J. Plasma Physics (2011), *vol.* 77, *part* 4, *pp.* 493–519. © Cambridge University Press 2010
doi:10.1017/S0022377810000644

Theory of azimuthal surface waves propagating in non-uniform waveguides

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(Received 31 July 2010; revised 14 October 2010; accepted 16 October 2010;
first published online 16 November 2010)

Abstract. This paper is devoted to the theory of surface waves propagating across axis of symmetry in non-uniform cylindrical metal waveguides with plasma filling. The presented results are devoted to: first, studying an influence of plasma density non-uniformity on the features of these waves; second, studying an influence of an external magnetic fields' non-uniformity on their dispersion properties; third, studying possibility to sustain gas discharge by propagation of these waves under different operating regimes. The problems have been solved both analytically and numerically. Plasma particles are described in the framework of hydrodynamics; fields of the studied waves are determined by a set of Maxwell equations. Analytical research of the obtained equations is carried out by the method of successive approximation; adequacy of such approach is proved here as well. Numerical evaluations of the possibilities to observe experimentally the phenomena, which accompany propagation of these waves, are carried out.

1. Introduction

Excitation of bounded plasma systems can be executed in the most effective manner nearby their eigen frequencies [1]. Here, we present the results of theoretical studying properties of electromagnetic waves, which propagate along plasma surface with finite value of its curvature (across the axis of symmetry, along azimuthal direction). That is why, these waves have been entitled as azimuthal surface waves (ASWs) [2]. ASW belong to assemblage of non-symmetrical electromagnetic perturbations, which features are interesting from practical point of view. When conducting experimental investigation into properties of bounded plasma, one deals frequently with non-uniform profiles of plasma density and of utilized external magnetic field. That is why, research into properties of ASW in non-uniform waveguides can be considered as an actual task of plasmas' electrodynamics.

Experimental studying of non-uniform plasma waveguides is important for solving different radio-physical problems. For instance, plasma produced during high frequency (HF) gas discharge affects efficiency of radiation of both the dipole antennae and spheroidal antennae; anisotropy of magnetoactive plasma, which coats slot antennae, strongly affects the spatial distribution of emissions' field of non-symmetric azimuthal waves and enhances their radiation. Results of a broad theoretical and experimental investigation into excitation of quasi-electrostatic and helicon waves by ring vibrator immersed into magnetoactive plasma are represented in [3]. It is found out there that power of their radiation depends on plasma density profile by resonant manner. Thus studying the ASW features in non-uniform waveguides is important for the development of new type of powerful antennae.

Dispersion properties of surface waves (SWs) are under consideration for a long time. Thus linear theory of the SW propagating in simple waveguide structures (i.e. uniform non-magnetized plasma waveguides) is developed good enough (see, for instance, review [4]). Related materials, which are devoted to the linear theory of plasma SW, are presented in [5]. It is shown there that frequency spectrum of electrostatic waves in a cold non-magnetized plasma is discrete and values of the eigen frequencies strongly depend on plasma density gradient. The paper [6] is devoted to studying the properties of solitary SW on a magnetoactive plasma cylinder. Propagation of electrostatic SW in cold non-magnetized plasma restricted by non-uniform dielectric is studied in [7]. There dispersion equation in a simple form is derived, and theory of solitary SW developed previously is generalized. Research into SW properties is underway at the present time. It would be useful to indicate, for instance, that spectrum of electromagnetic waves in a magnetoactive gaseous plasma layer is studied in detail in the paper [8]. Dispersion relation for electromagnetic SW in a magnetoactive annular plasma column located in a coaxial metal waveguide is under consideration in [9]. Taking into account the great interest in studying the dusty plasmas' properties, we would like to mention that propagation of SW on vacuum boundary with cold magnetoactive plasma containing the mobile dust grains is studied in [10]. It is found out there that two surface type modes can exist in the frequency range below ion-cyclotron one in such waveguide structure. Investigation of modulational instabilities of SWs, which propagate along the metal interface with the plasma containing the nanoparticles, is carried out in [11].

The SW properties are intensively studied as well with the purpose to sustain gas discharges. Microwave gas discharges sustained by SW have broad practical applications in different plasma technologies [12, 13]. Parameters of microwave gas discharges depend on different characteristics, such as geometry of discharge chamber, type of operating gas and its pressure, value of external magnetic field, type of operating electromagnetic mode, and so on. It is found that utilization of external magnetic field and non-symmetrical SW with azimuthal wave number $m \geqslant 1$ allows increasing the volume of the produced plasma and makes gas discharge longer [14–17]. Plasma producer based on utilization of transverse waves excited by slot antennae [16] seems to be an effective device because operating modes, which propagate along azimuthal direction, do not lose their energy from the discharge chamber. Application of SW has some advantages as compared with the case of bulk waves' application: - it allows one to obtain plasma with more uniform density profile because ionization of operating gas due to the SW propagation happens mostly at plasma periphery region where amplitude of SW is of maximum value; - it provides more effective transfer of electromagnetic power from external source into plasma because SW are slow modes as compared with bulk modes, then interaction of SW with plasma is more intensive, etc. That is why, studying the properties of SWs (including ASWs), which propagate in non-uniform plasma, is a typical problem of plasma electronics and plasma technologies.

