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Reliability analysis for mutative topology structure multi-AUV cooperative system based on interactive Markov chains model

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SUMMARY
In the real practice of multi-AUV (Autonomous Underwater Vehicle) cooperative systems, tasks or malfunctions will change the topology. The process of mutative topology structure will affect the reliability of multi-AUV cooperative system. The interactive Markov chains model, which is an intercurrent model of functional action and capability index, is selected to reflect the reliability of topology-changed multi-AUV cooperative systems. In this model, multi-AUV cooperative systems are described by the conception—"Action". The concept of "action transfer" is used to describe the topology-changed multi-AUV cooperative system, and model checking is used to solve the interactive Markov chains, giving the probability of reliability within a certain time for the system. The result shows that the method proposed in this paper has a practical value.

KEYWORDS: Multi-AUV cooperative system, Reliability, Mutative topology structure, Interactive Markov chains model

1. Introduction

With the increasing demands humanity places on its oceans, the tasks of Autonomous Underwater Vehicle (AUVs) are getting more and more complicated. When restricted to a single AUV, it becomes difficult to complete such increasingly precise, varied and complex tasks. Thus, the development of AUVs is tending toward miniaturization, structural simplification, intellectualization and multi-AUV cooperative systems.[1−8]

Reliability research on multi-AUV cooperative systems is gaining more and more attention with the increase in potential applications of these systems. At present, there are few data on the reliability of multi-AUV cooperative systems, but the reliability research on other cooperative systems (e.g. multi-Unmanned Aerial Vehicle cooperative system, multi-robot cooperative systems and multi-shipboard cooperative systems) has achieved some results, which can be used as a reference on multi-AUV systems.

Gerkey and Howard of USC's Robotics Research Lab Vaughan of its Information Sciences Laboratory[9] have studied the reliability of a cooperative system composed of a multi-robot/sensor network, and presented an improved program of communication protocol organized in a hub and spoke network structure, which improved the reliability of the system. In terms of mission reliability evaluation for a multi-UAV cooperative system, Andrews, Prescott and Remenyte of Loughborough University[10] divided the whole mission into periodic missions, analyzed the influence of the decision-making process on mission reliability of multi-UAV cooperative systems and found decision-making

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methods that can improve reliability. Rabbath and Léchevin of DRDC[11] researched the large number of faults and errors that influence the reliability of multi-UAV cooperative systems, and presented health state estimation methods that can improve reliability. A formation figure is used to describe the network topology of the UUV (Unmanned Underwater Vehicle) cooperative system in the paper.[12] Using a matrix representation of the graph, and working according to methods based on graph theory and matrix analysis, the network model of the UUV cooperative system is described, and a method of network vulnerability assessment, as the UUV cooperative system performs the exploration mission, is proposed. According to the characteristics of shipboard cooperative systems, eight major factors that influence the network reliability are summarized in the paper.[13] Then, the analytic hierarchy process (AHP) is used to evaluate the network reliability, and key influencing factors of network reliability are summarized.

In practical applications, the multi-AUV cooperative system has great flexibility: The AUVs in the system can have different functions, and multiple AUVs can be coordinated to conduct complicated underwater missions. When faced with a complex mission, a multi-AUV cooperative system may be required to change its network topology at some point. That is, the communication relationship between certain AUVs may be interrupted, and some new communication relationships may be built between other AUVs, which change the topological relationship between the AUVs.[14] The change of topological relationship will unavoidably influence the reliability of the system. The process of changing topology is vital to success in the actual task, given such changes can potentially cause a system-wide crash.[15] Thus, it is very important to study the reliability of a system's topology-changing process.

The reliability of the mutative topology structure of a multi-AUV cooperative system is very complicated, and can't simply be equated to the arithmetic product of two topology structures' reliability (the topology structure before and after mutation). Therefore, on the basis of the reliability analysis for a fixed topology structure multi-AUV cooperative system, a new method—interactive Markov chains (IMC)—is brought in. IMC is a combination of a process algebra model and a continuous-time Markov chains (CTMC) model. The process algebra model is a classical theory frame of combinational analysis for concurrent systems, and the CTMC model is a classical performance assessment model. IMC is a combination of the two models in the form of orthogonalization to build a modularization performance assessment model.

Recently, research on the IMC model has achieved much. Based on the IMC (with an added features index), a dynamic model of performance features was introduced in order to ensure the reliable service of computing platforms by Zhuang Lu, Cai Mian and Shen Changxiang from Beijing University of Technology.[16] Also based on the IMC, a features checker in order to visually verify the performance of concurrent system was proposed and implemented by Xu Zhenxing, Wu jinzhao and Chen jianfeng from the Chengdu Institute of Computer Application in Chinese Academy of Sciences,[17] and it can be used to validate the concurrent systems model, including sequential circuit design and communications protocol analysis.

The paper is organized as follows. In Section 2, the process of mutative topology structures of multi-AUV cooperative systems is discussed. In Section 3, IMC model and model verification are introduced. In Section 4, a model of a mutative topology structure multi-AUV cooperative system based on IMC model is set up and the system collapse probability is evaluated. In Section 5, an example is used to show how the method is operated. Section 5 draws some useful conclusions of the work.

