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FIRM DYNAMICS, ENDOGENOUS MARKUPS, AND THE LABOR SHARE OF INCOME

ANDREA COLCIAGO
De Nederlandsche Bank
and
University of Milan Bicocca

LORENZA ROSSI
University of Pavia

Recent U.S. evidence suggests that the response of labor share to a productivity shock is characterized by *countercyclicality* and *overshooting*. These findings cannot be reconciled easily with existing business cycle models. We extend the Diamond–Mortensen–Pissarides model of search in the labor market by considering strategic interactions among an endogenous number of producers, which leads to countercyclical price markups. Although Nash bargaining delivers a countercyclical labor share, we show that countercyclical markups are fundamental to address the overshooting. On the contrary, we find that real wage rigidity does not seem to play a crucial role in the dynamics of the labor share of income.

Keywords: Labor Share Overshooting, Endogenous Market Structures, Search and Matching Frictions

1. INTRODUCTION

Figure 1 shows the dynamics of the response of the labor share of income, the average product of labor, and the real wage to a one-standard-deviation orthogonalized productivity innovation for the United States in the period 1954.I–2004.IV. Each response function is obtained from a bivariate VAR of order 1 between the variable of interest and the Solow residual. The identification assumption is that the variable of interest has no contemporaneous effect on the Solow residual.

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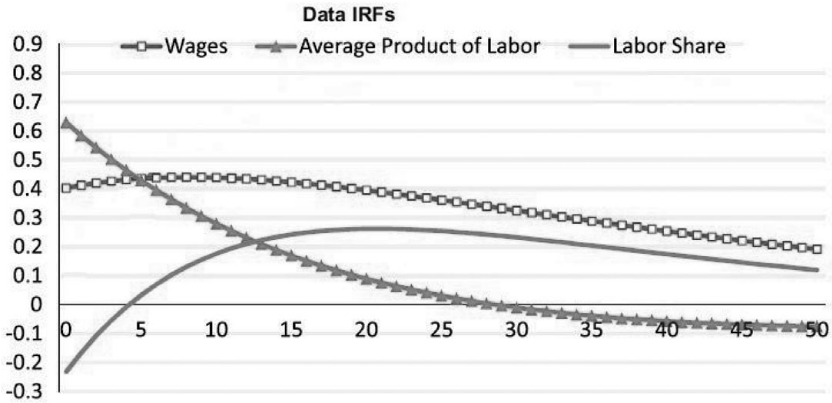

FIGURE 1. Empirical IRFs of wages, average product of labor, and labor share to productivity innovations in the United States. Percentage deviations from long-run averages. *Source*: Rios-Rull and Santaeulàlia-Llopis (2010).

As argued by Rios-Rull and Santaeulàlia-Llopis (2010), the response of the labor share is characterized by *countercyclicality* and *overshooting*. The labor share falls on impact in response to the shock and then shows a hump-shaped response, overshooting its long-run level after five quarters, and peaking at the fifth year at a level higher in absolute terms than the initial drop. Seven years after the peak the labor share is still halfway toward its steady state value.

A model should satisfy two desiderata to account for the response of the labor share to a technology shock displayed in the figure. The first is that the increase in the real wage on impact must be lower than that in average labor productivity. The second is the presence of a persistent wedge between average labor productivity and the real wage, such that the response of the latter rises above that of the former for several periods. The first property implies a countercyclical labor share, whereas the second is necessary for overshooting.

In this paper we build on Colciago and Rossi (2011) to develop a theory of the joint dynamics of the labor share and technology shocks that satisfies both desiderata and replicates the countercyclicality and the overshooting of the labor share.

Rios-Rull and Santaeulàlia-Llopis (2010) argue that standard business cycle models cannot explain these empirical regularities. The RBC model implies that the real wage and labor productivity move identically, so that the labor share of income displays no cyclical dynamics. The conventional Diamond–Mortensen–Pissarides model (DMP model, henceforth) of search in the labor market with Nash bargaining explains the countercyclicality of the labor share in response to a productivity shock, but cannot address the overshooting.[1] In the basic New Keynesian (NK) model, the labor share of income coincides with the real marginal cost. The latter responds countercyclically to an increase in labor productivity, but, as we discuss later, does not overshoot its long-run value for any empirically plausible calibration of the model. Thus, although the overshooting of the labor share is still unexplained, targeting the dynamics of the labor share in estimated DSGE models can help the identification of relevant parameters.

We outline a DMP model with Nash bargaining and endogenous market structures. Market structures are said to be endogenous because both the number of producers and the price markups are determined in each period. The model features firms' entry à la Bilbiie et al. (2012) (BGM 2012, henceforth) and oligopolistic competition between producers as in Jaimovich and Floetotto (2008) and Colciago and Etro (2010). We consider both Bertrand and Cournot competition. Nash bargaining delivers replication of the countercyclicality of the labor share, whereas the key ingredient to replicate the overshooting result is the countercyclicality of price markups originating from strategic interactions between an endogenous number of producers. To build intuition, consider the effect of a technology shock. It creates profit opportunities that attract firms into the market. This strengthens competition and, via strategic interactions, reduces the price markup persistently. A persistently lower markup acts as a shifter of the standard marginal product of labor and creates a wedge between average labor productivity and the real wage. Specifically, a persistently lower price markup implies that the real wage rises relative to the average productivity of labor for several periods. Besides being consistent with the dynamics displayed in Figure 1, this leads to the labor share overshooting. We find that in the Cournot model the initial drop of the labor share and the timing and amplitude of the overshooting are close to those displayed in Figure 1. The response of the real wage is also quantitatively and qualitatively similar to the empirical one. Differently, in the Bertrand model the magnitude of the overshooting is lower than in the data.

Aggregate real wages are characterized by a high degree of persistence. Hall (2005b), inter alia, points out that real wage rigidity is a feature needed to account for a number of labor market facts. For this reason, we study the effect of real wage rigidity on the dynamics of the labor share. Introducing real wage rigidity into the DMP framework with constant markups is not sufficient to match the empirical evidence on the dynamics of the labor share in response to a technology shock. We find that augmenting our framework with (a limited degree of) real wage rigidity does not alter the previous findings, but allows a better matching of the amplitude of the labor share overshoot observed in the data.

To the best of our knowledge we are the first to present a model addressing the labor share overshooting through countercyclical markups. Hornstein (1993) augments the neoclassical growth model with increasing returns to scale, a fixed number of firms, and constant markups. He finds a labor share that is half as volatile as is observed in the data, but does not address the overshooting. Also, the role of real wage rigidities in the dynamics of the labor share had not been explored yet.[2]

Rios-Rull and Santaeulàlia-Llopis (2010) obtain the overshooting by considering a model with putty-clay technology, decentralized noncompetitive wage setting (bilateral Nash bargaining), and an aggregate technological shock that has a stronger effect for newer hires. The technology process that we adopt is, instead, fully standard. Shao and Silos (2011) also consider an economy with costly entry of firms and a frictional labor market. However, their model is characterized by monopolistic competition between small firms and by constant price markups. In their framework the overshooting is due to the countercyclical value of vacancies. Nevertheless, this condition is difficult to test empirically. In contrast, our transmission mechanism is supported by two different strands of the empirical literature.