The objective of this paper is to summarize the results of a long-term research into the properties of ASWs, which propagate in different waveguide structures filled by plasma with non-uniform density profile immersed into an external non-uniform magnetic field. The paper is arranged as follows: Sec. 2 is devoted to considering the properties of ASW in non-uniform waveguides; application of ASW for sustaining gas discharges is studied in Sec. 3; results obtained are summarized in Sec. 4.

2. Azimuthal surface waves in non-uniform plasma waveguides

The model of plasma with uniform density describes in good enough manner only in the case of solid-state plasma [1]. Non-uniformity of its density affects the frequencies of both bulk and SWs. Often, laboratory plasma is immersed in the spatially non-uniform magnetic field, and this circumstance also affects the frequency spectrum of eigen modes. In this section, the results of research into influences both of plasma density and of external magnetic field non-uniformity on dispersion properties of ASW are represented.

2.1. Influence of non-uniform radial profile of plasma density on the ASW frequency spectrum

Let us consider a three-component waveguide structure, which is consisted of cylindrical metal waveguide of radius R_2 that has dielectric coating in the region $R_2 > r > R_1$ with dielectric permeability ε_d and coaxial plasma cylinder of radius R_1. Plasma density is supposed to be large so that inequality $\Omega_e^2 \gg \omega_e^2$ is valid, where Ω_e and ω_e are Langmuir and electron cyclotron frequencies, correspondingly. External magnetic field is directed along the axial direction $\vec{B}_0 \parallel \vec{z}$. Starting from the Maxwell set of equations, one can derive the following equation for the axial magnetic component of the ASW field in the region of non-uniform plasma:

$$\frac{1}{r}\frac{d}{dr}\frac{r}{k_\perp^2}\frac{dH_z}{dr} - \left[1 + \frac{m^2}{k_\perp^2 r^2} - \frac{m}{r}\frac{d}{dr}\left(\frac{\mu}{k_\perp^2}\right)\right] H_z = 0, \tag{1}$$

where $\mu = \varepsilon_2/\varepsilon_1$, $k_\perp^2 = k^2(\mu^2 - 1)\varepsilon_1$, $k = \omega/c$, ε_j are components of dielectric permeability tensor for magnetoactive plasma in hydrodynamic approximation [1], m is azimuthal wave number. Applying the method of etalon equations [18] for solving (1) in the plasma region, $r < R_1$, one can derive: $H_z = A(r)I_m(z)$, where $z' = k_\perp\sqrt{1 - (m/r)d(\mu/k_\perp^2)/dr}$, $I_m(z)$ is modified Bessel function [19], $A = \mathrm{const}\sqrt{zk_\perp^2/(rz')}$. Validity of such solution responds to the inequality:

$$1 \gg \left|\frac{m}{r}\frac{d}{dr}\left(\frac{\mu}{k_\perp^2}\right)\right| \tag{2}$$

which describes the case of weak non-uniformity of the plasma density.

Dispersion equation for ASW can be derived by the aid of the following boundary conditions:

- waves' fields can be of finite values on the axis of plasma cylinder;
- tangential components of the waves' field can be continued on the plasma–dielectric interface;
- waves' tangential electric field is equal to zero on the metal wall of the waveguide.

In this case, dispersion equation can be obtained as follows [18]:

$$\frac{m}{R_1}\mu + \frac{1}{H_z}\frac{dH_z}{dR_1} = \frac{k_\perp^2}{k}\frac{J_m'(kR_1)\,N_m'(kR_2) - J_m'(kR_2)\,N_m'(kR_1)}{J_m'(kR_2)\,N_m(kR_1) - J_m(kR_1)\,N_m'(kR_2)}, \tag{3}$$

where $J_m(z)$ and $J_n'(z)$ are Bessel function and its derivative over the argument, correspondingly, $N_m(z)$ and $N_n'(z)$ are Neumann functions, correspondingly. Analysis of (3) allows one to determine the frequency ranges, where ASW can propagate. There are two possible ranges, which can be referred here as the low frequency (LF) range:

$$\omega_{LH} < \omega < |\omega_e|,\ |\omega_e| < \omega < \omega_1, \tag{4}$$

and as the HF range, correspondingly:

$$\omega_{UH} < \omega < \omega_2. \tag{5}$$

Applying asymptotes of Bessel functions in the limiting case $|m| \gg 1$ [19], one can obtain analytical expressions for the ASW frequency at the HF and LF ranges, correspondingly:

$$\omega \approx \omega_1\left[1 + \frac{\delta^2}{2R_1^2}\left(\varDelta\frac{R_1^2}{\delta^2}\frac{|\omega_e|}{\omega_1} + \frac{m\Omega_e^2}{\omega_1^2} - \frac{\Omega_e^2\xi}{\omega_1|\omega_e|}\right)^2\right], \tag{6}$$

$$\omega \approx \frac{|\omega_e|}{\sqrt{1+k_0^{-2}}}\left[1 - \varDelta\frac{\Omega_e^2}{\omega_e^2}|m|\sqrt{1+k_0^{-2}} - \frac{\xi\delta^{-2}\omega_e^2\Omega_e^{-2}}{|m|\,m\sqrt[3]{1+k_0^{-2}}}\right]^{-1}, \tag{7}$$

where $\omega_{1,2} = \mp 0.5|\omega_e| + \sqrt{\Omega_e^2 + 4^{-1}\omega_e^2}$ are cutoff frequencies, ω_{LH} та ω_{UH} are low-hybrid and upper-hybrid frequencies, correspondingly, $k_0 = m\delta R_1^{-1}$, $\xi = (R_1\delta^{-1})(d\delta/dR_1)$ is parameter of plasma density's non-uniformity, and $\varDelta = R_2R_1^{-1} - 1$ is dimensionless thickness of the dielectric layer. Using solutions (6) and (7), it is possible to obtain approximate expressions for the ASW frequency ω in other limiting cases, namely, $z(R_1) \ll 1$ and $z(R_1) \gg |m|$. Results of numerical analysis of (3) at $\varDelta = 0,3$ are represented in [18]. There, it was found that condition of weak non-uniformity of plasma density weakly changes ASW frequency as compared with the case of uniform plasma. As these extraordinary polarized modes are non-reciprocal, then increasing $d\Omega_e^2/dR_1$ affects the ASW frequency in the cases of positive and negative meanings of azimuthal m wave numbers differently, for instance, frequency of LF ASW with $m < 0$ diminishes weakly as compared with the case $m > 0$.

It can be pointed out that dispersion equation (3) describes two more waveguide structures, namely, the waveguide, which is partially filled by a cold non-magnetized plasma ($B_0 = 0$), and the waveguide, which is completely ($\varDelta = 0$) filled by a cold magnetoactive plasma [2]. In the first case, one can obtain the ASW frequency expression within accuracy up to the summands of $\varDelta^2 \ll 1$ order of smallness:

$$\omega^2 \approx \frac{\varDelta(1-\varDelta/2)m^2\Omega_e^2}{\sqrt{m^2 + R_1^2/\delta^2 + \varDelta(1-\varDelta/2)R_1^2/\delta^2 - \xi}}. \tag{8}$$

In the second case, it is possible to derive the ASW frequencies just from expressions (6) and (7) putting there $\varDelta = 0$. But it can be noticed that if $\varDelta = 0$, then ASW become unidirectional modes (i.e. they can propagate only along one

https://doi.org/10.1017/S0022377810000644 Published online by Cambridge University Press

direction); thus only ASW with $m > 0$ can propagate at the LF range $\omega < \omega_1$ and only meaning $m < 0$ can be realized for ASW at the HF range $\omega > \omega_1$. Therefore, analyzing expressions (7) and (8), one can make the conclusion: if plasma density increases in the direction toward the plasmas' border then the ASW frequency value diminishes as compared with the case of uniform plasma density profile.

In the case of arbitrary type of the plasma density profile, there is unique method to determine the ASW spectrum; it is numerical analysis. But there is special limiting case, if ASWs propagate in a wide plasma cylinder (it means that the plasma radius is considerably larger than the ASWs' penetration depth into the plasma), then their property can be studied analytically. It is proved in the paper that if nearby the plasma border its density is characterized by linear profile, then dispersion equation (3) can be solved analytically.