2. The Process of Mutative Topology Structures of Multi-AUV Cooperative Systems

A multi-AUV cooperative system can change its topology structure. For example, Fig. 1 depicts a topology structure of a multi-AUV cooperative system that has one pilot.[18] That is, v_1 is the single-pilot AUV. v_2, v_3, v_4, v_5 and v_6 are connected with v_1 by a communication link, but have no communication links between themselves, as shown in Fig. 1(a). When the mission requirement or environment changes, the topology structure of multi-AUV cooperative system may change to a structure that has two pilots, as shown in Fig. 1(b). v_1 and v_2 are both pilot AUVs, v_5 and v_6 are connected with v_1, v_3 and v_4 are connected with v_2, and there is a communication link between v_1 and v_2. In this process, the topology structure has been changed.

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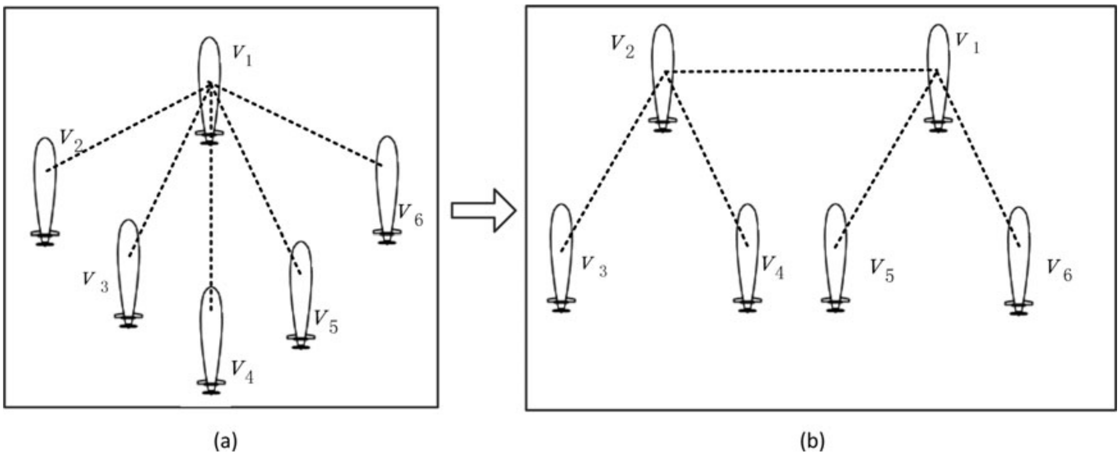

Fig. 1. The process of mutative topology structure.

3. Interactive Markov Chains Model and Model Verification

3.1. Interactive Markov chains model

The process algebra model (described by Labeled Transition Systems, LTS) is used to describe the model of concurrent systems behavior characteristics. It focuses on the actions when state transfer happens in a system, and disregards the information about the state itself. Conversely, the Markov chains model that is often used in performance evaluation field mainly deals with the state information of a system. As a concurrent system model, IMC was proposed by German scholar H. Hermanns in 1998.[19,20] It is composed of the process algebra model and the CTMC model. The two models are combined orthogonally. That is to say, the two kinds of transfer relationships are preserved in the IMC, not mixed into one. Therefore, there are two different transfer relationships in an IMC model at the same time.

Definition: An IMC is a five-element model ($S,\ Act,\ \longrightarrow,\ --\rightarrow,\ S_0$),

where S is a non-empty state set; Act $\subseteq A$ is an action set; $\longrightarrow \in S \times Act \times S$ is the action transfer relationship; $--\rightarrow \in S \times R_0 \times S$ is the Markov transfer relationship which satisfies

$$\forall s_1, s_2 \in S, |(--\rightarrow \cap (\{s_1\} \times R \times \{s_2\}))| \leq 1.$$

S_0 is the original state.

This definition contains two types of transfer relationships: action transfer in LTS and stochastic-distributed delay transfer in CTMC. According to the conditions of the Markov transfer relationship, there is no more than one Markov transfer relationship between two discrete states. Additionally, in a Markov transfer relationship $--\rightarrow$, a state transfer to itself is allowed, that is, $S \overset{\lambda}{--\rightarrow} S$ (where $\lambda > 0$) is allowed, which differs from transition rate matrix R. It means that the system still keeps in its original state or transfers to itself after the exponential distribution (parameter is λ) delay.

The definition of $R(s, s')$ is the transfer rate from state s to state s', so the total transfer rate in $(S,\ R(s, s'),\ S') \in --\rightarrow$. S is

$$E(s) = \sum_{s' \in S} R(s, s'). \tag{1}$$

The IMC model doesn't mark every state, but primarily focuses on the system transfer relationship, especially the action transfer relationship, so it inherits the characteristics of the LTS. That is, IMC is a system model based on actions, and it provides a combination model that can be used for performance evaluation.

