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A Data Association Algorithm for SLAM Based on Central Difference Joint Compatibility Criterion and Clustering

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SUMMARY

A data association algorithm for simultaneous localization and mapping (SLAM) based on central difference joint compatibility (CDJC) criterion and clustering is proposed to obtain the data association results. Firstly, CDJC criterion is designed to calculate joint Mahalanobis distance. Secondly, ordering points to identify the clustering structure is used to divide all observed features into several groups. Thirdly, CDJC branch and bound method is designed to be performed in each group. The results based on simulation data and benchmark dataset show that the proposed algorithm has low computational complexity and provide accurate association results for SLAM of mobile robot.

KEYWORDS: Mobile robot; Simultaneous localization and mapping; Data association; Central difference joint compatibility criterion; OPTICS.

1. Introduction

Simultaneous localization and mapping (SLAM) is the process that enables mobile robot to move in an unknown environment and then to incrementally build a map of this environment while simultaneously using this map to estimate its pose.[1,2] SLAM problem was first proposed by Smith et al. in 1987.[3] Since Smith et al. kicked off the study of SLAM, SLAM has gradually become a research focus in some fields, such as intelligent vehicle, intelligent robots, and unmanned aerial vehicle. SLAM is known as "Holy Grail" of autonomous mobile robot.[4,5]

SLAM is a concept and comprises many algorithms. In general, SLAM can be divided into two major steps: data association and state estimation.[6–8] The problem of state estimation contains the robot's state estimation and the location estimation of environment landmarks. The correct data associations are the pre-condition to realize the correct state estimation.

Originally data association is a problem that needs to be solved in target tracking.[9,10] It is used to determine the correspondence between the sensor measurements and the target. In SLAM, data association is the process to establish the correspondence between the sensor measurements and the map features to determine whether they originate from the same physical landmark of the environment; it also includes the process of determining the sensor measurements that do not match the map features as a new feature or noise data. Data association has always been a key issue for practical SLAM implementations. Since an incorrect data association often causes divergence into state estimation of the map, even more serious it can lead failure of the localization algorithm. Good data association method has two advantages: high accuracy and low computational complexity.

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In the early research, almost all solutions of data association problem are performed using the gated nearest neighbor (NN) algorithm,[11,12] which is inherited from the target-tracking literature. Based on the gated NN algorithm, the individual compatibility nearest neighbor (ICNN) algorithm and sequential compatibility nearest neighbor (SCNN) are proposed to solve the data association problem.[13,14] ICNN algorithm selects the best matchings according to the individual compatibility (IC) criterion. This algorithm is simple and easy to implement, but it only considers IC between a measurement and a feature. Hence, it is unreliable when the error of robot's pose grows with respect to sensor error.[13] SCNN algorithm is proposed to get the resulting hypothesis which contains jointly compatible pairings. In this algorithm, it is a random process to choose which measurement should be processed first, and the result is not affected by the sequence of the measurement processing. Hence, the likelihood of correct association is higher than ICNN. However, just like ICNN method, as a greedy algorithm, SCNN algorithm still get the error result of data association in ambiguous environments, because it neglects the correlations between observations.[13]

A lot of sophisticated algorithms called the multiple-hypothesis tracker method, such as the multi-hypothesis Kalman filter[15] and FastSLAM,[16] pursue multiple data association hypotheses at the same time to get more accurate data association in a large-scale complex environment. These algorithms are more robust, but the computational complexity still increases exponentially with the number of hypothesis. Neira put forward a gold standard method, namely joint compatibility branch and bound (JCBB) algorithm. Compared with other methods, it considers the association possibility between all measurements and map features to guarantee the robustness and accuracy of data association. However, JCBB algorithm has two drawbacks: (1) the joint compatibility (JC) criterion expands the covariance matrix at the initially estimated pose; hence, it is susceptible to linearization errors when the pose is estimated inaccurately; (2) the computation cost increases exponentially with the increase of the measurements because the JC should be constantly calculated. And branch and bound search algorithms require longer operation time. With the expansion of the environment, the environmental features increase obviously, and the run-time of JCBB algorithm will be significantly increased. Hence, JCBB algorithm is inappropriate to be applied to outdoor environment with a large number of environmental features.

In order to reduce the complexity of JCBB algorithm, Li et al. proposed a posterior-distribution-based JC test scheme.[17] Ref. [18] discovered a special structure in the feature cloud matching problem and proposed a fast JCBB method. Guo et al. proposed a fast JCBB algorithm, which reduces the computational complexity of JCBB by setting the upper bound on the number of joint compatible pairs.[19] The upper limit of the number of pairs is between 6 and 10, but the upper bound of the number of pairs is experiences data. To overcome this problem, Wu et al. proposed an optimized data association algorithm. In the optimized JCBB algorithm, the upper limit of pairings is determined according to the environment and batch order.[20] A data association algorithm based on K-means clustering and JCBB is proposed in ref. [21]. In this algorithm, the K-means clustering method is used to group the measurements, but the number of groups is determined by the environment. The above three methods ensure higher association accuracy; they also reduce the computational complexity of data association. However, the selection of the number of pairs depends on the distribution of environmental features in refs. [19] and [20]; they do not show how the measurements should be divided after the upper limit of pairings is set. In ref. [21], all measurements of the current moment are divided into K-means clustering; K value needs to be given in advance in the K-means algorithm; however, it is difficult to determine the K value in practical applications and the selection of K value has a great influence on the clustering results. Since measurements have a clear distribution for most environments, a density-based clustering algorithm, called OPTICS, is used to group measurements in the proposed algorithm.

In order to reduce the influence of linearization error on JC test, ref. [17] uses the iterated extended Kalman filter (IEKF) to compute the posterior JC. For the obvious nonlinear observation equation, IEKF is more effective than extended Kalman filter (EKF) due to iterative computation. But it also suffers from a number of drawbacks, namely the problem caused by the calculation of the Jacobian matrices and the linear approximations of the nonlinear functions. Hence, central difference joint compatibility (CDJC) criterion is set without the calculation of Jacobian matrices and the linearization error.

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In summary, a SLAM data association algorithm based on CDJC criterion and OPTICS method is proposed. It will be abbreviated as OCDJCBB. OCDJCBB algorithm has three characteristics.

(1) The local associated strategy is introduced, that is, the local map is divided according to the pose of the mobile robot and the effective observation range of the sensor, and map features that need to be associated are limited to the local map.
(2) All measurements of the current moment are divided into groups with small correlation degree based on the OPTICS method, CDJCBB algorithm is executed at each group, and CDJC criterion is designed to calculate joint Mahalanobis distance.
(3) The association solutions of each group are combined to get the final association results at the current moment.

The simulation results based on the simulator and a benchmark dataset verify that the association performance and computational efficiency of the proposed association algorithm are better than those of JCBB and SCNN.

The rest of this paper is organized as follows. In Section 2, the data association problem in SLAM and the computational complexity analysis of JCBB are reviewed. The proposed association algorithm is presented in Section 3. The simulation results based on the simulator and a benchmark dataset are presented in Sections 4 and 5, respectively. The conclusions are given in the final section.