Early works in the new empirical industrial organization literature, starting with Bresnahan and Reiss (1987), provide direct support to the fact that stronger competition leads to lower markups. More recent research by Manuszak (2002), Campbell and Hopenhayn (2005), and Manuszak and Moul (2008) also provides convincing evidence that markups fall after the entry of new competitors into the market. These results, in conjunction with the evidence that net entry of firms is procyclical, support the propagation mechanism purported in our model.[3]

Bils (1987), Rotemberg and Woodford (2000), and Galì et al. (2007) provide direct evidence of price markup countercyclicality using a different approach. Under the assumptions of a competitive labor market coupled with a Cobb–Douglas production function, the labor share of income delivers a measure of real marginal costs. Because the price markup is the inverse of the marginal cost, a countercyclical labor share would lead to a procyclical markup. However, in a labor-search framework, the inverse labor share is no longer an appropriate proxy for marginal costs and hence for the price markup. As discussed by Krause and Lubik (2007), the presence of labor market frictions reduces the allocative role of current real wages. In this case marginal costs could change even if the wage did not move. Rotemberg and Woodford (2000) suggest a number of corrections to the baseline measure to obtain a more realistic measure of marginal costs. Among them, they propose overhead labor, a CES production function, and convex costs of adjusting labor.[4] Under each of these alternative formulations, marginal costs are more procyclical than in the baseline case and imply a countercyclical movement in the price markup, in line with the transmission mechanism supported by this paper.

The remainder of the paper is organized as follows. Section 2 provides a decomposition of the labor share of income. Section 3 outlines the model economy. Section 4 is devoted to calibration. Sections 5 and 6 contain the main results. Section 7 concludes. Technical details are left to the Appendix.

2. THE LABOR SHARE AND ITS COMPONENTS

Independent of the specification of the model considered, the labor share is defined as $\mathrm{ls}_t = w_t H_t / Y_t = w_t / \mathcal{A}_t$, where H_t is total hours worked and $\mathcal{A}_t = Y_t / H_t$ is the average productivity of labor. In log-deviations,

$$\widehat{\mathrm{ls}}_t = \hat{w}_t - \left(\hat{y}_t - \hat{H}_t\right) = \hat{w}_t - \widehat{\mathcal{A}}_t, \quad (1)$$

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where a circumflex over a variable denotes the log-deviation from the steady state. Equation (1) simply states that the log-deviation of the labor share is the difference between the log-deviation of the real wage and that of the average labor productivity. In the standard RBC model the real wage equals the marginal product of labor. In log-deviation this amounts to

$$\hat{w}_t = \hat{y}_t - \hat{H}_t = \widehat{\mathcal{A}}_t. \quad (2)$$

As a result, the labor share is constant and does not deviate from its steady state; that is, $\widehat{\mathrm{ls}}_t = 0$. Equations (1) and (2) suggest that to obtain a nonconstant labor share the allocative role of the real wage has to be broken.

In the search and matching framework this is accomplished through Nash bargaining, which implies that workers and firms split the total surplus originating from a match. The equilibrium real wage maximizes the joint surplus of the parties and depends on their relative bargaining power. Thus, in the aftermath of a productivity increase, just a fraction of the increase is distributed to workers. Differently from the standard RBC model, this implies that the real wage rises by less than the increment in labor productivity. Hence, Nash bargaining helps in explaining the countercyclicality of the labor share. However, we show below that in the standard DMP framework with Nash bargaining the real wage never rises relative to labor productivity in response to a technology shock. This goes against the evidence reported in Figure 1 and, importantly, prevents the standard DMP model from addressing the overshooting of the labor share.

In the baseline NK model with Calvo pricing, just a fraction of firms can cut prices in response to a positive technology shock that lowers marginal costs. Those firms that cannot optimally reset prices reduce labor demand, which implies that the real wage increases by less than productivity in the aftermath of the shock. As a result, the labor share of income is countercyclical. Nevertheless, for any empirically plausible calibration of the model parameters, the labor share never overshoots its long-run level. We find that overshooting can be obtained assuming an extremely low persistency of the technology process, lower than suggested by available estimates, coupled either with a high degree of interest rate smoothing in the monetary policy rule or with a hybrid form of the NK Phillips curve. Under these conditions, the real wage response to the productivity shock is more persistent than productivity itself. As a consequence, after a few periods, the real wage rises relative to productivity and the labor share overshoots its long-run value.[5] However, the labor share overshoot is quantitatively smaller than that depicted in Figure 1 and is short-lived.[6] Adding real wage rigidities into the model does not alter these findings.[7]

To sum up, in order to reproduce the overshooting, the real wage most rise relative to labor productivity for several periods. We show that the countercyclical and inertial dynamics of the price markup that characterizes our approach delivers this mechanism.

3. THE MODEL

3.1. Labor and Goods Markets

There are two main building blocks in the model: oligopolistic competition with endogenous entry in the goods market and search and matching frictions in the labor market. In this section we outline their main features.

As in Colciago and Etro (2010), the economy features a continuum of sectors, or industries, on the unit interval. Sectors are indexed with $j \in (0, 1)$. Each sector j is characterized by different firms $i = 1, 2, \ldots, N_{jt}$ producing the same good in different varieties. At the beginning of each period, N_{jt}^e new firms enter into sector j, whereas at the end of the period, a fraction $\delta \in (0, 1)$ of market participants exit from the market for exogenous reasons.

The labor market is characterized by search and matching frictions, as in Merz (1995) and Andolfatto (1996). A fraction u_t of the unit mass population is unemployed at time t and searches for a job. Firms producing at time t need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS matching function and deliver m_t new hires, or matches, in each period. The matching function reads as $m_t = \gamma_m (v_t^{\text{tot}})^{1-\gamma} u_t^{\gamma}$, where γ_m reflects the efficiency of the matching process, v_t^{tot} is the total number of vacancies created at time t, and u_t is the unemployment rate. The probability that a firm fills a vacancy is given by $q_t = m_t/v_t^{\text{tot}}$, whereas the probability of finding a job for an unemployed worker is $z_t = m_t/u_t$. Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction $1-\varrho$ of existing workers each period, where ϱ is the probability that a worker stays with a firm until the next period.

As a result, a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Because these sources of separation are independent, the evolution of aggregate employment, L_t, is given by $L_t = (1-\delta)\varrho L_{t-1} + m_t$. Thus, the number of unemployed workers searching for a job at time t is $u_t = 1 - L_{t-1}$.

3.2. Households and Firms

Using the family construct of Merz (1995), we can refer to a representative household consisting of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln C_{jt} dj - \chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\}, \quad \chi, \varphi \geq 0, \tag{3}$$

where $\beta \in (0, 1)$ is the discount factor and the variable h_t represents individual hours worked. Note that C_{jt} is a consumption index for a set of goods produced

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in sectors $j \in [0, 1]$, defined as

$$C_{jt} = N_{jt}^{\frac{1}{1-\varepsilon}} \left[\sum_{i=1}^{N_{jt}} C_{jt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \mathbf{(4)}$$

where $C_{jt}(i)$ is the production of firm i of this sector, and $\varepsilon > 1$ is the elasticity of substitution between the goods produced in each pair of sectors.[8] The distinction between different sectors and different goods within a sector allows us to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. The family receives real labor income $w_t h_t L_t$ and profits from the ownership of firms. Further, we assume that unemployed individuals receive an unemployment benefit b in real terms, leading to an overall benefit for the household equal to $b(1 - L_t)$. This is financed through lump sum taxation by the government. Notice that the household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1 - \delta)\varrho L_{t-1} + z_t u_t. \quad \mathbf{(5)}$$

Households choose how much to save in riskless bonds and in the creation of new firms through the stock market according to standard Euler and asset pricing equations.[9]

Each firm i in sector j produces a good with a linear production function. We abstract from capital accumulation issues and assume that labor is the only input. The output of firm i in sector j is then

$$y_{jt}(i) = A_t n_{jt}(i) h_{jt}(i), \quad \mathbf{(6)}$$

where A_t is the total factor productivity, common to all sectors, at time t, $n_{jt}(i)$ is firm i's time-t workforce, and $h_{jt}(i)$ represents hours per employee. Because each sector can be characterized in the same way, in what follows we will drop the index j and refer to the representative sector.