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3.2. Continuous stochastic logic based on the action (aCSL)

Continuous Stochastic Logic based on action (aCSL) describes the temporal logic of IMC. Using aCSL, characteristics such as "The probability of action a occurs at least 0.9 in 4 time units." can be expressed. A system model and logic based on action are suitable to validate the system.[21]

Definition: $p \in [0, 1]$, $\Delta \in \{\leq, <, \geq, >\}$, the syntax for state formula of aCSL is

$$\Phi ::= \text{true}|\Phi \wedge \Phi|\neg\Phi|P_{\Delta p}(\varphi) \tag{2}$$

and $t \in R_{>0}$, $A \subseteq \text{Act}$, $B \subseteq \text{Act}$, the syntax for path formula of aCSL is

$$\varphi ::= \Phi_A U^{<t} \Phi|\Phi_A U^{<t}{}_B \Phi. \tag{3}$$

The state formula shows the character of the system state, and the path formula shows the character of the execution path. The most important part in the state formula is $P_{\Delta p}(\varphi)$. It means that the probability measure of the path set which satisfies path φ is limited to Δp range. The operator $U^{<t}$ in the path formula has a time factor, which means such a path must be completed within the time t.

3.3. Model checking

Model checking is an approach to formal verification for models; it was proposed, respectively, by Clarke[22,23] and Queille.[24] Model checking refers to the following problem: A model of a system is checked exhaustively and automatically, as to whether it meets a given specification. So, whether an IMC model satisfies certain properties aimed at a finite-state system can be determined via this technology.

Assumption: if a system is described in IMC model, it includes the original state Φ_0 and the action transfer a, so the logical characterization for the system's reliability evaluation is the "future" property. That is to say, if Φ_{down} means the system is in the state of collapse and the probability of collapse is less than η in t unit time in the future, the IMC model will be

$$P_{<\eta}(F^{<t}\Phi_{\text{down}}) = P_{<\eta}(\text{true}_{\text{Act}} U^{<t}\Phi_{\text{down}}). \tag{4}$$

While performing model checking for IMC model, the aim is to check whether a given IMC model satisfies a given aCSL state formula Φ, or whether M is a model for Φ. That is to say, whether $M| = \Phi$ holds.[25]

According to the analysis of the literature,[25] the calculating method of probability operator P can be obtained by dividing into different particular cases as follows:

1. When the form of the path formula is $\varphi = \Phi_{1A}U^{<t}\Phi_2$, the following cases can be discussed, respectively: If $F(s,t) = \Pr ob(s, \Phi_{1A}U^{<t}\Phi_2)$, $F(s,t)$ should be

$$F(s,t) = \begin{cases} 1 & \text{case 1} \\ \sum\limits_{s' \in S} \int_0^{t} R(s,s') \cdot e^{-E(s)\cdot x} \cdot F(s', t-x)dx & \text{case 2} \\ \sum\limits_{R(s,s')\geq 0} \int_0^{\delta_A} R(s,s') \cdot e^{-E(s)\cdot x} \cdot F(s', t-x)dx + \sum\limits_{I(s,s')\in A} F(s', t-\delta_A(s)) & \text{case 3} \\ 0 & \text{case 4} \end{cases}. \tag{5}$$

Case 1: $s| = \Phi_2$.

Case 2: $(s| = \Phi_1 \wedge \neg\Phi_2) \wedge (s \in PS)$. State s is a probabilistic state (system only executes a Markov transfer). It only executes a Markov transfer, where $R(s,s') \cdot \mathrm{e}^{-E(s)\cdot x}$ is the transfer probability density from state s to s' in x time unit.

Case 3: $(s| = \Phi_1 \wedge \neg\Phi_2) \wedge (s \in NS)$. State s is a non-deterministic state. The system may exhibit two possible behaviors starting from state s, depending on the standing time of state s (determined by Markov transfer) and the transfer time of action A.

Case 4: All other cases.

2. When the form of the path formula is $\varphi = \Phi_{1A}U^{<t}{}_B\Phi_2$, the following cases can be discussed, respectively. If $G(s,t) = \mathrm{Pr}ob(s, \Phi_{1A}U^{<t}{}_B\Phi_2)$, $G(s,t)$ should be

$$G(s,t) = \begin{cases} 1 & \text{case 5} \\ \sum_{s' \in S} \int_0^t R(s,s') \cdot \mathrm{e}^{-E(s)\cdot x} \cdot G(s', t-x)\mathrm{d}x & \text{case 6} \\ \sum_{R(s,s')\geq 0} \int_0^{\delta_A/B^{(s)}} R(s,s') \cdot \mathrm{e}^{-E(s)\cdot x} \cdot G(s', t-x)\mathrm{d}x & \\ \quad + \sum_{I(s,s')\in A/B} G(s', t - \delta_{A/B}(s)) & \text{case 7} \\ 0 & \text{case 8} \end{cases} \tag{6}$$

Case 5: When $(s| = \Phi_1) \wedge (\exists s'((s'| = \Phi_2) \wedge (\delta_B(s) \leq \delta_{A/B}(s) < t)))$, because of $\delta_B(s) \leq \delta_{A/B}(s)$, the transfer action B can be performed, but the transfer action A/B can't be performed (there may be intersection in A and B). Then there is a next state s' which satisfies state Φ_2, and this state must meet the path formula. Therefore, $\mathrm{Pr}\,ob(s, \Phi_{1A}U^{<t}{}_B\Phi_2) = 1$.