2. Related Work

2.1. *Description of data association problem in SLAM*

As a difficult problem in SLAM, the data association problem occurs at the prediction stage. If the data association is known, the SLAM problem will become simple. But in the real-world environment, the correspondence between measurements and environmental features is rarely known. Suppose there are n environmental features in the constructed map, which are the $\{F_1, F_2, ..., F_n\}$. There are m measurements from the latest scan at the current time, which are $\{O_1, O_2, ..., O_m\}$. The association hypothesis is expressed as follows:

$$C_t = \{c_1, c_2, ..., c_m\} \tag{1}$$

If the observed feature O_i is matched with the feature F_j of the constructed map, the association can be obtained, namely $c_i = j$. If the observed feature is not matched with all the features of the constructed map, then $c_i = 0$, it means that new environmental features have been observed or the observed feature is false alarm. The data association problem of SLAM is defined as:

$$\hat{d}_t \triangleq \text{argmax}(P(d_t|\boldsymbol{x}_t, M, \boldsymbol{z}_{1:t}, \boldsymbol{u}_{1:t})),\ d_t \subseteq C_t \tag{2}$$

where d_t represents a set of decisions on data association obtained at time t. $\hat{d}_t$ is the optimal association set which is obtained by an association algorithm. $\boldsymbol{x}_t$ stands for the pose of mobile robot and M describes the constructed map. $\boldsymbol{z}_{1:t}$ is the set of all measurements $\{z_1, ..., z_t\}$ up to time t, $\boldsymbol{u}_{1:t} = \{u_1, ..., u_t\}$ is the control vectors up to time $t.C_t$ is a set of hypotheses at time t.

In the process of data association, a map feature can produce only one observation, and an observation can only be associated with a map feature.

2.2. *JCBB algorithm*

JCBB algorithm is an association algorithm which combines the JC criterion with the branch and bound method. It traverses the interpretation tree in search of the hypothesis with the largest number of non-null jointly compatible pairings and the optimal solution of data association can be obtained by combining branch and bound method. The core idea of JCBB is to extend the idea of IC to JC of multiple measurements and features. In JCBB, the compatibility between all measurements and map features is taken into account, and the branch and bound search algorithm is used to search the

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association solution space C_t, and then the optimal data association set $\hat{d}_t$ is obtained. The definition of JC criterion is as follows:

$$D_{C_t} = \boldsymbol{v}_{C_t}^T \cdot \boldsymbol{S}_{C_t}^{-1} \cdot \boldsymbol{v}_{C_t} \leq \chi^2_{\dim(\boldsymbol{v}_{C_t}),1-\alpha} \tag{3}$$

$$\boldsymbol{v}_{C_t} = \boldsymbol{z}_{C_t} - \hat{\boldsymbol{z}}_{C_t} \tag{4}$$

$$\boldsymbol{S}_{C_t} = \nabla \boldsymbol{h}_{C_t} \cdot \boldsymbol{P}_{C_t} \cdot \nabla \boldsymbol{h}_{C_t}^T + \boldsymbol{R}_{C_t} \tag{5}$$

$$\hat{\boldsymbol{z}}_{\boldsymbol{H}_t} = h_{\boldsymbol{H}_t}\left(\hat{\boldsymbol{x}}_v, \hat{\boldsymbol{F}}_{\boldsymbol{H}_t}\right) = \left[h\left(\hat{\boldsymbol{x}}_v, \hat{f}_{h_1}\right) \ldots h\left(\hat{\boldsymbol{x}}_v, \hat{f}_{h_m}\right)\right]^T \tag{6}$$

where $\hat{\boldsymbol{x}}_v$ is the predictive pose of mobile robot. The jointly predicted locations of the observed map features are $\hat{z}_{C_t}$. $\boldsymbol{R}_{C_t}$ is the covariance of measurement noise. $\boldsymbol{v}_{C_t}$ is the joint innovation. $\boldsymbol{S}_{C_t}$ is the innovation covariance. D_{C_t} is the joint Mahalanobis distance. It obeys the distribution of degrees of freedom for $\dim(\boldsymbol{v}_{C_t})$, $1-\alpha$ is a confidence level, and it usually takes 0.95 or 0.99. When (3) is established, it is shown that the associated solution C_t satisfies the JC condition.

The key of the JCBB algorithm is to use the JC condition as the criterion of traversal of association interpretation tree branch to enlighten the new nodes; monotone non-decreasing rule of number of pairs is considered as boundary condition to discard old nodes that cannot export optimal solutions. Finally, the maximum association hypothesis is searched as the optimal association solution.

2.3. Computational complexity analysis of JCBB

The process of searching association solutions with JCBB algorithm is actually a graph searching process of association tree. Suppose there are m sensor measurements at time t, and the constructed map contains n features. The complexity of JCBB algorithm can be divided into the following two parts:

(1) The time complexity of JCBB algorithm is considered as a whole, and it is mainly determined by the size of the associated search space. The search space size is $\prod_{i=1}^{m}(n_i+1)$, where n_i expresses the number of map features that may be associated with the ith measurement after filtering through the threshold ($n_i \leq n$). Hence, the time complexity of JCBB is defined as $O(mn + \prod_{i=1}^{m}(n_i+1))$.[7]

(2) When performing JC calculations, the computational quantities of matrices such as $\hat{z}_{C_t}$. $\boldsymbol{v}_{C_t}$ and $\boldsymbol{S}_{C_t}$ are related to the number of measurements (m). Especially, the computational complexity of the inverse operation of $\boldsymbol{S}_{C_t}$ is $O(m^3)$ in the calculation of joint Mahalanobis distance.[13]

In conclusion, the time complexity of the JCBB algorithm depends not only on the number of measurements (m) at the current moment but also on the number of features (n) of the constructed map.

3. Proposed Data Association Method

As the core of JCBB algorithm, the JC criterion takes into account the compatibility between all measurements and map features. Therefore, when the environmental features are dense and the observation noise is large, JCBB algorithm can obtain reliable association results. The correct association results can be obtained even when there is no odometer data and the location error is large. However, the time complexity of the JCBB algorithm has an exponential relation with the number of observations, which affects its real-time application in the SLAM process. But beyond that, the JC criterion used by JCBB is easy to be affected by the linear approximations.

In order to solve the problem of JCBB algorithm, four improvements are made to the JCBB algorithm. The improvements are as follows: (1) the local association region is set to reduce the dimensionality of the map features involved in the association, (2) the CDJC criterion is designed to calculate joint Mahalanobis distance, (3) OPTICS method is used to group measurements, and (4) the augmented threshold is set up to distinguish the newly visited features and the noise data.

3.1. Local associated strategy

According to the pose of the mobile robot and the effective observation range of the sensor, the local map involved in the data association is set. The definition of the local association region can be expressed as follows:

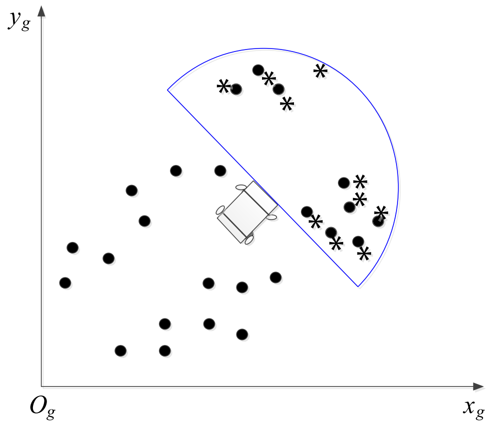

Fig. 1. Schematic diagram of local association.

$$\begin{cases} abs(x_i - x_v) < (r + d) \\ abs(y_i - y_v) < (r + d) \\ (x_i - x_v) \cdot \cos\theta + (y_i - y_v) \cdot \sin\theta > 0 \\ (x_i - x_v)^2 + (y_i - y_v)^2 < (r + d)^2 \end{cases} \tag{7}$$

where (x_v, y_v, θ) represents the pose of mobile robot, represents the coordinates of a feature in a constructed map, r is the effective scanning sensor distance, and d represents the compensation distance. The significance of adding the compensation distance is that the radius of the local map is slightly larger than the scanning distance of the sensor so as to cover all the features that may be matched with the measurements. The purpose of local association is that, regardless of the number of features in the global map, the complexity of the data association is only related to the number of features present in the local map.

