PROPAGATION OF ELECTROMAGNETIC WAVES IN PULSAR MAGNETOSPHERES

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Abstract. We have developed a numerical code for the propagation of different electromagnetic modes in a pulsar magnetosphere filled by a relativistic, streaming electron-positron plasma in a strong, curved magnetic field. We determine the trajectories, limiting polarization and damping of the waves leaving the magnetosphere.

There is no doubt that the source of the radio emission, received from a radio pulsar, lies in the pulsar magnetosphere (Taylor and Stinebring, 1986), (Lyne and Smith, 1990). Radio signals from the source to the receiver on Earth propagate mainly in the interstellar matter (ISM). The dispersion of the electromagnetic waves in the ISM is simply, $N^2 = 1 - \omega_p^2/\omega^2$, which for a cold nonmagnetized plasma gives many observed physical effects such as the dispersion delay of the signal, scattering, diffraction and interference if the plasma frequency varies in space and time. Also, the small gyrotropy of the ISM, due to the presence of the Galactic magnetic field, results in Faraday rotation of the polarization of an observed signal. The distance the radio wave propagates in the pulsar magnetosphere is relatively small, however. The aim of this paper to show that the effects of the propagation of electromagnetic waves in the neutron star magnetosphere are not less important than in the ISM due to the large plasma density in the magnetosphere and its complex electromagnetic properties.

The dimension of the magnetosphere is the radius of the light cylinder $R_L = c/\Omega$ inside of which the charged particles corotate with the same angular velocity Ω as the neutron star. The characteristic scale is $10^9 - 10^{10}$ cm, or $\simeq 10^3 - 10^4$ times the neutron star radius of, $R \approx 10^6$ cm. For the millisecond pulsars this scale is one hundred times less. The pulsar magnetosphere is formed by the strong magnetic field frozen to the neutron star matter and rotating together with it. The typical value of the magnetic field strength on the neutron star surface is, $B_0 \approx 10^{12}$ G. At larger distances from the surface the magnetic field decreases as, $B \propto r^{-3}$, and at the light cylinder the magnetic field strength is, $1 - 10^3$ G. Due to this large value of magnetic field the magnetospheric plasma is strongly magnetized and moreover the plasma particles do not possess transverse momentum owing to the fast synchrotron losses. The plasma particles themselves are generated in the magnetosphere in the strong magnetic field and are electrons and positrons. They are relativistic and more along the magnetic field. The density of the magnetospheric plasma is proportional to the Goldriech-Julian density $n_e = \lambda n_{GJ}$. The value of $n_{GJ} = -(\mathbf{B}\Omega)/2\pi ce$ is

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the minimum plasma density in the pulsar magnetosphere required to screen the electric field induced by the rotating magnetic field. The coefficient λ is a so-called multiplication parameter $\simeq 10^3 - 10^5$. So, the plasma density near the star surface can be as high as 10^{16} cm^{-3}, and correspondingly the plasma frequency will be $\omega_p^2 = (6 \cdot 10^{12}\ \mathrm{sec}^{-1})^2$.

One might think that the electromagnetic waves of frequency ω of 10^9 sec^{-1} cannot propagate in the inner parts of the pulsar magnetosphere. But this ignores the relativistic nature of the magnetospheric plasma. Indeed, the distribution functions of electrons and positrons have maxima at the large energies of, $E \gg m_e c^2$. The shapes of the distribution functions over the Lorenz factor $\gamma = E/m_e c^2$ have a power low dependence $F^{\pm}(\gamma) \propto \gamma^{-\beta}$ ($\beta \simeq 2$) with a minimum value of $\gamma_{\mathrm{min}} \simeq 10^2$. It is important that the distributions of electrons and positrons are not identical, as they are shifted in energy. This shift appears in order to satisfy the stationary condition $n^+ - n^- = n_{GJ}$. As a result, the induced electric field directed along the magnetic field accelerates one component of the plasma and decelerates the other. This nonidentity of electrons and positrons is responsible for the gyrotropy properties of the magnetospheric plasma. For the one dimensional (motion only along the magnetic field) relativistic plasma, the response to the wave electric field is γ^3 times less than for the nonrelativistic case. As a result the plasma dispersion is characterized by the effective plasma frequency $\langle \omega_p^2/\gamma^3 \rangle \ll \omega_p^2$. Here the angular brackets $\langle \cdots \rangle$ mean summing and averaging over the distribution functions of electrons and positrons. Usually $\langle \omega_p^2/\omega^2\gamma^3 \rangle < 1$ and the electromagnetic waves at the observed frequencies can propagate in the pulsar magnetosphere.

The general expression for the dielectric permittivity tensor $\varepsilon_{\alpha\beta}(\omega, \mathbf{k})$ is rather complicated (Hardee and Rose, 1976). All nine components of the permittivity tensor $\varepsilon_{\alpha\beta}$ are nonzero. In the nondiagonal terms the electrons and positrons do not compensate each other, though the cyclotron frequencies have opposite signs: the distribution functions are different. This circumstance results in Faraday rotation of the wave polarization. The solution of the dispersion equation is rather complicated, but we can draw some conclusions only looking at the permittivity tensor. There are two singularities in the expressions for $\hat{\varepsilon}$.