3.3. Endogenous Market Structures

Following BGM (2012), we assume that new entrants at time t will only start producing at time $t + 1$. Given the exogenous exit probability δ, the average number of firms per sector, N_t, follows the equation of motion

$$N_{t+1} = (1 - \delta)(N_t + N_t^e), \quad \mathbf{(7)}$$

where N_t^e is the average number of new entrants at time t. In each period, the same nominal expenditure for each sector EXP_t is allocated across the available goods according to the direct demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\varepsilon} \frac{Y_t}{N_t} = \frac{p_t(i)^{-\varepsilon}}{P_t^{1-\varepsilon}} \frac{\text{EXP}_t}{N_t}, \quad i = 1, 2, \ldots, N_{jt}, \quad \mathbf{(8)}$$

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where P_t is the price index

$$P_t = N_{jt}^{\varepsilon-1}\left[\sum_{i=1}^{N_t}(p_t(i))^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{9}$$

such that total expenditure, EXP_t, satisfies $\mathrm{EXP}_t = \sum_{j=1}^{N_t} p_t(j)y_t(j) = P_t Y_t$.[10] Inverting the direct demand functions, we can derive the system of inverse demand functions

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}}\,\mathrm{EXP}_t, \qquad i = 1, 2, \ldots, N_{jt}, \tag{10}$$

which will be useful for the derivation of the Cournot equilibrium. Period-t real profits of an incumbent producer are defined as

$$\pi_t(i) = \frac{p_t(i)}{P_t} y_t(i) - w_t(i)n_t(i)h_t(i) - \kappa v_t(i), \tag{11}$$

where $w_t(i)$ is the real wage paid by firm i, $v_t(i)$ represents the number of vacancies posted at time t, and κ is the output cost of keeping a vacancy open. The value of a firm is the expected discounted value of its future profits,

$$V_t(i) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s}\pi_s(i), \tag{12}$$

where $\Lambda_{t,t+1} = (1-\delta)\beta(C_{t+1}/C_t)^{-1}$ is the households' stochastic discount factor, which takes into account that firms' survival probability is $1-\delta$. Incumbent firms that do not exit from the market have a time-t individual workforce given by

$$n_t(i) = \varrho n_{t-1}(i) + v_t(i)q_t. \tag{13}$$

Under different forms of competition between firms, we obtain prices satisfying

$$\frac{p_t(i)}{P_t} = \mu(\varepsilon, N_t)\mathrm{mc}_t(i), \tag{14}$$

where $\mu(\theta, N_t) > 1$ is the function depending on the degree of substitutability between goods, ε, and on the number of firms, N_t, and $\mathrm{mc}_t(i)$ is the real marginal cost. In the remainder of this section we characterize the markup function under Bertrand and Cournot competition, taking strategic interactions into account.

3.3.1. Bertrand Competition. Each firm chooses $p_t(i)$, $n_t(i)$, and $v_t(i)$ to maximize $\pi_t(i) + V_t(i)$, taking as given the price of the other firms in the sector. The problem is subject to two constraints, namely equations (8) and (13).[11] The

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symmetric Bertrand equilibrium generates an equilibrium markup

$$\mu_t^P(\varepsilon, N_t) = \frac{\varepsilon (N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)}. \quad (15)$$

The markup μ_t^P is decreasing in the degree of substitutability between products ε, with an elasticity $\epsilon_\varepsilon^P = \varepsilon N_t/(1 - \varepsilon + \varepsilon N_t)(\varepsilon - 1)$. Moreover, the markup vanishes in the case of perfect substitutability: $\lim_{\varepsilon \to \infty} \mu^P(\theta, N_t) = 1$. Finally, the markup is decreasing in the number of firms, with an elasticity $\epsilon_N^P = N/[1 + \varepsilon(N - 1)](N - 1)$. Notice that the elasticity of the markup to entry under competition in prices is decreasing in the level of substitutability between goods, and it tends to zero when the goods are approximately homogenous. When $N_t \to \infty$, the markup tends to $\varepsilon/(\varepsilon - 1)$, the traditional one under monopolistic competition. As is well known, strategic interactions between a finite number of firms lead to a higher markup than under monopolistic competition.

3.3.2. Cournot Competition. In this case firms maximize $\pi_t(i) + V_t(i)$, choosing their production $y_t(i)$ as well as $n_t(i)$ and $v_t(i)$, taking as given the production of the other firms. The profit maximization problem is constrained by the inverse demand function (10) and by equation (13). The symmetric Cournot equilibrium generates an equilibrium markup

$$\mu^Q(\varepsilon, N_t) = \frac{\varepsilon N_t}{(\varepsilon - 1)(N_t - 1)}. \quad (16)$$

First, notice that for a given number of firms, the markup under competition in quantities is always larger than the one obtained under competition in prices.[12] Further, in this case the markup is also decreasing in the degree of substitutability between products ε, with an elasticity $\epsilon_\varepsilon^Q = 1/(\varepsilon - 1)$, which is always smaller than ϵ_ε^P: higher substitutability reduces markups faster under competition in prices. In the Cournot equilibrium, the markup remains positive for any degree of substitutability, because even in the case of homogenous goods, we have $\lim_{\varepsilon \to \infty} \mu^Q(\varepsilon, N_t) = N_t/(N_t - 1)$. The markup $\mu^Q(\varepsilon, N_t)$ is decreasing and convex in the number of firms with elasticity $\epsilon_N^Q = 1/(N_t - 1)$, which is decreasing in N_t (the markup decreases with entry at an increasing rate) and independent of the degree of substitutability between goods. Because $\epsilon_N^Q > \epsilon_N^P$ for any number of firms or degree of substitutability, entry decreases markups faster under competition in quantities than under competition in prices, a result that will impact the relative behavior of the economy under the two forms of competition. Only when $N_t \to \infty$ does the markup tend to $\varepsilon/(\varepsilon - 1)$, which is the traditional markup under monopolistic competition.

3.4. Entry and Job Creation

We assume that entry requires a fixed cost ψ, which is measured in units of output. Define V_t^e as the value at time t of a prospective entrant. Given our timing assumption, this represents the value of a firm that will start producing at time $t+1$. In each period, the level of entry is determined endogenously to equate the value of a prospective entrant to the entry cost,[13]

$$V_t^e = \psi. \tag{17}$$

Profits maximization implies the following job creation condition (JCC):

$$\frac{\kappa}{q_t} = \left(\frac{1}{\mu_t^j} - \frac{w_t}{A_t}\right) A_t h_t + \varrho E_t \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}}.$$

The JCC equates the real marginal cost of hiring a worker, the left-hand side, with the marginal benefit, the right-hand side. Importantly, the marginal benefit depends positively on the ratio $1/\mu_t^J$ (with J equal either to P or to Q), which is a positive function of the number of firms in the market, N_t. Stronger competition leads to a lower markup, which stimulates demand by consumers and hence has a positive effect on output and ultimately on employment.

