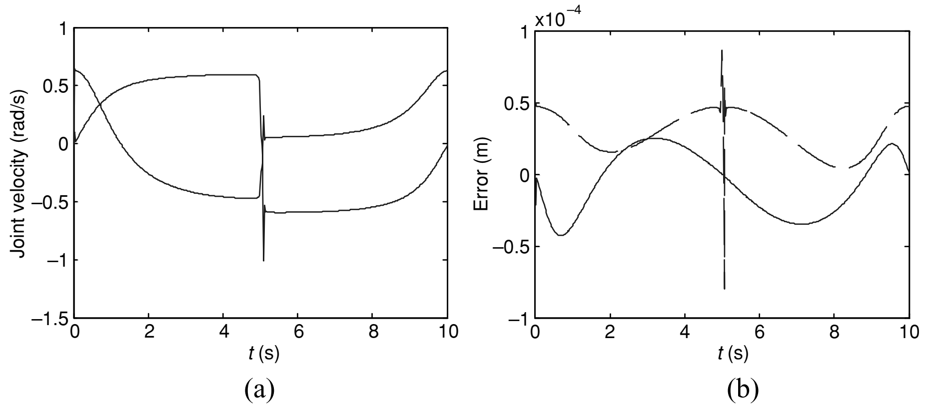

Fig. 3. The results for a 2-link planar manipulator following a circular trajectory using SRI method with adaptive damp ratio adjustment.

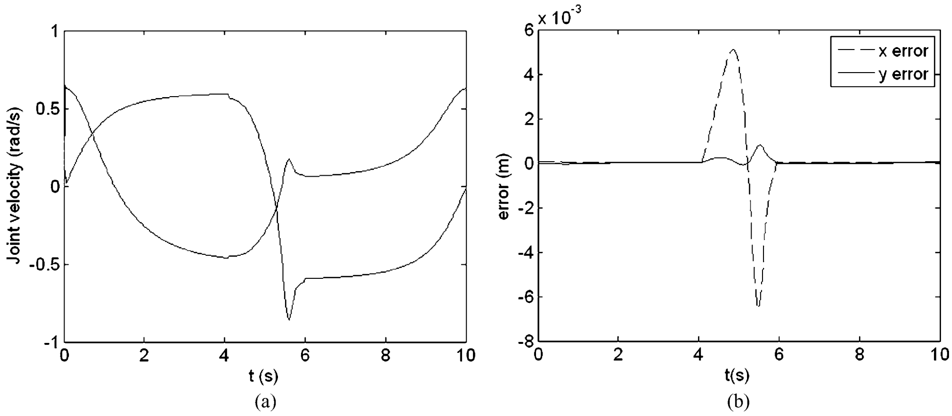

Fig. 4. The results for a 2-link planar manipulator following a circular trajectory using the OTMM with $p(\sigma_{\min}) = 1 - \sigma_{\min}/\sigma_S$.

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

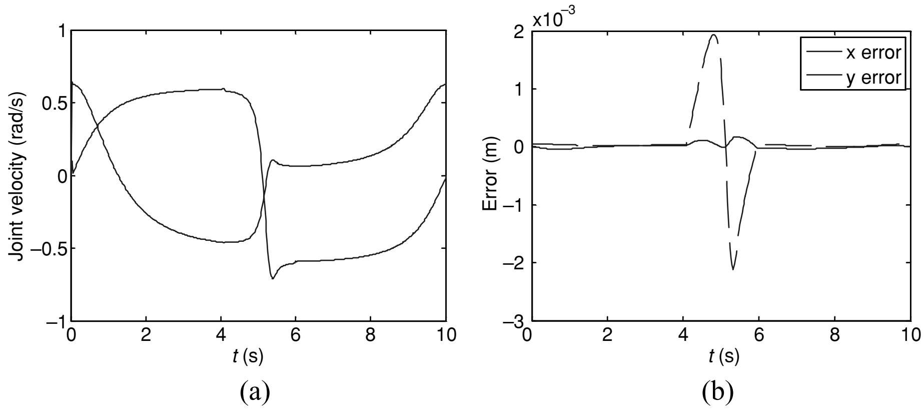

Fig. 5. The results for a 2-link planar manipulator following a circular trajectory using the OTMM with $p(\sigma_{\min}) = 1 - \sqrt{\sigma_{\min}/\sigma_S}$.