Case 6: This case is similar to case 2, $(s| = \Phi_1) \wedge (s \in PS)$. State s is a probabilistic state, it only executes Markov transfer.

Case 7: When $(s| = \Phi_1) \wedge (s \in NS) \wedge (\delta_{A/B}(s) < \delta_B(s) < t)$, because of $\delta_B(s) > \delta_{A/B}(s)$, the transfer action B can't be performed, only the transfer action A/B can be performed, or the system will transfer to next state s' which still satisfies the state Φ_1. Additionally, the system will perform an action transfer or a Markov transfer, depending on the standing time $\delta(s)$ of state s and the transfer time $\delta_{A/B}(s)$ of action A/B.

Case 8: All other cases.

4. Reliability of Mutative Topology Structures of Multi-AUV Cooperative Systems

In essence, the reliability of a multi-AUV cooperative system is a system performance evaluation. So the IMC model could be used as model of a mutative topology structure multi-AUV cooperative system to evaluate the system performance.

4.1. Markov transfer parameter λ_i

Two typical reliability indexes are selected: the System Reliability ϑ and the All-terminal Reliability R_U. These two indexes measure the reliability of state s, respectively, from different angles; they show not only the topology structure reliability of a multi-AUV cooperative system, but also the reliability of the communication links in the system. The Markov transfer parameter λ_i in IMC model can be obtained thus.

After transfer, if the system doesn't collapse, we have

$$\lambda_i = a \cdot \vartheta + b \cdot R_U. \tag{7}$$

Else,

$$\lambda_i = 1 - [a \cdot \vartheta + b \cdot R_U], \tag{8}$$

where a, b are the normalized weights. The two indexes (probability of collapse and the all-terminal reliability) are of same importance, so $a = 0.5$, $b = 0.5$ are adopted.

4.2. The occurrence time $\delta_A(s)$ of action transfer A

In the IMC model, if the action transfer A means transferring from state s to state s', that is $s \xrightarrow{A} s'$, then the time $\delta_A(s)$ of action transfer A is determined by state s and state s'. Specifically, a communication

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Table I. The AUV coordinates in the system.

AUV	v_1	v_2	v_3	v_4	v_5	v_6
Coordinate (m)	(4000,9000)	(9500,8500)	(1000,6000)	(6000,1100)	(5000,10000)	(1000,2000)

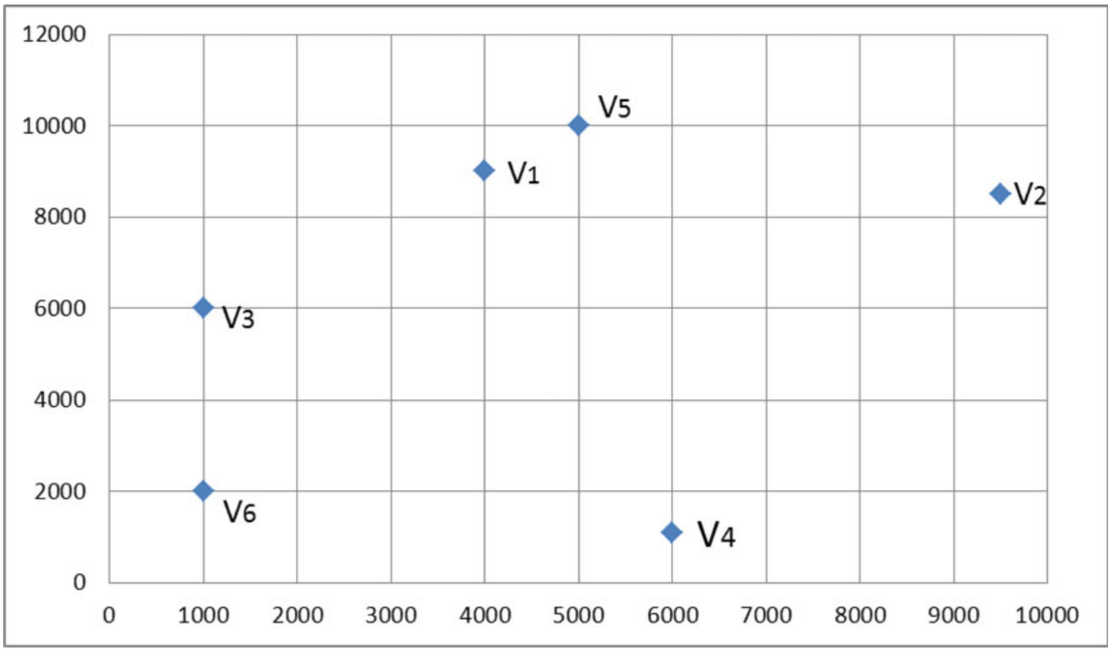


Fig. 2. AUV distribution in the system.

link increases or disappears when state s transfers to state s'. $\delta_A(s)$ is the time of transmitting a signal underwater through the communication link. If there is more than one increasing or disappearing communication link, the longest transmitting time is selected as $\delta_A(s)$. Then the time $\delta_A(s)$ of action transfer A can be obtained:

$$\delta_A(s) = \max\left\{\frac{l_i}{c}\right\}, \tag{9}$$

where, l_i is the length of the communication link, c is sound velocity.