As shown in Fig. 1, the semicircular region is plotted by taking the pose of mobile robot as the center of the circle and taking the sensor scanning distance as the radius. The dot (.) represents the environmental features of the constructed map, and the asterisk (*) indicates the environmental features observed by sensor at current time.

3.2. CDJC criterion

In traditional JCBB algorithm, the JC criterion is usually designed by the EKF. In our paper, central difference transform (CDT) is used to compute the joint innovation and innovation covariance without the calculation of Jacobian matrices of an observation model and without linearization error. The joint Mahalanobis distance is calculated as follows.

At time t, there are n environmental features, which are the $\{F_1, F_2, \ldots F_j, \ldots F_n\}$. And they are involved in joint compatible computing. An augmented stochastic state vector $\boldsymbol{x}_{t|t-1}^{M,[j]}$ is defined as follows:

$$\boldsymbol{x}_{t|t-1}^{M,[j]} = \begin{bmatrix} \boldsymbol{x}_{V,t|t-1}^{M} \\ \boldsymbol{x}_{F_j,t|t-1}^{M} \end{bmatrix} \tag{8}$$

and the covariance matrix:

$$\boldsymbol{P}_{t|t-1}^{M,[j]} = \begin{bmatrix} \boldsymbol{P}_{V,t|t-1}^{M} & 0 \\ 0 & \boldsymbol{P}_{F_j,t|t-1}^{M} \end{bmatrix} \tag{9}$$

where $\boldsymbol{x}_{V,t|t-1}^{M}$ is the predicted pose of mobile robot with respect to a base reference frame M, $\boldsymbol{x}_{F_j,t|t-1}^{M}$ is the predicted location of the jth feature, $\boldsymbol{P}_{V,t|t-1}^{M}$ is the predicted error covariance of the pose of mobile robot, and $\boldsymbol{P}_{F_j,t|t-1}^{M}$ is the predicted error covariance of the location of the jth feature.

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The CDT has been proposed which allows the sigma points to be scaled to an arbitrary dimension.[22] A symmetric set of $2L+1$ sigma points for the augmented state vector with L-dimensional state vector is given by:

$$\chi_{t|t-1}^{M,[j],[i]} = \begin{bmatrix} \chi_{V,t|t-1}^{M,[j],[i]} \\ \chi_{F_j,t|t-1}^{M,[j],[i]} \end{bmatrix} \tag{10}$$

$$\chi_{t|t-1}^{M,[j]} = \begin{cases} \boldsymbol{x}_{t|t-1}^{M,[j]} & i = 0 \\ \boldsymbol{x}_{t|t-1}^{M,[j]} + \left(\boldsymbol{h} \cdot \sqrt{\boldsymbol{P}_{t|t-1}^{M,[j]}}\right)_i & i = 1, 2, ..., L \\ \boldsymbol{x}_{t|t-1}^{M,[j]} - \left(\boldsymbol{h} \cdot \sqrt{\boldsymbol{P}_{t|t-1}^{M,[j]}}\right)_i & i = L+1, ..., 2L \end{cases} \tag{11}$$

where the random variable $\chi_{t|t-1}^{M,[j],[i]}$ contains two components: state component of mobile robot $\chi_{V,t|t-1}^{M,[j],[i]}$ and observed component $\chi_{F_j,t|t-1}^{M,[j],[i]}$. h is the central difference half step length. For Gaussian distribution, the optimal value of h is $\sqrt{3}$. Then, the measurement sigma points are transformed through the nonlinear observation model:

$$\bar{N}_t^{[j],[i]} = y\left(\chi_{F_j,t|t-1}^{M,[j],[i]}\right) + \chi_{F_j,t|t-1}^{M,[j],[i]} \tag{12}$$

where $y(\cdot)$ is the nonlinear observation equation.

The predicted measurement $\bar{z}_t^{[j]}$ and the innovation covariance S_t are given by:

$$\bar{z}_t^{[j]} = \sum_{i=0}^{2L} \omega_i^m \bar{N}_t^{[j],[i]} \tag{13}$$

$$\begin{aligned} \boldsymbol{\zeta}_{1i}^{[j]} &= \sqrt{\omega_i^{c_1}} \cdot \left(\bar{N}_t^{[j],[i]} - \bar{N}_t^{[j],[i+L]}\right) \\ \boldsymbol{\zeta}_{2i}^{[j]} &= \sqrt{\omega_i^{c_2}} \left(\bar{N}_t^{[j],[i]} + \bar{N}_t^{[j],[i+L]} - 2 \cdot \bar{N}_t^{[0]}\right) \end{aligned} \tag{14}$$

$$\begin{aligned} \boldsymbol{C}_t &= \left[\boldsymbol{\zeta}_{1i}^{[1]} \ldots \boldsymbol{\zeta}_{1i}^{[j]} \ldots \boldsymbol{\zeta}_{1i}^{[n]}\right]^T \\ \boldsymbol{D}_t &= \left[\boldsymbol{\zeta}_{2i}^{[1]} \ldots \boldsymbol{\zeta}_{2i}^{[j]} \ldots \boldsymbol{\zeta}_{2i}^{[n]}\right]^T \end{aligned} \tag{15}$$

The corresponding mean sequence weight ω_i^m and variance sequence weight $\omega_i^{c_1}$, $\omega_i^{c_2}$ of each sampling point are defined as:

$$\omega_0^m = \frac{(h^2 - L)}{h^2}, \omega_i^m = \frac{1}{2 \cdot h^2} \quad i = 1, ...2L \tag{16}$$

$$\omega_i^{c_1} = \frac{1}{4 \cdot h^2}, \omega_i^{c_2} = \frac{(h^2 - 1)}{4 \cdot h^4} \quad i = 1, ...2L \tag{17}$$

The joint innovation and joint innovation covariance are calculated as follows:

$$\boldsymbol{v} = \begin{bmatrix} z_t(1) \\ \vdots \\ z_t(j) \\ \vdots \\ z_t(n) \end{bmatrix} - \begin{bmatrix} \bar{z}_t^{[1]} \\ \vdots \\ \bar{z}_t^{[j]} \\ \vdots \\ \bar{z}_t^{[n]} \end{bmatrix} \tag{18}$$

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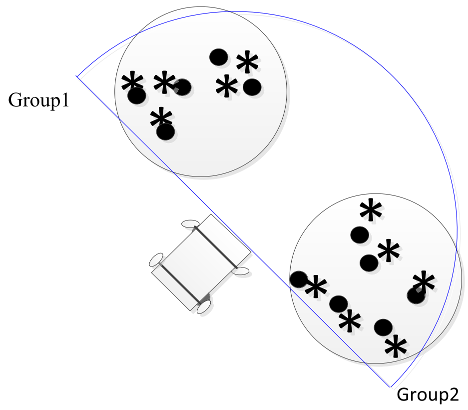

Fig. 2. Schematic diagram of measurements grouping.

$$\boldsymbol{S}_t = C_t \cdot C_t^T + D_t \cdot D_t^T + \begin{bmatrix} R & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R \end{bmatrix}_{n\times n} \tag{19}$$

The central difference joint compatibility (CDJC) criterion is defined as follows:

$$D_{\boldsymbol{H}_k} = \boldsymbol{v}^T \cdot \boldsymbol{S}_t^{-1} \cdot \boldsymbol{v} \leq \chi^2_{\dim(v_{H_t}),1-\alpha} \tag{20}$$

3.3. OPTICS method is used to group measurements

The measurements obtained by sensors are usually clearly distributed when mobile robot travels autonomously in outdoor large-scale environment. As shown in Fig. 2, the current measurements in group 1 and group 2 are distributed on both sides of mobile robot, and they are not significant correlation. In the traditional JCBB algorithm, the JC between all measurements and map features will be computed, and this makes computation extremely large. If the current measurements are divided into two groups for batch association, then the measurement dimension of each JCBB will be reduced, and the computational complexity of the algorithm will be reduced. Therefore, the clustering method is used to group the measurements at each moment in proposed association algorithm.