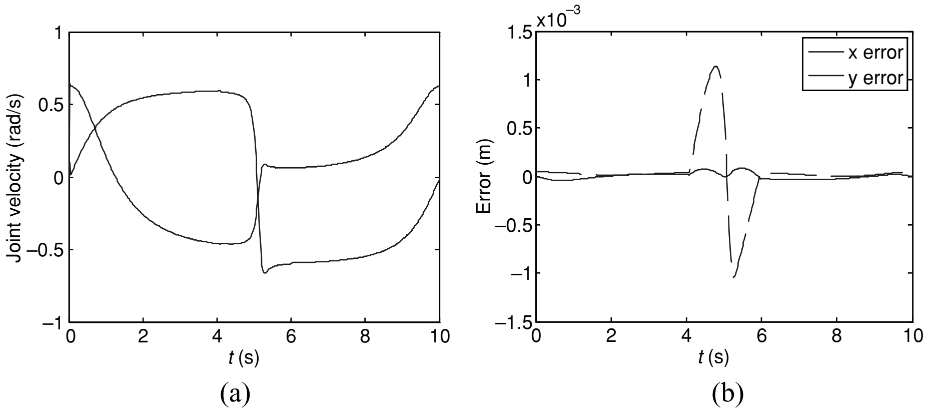

Fig. 6. The results for a 2-link planar manipulator following a circular trajectory using the OTMM with $p(\sigma_{\min}) = 1 - \sqrt[3]{\sigma_{\min}/\sigma_S}$.

conditions of joint angle and joint velocity are same to the former simulation tests also. For length consideration, the robot motion and joint angle history is omitted. The results in Figs. 4(a), 5(a), and 6(a) indicate that the OTMM is effective and robust to avoid singularity with different function $p(\sigma_{\min})$ and the OTMM is successful in avoiding the joint velocity chattering or break in the vicinity of singular configuration. Contrasting the tracking error shown in Figs. 4(b), 5(b), and 6(b), the tracking error caused by task modification can be decreased by properly choosing the scalar adjusting function $p(\sigma_{\min})$. The function $p(\sigma_{\min})$ is the only factor to impact the performance of the algorithm.

These simulation tests show the effectiveness of the OTMM in avoiding singularity when robot manipulators transverse singularity. However, the initial configuration of the robot manipulator being at a singular configuration is not considered. This situation is not treated in existed literature. We design a simulation experiment to check if the OTMM is effective when the initial configuration of the robot manipulator is at singularity. Figure 7 shows the simulation results with the OTMM for a 2-link manipulator with the end-effector following a straight line trajectory within 5.2 s, which is from point (1.8, 0) to point (0.6, −1). The trajectory is defined with bell-shape acceleration law as $x(t) = -1.2(10(t/T)^3 - 15(t/T)^4 + 6(t/T)^5) + 1.8$, $y(t) = -10(t/T)^3 + 15(t/T)^4 - 6(t/T)^5)$. The parameters of the manipulator and its initial states are $[l_1, l_2] = [1, 0.8]$ m and $[q_1, q_2] = [0, 0]$ rad. The initial joint velocity of the manipulator is $[\dot{q}_1, \dot{q}_2] = [0, 0]$ rad/s and the sample time interval is 20 ms. The initial configuration of the manipulator in this simulation test is a singular configuration. With traditional Jacobian inverse based resolve motion ration control algorithm, the inverse kinematics cannot find a feasible solution at the beginning of the control loop. The velocity constraints for the mission were modified as $(0, \dot{y}(t))^T$ with the OTMM formulated in (8). Therefore, only the second row of the Jacobian was considered in the control algorithm. When the manipulator escapes the singular configuration, the whole Jacobian matrix is applied again. Observing the joint velocity history, we can find that the OTMM is effective.

Figure 8 shows the simulation results with the OTMM for a 4-link planar manipulator tracking a circle with radius 0.6 m and center (2.6, 0) in clockwise direction and even velocity movement within 6 s. The parameters of the manipulator and its initial states are $[l_1, l_2, l_3, l_4] = [1, 0.8, 0.8, 0.6]$ m,$[q_1, q_2, q_3, q_4] = [-1.2763, 1.1927, 0.7517, 0.4086]$ rad, and $[\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4] = [0, 0, 0, 0]$ rad/s. The sample time interval is 20 ms and the low limit of singular value is $\sigma_S = 0.05$. The scalar adjusting function is chosen as $p(\sigma_{\min}) = 1 - \sqrt[3]{\sigma_{\min}/\sigma_S}$. Observing Fig. 8(c), the joint velocity chattering or break phenomenon is avoided with the OTMM successfully.