As shown by Colciago and Rossi (2011), a positive technology shock leads to entry of new firms and thus to an increase in $1/\mu_t^J$. At equilibrium, because hiring depends on the current and expected future values of the marginal product of labor, this boosts hiring and employment with respect to a model with constant markups.

The JCC is common across firms, independent of their period of entry. Thus, the optimal hiring policy of new producers, i.e., firms that at time t are producing for the first time and have no initial workforce, consists in posting as many vacancies as required to reach the size of firms that started production in earlier periods. This has two implications. The first is that the size gap between new producers and incumbent firms is closed in a single period. The second is that new producers grow faster than more mature firms. This is consistent with the U.S. empirical evidence discussed in Haltiwanger et al. (2010), which suggests that a start-up creates on the average more new jobs than an incumbent firm. Given that vacancy posting is costly, new producers will suffer lower profits and pay lower dividends in their first period of activity than firms that entered into the market in earlier periods. This is consistent with the evidence on the financial behavior of firms discussed by Cooley and Quadrini (2001).

3.5. Bargaining over Wages and Hours

In the Appendix it is shown that Nash wage bargaining results in the wage equation

$$w_t = \eta\left(\frac{1}{\mu_t^J} A_t + \frac{\kappa}{(1-\delta)} \frac{1}{h_t} E_t \Lambda_{t,t+1} \theta_{t+1}\right) + (1-\eta)\left(\chi C_t \frac{h_t^{1/\varphi}}{1+1/\varphi} + \frac{b}{h_t}\right), \tag{18}$$

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where μ_t^J is the markup function, $\theta_t = v_t^{\text{tot}}/u_t$ is the tightness of the job market, and the parameter η reflects the relative bargaining power of workers. The wage shares costs and benefits associated with the match. The worker is rewarded for a fraction η of the firm's revenues and savings of hiring costs and compensated for a fraction $1 - \eta$ of the disutility he suffers from supplying labor and the foregone unemployment benefits. The direct effect of competition on the real wage is captured through the term $\eta \frac{1}{\mu_t^j} A_t$, which represents the share of the marginal revenue product (MRP) that goes to workers. As discussed previously, entry leads to an increase in the ratio $1/\mu_t^j$ and hence in the MRP. Thus, everything else being equal, stronger competition shifts the wage curve up. This result is similar to that in Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage. Hours are set to maximize the joint surplus of the match. This is obtained when the marginal rate of substitution between hours and consumption equals the MRP of labor, that is,

$$\chi C_t h_t^{1/\varphi} = \frac{1}{\mu_t^J} A_t. \tag{19}$$

Stronger competition leads to an increase in hours bargained between workers and firms for the same reasons for which competition positively affects the wage schedule.

3.6. Aggregation and Market Clearing

Since the individual workforce, n_t, is identical across producers, it follows that

$$L_t = n_t N_t. \tag{20}$$

To obtain aggregate output, notice that $P_t Y_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t$; further, given $p_t/P_t = 1$ and the individual production function, it follows that

$$Y_t = N_t y_t = A_t L_t h_t = A_t H_t, \tag{21}$$

where H_t is the total number of hours worked. As a consequence, A_t amounts to average labor productivity, which is assumed to follow a first-order autoregressive process given by $\ln(A_t/A) = \rho_A \ln(A_{t-1}/A) + \varepsilon_{At}$, where $\rho_A \in (0, 1)$ and ε_{At} is a white noise disturbance, with zero expected value and standard deviation σ_A.

Aggregating the budget constraints of households, we obtain the aggregate resource constraint of the economy,

$$C_t + \psi N_t^e = W_t h_t L_t + \Pi_t, \tag{22}$$

which states that the sum of consumption and investment in new entrants must equal the sum of labor income and aggregate profits, Π_t, distributed to households at time t. Goods market clearing requires

$$Y_t = C_t + N_t^E \psi + \kappa v_t^{\text{tot}}, \tag{23}$$

where v_t^{tot} is the sum of vacancies posted by new entrants and by firms which entered in earlier periods. Finally, the dynamics of aggregate employment reads as

$$L_t = (1 - \delta)\varrho L_{t-1} + q_t v_t^{\text{tot}}, \tag{24}$$

which shows that workers employed by a firm that exits the market join the mass of unemployed.

4. CALIBRATION

To solve the model described in the previous section, the equations are linearized around the model's steady state.[14] The calibration of parameters aims at matching key features of the U.S. economy. The discount factor, β, is set to 0.99. As in BGM (2012), the rate of business destruction, δ, equals 0.025. This means that roughly 10% of firms disappear from the market every year, independent of firm age. The entry cost is $\psi = 1$ and is held constant along the cycle. With no loss of generality, the value of χ is such that steady state labor supply equals one. The Frisch elasticity of labor supply is $\varphi = 1$. The intersectoral elasticity of substitution is $\varepsilon = 6$, as estimated by Christiano et al. (2005). As is standard in the literature, we set the steady state marginal productivity of labor, A, to 1. We calibrate the parameters of the productivity process as estimated by Rios-Rull and Santaeulàlia-Llopis (2010), with persistence $\rho_A = 0.958$ and standard deviation $\sigma_A = 0.0067$. We set the separation rate ϱ equal to 0.1, as suggested by estimates provided by Hall (2005a) and Davis et al. (1996). The elasticity of matches to unemployment, γ, is set equal to the worker bargaining power η and is equal to $\frac{1}{2}$, as in the bulk of the literature. The efficiency parameter in matching, γ_m, and the steady state job market tightness are calibrated to target an average job-finding rate, z, equal to 0.7 and a vacancy-filling rate, q, equal to 0.9. We draw the latter value from Andolfatto (1996) and Den Haan et al. (2000), whereas the former is from Blanchard and Galì (2010).[15] Finally, we calibrate the unemployment benefit in real terms, b, so that the monetary replacement rate, b/wh, equals 0.60. This value is consistent with that reported in the OECD Economic Outlook of 1996 for the United States. Given these parameters, we can recover the cost of posting a vacancy, κ, by equating the steady state version of the JCC and the steady state wage-setting equation. Notice that none of the qualitative results are affected by the calibration strategy.