Up to now, the academic does not have a generally agreed definition of the clustering. The definition of clustering given by ref. [23] is given here: the entities within a cluster are similar, entities of different clusters are not alike. A cluster is the convergence of the midpoint in the test space, and the distance between any two points of the same cluster is less than the distance between any two points of different clusters. A class cluster can be described as a connected region in a multidimensional, and this connected region contains a set of points with higher density. Clustering algorithms are broadly divided into several categories, such as hierarchical clustering algorithm, partitioning clustering algorithm, a clustering algorithm based on density and grid, and a clustering algorithm based on model. In ref. [21], the measurements are grouped by K-means clustering method. K-means clustering algorithm is a typical partition clustering approach and it often finds spherical shaped clusters only. However, the size, shape, and number of environmental features observed by sensors at each moment are different in SLAM; the K value in this method is difficult to determine. Hence, the density-based clustering algorithm is used to classify the measurements in this paper.

Density-based clustering algorithm is fundamentally different from other clustering algorithms. Density-based clustering can discover clusters with arbitrary shapes. DBSCAN[24] and OPTICS[25] are two typical density-based clustering algorithms. The basic idea of the DBSCAN can be simply described as: define the ε-neighborhood of a point and minimum number of data points *MinPts* in one cluster. Picking an arbitrary unvisited point p, if its ε-neighborhood contains at least *MinPts* points, a cluster is formed and the points in the ε-neighborhood are added into the cluster. The above processes have been carried out until all of the points have been processed. There are two initial parameters, ε (neighborhood radius) and *MinPts* that require the user to manually set up in DBSCAN. Moreover, the clustering results are very sensitive to the values of these two parameters, and different values will lead to different clustering results. In order to overcome this shortcoming, OPTICS was proposed.

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Table I. Algorithm for grouping measurements based on optics.

Algorithm. Grouping measurements for time t based on OPTICS.

1: Begin
2: Input:
x - data matrix; namely the measurements at time t $\boldsymbol{O}_t = \{o_i | i = 1....m\}$.
MinPts - minimal number of objects considered as a cluster
3: The distance curve is obtained according to x and *MinPts*, and ε is determined.
4: All noise points are extracted and they are divided into a group.
5: Variables that belong to noise points are removed, and then the matrix x_1 is formed by the remaining variables.
6: $[RD, CD, order] = \text{OPTICS}(x_1, MinPts)$, RD represents the reachability distances, CD represents the core distances, *order* represents the specified ordering of objects.
7: The reachability-plot of search results is got based on RD and *order*. n clusters are finally achieved by recognizing the dented regions according to peak values of the reachability-plot.
8: The measurements for time t are divided into $n + 1$ groups.
9: End

While expanding a cluster, OPTICS selects each point to be expanded in increasing order of its distance to the current cluster and changes ε for identifying ε-neighborhood gradually. OPTICS does not display clustering result; it generates an augmented cluster sorting for clustering analysis. Hence, the measurements of each moment are divided into several groups by OPTICS algorithm to solve data association problem of SLAM. The algorithm for grouping measurements based on OPTICS is described in Table I.

3.4. Setting up the augmented threshold

In the traditional JCBB algorithm, the identity of revisited features and newly visited features is obtained by optimal association set C_{best}. If $c_i = j$, it indicates that the observed feature O_i is matched with the feature F_j of the constructed map. If $c_i = 0$, it indicates that the observed feature is a newly visited feature, but this observed feature may be a noise data. Therefore, an augmented threshold G_a is set up in proposed association algorithm. When $c_i = 0$, the Mahalanobis distance between O_i and each feature of the constructed map is calculated, and the smallest distance $nis_{i,j}$ of these distances is selected. If $nis_{i,j} > G_a$, O_i is regarded as a newly visited feature. If $G_d < nis_{i,j} < G_a$, O_i is regarded as a noise data, where G_d represents the association threshold. In summary, the description diagram of the proposed association algorithm is shown in Fig. 3.

The proposed association algorithm has several improvements compared to the traditional JCBB algorithm.

(1) Based on the local association strategy, the set of map features involved in the association is changed from $F_t = \left\{f_j | j = 1...n\right\}$ to $\boldsymbol{F}_t^{'} = \left\{f_j | j = 1...N\right\}$. N represents the number of map features in the local association region ($N <$ n).
(2) The measurements $\boldsymbol{O}_t = \{o_i | i = 1...m\}$ of the time t are divided into M groups, which can be represented as: $\boldsymbol{O}_t^1 = \{o_i | i = 1...m_1\} \ldots \boldsymbol{O}_t^l = \{o_i | i = 1...m_l\} \ldots \boldsymbol{O}_t^M = \{o_i | i = 1...m_M\}$, where l indicates the group number, $m = m_1 + ... + m_l + ... + m_M$.
(4) The size of the search space is changed from $\prod_{i=1}^{m}(n_i+1)$ to $\prod_{i=1}^{m_1}(N_i+1)+...+\prod_{i=1}^{m_M}(N_i+1)$, and the overall complexity of the proposed association algorithm is changed from $O(mn + \prod_{i=1}^{m}(n_i+1))$ to $O(mN + \prod_{i=1}^{m_1}(N_i+1)+...+\prod_{i=1}^{m_M}(N_i+1))$. N_i represents the number of map features associated with the ith observed feature after threshold filtering in the local association region. The computational complexity of the joint Mahalanobis distance in each association group is reduced because it depends on m_l and $m_l < m$.
(4) The accuracy of the JC criterion has improved by CDT.

In conclusion, the OCDJCBB algorithm can improve the computation efficiency and reduce the time complexity while ensuring reliable association results.

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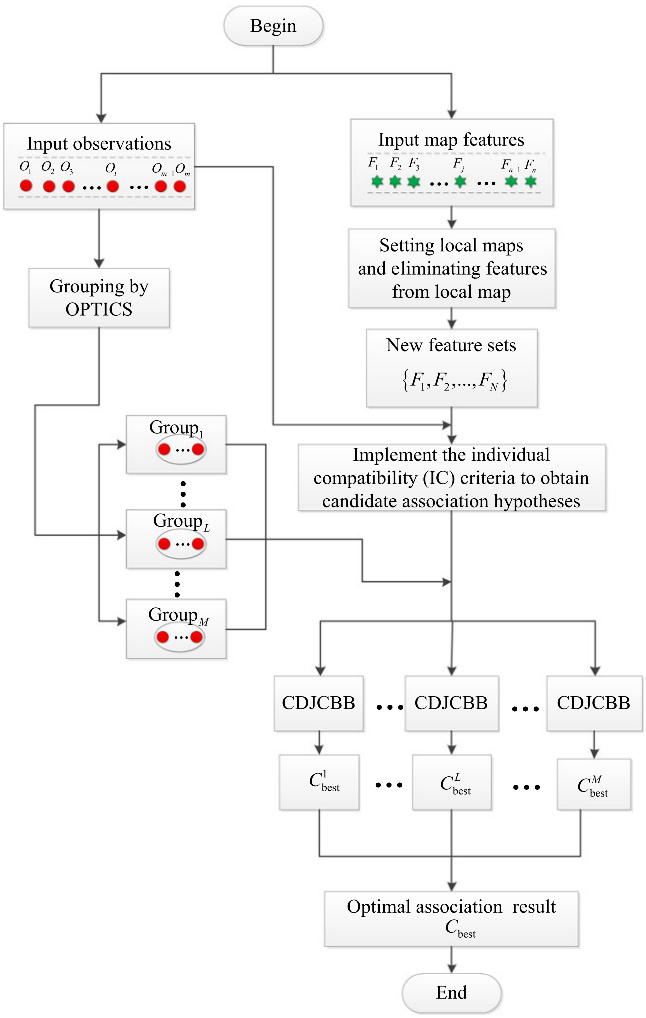

Fig. 3. Description diagram of the proposed associated algorithm.