4.3. *The collapse probability of a system*

Using the methods of model verification in Section 3.3, and judging whether the action transfer has been performed in the mutative topology process, the collapse probability of system can be obtained corresponding to the particular cases in formulas (5) and (6).

5. Example

5.1. *Example 1*

The mutative topology process of a multi-AUV cooperative system which is shown in Fig. 1 is analyzed. The reliability of multi-AUV cooperative system in 3D space is more complicated, and the number of states in the model is greatly increased by the addition of ocean depth dimension. In this paper, the topological structure of multi-AUV cooperative system is limited in the same depth of the sea level, which is simplified to the two-dimensional plane.

The AUV distribution in the system is shown in Fig. 2.

The AUV coordinates are in Table I.

Table II. The system reliability and the all-terminal reliability in every state.

State	s_0	s_1	s_2	s_3	s_4	s_5	s_6
System reliability	0.888	0.888	0	0.833	0.833	0.667	0
All-terminal reliability	0.9621	0.9621	0	0.9569	0.9848	0.9795	0

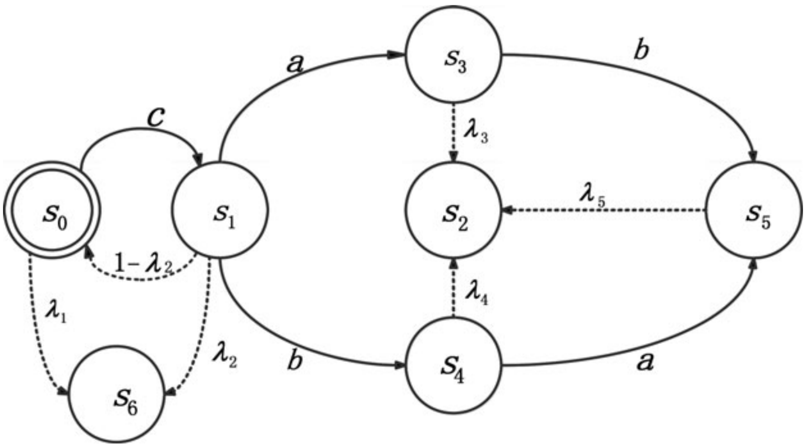
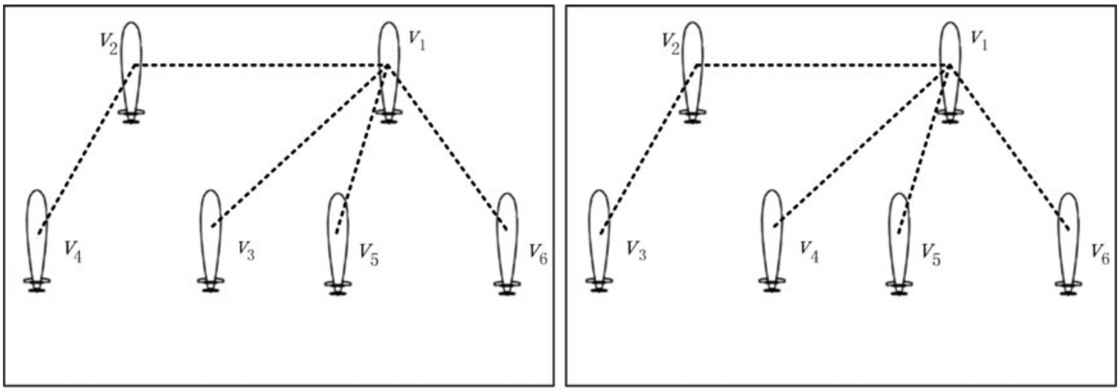


Fig. 3. The IMC model for the process of mutative topology structure of Fig. 1.



(a) (b)

Fig. 4. Intermediate states of Example 1.

According to the mutative topology process of the multi-AUV cooperative system, which is shown in Fig. 1, the IMC model can be obtained, which is shown in Fig. 3.