1. $\tilde{\omega} = \omega - k_z v_{\parallel} \simeq 0$. This is the condition for Cherenkov resonance. For the relativistic, streaming plasma the resonance condition is not sensitive to the particle distribution function. All particles move almost with the speed of light. As a result, the influence of the Cherenkov resonance on the real part of ε_{zz} is significant. Thus, there exists an eigen mode, $\omega = k_z c$. It can be called an Alfvén mode because it propagates along the magnetic field. Indeed, the group velocity $\mathbf{v}_{gr} = \partial\omega/\partial\mathbf{k} = c\mathbf{B}/B$ only has a component along the magnetic field.

2. $\tilde{\omega} = \pm\omega_B/\gamma$. This is the condition of cyclotron resonance. But this resonance condition is very sensitive to the particle energy. At a fixed coordinate the cyclotron resonance takes place only for a small group of the particles $\gamma \simeq |\omega_B/\tilde{\omega}|$. So the particle distribution function smooths the contribution of the resonance to the real part of the dispersion equation, and there are no cyclotron modes. The cyclotron

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resonance defines only the cyclotron absorption of the eigen modes. Note also that the cyclotron resonance depends strongly not only on the particle energy γ but also on the orientation of the wave vector **k** with respect to the magnetic field **B**. $\tilde{\omega} = \omega[1 - N\cos\theta(1 - 1/2\gamma^2)]$, $\cos\theta = (\mathbf{kB})/kB$. And for the Alfvén mode $N_A = 1/\cos\theta$ the cyclotron resonance takes place in the outer region of the magnetosphere, where $\omega_B = \omega/2\gamma \ll \omega$. But for other modes, $N \simeq 1$, the point of resonance is closer to the neutron star surface $\omega_B = \omega\gamma\theta^2/2 \gg \omega/2\gamma$ ($\gamma^{-1} \ll \theta \ll 1$). Because the plasma density is proportional to the magnetic field strength, the wave absorption is stronger for denser plasma, the Alfvén mode experiences weaker cyclotron absorption than another modes.

It is convenient to begin mode classification in the inner magnetosphere where $\omega_B \gg \tilde{\omega}\gamma$ (Beskin, Gurevich and Istomin, 1993). Apart from the Alfvén mode there exists the extraordinary transverse mode $N_t = 1$. It is polarized so that the wave electric field is orthogonal to the wave vector, as the ambient magnetic field $\mathbf{E}\perp\mathbf{k}, \mathbf{B}$. Also the normal modes are two plasma waves $N_p = 1 + \theta^2/4 \pm [\langle\omega_p^2/\omega^2\gamma^3\rangle + \theta^4/16]^{1/2}$. At $\theta = 0$ they are longitudinal $\mathbf{E} = \mathbf{E}_z$, but at larger angles $\theta > \theta^*$, $\theta^* = \langle\omega_p^2/\omega^2\gamma^3\rangle^{1/4}$, the plasma waves become transverse. The upper mode approaches the Alfvén one, $N_p^+ \rightarrow 1 + \theta^2/2 = 1/\cos\theta$, and the lower plasma wave approaches the extraordinary one, $N_p^- \rightarrow 1$. Due to this, the propagation of different waves are essentially different. The extraordinary wave propagates along a straight line. Conversely, the Alfvén mode propagates along the curved magnetic field. The propagation of the plasma modes N_p are intermediate. The mode N_p^- initially deflected from the magnetic field except at $\theta > \theta^*$, begins to move as in a vacuum. The mode N_p^+ at $\theta > \theta^*$ is trapped by the magnetic field lines. The different modes have different directivity patterns. For example, the Alfvén waves spread more widely than the extraordinary ones.

For the investigation of mode propagation, absorption and polarization, we developed a numerical code for electromagnetic wave propagation in pulsar magnetospheres. This code is based on that for the same problem in toroidal thermonuclear devices with the aim of plasma heating and creating electric current (Smirnov and Harvey, 1995), strongly modifying this code for the specifics of pulsar magnetospheres. First of all the numerical analysis shows that the Alfvén mode splits into four separate modes because they correspond to $\tilde{\omega} \simeq 0$. $\tilde{\omega}$ itself contains the value of the refractive index N and depends on the particle energy γ. Thus, the equation for N is not algebraic, but is an integral one. The splitting of the Alfvén mode corresponds to the result obtained in (Beskin, Gurevich and Istomin, 1993), where curvature effects gave an analogous splitting. The propagation of different modes through the pulsar magnetosphere was calculated using ray optics. We used the dipole magnetic field with amplitude B_0 given by the magnetic pole of the pulsar. The plasma density $n(\mathbf{r})$ was proportional to the magnetic field $n(\mathbf{r}) = n_0 B(\mathbf{r})/B_0$. An example of a ray trajectory for a plasma wave of frequency 1 GHz propagating from the star's surface, where $B = 10^{12}$ G, $\lambda = 10^4$ is shown in the figure. The plasma modes are practically all damped in the pulsar magnetosphere and cannot escape. Only two Alfvén modes can escape with a small amount of damping.

We consider that the code developed is able to reveal the source of the radio emission in the pulsar magnetosphere if we can compare observed emission parameters with calculated ones. To realize this program we need to solve the reverse problem, simulating ray propagation in reversed time inside of the pulsar magnetosphere. We must start from the outer magnetosphere for the different modes with parameters corresponding to the observed ones, and the reverse trajectories will indicate the locations and characteristics of emission regions.

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