4. Simulation Results Based on Simulator

4.1. Simulation model

The simulation model adopts the motion model of intelligent vehicle. It can be represented as:

$$\boldsymbol{x}_v(t) = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v \cdot \Delta t \cdot \cos(\theta_{t-1} + G) \\ y_{t-1} + v \cdot \Delta t \cdot \sin(\theta_{t-1} + G) \\ \theta_{t-1} + v \cdot \Delta t \cdot \tan(G)/L \end{bmatrix} + \boldsymbol{w}_{t-1} \tag{21}$$

where (x_t, y_t, θ_t) is the vehicle states, Δt is sampling time of the sensor, L is the distance between wheel axles, $\boldsymbol{v}$ is the velocity, and G is the steering angle. $\boldsymbol{w}_{t-1}$ refers to various errors in the process of vehicle movement, which is caused by control noise. The control noise of Eq. (21) is described in Table II.

The observation model is described as follows:

$$z_{j,t} = \begin{bmatrix} \sqrt{(\boldsymbol{x}_t - \boldsymbol{x}_j)^2 + (\boldsymbol{y}_t - \boldsymbol{y}_j)^2} \\ \arctan[(\boldsymbol{y}_t - \boldsymbol{y}_j) / (\boldsymbol{x}_t - \boldsymbol{x}_j) - \theta_t \end{bmatrix} + v_t \tag{22}$$

where $z_{j,t}$ is the observation vector, (x_j, y_j) is coordinates of the j th map feature. $\boldsymbol{v}_t$ is Gaussian white noise with covariance of R. The observation noise of Eq. (22) is described in Table II.

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Table II. Parameters used in simulation.

Parameters	**Values**
Vehicle speed	3 m/s
Maximum steering angle	30°
Vehicle wheel-base	2.25 m
Maximum range	30 m
Control noise	(0.3 m/s, 3°)
Observation noise	(0.1 m, 1°)
Control frequency	40 Hz
Observation frequency	5 Hz

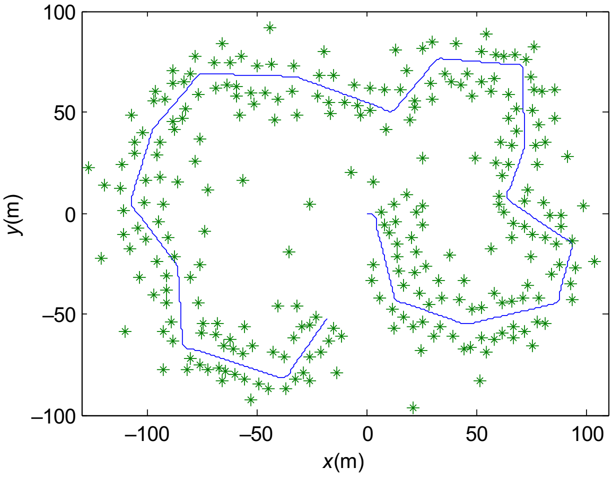

Fig. 4. Simulation environment.

4.2. *Simulation environment*

Bailey et al. developed the SLAM simulator and opened it to the public on a Web site.[26] This simulator makes the comparison of different SLAM algorithms possible. As shown in Fig. 4, a simulation environment is designed based on this simulator. The SLAM algorithm based on OCDJCBB association method (OCDJCBB-SLAM) is compared with the SLAM algorithm based on JCBB (JCBB-SLAM) and the SLAM algorithm based on SCNN (SCNN-SLAM) in order to verify the performance of OCDJCBB association method in this simulation environment. In the three SLAM algorithms, EKF is used to estimate the pose of intelligent vehicle and feature position in state estimation stage.

In Fig. 4, suppose that the intelligent vehicle starts at the initial state of $X_v(0)=[0, 0, 0]^T$ and moves uniformly around an irregular region of the 240*200 m^2. The blue line represents vehicle trajectory, "*" denotes the landmark location. Simulation parameters are shown in Table II.

4.3. *Computational complexity*

In the same simulation environment, 50 experiments are performed on three SLAM algorithms. The computational cost of the three SLAM algorithms is compared using Matlab simulations on an Intel(R) Core(TM)i5-3470 CPU@3.2GHz PC. The CPU time of implementing each SLAM algorithm and the CPU time of the three association algorithms performed in the SLAM process are utilized to evaluate computational complexity. The average CPU time calculated over 50 simulation runs with same simulation parameters is shown in Table III.

As shown in Table III, the associated process of the OCDJCBB algorithm is less than that of the other two algorithms, because the OCDJCBB association algorithm is performed only in the local map and the time complexity of the algorithm is only related to the number of features in the local map. In addition, the OPTICS method is used to group the measurements of the current moment in

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Table III. Running time of three algorithms.

Algorithm	CPU time of SLAM algorithm (s)	CPU time of associated process(s)
OCDJCBB-SLAM	62.661	32.016
SCNN-SLAM	262.322	234.256
JCBB-SLAM	286.626	259.289

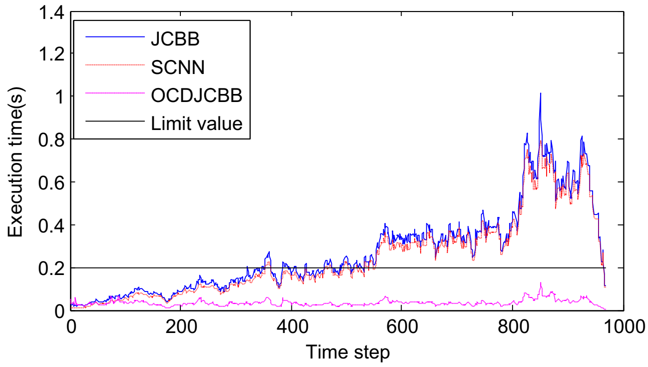

Fig. 5. Execution time of each step with three data association algorithms.

the proposed association algorithm, and then the dimensionality of the observation vectors is reduced when the CDJCBB is executed. Since the proposed association algorithm has the least execution time, the execution time of OCDJCBB-SLAM algorithm based on OCDJCBB is much less than the execution time of other two SLAM algorithms.

Figure 5 shows the execution time of each step with the OCDJCBB algorithm, JCBB algorithm, and SCNN algorithm. As shown in Fig. 5, the execution time of both the SCNN algorithm and the JCBB algorithm increases with the simultaneous localization and building of the intelligent vehicle, but the execution time of each step with SCNN algorithm is less than that of JCBB algorithm. After the 500th step, the execution time of each step with JCBB algorithm and SCNN algorithm is all over 0.2 s. Hence, these two algorithms could not meet the real-time requirement in the simulation environment shown in Fig. 4, because the observation frequency is 0.2 s.