5. PRODUCTIVITY SHOCKS AND DYNAMICS OF THE LABOR SHARE

In what follows we study the impulse response of the labor share and its components to a one-standard-deviation increase in technology.[16] To isolate the role of

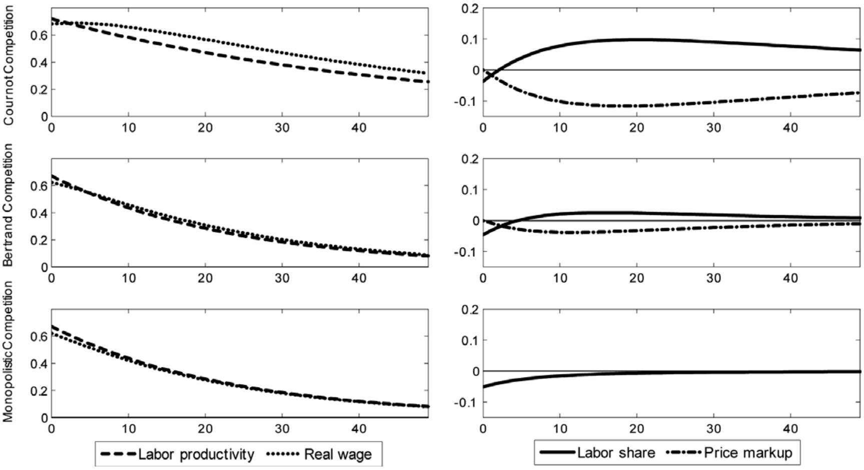

FIGURE 2. IRFs to a technology shock. Top panel: Cournot competition; middle panel: Bertrand competition; bottom panel: monopolistic competition.

endogenous markup variability in the dynamics of the labor share, we compare the performance of the models with Bertrand and Cournot competition with that of a model characterized by monopolistic competition. Under monopolistic competition firms do not interact strategically, and they set a constant markup over marginal costs equal to $\mu = \frac{\varepsilon}{\varepsilon-1}$.

Figure 2 shows that, on impact, the real wage increases less than average labor productivity, no matter the form of competition in the goods market. As argued previously, Nash bargaining delivers the countercyclicality of the labor share of income. Under monopolistic competition, after peaking on impact, the real wage returns monotonically to its initial level. Further, it never rises relative to labor productivity. As a result, the labor share does not overshoot.

This is not the case when the goods market is characterized by oligopolistic competition. Under both Bertrand and Cournot, the labor share is countercyclical because of Nash bargaining. Moreover, the labor share overshoots its long-run level after about five quarters, it peaks at about the fifth year at a level higher than its long-run value, and seven years after the shock has hit, the economy is still halfway toward its average. The key lies in the countercyclical and inertial response of the price markup. To see this, consider the log-deviations of the real wage and labor hours from their steady state. These are, respectively,

$$\hat{w}_t = \Upsilon_1 \left(\hat{A}_t - \hat{\mu}_t\right) - \Upsilon_2 \hat{h}_t + \Upsilon_3 E_t \hat{\Theta}_{t+1} \tag{25}$$

and

$$\hat{h}_t = \varphi \left(\hat{A}_t - \hat{\mu}_t - \hat{c}_t\right), \tag{26}$$

where $\Upsilon_1 = \frac{1}{\mu w}(\frac{\eta+\varphi}{1+\varphi})$, $\Upsilon_2 = 1 - \Upsilon_1$, $\Upsilon_3 = \eta\kappa\theta/w$, and $\hat{\Theta}_{t+1} = \widehat{\Lambda}_{t,t+1} + \widehat{\theta}_{t+1}$. Under all plausible parameterizations, we find that Υ_1 is lower than one. As

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a result, only a fraction $\Upsilon_1 < 1$ of the impact of increase in productivity $\hat{A}_t$ goes to workers. Further, equation (26) shows that labor hours increase with productivity and contribute to dampening the positive effect of productivity on real wages. Hence, the impact of increase in real wages is smaller than that of labor productivity and the labor share is countercyclical. In a model with endogenous market structures, these are just partial effects. Technology shocks create expectations of future profits, which lead to the entry of new firms. Stronger competition leads to lower price markups. Given that entry is subject to a one-period time-to-build lag, the total number of firms, N_t, does not change on impact, but builds up gradually. As shown in Figure 2, in the Cournot and in the Bertrand model, this translates into an initially muted response of the markup. As entry increases the number of firms, however, the price markup starts declining. In particular, it finds its negative peak after a few periods and then gradually reverts to its long-run value.[17] Equation (25) shows that a persistently lower markup acts as a shifter of the standard marginal product of labor, allowing the real wage to rise relative to the average productivity of labor for several periods. Because $\widehat{\text{ls}}_t = \hat{w}_t - \hat{A}_t$, this explains the overshooting of the labor share. Thus, we can state that the dynamic response of the markup to technology shocks is fundamental for the overshoot.[18]

In the Cournot model, the initial drop of the labor share, as well as the timing and amplitude of the overshoot, are very close to their data counterpart (see Figure 1). The response of the real wage is also quantitatively and qualitatively similar to the empirical one. Differently, in the Bertrand model, the magnitude of the overshoot is less than in the data. The reason is the stronger markup variation under Cournot, which is reflected in a larger wedge between the real wage and average labor productivity.

6. THE ROLE OF REAL WAGE RIGIDITY

Aggregate wages are characterized by a high degree of persistence, so that sudden and large shifts in the aggregate wage level are not observed. The existence of real wage rigidities has been pointed to by many authors as a feature needed to account for a number of labor market facts [see, e.g., Hall (2005b)].

Real wage rigidity leads to slow adjustment of wages to labor market conditions. In particular, in response to a productivity shock, it leads to a smoother and more inertial dynamics of the real wage than the average labor productivity. As emphasized previously, this is the key feature a model should satisfy to address the overshooting of the labor share in response to a technology shock. For this reason, we study the effect of real wage rigidity on the dynamics of the labor share. Following Hall (2005b), we model real wage rigidity in the form of a backward-looking social norm,[19]

$$w_t = \phi_w w_{t-1} + (1 - \phi_w)\, w_t^{\text{nash}}, \tag{27}$$

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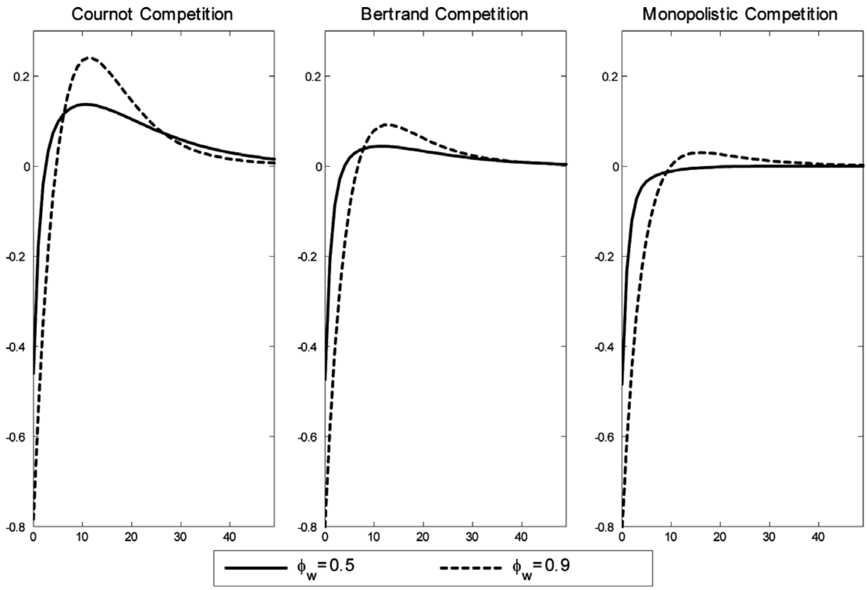

FIGURE 3. Labor share response to a technology shock under alternate degrees of real wage rigidity.

where ϕ_w is an index reflecting the degree of real wage rigidity and w_t^{nash} is the wage obtained under Nash bargaining, i.e., that in equation (18). Notice that $\phi_w = 1$ implies a fixed real wage, whereas $\phi_w = 0$ corresponds to the case of Nash bargaining analyzed earlier. As observed by Blanchard and Galì (2007), equation (27), even though admittedly ad hoc, is a parsimonious way of introducing a slow adjustment of real wages to labor market conditions.[20]

Figure 3 displays the response of the labor share to a one-standard-deviation increase in technology in the Bertrand and Cournot models, as well as in the model with monopolistic competition. Because there is no evidence on the degree of real wage rigidities, we consider two alternative values of the parameter ϕ_w. Dashed lines refer to the case $\phi_w = 0.5$, the midpoint of the admissible range. Solid lines depict the extreme case where $\phi_w = 0.9$.[21]

In the model with constant price markups, the labor share overshoots its long-run level just in the case of extreme real wage rigidity. Nevertheless the overshoot is negligible. This confirms that countercyclical price markups are the key to the overshooting of the labor share.