In the above simulation experiments, we realize the singularity avoidance by directly modifying the task trajectory with the task velocity and the singular vector. The OTMM

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

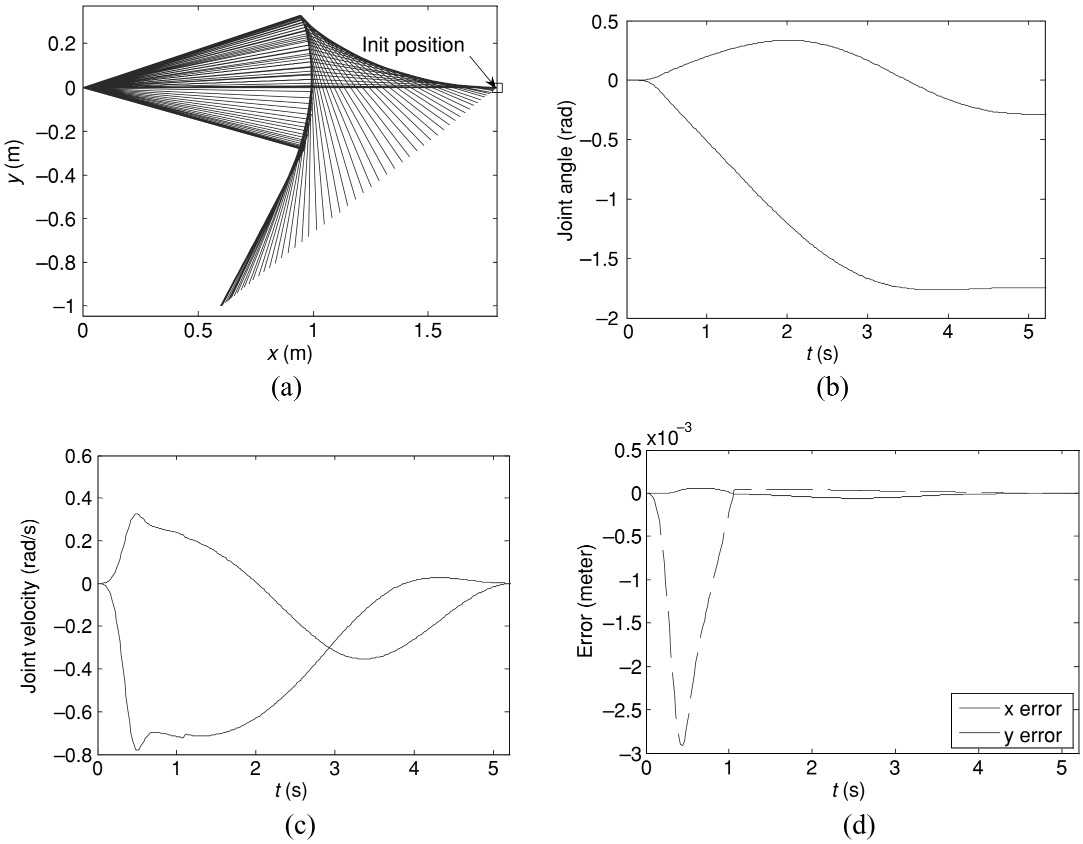

Fig. 7. The results for a 2-link planar manipulator following a line trajectory using the OTMM with $p(\sigma_{\min}) = 1 - \sqrt[3]{\sigma_{\min}/\sigma_S}$.