Because of the restriction of local association region and the use of clustering algorithm, the execution time of OCDJCBB algorithm changes smoothly in comparison with JCBB algorithm and SCNN algorithm. The execution time of each step with OCDJCBB algorithm is stable at about 0.04 s, which is far less than the observation frequency. Therefore, the improved association algorithm can meet the real-time requirement of SLAM in large-scale dense environment.

4.4. Association performance

Figure 6 shows the data association performance of OCDJCBB algorithm, JCBB algorithm, and SCNN algorithm, respectively. There are four indexes for evaluating the performance of data association, namely true positive rate (TPR), true negative rate (TNR), false positive rate (FPR), and false negative rate (FNR).[27–28] They are defined as follows:

$$\mathrm{TPR} = \mathrm{TP}/\,(\mathrm{TP}+\mathrm{TN}+\mathrm{FP}+\mathrm{FN}) \tag{23}$$

$$\mathrm{TNR} = \mathrm{TN}/\,(\mathrm{TP}+\mathrm{TN}+\mathrm{FP}+\mathrm{FN}) \tag{24}$$

$$\mathrm{FPR} = \mathrm{FP}/\,(\mathrm{TP}+\mathrm{TN}+\mathrm{FP}+\mathrm{FN}) \tag{25}$$

$$\mathrm{FNR} = \mathrm{FN}/\,(\mathrm{TP}+\mathrm{TN}+\mathrm{FP}+\mathrm{FN}) \tag{26}$$

where true positive(TP) denotes the number of association pairs detected by the association algorithms. True negative (TN) denotes the number of new environment features that are detected. False positive (FP) denotes the number of association pairs that are wrongly checked by the associated

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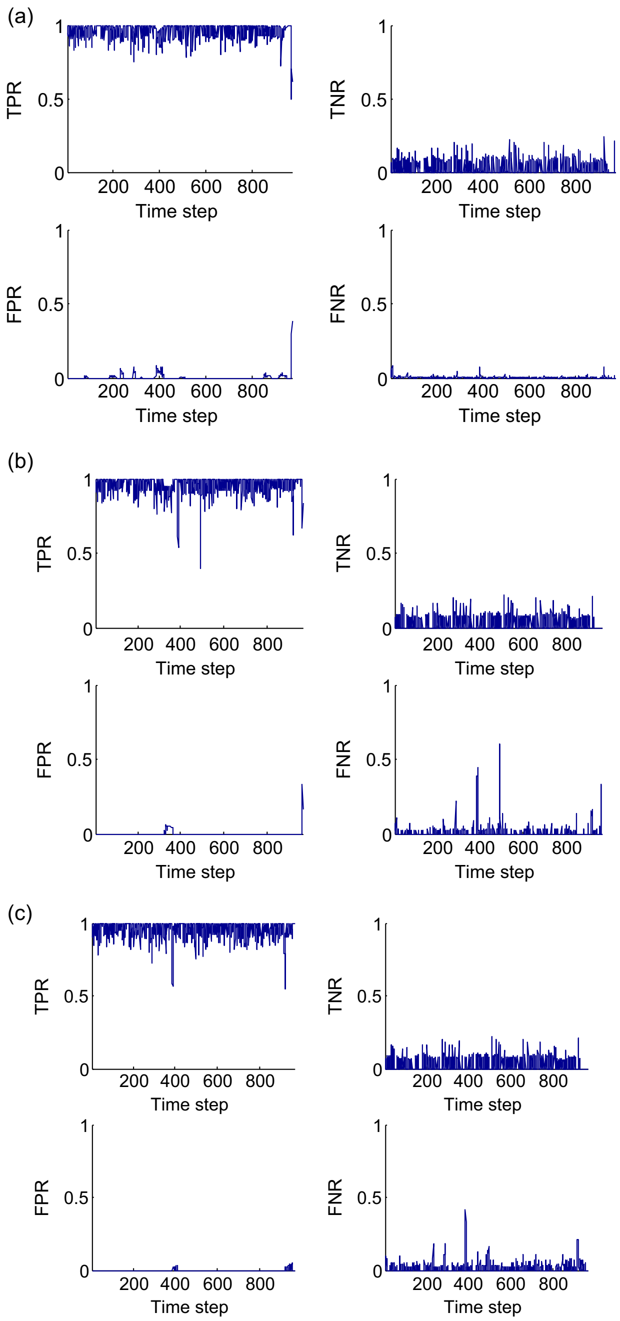

Fig. 6. Association performance of three algorithms. (a) OCDJCBB, (b) JCBB, and (c) SCNN.

algorithm and FNR denotes the number of association pairs that are omitted by the associated algorithm. The sum of TP, TN, FP, and FN denotes the number of all association pairs.

Figure 6 shows the association performance of three algorithms. In order to compare the association performance of each algorithm more easily, the average TPR, TNR, FPR, and FNR of each algorithm are calculated, respectively, as shown in Table IV. Since the proposed association algorithm is executed within the local association area, the FPR is slightly higher than the other two algorithms. But the TPR of OCDJCBB algorithm is higher than that of JCBB and SCNN, and it can correctly judge the new landmarks observed at each moment and add them to the map. Not only that, the FNR of OCDJCBB is obviously smaller than the other two algorithms. Table IV and Fig. 6 show that the proposed algorithm outperforms the other two algorithms in association performance.

In order to evaluate the performance of OCDJCBB association algorithm more intuitively, the average association accuracy (AA) of three SLAM algorithms is obtained over 50 Monte Carlo runs

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Table IV. Association performance of three algorithms.

Algorithm	TPR	TNR	FPR	FNR
OCDJCBB	0.9678	0.0233	0.0052	0.0037
JCBB	0.9653	0.0213	0.0024	0.0113
SCNN	0.9613	0.0212	0.0013	0.0162

Table V. Accuracy of three algorithms.

Algorithm	AA
OCDJCBB-SLAM	0.9911
JCBB-SLAM	0.9866
SCNN-SLAM	0.9825

Table VI. Average error for vehicle pose.

Algorithm	X(m)	Y(m)	Heading angle (rad)
OCDJCBB-SLAM	0.2297	0.5321	0.0137
JCBB-SLAM	0.4632	0.6391	0.0141
SCNN-SLAM	0.6212	0.7285	0.0147

under the simulation environment shown in Fig. 4. AA reflects whether the association algorithm can correctly detect the association pairs and find the new environment features, its expression is as follows:

$$\mathrm{AA} = (\mathrm{TP} + \mathrm{TN})/(\mathrm{TP} + \mathrm{TN} + \mathrm{FP} + \mathrm{FN}) \tag{27}$$

The results of Table V show that the accuracy of the three association algorithms is all over 0.98. However, the AA of OCDJCBB algorithm is higher than that of the other two algorithms. Furthermore, the proposed association algorithm has the advantage of low computational complexity. Hence, it is more suitable for SLAM of intelligent vehicle.

4.5. Comparison and analysis of SLAM accuracy

Figure 7 shows the estimation results of SLAM based on three different association algorithms. The blue dotted line represents the estimation path, and the red solid line represents the ideal path for vehicle planning. The green "*" represents the landmark in the environment, and the red circle represents the estimated landmark. As shown in Fig. 7, the vehicle path estimated by the SLAM based on OCDJCBB and JCBB algorithms is more close to the ideal path and the location of the estimated environmental features is more accurate, in comparison with the SLAM based on the SCNN algorithm. As shown in Fig. 8, the pose accuracy of the three algorithms is more intuitively contrasted based on the error curve. Table VI shows the average error for vehicle pose in the X axis, Y axis, and heading angle. Results show that the vehicle's pose error of OCDJCBB-SLAM is less than that of JCBB-SLAM and SCNN-SLAM. The reason is that the association results of the proposed algorithm are more accurate than the other two algorithms, and then the vehicle path and the location of the environmental features are correctly estimated in the estimation stage of SLAM.