Action set is $\text{Act} = \{a, b, c\}$. Action a means that v_3 interrupts communication with v_1, and establishes communication with v_2. Action b means that v_4 interrupts communication with v_1, and establishes communication with v_2. Action c, as an internal action, means that the system will change its topology structure. '$\longrightarrow$' means action transfer, and '$-- \rightarrow$' means Markov transfer. State s_0 is the original state, means that the topology structure hasn't been changed, as shown in Fig. 1(a). State s_1 is the state to which the system topology structure is about to change. Notice from Fig. 3, when the system is in state s_1, it means the system will change its topology structure by performing action a or action b at this time, and also may return to state s_0 in some cases. State s_3 means v_3 has communication with new pilot v_2, but v_4 still communicates with v_1, as shown in Fig. 4(a). State s_4 means v_4 has communication with new pilot v_2, but v_3 still communicates with v_1, as shown in Fig. 4(b). State s_5 means there are two pilots v_1 and v_2 in the system, that is to say, the topology structure has been changed, as shown in Fig. 1(b). State s_2 and state s_6 mean that the system can't change its topology structure successfully, and the system is in a state of collapse. Through the above process, the IMC model for the mutative topology structure of a multi-AUV cooperative system can be built.

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Table III. Results of property Φ.

State	s_0	s_1	s_2	s_3	s_4	s_5	s_6
$\Pr ob$	0.0037	0.0063	1	0.0151	0.0098	0.0876	1

The system reliability and the all-terminal reliability in every state are shown in Table II. So, the Markov transfer parameter can be obtained.

$$\lambda_1 = \lambda_2 = 0.075, \quad \lambda_3 = 0.105, \quad \lambda_4 = 0.091, \quad \lambda_5 = 0.177.$$

The occurrence time of action transfer a and b is as follows:

$$\delta_a(s) = \frac{l(v_2, v_3)}{c}, \quad \delta_b(s) = \frac{l(v_2, v_4)}{c},$$

where, $l(v_2, v_3)$ means the distance from v_2 to v_3. $l(v_2, v_4)$ means the distance from v_2 to v_4, c is sound velocity, here $c = 1500\,\mathrm{m/s}$.

So, $\delta_a(s) = 5.91s$, $\delta_b(s) = 5.46s$.

Using the IMC model, the mutative topology structure system can be evaluated by the following property.

The collapse probability of the system in 30 min is $\Pr ob$, which is shown in Table III. And

$$\Phi = P_{<\eta}\left(\mathrm{true}_{\mathrm{Act}} u^{<30}\{s_2, s_6\}\right).$$

Comparing the value of $\Pr ob$ in Table III with the collapse probability upper bound $\eta = 0.01$, the state set which satisfies property $\Pr ob < 0.01$ can be obtained.

$$\mathrm{Sat}(\Phi)_{\eta=0.01} = \{s_0, s_1, s_4\}.$$

In this way, the solution set of global model verification for property Φ can be obtained. That is, all states which satisfy the property have been calculated. By checking whether the state is in this solution set, the solution of partial model can be obtained. Because of $s_0 \in \mathrm{Sat}(\Phi)_{\eta=0.01}$, so $s_0 |= \Phi$. That is to say, if the system begins changing its topology structure from state s_0, the collapse probability in 30 min is less than 0.01. In other words, in this AUV distribution and set of environmental conditions, if state s_0 is the original state for the mutative topology structure system, the multi-AUV cooperative system wouldn't collapse in 30 min.

At the same time, $s_4 \in \mathrm{Sat}(\Phi)$ and $s_3 \notin \mathrm{Sat}(\Phi)$. That is to say, the reliability in state s_4 is higher than the reliability in state s_3. So, path $s_1 \xrightarrow{b} s_4 \xrightarrow{a} s_5$ is better than path $s_1 \xrightarrow{a} s_3 \xrightarrow{b} s_5$, and it should be selected preferentially in the process of changing topology structures for multi-AUV cooperative systems.

5.2. Example 2

For the same multi-AUV cooperative system shown in Fig. 2, if the mutative topology structure process is different from Fig. 1 (shown in Fig. 5), the IMC model can be obtained, which is shown in Fig. 6.

Similar to the situation shown in Fig. 1(a), the multi-AUV cooperative system in Fig. 5(a) has one pilot v_1 in initial state. When mission requirement or environment changes, the topology structure of multi-AUV cooperative system may change to a hierarchical structure, as shown in Fig. 5(b). Obviously, v_5 is the bottom stratum, it follows v_4. v_4 and v_6 are the second stratum, they all follow v_3. v_3 and v_2 are the third stratum, they all follow v_1. v_1 is the top stratum, it leads the whole formation.

According to the mutative topology process of the multi-AUV cooperative system, which is shown in Fig. 5, the IMC model can be obtained, which is shown in Fig. 6. Where S_0 is the initial state. S_1, S_2 and S_3 are the intermediate states. S_4 is the final state.