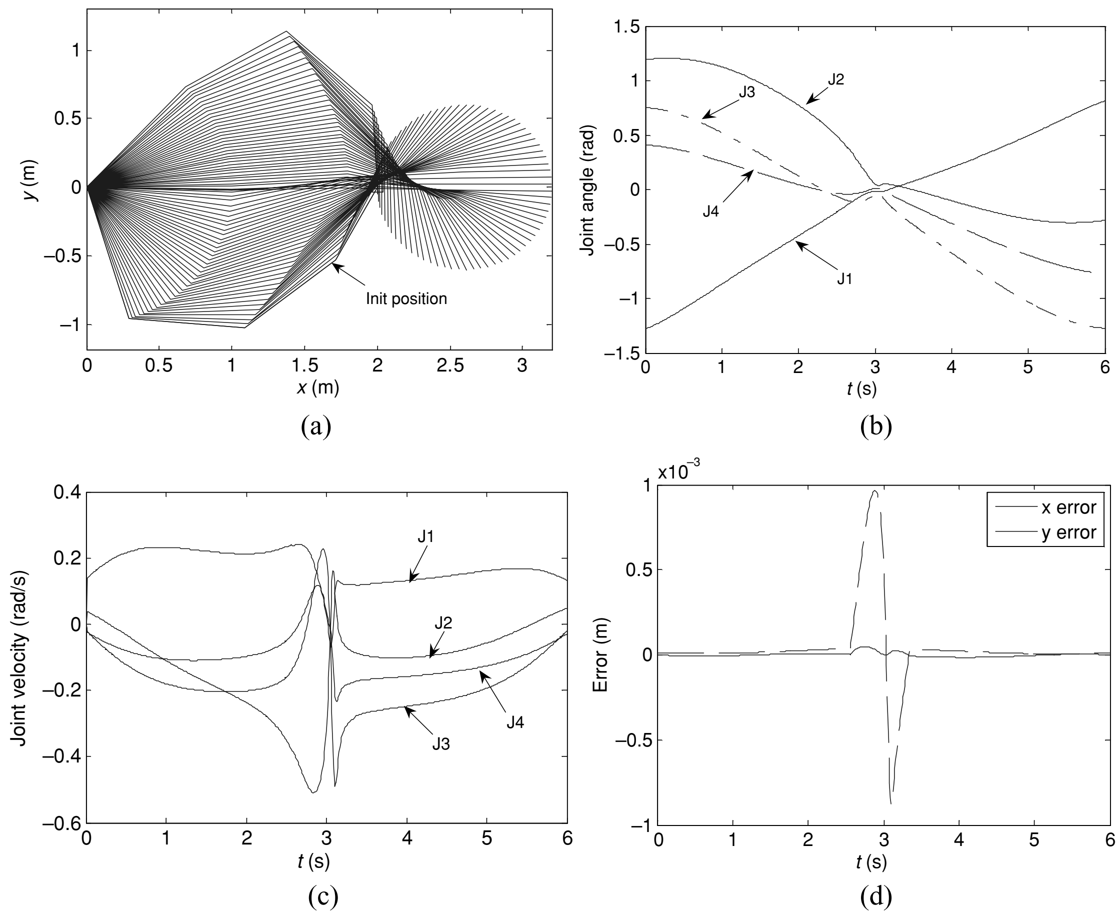

Fig. 8. The results for a 4-link planar manipulator following a circular trajectory using the OTMM with $p(\sigma_{\min}) = 1 - \sqrt[3]{\sigma_{\min}/\sigma_S}$.

https://doi.org/10.1017/S026357470800492X Published online by Cambridge University Press

supervises the change of the minimum singular value of the matrix JJ^T and is a general task modification method for nonredundant and redundant manipulators. By applying the OTMM, the resolved motion rate control algorithm or system can be designed without consideration of the singularity.

We validated the usefulness of the OTMM for singularity avoidance of nonredundant and redundant manipulators with the above simulation tests. The tracking error of the end-effector can be decreased further by choosing a smaller low limit of the singular value σ_S and more refined scalar adjusting function $p(\sigma_{\min})$.

5. Conclusions

This paper proposes an on-line singularity avoidance method, OTMM, to realize singularity avoidance for nonredundant and redundant manipulators by introducing a corrected vector into the object task velocity. The method is developed based on the observation of the geometry relation between the task velocity and the singular vector at singular configurations. With this intuitive geometry approach, there is no need to differentiate the manipulator types (nonredundant or redundant) and the escapability of the singularities for redundant manipulator. With the OTMM, the switching mechanism of control algorithm design can be eliminated.

The OTMM is a general method to avoid singularity for nonredundant or redundant robot manipulators. On-line modification of the task velocity to avoid singularity alleviates the effort in path planning and the design of resolved motion rate control system. It makes unnecessary singularity avoidance in path planning. Several simulation experiments for nonredundant and redundant manipulators validate the effectiveness of the proposed method.