Augmenting the Cournot and Bertrand competitive frameworks with a limited degree of real wage rigidity does not alter the previous findings substantially; nevertheless, it improves the matching of the amplitude of the overshooting from a quantitative point of view.

To conclude this section, let us point out that in Colciago and Rossi (2011), we compare, for selected variables, the second moments delivered by our model in response to a technology shock with the unconditional empirical moments. We consider quarterly U.S. postwar data. We show that considering the extensive margin of job creation and destruction due to firms' entry and exit improves the performance of the DMP framework along various dimensions. The model addresses the procyclicality of job creation by new firms. Further, the negative contemporaneous correlation between output and the markup is not too far in magnitude from the empirical one. Despite this, aggregate profits remain strongly procyclical, as in the data.

7. CONCLUSIONS

Recent U.S. evidence suggests that the response of labor share to a productivity shock is characterized by *countercyclicality* and *overshooting*. To account for these empirical findings, a model should satisfy two desiderata. The first one is that the impact increase in the real wage must be lower than that of average labor productivity. The second one is the presence of a persistent wedge between average labor productivity and the real wage such that, in the aftermath of the shock, the response of the latter rises above that of the former for several periods.

We propose a DMP model that addresses this evidence, characterized by firms' entry and oligopolistic competition between producers. Nash bargaining delivers the countercyclicality of the labor share in response to a technology shock. The countercyclicality of price markup originating from strategic interactions in the goods market acts as a shifter of the standard marginal product of labor and allows the labor share of income to overshoot.

Although real wage rigidity helps in accounting for a number of labor market facts, such as the variability of unemployment in response to a technology shock and the slow response of real wages to labor market conditions, it does not seem to play a crucial role in the dynamics of the labor share of income.

NOTES

1. Chois and Rios-Rull (2009) consider alternative search and matching models with Nash bargaining and show that none of these models can replicate the labor share overshooting. Further, Rios-Rull and Santaeulàlia-Llopis (2010) notice that the departure from a Cobb–Douglas technology is a necessary but not sufficient condition to get the labor share overshooting.

2. Reicher (2011) considers a DMP framework with physical capital accumulation and staggered wage bargaining as in Gertler and Trigari (2009). He shows that nominal wage stickiness does not lead the labor share to overshoot.

3. Early references on the procyclicality of firms' entry are Chatterjee and Cooper (1993) for the United States and Portier (1995) for France. More recent evidence is provided by BGM (2012).

4. Considering measures of hiring costs or of the asset values of workers could represent alternative ways to measure marginal costs in the presence of labor market frictions. Merz and Yashiv (2007) estimate hiring and investment costs from the asset value of firms.

5. Notice, however, that a high degree of interest rate smoothing combined with a low persistency of the technology process leads to countercyclical real wages. This goes against the evidence in Figure 1.

6. Notice also that, because the average price markup is the inverse of the average real marginal cost, the NK model is characterized by procyclical markups in response to productivity shocks. This contrasts with the unconditional evidence discussed previously.

7. Results concerning the dynamics of the NK model are available upon request.

8. The term $N_{jt}^{1/(1-\varepsilon)}$ in (4) implies that there is no variety effect in the model. However, allowing a variety effect would not change our results.

9. These conditions are in the Appendix.

10. The demand for the individual good and the price index are the solution to the usual consumption expenditure minimization problem.

11. Details concerning the firm maximization problem under Bertrand and Cournot competition are in the Appendix.

12. This is well known for models of product differentiation [see for instance Vives (1999)].

13. This condition holds as long as the mass of new entrants N_t^e is positive. Like BGM (2012), we assume that macroeconomic shocks are small enough for this condition to hold in each period.

14. The resulting linearized system is solved using DYNARE.

15. A job-finding rate equal to 0.7 corresponds, approximately, to a monthly rate of 0.3, consistent with U.S. evidence.

16. This is for consistency with the evidence displayed in Figure 1.

17. Notice that the shape of the response of the price markup to a technology shock is consistent with the evidence in Rotember and Woodford (1999) and the VAR evidence in Colciago and Etro (2010).

18. We experimented with alternative values of the parameters η, ε, and δ. Our results are qualitatively unaffected. In particular: (i) an increase in worker bargaining power, η, dampens the impact response of the labor share, leaving the size of the labor share overshooting unchanged; (ii) an increase in the elasticity of substitution between goods, ε, slightly amplifies the magnitude of the overshooting; (iii) a reduction of the rate of business destruction, δ, implies negligible changes in both labor share countercyclicality and overshooting. Our results are robust also to alternative values of the Frisch elasticity of labor supply, φ. In particular, even in the case of fixed individual hours, $\varphi = 0$, the labor share overshoots its long-run level.

19. Blanchard and Galì (2007), Christoffel and Linzert (2010), Ascari and Rossi (2011), and Faia and Rossi (2013) take a similar approach.

20. The authors consider alternative formalizations, explicitly derived from staggering of real wage decisions. Although the algebra is more involved, the basic conclusions are the same as those obtained with the ad-hoc formulation.

21. A value of $\phi_w = 0.9$ implies a real wage adjustment of about six quarters.

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APPENDIX

Let us provide some terminology before starting the analysis. The term "new entrants" refers to the firms that enter the market at time t. The value of these firms is denoted by V_t^e. The term "new producers" refers to firms that entered the market at time $t-1$ and at time t produce for the first time (these firms are a fraction $(1-\delta)$ of time-$(t-1)$ new entrants). The term "incumbent firms" refer to firms that entered the market in period $t-2$ or earlier. Notice that new producers and incumbent firms have the same value, which we denote as V_t. This is so because new producers close their size gap with incumbent firms in their first period of activity. For this reason, after their first period of activity, new producers are indistinguishable from firms that entered at time $t-2$ or earlier.