The simulation results show that the proposed association algorithm reduces the computational complexity of the whole SLAM system and improves the estimation accuracy of SLAM.

5. Outdoor Environments

The Victoria dataset is the popular and standard dataset in the SLAM community.[29] The dataset was collected by a four-wheeled vehicle, as shown in Fig. 9(a). This vehicle is equipped with Sick

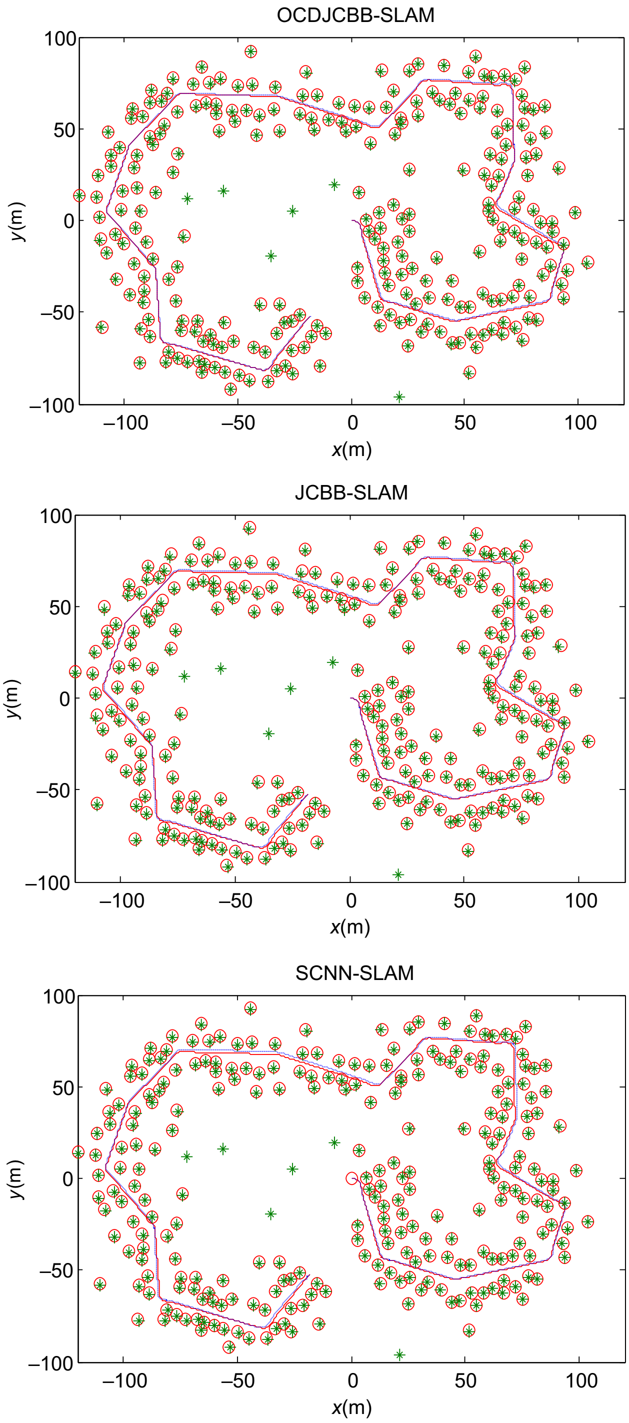

Fig. 7. Estimation results of SLAM based on three association methods.

laser range and bearing sensor, linear variable differential transformer sensor for the steering, back wheel velocity encoder, and GPS. The benchmark Sydney Victoria Park dataset was collected in the Victoria Park; the vehicle was driven around about 30 min and moved over 4 km in mild uneven terrain and different types of surfaces. The environmental landmarks were mostly nearby trees in the Victoria Park. GPS gave intermittent data due to limited satellite availability and these data were used to evaluate the ground truth. Figure 9(b) shows the Victoria Park and trees in Google Earth; the yellow line in the picture represents the ground truth of the vehicle recorded by the GPS.

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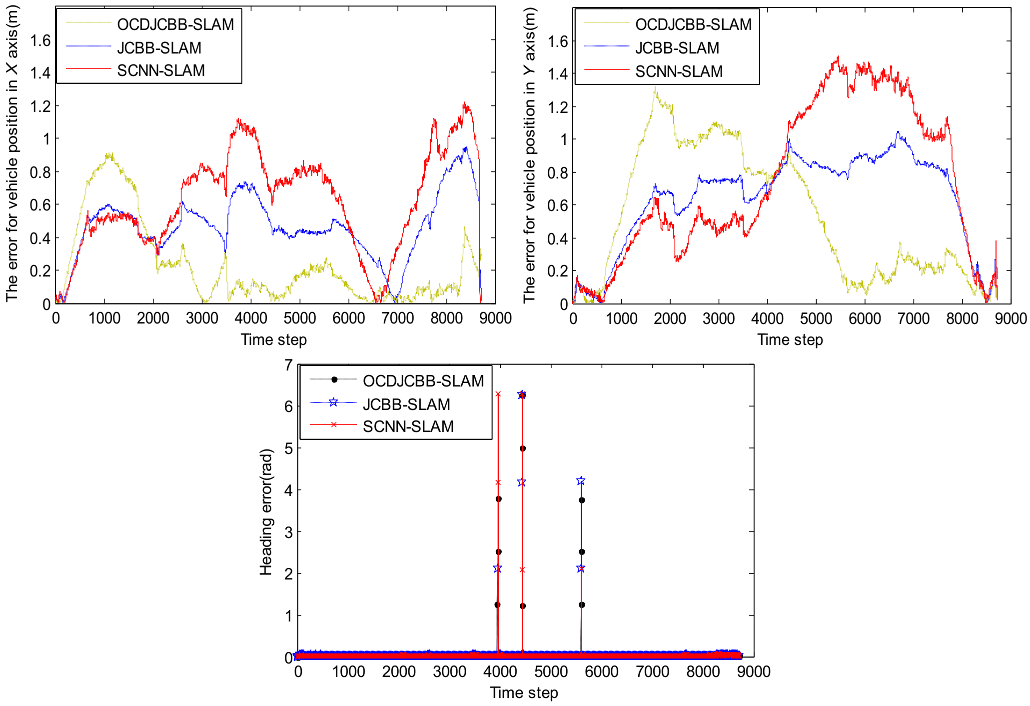

Fig. 8. Comparison of the vehicle's pose error.

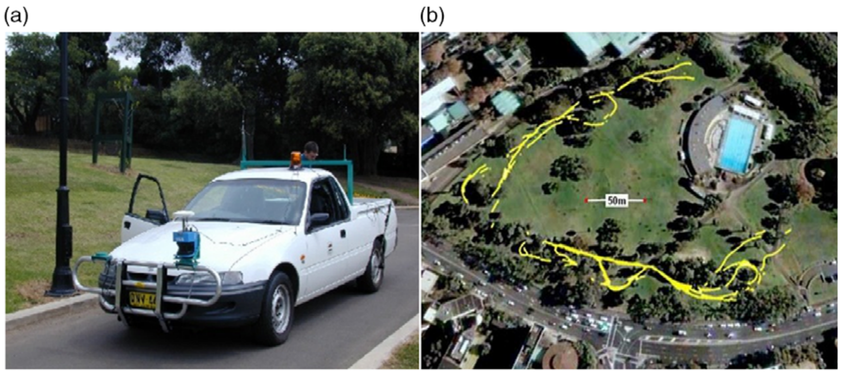

Fig. 9. The four-wheeled vehicle and Victoria Park.