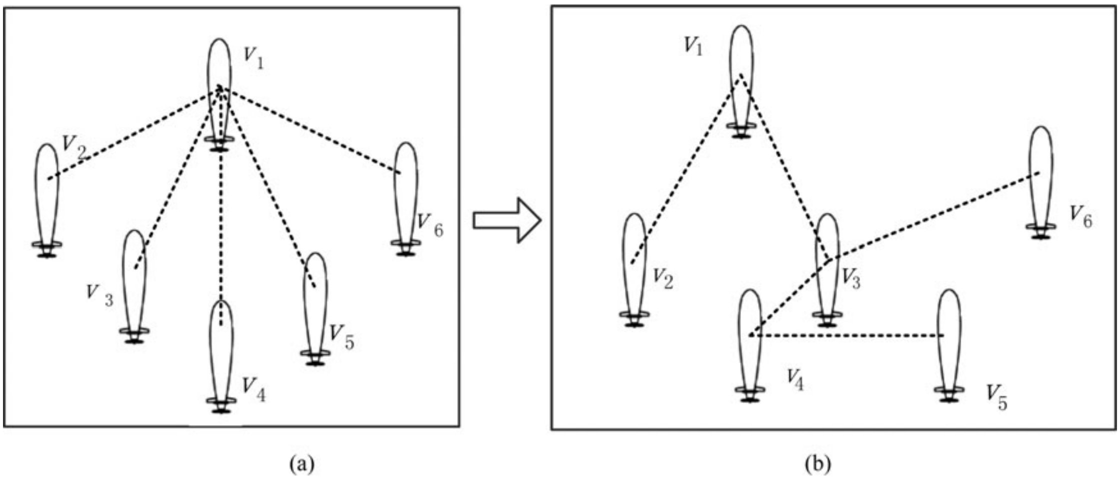

Fig. 5. The process of the variable topology.

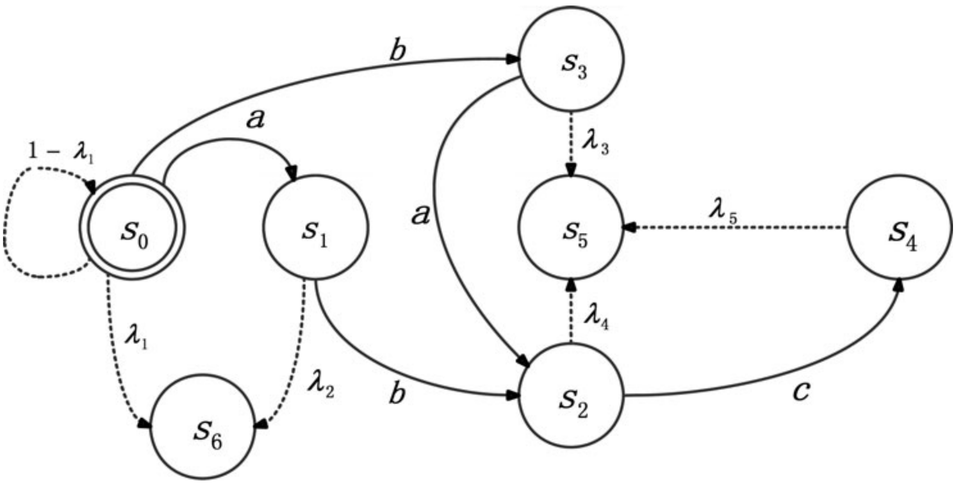

Fig. 6. The IMC model for the process of mutative topology structure of Fig. 5.

Action set is $\mathrm{Act} = \{a, b, c\}$. Action a means that v_4 interrupts communication with v_1, and establishes communication with v_3. Action b means that v_5 interrupts communication with v_1, and establishes communication with v_4. Action c means that v_6 interrupts communication with v_1, and establishes communication with v_3. State S_1 means v_4 has communication with v_3, and v_2,v_3, v_5, v_6 still have communication with v_1, as shown in Fig. 7(a). State S_2 means v_5 has communication with v_4, v_4 have communication with v_3, and v_2, v_3, v_6 still have communication with v_1, as shown in Fig. 7(b). State S_3 means v_5 has communication with v_3, and v_2, v_3, v_4, v_6 still have communication with v_1, as shown in Fig. 7(c). State s_5 and s_6 mean that the system can't change its topology structure successfully, and the system is in a state of collapse.

The system reliability and the all-terminal reliability in every state are shown in Table IV.

So, the Markov transfer parameter can be obtained.

$$\lambda_1 = \lambda_2 = 0.075, \quad \lambda_3 = 0.111, \quad \lambda_4 = 0.096, \quad \lambda_5 = 0.118.$$

The occurrence time of action transfer a and b is as follows:

$$\delta_a(s) = \frac{l(v_3, v_4)}{c}, \quad \delta_b(s) = \frac{l(v_4, v_5)}{c}, \quad \delta_c(s) = \frac{l(v_3, v_6)}{c},$$

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Table IV. The system reliability and the all-terminal reliability in every state.

State	s_0	s_1	s_2	s_3	s_4	s_5	s_6
System reliability	0.888	0.888	0.821	0.845	0.813	0	0
All-terminal reliability	0.9621	0.9621	0.957	0.9623	0.95	0	0

Table V. Results of property Φ.