A.1. HOUSEHOLDS

We assume that households invest in both incumbent firms and new entrants. Bonds and stocks are denominated in terms of the final good. The budget constraint expressed in nominal terms is

$$\begin{aligned} &P_t B_{t+1} + P_t C_t + P_t \int_0^1 V_{jt} N_{jt} s_{jt+1} dj + P_t \int_0^1 V_{jt}^e N_{jt}^e s_{jt+1}^e dj \\ &\quad = W_t L_t h_t + (1-L_t)\, P_t b + (1+r_t) P_t B_t \\ &\qquad + (1-\delta) P_t \int_0^1 \left[\pi_{jt}(\varepsilon, N_{jt}) + V_{jt}\right] N_{jt-1} s_{jt} dj \\ &\qquad + (1-\delta) P_t \int_0^1 \left[\pi_{jt}^{\text{new}}(\varepsilon, N_{jt}) + V_{jt}\right] N_{jt-1}^e s_{jt}^e dj - P_t T_t, \end{aligned} \quad \textbf{(A.1)}$$

where B_t is net bond holdings with interest rate r_t , V_{jt} is the value of an incumbent firm in sector j, and V_{jt}^e is the value of a new entrant in the same sector. The variables N_{jt} and N_{jt}^e represent the number of active firms in sector $j$and the new entrants in this sector at the end of the period, respectively. The variable s_{jt} represents the share of the portfolio of incumbent firms belonging to sector j that is owned by the household, whereas s_{jt}^e is the share of portfolio of new entrants held by the household. The term $(1-\delta) P_t \int_0^1 [\pi_{jt}(\varepsilon, N_{jt}) + V_{jt}] N_{jt-1} s_{jt}$ represents the sum of the value of the portfolio of firms that entered the market in period $t-2$ or earlier held by the household and the profits distributed by these firms. Notice that the number of these firms is equal to $(1-\delta)N_{jt-1}$ in each sector. The term $(1-\delta) P_t \int_0^1 [\pi_{jt}^{\text{new}}(\varepsilon, N_{jt}) + V_{jt}] N_{jt-1}^e s_{jt}^e$denotes the sum of the value of the portfolio of new producers, where $(1-\delta) N_{jt-1}^e$ is the number of firms that produce for the first time at time t. In the budget constraint we have imposed symmetry on the value of new firms and incumbent firms. Finally, $P_t T_t$ represent nominal lump sum taxes imposed to finance unemployment benefits. The household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1-\delta)\varrho L_{t-1} + z_t u_t. \quad \textbf{(A.2)}$$

Equations (A.1) and (A.2) represent the constraint on the utility maximization problem. We denote by ξ_t the Lagrangian multiplier of the first constraint, whereas Γ_t is that of the second constraint. The intertemporal optimality conditions with respect to s_{jt+1}, s^e_{jt+1} for each sector and with respect to $_{t+1}$ are, respectively,

$$P_t V_{jt} = \beta E_t (1-\delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1} \left[\pi_{jt+1}(\varepsilon, N_{jt+1}) + V_{jt+1}\right], \tag{A.3}$$

$$P_t V^e_{jt} = \beta E_t \left(1-\delta\right) \frac{\xi_{t+1}}{\xi_t} P_{t+1} \left[\pi^{\text{new}}_{jt+1}(\varepsilon, N_{jt+1}) + V_{jt+1}\right], \tag{A.4}$$

$$P_t \xi_t = \beta E_t (1 + r_{t+1}) P_{t+1} \xi_{t+1}. \tag{A.5}$$

The optimal choice of consumption requires

$$\frac{1}{P_t C_t} = \xi_t. \tag{A.6}$$

Notice that Γ_t has the meaning of the marginal value to the household of having a member employed rather than unemployed. This affects bargaining over the real wage and individual hours and is given by

$$\Gamma_t = \frac{1}{C_t}\left(w_t h_t - b\right) - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} + \beta E_t \left[(1-\delta)\rho - z_{t+1}\right] \Gamma_{t+1}, \tag{A.7}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage.

A.2. PROFIT MAXIMIZATION PROBLEM

Consider Bertrand competition. We initially consider the problem of an incumbent firm. Substituting the direct demand for the individual good into period-t real profits, we obtain

$$\pi_t = \frac{p_t(i)^{1-\varepsilon}}{\left[\sum_{i=1}^{N_t} p_t(i)^{-(\varepsilon-1)}\right]} \frac{\text{EXP}_t}{P_t} - w_t(i) n_t(i) h_t(i) - \kappa v_t(i). \tag{A.8}$$

The profit maximization problem of an incumbent firm reads as

$$\max_{\{p_t(i), n_t(i), v_t(i)\}_t^\infty} \pi_t + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_s, \tag{A.9}$$

subject to

$$A_t n_t(i) h_t(i) = \frac{p_t(i)^{-\varepsilon} \text{EXP}_t}{\left[\sum_{i=1}^{N_t} p_t(i)^{(1-\varepsilon)}\right]} \tag{A.10}$$

and

$$n_t(i) = \rho n_{t-1}(i) + v_t(i) q_t. \tag{A.11}$$

Lagrangian multipliers on constraints $(A.10)$ and $(A.11)$ are respectively $\text{mc}_t(i)$ and $\phi_t(i)$. Setting up the Lagrangian $\mathbf{L}$, the FOCs with respect to $n_t(i)$, $v_t(i)$, and $p_t(i)$ are, respectively,

$$\frac{\partial \mathbf{L}}{\partial n_t(i)} = 0 : w_t(i) h_t(i) + \phi_t(i) - \text{mc}_t(i) A_t h_t(i) = \varrho E_t \Lambda_{t,t+1} \phi_{t+1}(i), \tag{A.12}$$

$$\frac{\partial \mathbf{L}}{\partial v_t(i)} = 0 : \kappa = \phi_t(i) q_t, \tag{A.13}$$

and

$$\frac{\partial \mathbf{L}}{\partial p_t(i)} = 0 : \frac{(1-\varepsilon)\left[\sum_{i=1}^{N_t} p_t(i)^{(1-\varepsilon)}\right] - (1-\varepsilon)\, p_t(i)^{1-\varepsilon}}{\left[\sum_{i=1}^{N_t} p_t(i)^{1-\varepsilon}\right]^2} p_t(i)^{-\varepsilon} \frac{\mathrm{EXP}_t}{P_t} \tag{A.14}$$

$$\mathrm{mc}_t(i) \frac{\varepsilon p_t(i)^{-1}\left[\sum_{i=1}^{N_t} p_t(i)^{(1-\varepsilon)}\right] + (1-\varepsilon)\, p_t(i)^{-\varepsilon}}{\left[\sum_{i=1}^{N_t} p_t(i)^{1-\varepsilon}\right]^2} p_t(i)^{-\varepsilon} \mathrm{EXP}_t$$

$$= 0.$$

Notice that we assume that firms take individual wages as given when choosing employment. Also notice that because there is a continuum of sectors, the individual firm takes the aggregate price level as given. The second condition shows that $\phi_t(i)$, the surplus created by a match, is identical across incumbent firms. Before providing an explicit formula for the individual price level and the price markup, we turn to the profit maximization problem of a first-period producer, which sets the price for the first time. The relevant difference from the previous case is represented by the form of constraint $(A.11)$, which reads as $v_t(i)q_t = n_t(i)$, because producers in their first period of activity have no initial workforce. However, FOCs with respect to $p_t(i)$, $n_t(i)$, and $v_t(i)$ are identical to those reported previously. Because the surplus ϕ_t created by a match is identical across all producers, they will face the same wage bargaining problem, and thus will face the same wage, $w_t(i) = w_t$ and the same marginal cost, $\mathrm{mc}_t(i) = \mathrm{mc}_t$, and will demand the same number of hours, $h_t(i) = h_t$. As a result, the third condition can be written as

