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CORRECTION

TSUKUDA, K. (2017). Estimating the large mutation parameter of the Ewens sampling formula. *J. Appl. Prob.* **54,** 42–54.

In the above paper we demonstrated the asymptotic normality of K_n in cases (A), (B), and (C1). The proof of Theorem 2 holds only in cases (A) and (B), and the error is contained in case (C1). In this correction, we provide the correct proof when case (C1) is considered (using the same notation as in the above paper).

Description of the error. In the second and fourth equations in (15) of the above paper, '$\theta/\sigma^3 \to 0$ as $n \to \infty$' was used. However, in case (C1), θ/σ^3 does not always converge to 0, since $\sigma \sim \sqrt{n^2/(2\theta)}$ yields $\theta/\sigma^3 \sim \sqrt{8\theta^5/n^6}$. Therefore, the proof of Theorem 2 after equation (14) needs to be corrected.

Proof of Theorem 2 in case (C1). As stated on the right-hand side of (14) of the above paper, it holds that $\log \mathbb{E}[\mathrm{e}^{Z_n t}] = -\sigma t + (\mathrm{e}^{t/\sigma} - 1)\sigma^2 + A + o(1)$, where A is

$$A = \theta\left(\frac{\theta}{n+\theta} - 1\right)\left\{\frac{t}{\sigma} - (\mathrm{e}^{t/\sigma} - 1)\right\} - \frac{t}{\sigma}\theta \mathrm{e}^{t/\sigma} + (\theta \mathrm{e}^{t/\sigma} + n)\log\left(1 + \frac{\theta}{n+\theta}(\mathrm{e}^{t/\sigma} - 1)\right). \tag{1}$$

In case (C1), since $\sigma \sim \sqrt{n^2/(2\theta)}$, it holds that $\theta\{n/(n+\theta)\}^3(-t/\sigma)^3 = O(1/\sqrt{\theta})$ and, hence, the third term on the right-hand side of (1) is equal to

$$\begin{aligned}
&(\theta \mathrm{e}^{t/\sigma} + n)\log\left(\frac{n + \theta \mathrm{e}^{t/\sigma}}{n+\theta}\right)\\
&\quad = (\theta \mathrm{e}^{t/\sigma} + n)\log\left(\mathrm{e}^{t/\sigma} + \frac{n}{n+\theta}(1 - \mathrm{e}^{t/\sigma})\right)\\
&\quad = (\theta \mathrm{e}^{t/\sigma} + n)\left(\frac{t}{\sigma} + \log\left(1 + \frac{n}{n+\theta}(\mathrm{e}^{-t/\sigma} - 1)\right)\right)\\
&\quad = (\theta \mathrm{e}^{t/\sigma} + n)\left\{\frac{t}{\sigma} + \frac{n}{n+\theta}(\mathrm{e}^{-t/\sigma} - 1) - \frac{1}{2}\left(\frac{n}{n+\theta}\right)^2(\mathrm{e}^{-t/\sigma} - 1)^2\right\} + o(1).
\end{aligned}$$

We thus have

$$\begin{aligned}
A &= -\frac{\theta n}{n+\theta}\frac{t}{\sigma} + \frac{n}{\sigma}t + \frac{n^2}{n+\theta}(\mathrm{e}^{-t/\sigma} - 1) - \frac{1}{2}\left(\frac{n}{n+\theta}\right)^2(\theta \mathrm{e}^{t/\sigma} + n)\left(\mathrm{e}^{-t/\sigma} - 1\right)^2 + o(1)\\
&= \frac{n}{\sigma}t\left(1 - \frac{\theta}{n+\theta}\right) + n\frac{n}{n+\theta}(\mathrm{e}^{-t/\sigma} - 1) - \frac{1}{2}\left(\frac{n}{n+\theta}\right)^2(\theta \mathrm{e}^{t/\sigma} + n)(\mathrm{e}^{-t/\sigma} - 1)^2 + o(1)\\
&= n\left(\frac{n}{n+\theta}\right)\left(\frac{t}{\sigma} + \mathrm{e}^{-t/\sigma} - 1\right) - \frac{n^2}{2\theta}\frac{\mathrm{e}^{t/\sigma}}{(1+n/\theta)^2}\left(1 + \frac{n\mathrm{e}^{-t/\sigma}}{\theta}\right)(\mathrm{e}^{-t/\sigma} - 1)^2 + o(1)\\
&= n\left(\frac{n}{n+\theta}\right)\left(\frac{t}{\sigma} + \mathrm{e}^{-t/\sigma} - 1\right) - \frac{n^2}{2\theta}(\mathrm{e}^{-t/\sigma} - 1)(1 - \mathrm{e}^{t/\sigma})\frac{1 + n\mathrm{e}^{-t/\sigma}/\theta}{(1+n/\theta)^2} + o(1).
\end{aligned} \tag{2}$$

Using $n/(n+\theta) = n/\theta - n^2/\theta^2 + O(n^3/\theta^3)$, the first term of (2) is

$$n\left(\frac{n}{\theta} - \frac{n^2}{\theta^2} + O\left(\frac{n^3}{\theta^3}\right)\right)\left(\frac{t^2}{2\sigma^2} + O\left(\frac{1}{\sigma^3}\right)\right) = \frac{n^2}{\theta}\frac{t^2}{2\sigma^2} + o(1).$$

The second term of (2) is $-n^2t^2/(2\theta\sigma^2) + o(1)$ since it holds that

$$\begin{aligned}(\mathrm{e}^{-t/\sigma} - 1)(1 - \mathrm{e}^{t/\sigma}) &= \left(-\frac{t}{\sigma} + \frac{t^2}{2\sigma^2} + o\left(\frac{1}{\sigma^2}\right)\right)\left(-\frac{t}{\sigma} - \frac{t^2}{2\sigma^2} + o\left(\frac{1}{\sigma^2}\right)\right) \\ &= \frac{t^2}{\sigma^2} + O\left(\frac{\theta^2}{n^4}\right)\end{aligned}$$

and

$$\frac{1 + n\mathrm{e}^{-t/\sigma}/\theta}{(1 + n/\theta)^2} = 1 + O\left(\frac{n}{\theta}\right).$$

Therefore, $A = o(1)$ and, consequently, $\log \mathbb{E}[\mathrm{e}^{Z_n t}] = -\sigma t + (\mathrm{e}^{t/\sigma} - 1)\sigma^2 + o(1) \to \frac{1}{2}t^2$ also in case (C1). □

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