References

1. C. W. Wampler, "Manipulator inverse kinematic solutions based on vector formulations and damped least-squares methods," *IEEE Trans. Sys. Man, Cybernetics*, SMC-16(1), 93–101 (1986).
2. Y. Nakamura and H. Hanafusa, "Inverse kinematic solutions with singularity robustness for robot manipulator control," *J. Dynamic Sys. Meas. Control* **108**, 163–171 (1986).
3. O. Egeland, J. R. Sagli and I. Spangelo, "A Damped Least-Squares Solution to Redundancy Resolution," *IEEE International Conference on Robotics and Automation*, Sacramento, CA, (1991) pp. 945–950.
4. S. Chiaverini, "Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators," *IEEE Trans. Robot. Automation* **13**(3), 398–410 (1997).
5. T. S. Liu and S. Y. Tsay, "Singularity of robotic kinematics: A differential motion approach," *Mech. Mach. Theory* **25**(4), 439–448 (1990).
6. F. T. Cheng, T. L. Hour and Y. Y. Sun, "Study and resolution of singularities for a 6-DOF PUMA manipulator," *IEEE Trans. Sys. Man Cybernetics* **27**(2), 165–178 (1997).
7. E. W. Aboaf and R. P. Paul, "Living with the Singularity of Robot Wrists," *IEEE International Conference on Robotics and Automation*, Raleigh, North Carolina (1987), pp. 1713–1717.
8. S. Chiaverini and O. Egeland, "A Solution to the Singularity Problem for Six-Joint Manipulators," *IEEE International Conference on Robot and Automation*, Cincinnati, OH (1990), pp. 644–649.
9. I. Duleba and J. Z. Sasiadek, "Modified Jacobian Method of Transversal Passing through Singularities of Nonredundant Manipulators," *Robot.* **20**, 405–415 (2002).
10. R. V. Mayorga, A. K. C. Wong and K. S. Ma, "An efficient local approach for the path generation of robot manipulators," *J. Robot. Sys.* **7**(1), 23–55 (1990).
11. R. V. Mayorga and A. K. C. Wong, "A Singularities Prevention Approach for Redundant Robot Manipulators," *IEEE International Conference on Robotics and Automation*, Cincinnati, OH, (1990) **2** pp. 812–817.
12. R. Muszyński and K. Tchoń, "Singularities of nonredundant robot kinematics," *Int. J. Robot. Res.* **16**(1), 60–76 (1997).
13. K. Tchoń and R. Muszyński, "Singular inverse kinematic problem for robotic manipulators: A normal form approach," *IEEE Trans. Robot. Automation* **14**(1), 93–104 (1998).
14. N. S. Bedrossian, "Classification of Singular Configurations for Redundant Manipulators," *IEEE International Conference on Robotics and Automation*, New York (1990), pp. 818–823.
15. J. Seng, K. A. O'Neil and Y. C. Chen, "Escapability of Singular Configuration for Redundant Manipulators via Self-Motion," *IEEE/RSJ International Conference on Intelligent Robots and Systems*, New York (1995), Part 3, pp. 78–83.
16. P. S. Donelan, "Singularity-theoretic methods in robot kinematics," *Robot.* **25**, 641–659 (2007).
17. Y. Nakamura, H. Hanafusa and T. Yoshikawa, "Task priority based redundancy control of robot manipulators," *Int. J. Robot. Res.* **6**(2), 3–15 (1987).
18. F.-T. Cheng and M. S. Shih, "Multiple-goal priority considerations of redundant manipulators," *Robot.* **15**(6), 675–691 (1997).
19. G. Marani, J. Kim, J. Yuhl and W. K. Chung, "A Real-Time Approach for Singularity Avoidance in Resolved Motion Rate Control of Robotic Manipulators," *IEEE International Conference on Robotics and Automation*, Washington, DC (2002), pp. 1973–1978.
20. J. Tan, N. Xi and Y. Wang, "A singularity-free motion control algorithm for robot manipulators—a hybrid system approach," *Automatica* **40**, 1239–1245 (2004).
21. A. Maciejewski and C. A. KBein, "The singular value decomposition: Computation and applications to robotics," *Int. J. Robot. Res.* **8**(6), 63–79 (1989).
22. A. Maciejewski, "Real-Time SVD for the Control of Redundant Robotic Manipulators," *IEEE International Conference on Systems Engineering*, Fairborn, OH (1989), pp. 549–552.
23. M. Kirćanski, N. Kirćanshi, D. Leković and M. Vukobratović, "An experimental study of resolved acceleration control of robots at singularities: Damped least-squares approach," *J. Dynamic Sys. Measurement Control* **119**, 97–101 (1997).
24. K. A. O'Neil and Y. C. Chen, "Instability of pseudoinverse acceleration control of redundant mechanisms," *IEEE International Conference on Robotics and Automation*, San Francisco, CA (2000), pp. 2575–2582.