Victoria Park dataset is a challenge for some data association algorithms. In this paper, the SLAM algorithm based on OCDJCBB is compared with the SLAM algorithm based on JCBB and the SLAM algorithms based on SCNN by the Victoria Park dataset. The control noises in the experiment are $\delta_v = 0.8\text{m/s}$, $\delta_G = 1.8°$, and the measurement noises are $\delta_r = 1.5$ m, $\delta_\theta = 2.8°$. The control frequency is 25 ms and the observation frequency is 214 ms. The confidence level is 0.99.

As shown in Fig. 10, the estimated vehicle state and the estimated positions of trees are obtained based on three algorithms. Figure 10 shows that the estimation errors of the OCDJCBB-SLAM algorithm are lower than the two other algorithms because the result of the proposed algorithm (such as the estimated path marked by circles in the figure below) matches with the GPS ground truth better.

The run-time of the three SLAM algorithms and the execution time of the data association method in SLAM are shown in Fig. 11. In this figure, the execution time of the data association based on OCDJCBB algorithm is significantly smaller than the other two association algorithms, and then OCDJCBB-SLAM algorithm cuts the execution time by half compared to the other two SLAM algorithms.

Figure 12 shows the execution time of each association step with three algorithms in experiment on the Victoria Park dataset. As shown in Fig. 12, there are a lot of association moments running longer

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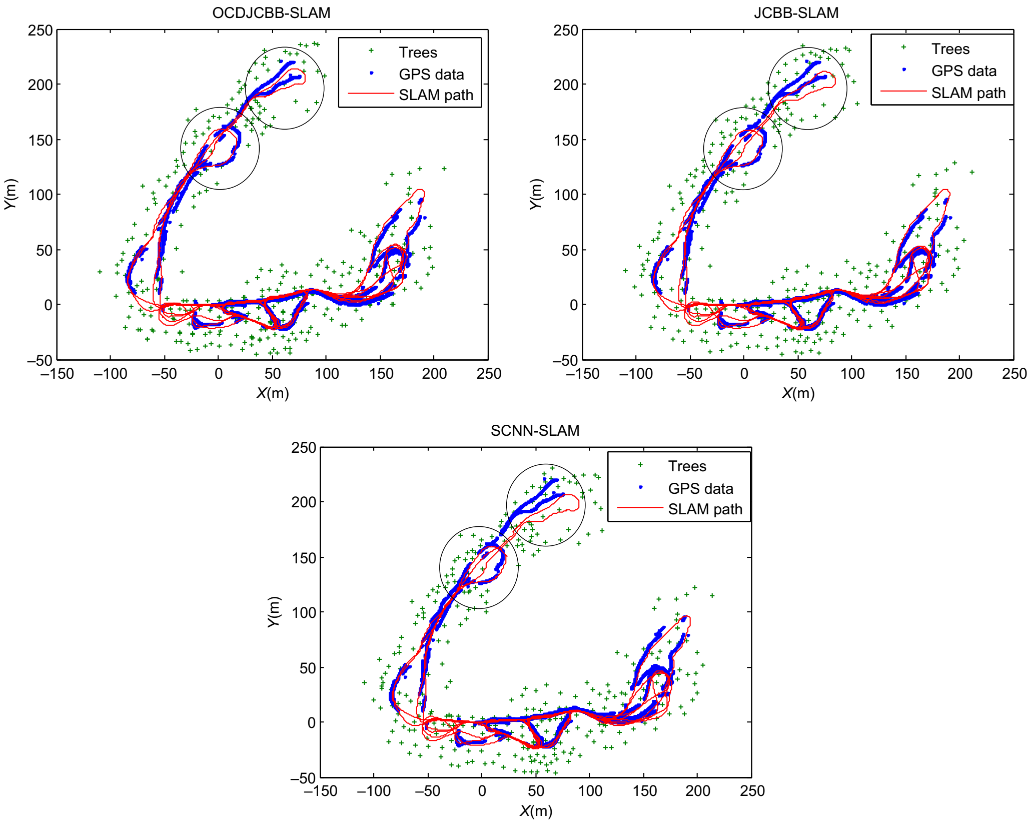

Fig. 10. The estimation results based on three SLAM algorithms. Three different algorithms, OCDJCBB, JCBB, and SCNN, are applied to get association set in SLAM.

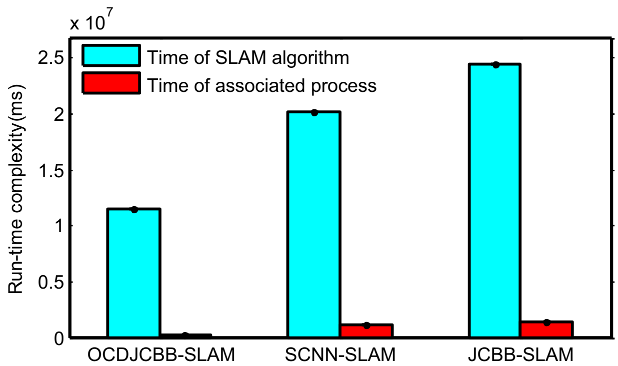

Fig. 11. Running time of three algorithms.

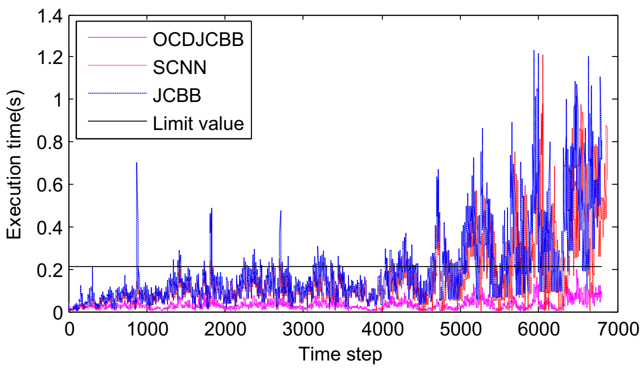

Fig. 12. Execution time of each step with three data association methods.

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than observation frequency (214 ms) with JCBB and SCNN; they are not suitable for EKFSLAM with the Victoria Park dataset. But the execution time of each step with OCDJCBB algorithm is less than 0.214 s. It can meet the real-time requirement in experiment on the Victoria Park dataset.

In summary, the experimental results show that the proposed algorithm can obtain the reliable localization and mapping and greatly reduces the computational complexity. It is suitable for SLAM of intelligent vehicle in large-scale environments.

6. Conclusion

Data association is a difficult problem in SLAM for mobile robot. JCBB algorithm can obtain very reliable association results, but its computational complexity is exponentially related to the number of measurements, which seriously limits its real-time application. A SLAM data association algorithm based on CDJC criterion and clustering is proposed. Partial batch data association and CDJC criterion are adopted in the proposed algorithm. The proposed algorithm has three advantages. Firstly, the dimensionality of the associated map feature is reduced at each moment. Thus, the problem that the computational complexity of the algorithm increases gradually with the expansion of the constructed map is solved. Secondly, it is significantly more accurate on nonlinear problems based on CDT. Finally, the clustering method is used to group the measurements to solve the problem that the complexity of the JCBB algorithm increases exponentially with the increase of the measurements. Experiments based on simulator and benchmark dataset verify these advantages. The results show that the proposed algorithm outperforms JCBB and SCNN on both efficiency and association performance. It can provide reliable guarantee for the real-time and accuracy of SLAM for mobile robot in complex large-scale environments.

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