$$(1-\varepsilon)\, N_t P_t^{1-\varepsilon} - (1-\varepsilon)\, p_t(i)^{1-\varepsilon} = \mathrm{MC}_t \left[(\varepsilon - 1)\, p_t(i)^{-\varepsilon} - \varepsilon p_t(i)^{-1} N_t P_t^{1-\varepsilon}\right], \tag{A.15}$$

where MC_t is the nominal marginal cost, which shows that $p_t(i)$ does not depend on any firm-specific variable. In other words, all firms that are active at time t, no matter the period of entry, choose the same price. Because firms face the same demand function and adopt the same technology, it follows that $y_t(i) = y_t$ and $n_t(i) = n_t$. We are now ready to provide an expression for the common price chosen by firms. Given that firms choose the same price level, it follows that $p(i) = p_t = P_t$. Imposing symmetry and rearranging, condition (14) can be rewritten as

$$\frac{1}{\mathrm{mc}_t} = \mu_t, \tag{A.16}$$

where

$$\mu_t = \frac{\varepsilon\,(N_t - 1) + 1}{(\varepsilon - 1)\,(N_t - 1)}. \tag{A.17}$$

Further, notice that, after imposing symmetry, by combining equations (A.12) and (A.13), we get the JCC reported in the main text. Under Cournot competition, profit maximization must take the inverse demand function as a constraint,

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \mathrm{EXP}_t,$$

which implies that period profits can be written as

$$\pi_t = \frac{y_t(i)^{1-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \frac{\text{EXP}_t}{P_t} - w_t(i) n_t(i) h_t(i) - k v_t(i).$$

By setting up a Lagrangian function as in the previous case and differencing with respect to $y_t(i)$, $n_t(i)$, $v_t(i)$, it can easily be verified that the FOCs with respect to $n_t(i)$, $v_t(i)$ are unchanged from the Bertrand case.

A.3. WAGE SETTING

The real wage and hours worked are set to maximize the product

$$(\phi_t)^{1-\eta} (\Gamma_t C_t)^{\eta}, \quad \textbf{(A.18)}$$

where the term in the first set of parentheses, ϕ_t, is the value to the firm of having an additional worker, i.e.,

$$\phi_t = \frac{1}{\mu_t} A_t h_t - w_t h_t + \varrho E_t \Lambda_{t,t+1} \phi_{t+1}, \quad \textbf{(A.19)}$$

and the second term, Γ_t, is the household's surplus expressed in units of consumption,

$$\Gamma_t = \frac{1}{C_t} w_t h_t - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} - \frac{b}{C_t} + \beta E_t \left[(1-\delta)\rho - z_{t+1}\right] \Gamma_{t+1}. \quad \textbf{(A.20)}$$

The FOC with respect to the wage is

$$(1-\eta)(\phi_t)^{-\eta} (\Gamma_t C_t)^{\eta} \frac{d\phi}{d_w} + \eta (\Gamma_t C_t)^{\eta-1} (\phi_t)^{1-\eta} \frac{d\Gamma_t}{d_w} C_t = 0. \quad \textbf{(A.21)}$$

Notice that $\frac{d\Gamma_t}{dw_t} C_t = -\frac{d\phi_t}{dw_t} = h_t$; thus (A.21) can be simplified as follows:

$$\eta \phi_t = (1-\eta) \Gamma_t C_t. \quad \textbf{(A.22)}$$

Multiplying both sides of equation $(A.22)$by $\varrho\beta(1-\delta)\frac{C_{t-1}}{C_t}$ yields

$$\eta \varrho \beta (1-\delta) \frac{C_{t-1}}{C_t} \phi_t = (1-\eta) \varrho \beta (1-\delta) C_{t-1} \Gamma_t; \quad \textbf{(A.23)}$$

leading one period and taking expectations as of time t leads to

$$\eta \varrho E_t \Lambda_{t,t+1} \phi_{t+1} = (1-\eta) \varrho \beta (1-\delta) C_t E_t \Gamma_{t+1}; \quad \textbf{(A.24)}$$

substituting for ϕ_t and $\Gamma_t C_t$ and simplifying gives

$$\eta \frac{1}{\mu_t} A_t h_t = w_t h_t - (1-\eta) \left(\chi \frac{h_t^{1+1/\varphi} C_t}{1+1/\varphi} + b + \beta E_t z_{t+1} \Gamma_{t+1} C_t \right). \quad \textbf{(A.25)}$$

Multiplying both sides of (A.22) by $z_t \frac{C_{t-1}}{C_t}$, leading one period, and taking the expectation as of time t, we can rewrite this as

$$\eta z_{t+1} \frac{C_t}{C_{t+1}} \phi_{t+1} = (1-\eta) z_{t+1} C_t \Gamma_{t+1}. \quad \textbf{(A.26)}$$

It follows that

$$(1-\eta)\,\beta C_t E_t z_{t+1}\Gamma_{t+1}= \eta\beta E_t \frac{C_t}{C_{t+1}} z_{t+1}\phi_{t+1}=\frac{\eta}{(1-\delta)}\Lambda_{t,t+1}z_{t+1}\phi_{t+1}. \qquad \textbf{(A.27)}$$

Substituting into (A.25) yields

$$\eta\frac{1}{\mu_t}A_t h_t= w_t h_t-(1-\eta)\,\chi\frac{h_t^{1+1/\varphi}C_t}{1+1/\varphi}+(1-\eta)\,b+\frac{\eta}{(1-\delta)}\Lambda_{t,t+1}z_{t+1}\phi_{t+1}. \qquad \textbf{(A.28)}$$

Finally, using $\phi_t=\frac{\kappa}{q_t}$ and $\frac{z_t}{q_t}=\theta_t$ and rearranging, we get

$$w_t h_t=(1-\eta)\,b+\eta A_t\frac{1}{\mu_t}h_t+(1-\eta)\,\chi\frac{h_t^{1+1/\varphi}}{1+1/\varphi}C_t+\frac{\eta\kappa}{(1-\delta)}E_t\Lambda_{t,t+1}\theta_{t+1}, \qquad \textbf{(A.29)}$$

which is the wage equation in the text. Similarly, the FOC for hours Nash bargaining is

$$(1-\eta)\,(\phi_t)^{-\eta}\,(\Gamma_t C_t)^{\eta}\,\frac{d\phi}{dh}+\eta\,(\Gamma_t C_t)^{\eta-1}\,(\phi_t)^{1-\eta}\,\frac{d\Gamma_t}{dh}C_t=0. \qquad \textbf{(A.30)}$$

Considering that $\frac{d\phi_t}{dh_t}=\frac{1}{\mu_t}A_t-w_t$ and that $\frac{d\Gamma_t}{dh_t}C_t=w_t-\chi h_t^{1/\varphi}C_t$, equation (A.30) can be written as

$$(1-\eta)\,\Gamma_t C_t\left(\frac{1}{\mu_t}A_t-w_t\right)+\eta\phi_t\left(w_t-\chi h_t^{1/\varphi}C_t\right)=0. \qquad \textbf{(A.31)}$$

Finally, using equation (A.22), equation (A.31) simplifies to

$$h_t=\left(\frac{1}{\chi}\frac{\rho_t}{\mu_t}\frac{A_t}{C_t}\right)^{\varphi}, \qquad \textbf{(A.32)}$$

which is the equation for hours worked.

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