State	s_0	s_1	s_2	s_3	s_4	s_5	s_6
Pr*ob*	0.0105	0.0640	0.0074	0.0061	0.0890	1	1

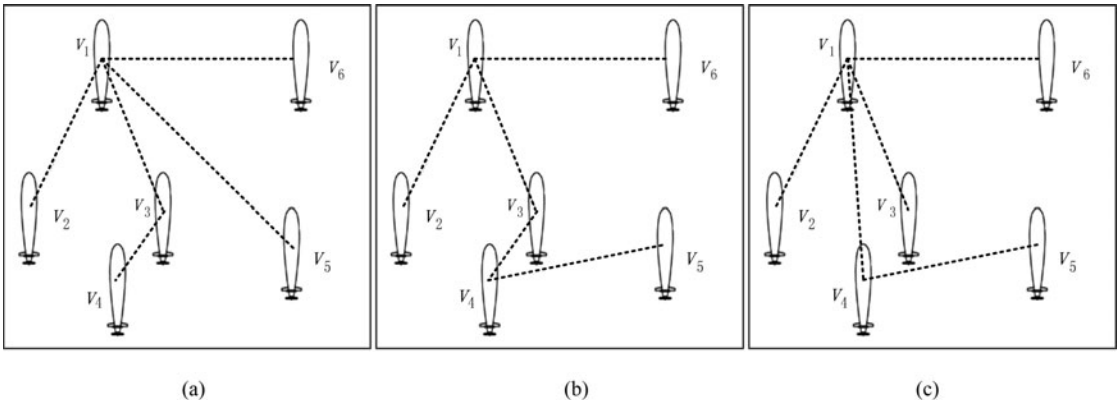

Fig. 7. Intermediate states of Example 2.

where, $l(v_3, v_4)$ means the distance from v_3 to v_4. $l(v_4, v_5)$ means the distance from v_4 to v_5, $l(v_3, v_6)$ means the distance from v_3 to v_6, c is sound velocity, here $c = 1500\,\mathrm{m/s}$.

So, $\delta_a(s) = 4.67s,\ \delta_b(s) = 0.94s, \delta_c(s) = 2.67s$.

Using the IMC model, the mutative topology structure system can be evaluated by the following property.

The collapse probability of the system in 30 min is Pr*ob*, which is shown in Table V. And

$$\Phi = P_{<\eta}\left(\mathrm{true}_{\mathrm{Act}} u^{<30}\{s_5, s_6\}\right)$$

The value of Φ in Table V is compared with the collapse probability upper bound 0.01, the state set which satisfies property Φ can be obtained.

$$\mathrm{Sat}(\Phi)_{\eta=0.01} = \{s_2, s_3\}$$

It means $s_0 \notin \mathrm{Sat}(\Phi)_{\eta=0.01}$. That is to say, if the system begins changing its topology structure from state s_0, the collapse probability in 30 min is more than 0.01.

If the benchmark is selected as 0.02, then

$$\mathrm{Sat}(\Phi)_{\eta=0.02} = \{s_0, s_2, s_3\}.$$

It means $s_0 \in \mathrm{Sat}(\Phi)_{\eta=0.02}$. That is to say, if the system begins changing its topology structure from state s_0, the collapse probability in 30 min is less than 0.02. Similar as Example 1, path $S_0 \xrightarrow{b} S_3 \xrightarrow{a} S_2 \xrightarrow{c} S_4$ is better than path $S_0 \xrightarrow{a} S_1 \xrightarrow{b} S_2 \xrightarrow{c} S_4$.

5.3. Analysis of two examples

The two examples illustrate that mutative topology structure reliability is different when final topology structures are different with same initial topology structure (from initial topology structure, for Example 1, the collapse probability in 30 min can less than 0.01. but for Example 2, the collapse probability in 30 min will be more than 0.01). They also illustrate that, for same initial topology structure and same final topology structure, different mutative topology structure process can lead to different reliability of the system. So the study of reliability for mutative topology structure can provide reference for mutative topology structure process of multi-AUV cooperative system.

The two examples are simulation result. If a multi-AUV cooperative system composed of six AUVs can be build up, then the mutative topology structure procedure can be performed. The mutative topology structure procedure of Fig. 4 in path $s_1 \xrightarrow{b} s_4 \xrightarrow{a} s_5$ and in path $s_1 \xrightarrow{a} s_3 \xrightarrow{b} s_5$ can be performed, respectively, and the reliability of these two procedures can be contrasted. And the reliability of procedure Fig. 5 in path $S_0 \xrightarrow{b} S_3 \xrightarrow{a} S_2 \xrightarrow{c} S_4$ can be contrasted with the reliability of procedure Fig. 4 in path $s_1 \xrightarrow{a} s_3 \xrightarrow{b} s_5$. The result can validate this paper.

6. Conclusions

For multi-AUV cooperative system, there is no effective maturity model to describe the reliability of the process of the variable topology. The existing reliability analysis methods, such as neural network method, Fault Tree method, AHP, and so on, are suitable for the static topology structure. With these methods, the process of the variable topology cannot be described and analyzed. This paper put forward using the IMC, which is a concurrent system model composed of the functional behavior and performance index, to analyze the mutative topology structure of multi-AUV cooperative system. And then model checking method is used to obtain the reliable probability of the system. This provides a method of reliability analysis for a mutative topology structure multi-AUV cooperative system.

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