Let us restrict our consideration by the case of LF ASW, because at the HF range their phase speed can be large enough so their application in radio engineering is abridged. It is assumed that plasma, which is located in the region $r \leqslant R_1$, is characterized by the following density profile [20]:

$$n(r) = n(R_1) + (r - R_1)dn/dr|_{r=R_1}, \tag{9}$$

where collisions between the plasma particles happen rarely, so that collisional frequency of electrons $\nu \ll \omega$. In the case of large plasma cylinder ($R_1 \gg |m|\delta_1$, where $\delta_1 = c\Omega_e^{-1}(R_1)$), the inequality $|m^2 k_\perp^{-2} - (m/r)d(qk_\perp^{-2})/dr| \ll 1$ is valid. Then applying representation of the ASW field as a series: $H_z = H_z^{(0)} + H_z^{(1)} + \ldots$, where $|H_z^{(n+1)}| \ll |H_z^{(n)}|$, one can solve (1) by the aid of the method of successive approximations. Let us write down here the expressions for the zeroth and the first approximations accordingly:

$$H_z^{(0)} = C_1 U'(a) + C_2 V'(a), \tag{10}$$

$$H_z^{(1)} = U'(a)\int_{a_1}^{a} V'(a) g da - V'(a)\int_{a_2}^{a} U(a) g da. \tag{11}$$

where

$$g = -\frac{\delta_0^2}{r}\frac{dH_z^{(0)}}{dr} + q(r)H_z^{(0)} - \frac{k^2}{r}\frac{d}{dr}\left[\delta_0^4 r\left(1+\frac{\omega_e^2}{\omega^2}\right)\frac{dH_z^{(0)}}{dr}\right], \quad |\frac{g}{H_z^{(0)}}| \ll 1,$$

$$a(r) = (\delta_1 k_0)^{-2} + k_0(R_1 - r), \quad k_0^3 = -d\delta_0^{-2}/dr|_{R_1},$$

$U'(a)$ and $V'(a)$ are derivative of Airy functions over their arguments [19]. Applying asymptotes of Airy function of large arguments and choosing the solution of (1) in the form of SW, one can find that $C_1 = 0$, $a_1 = a(0)$, and $a_2 = a(R_1)$ if value of plasma density increases with going from the plasma border toward waveguides' wall (it means that $k_0 > 0$). In opposite case, if $k_0 < 0$ then $C_2 = 0$, $a_1 = a(R_1)$, and $a_2 = a(0)$. To represent solution of (1) in the form of expressions (10) and (11), it is not enough to satisfy inequalities $R_1 \gg |m|\delta$ and $\Omega_e^2 \gg \omega_e^2$, it is also necessary to provide smooth changing of the plasma density so that the following inequality can be realized:

$$|\delta_0 d\ln[n(r)]/dr|_{|r=R_1} \ll 1. \tag{12}$$

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The condition (12) is satisfied if the inequality $k_0^3 \leqslant (R_1\delta_1^2)^{-1}$ is valid. In this case, dispersion equation for the ASW looks like in the previous case [see (3)]. Let us substitute solutions of Maxwell equations in the forms (10) and (11) to (3) and apply asymptotes of Airy functions of large argument ($a \gg 1$) [19], then one can derive the following expression for the LF ASW in approximation of narrow vacuum layer $k(R_2 - R_1) \ll 1$:

$$\omega \approx \frac{0.5m|\omega_e|}{R_1/\delta_2 + \Delta R_1^2/\delta_2^2} + \sqrt{\frac{0.25m^2\omega_e^2}{\left(R_1/\delta + \Delta R_1^2/\delta^2\right)^2} + \frac{\Delta m^2 \Omega_e^2(R_1)}{R_1/\delta + \Delta R_1^2/\delta^2}}, \tag{13}$$

where $\delta_2 = \delta_1(1 + k_0^3\delta_1^3/4)$.

If $\delta = \delta_1$, then the formula (13) coincides with the corresponding expression, which was obtained for the case of uniform plasma [21]. Analyzing (13) one can see that if density of plasma diminishes with going from the plasma border toward the waveguides' wall, then effective δ penetration depth of the ASW into the plasma (the same conclusion is valid for their frequency as well) is less than the corresponding value in the case of the uniform plasma. It can be underlined that the type of plasma density profile can be nonlinear in the plasma depth, for example, it can pass from linear form nearby the plasma periphery into the parabolic form in the plasma depth, but such behavior of the plasma density will not affect the obtained solution.

Studying tasks on propagation of electromagnetic waves in thermonuclear devices filled by gaseous plasma, the following form of radial dependence for the plasma density nearby the plasmas' border is often applied:

$$n(r) = \begin{cases} (r - R_1)dn/dr_{|r=R_1}, & r \leqslant R_1, \\ 0, & R_1 < r < R_2. \end{cases} \tag{14}$$

Let us assume that plasma density gradients' value is as follows: $|dn/dr|_{|r=R_1} \sim n(0)R_1^{-1}$, and that density becomes uniform nearby the axis of the plasma column, where $|\varepsilon_1(0)| \gg 1$. Then, for the plasma region $R_1 - r \gg k^2k_0^{-3}$, it is possible to apply the solution of (1) in the form of (10) and (11) using the following replacement $a \to k_0(R_1 - r)$, if the following inequalities: $k_0^2 \gg k^2$, $k_0R_1 \gg |m\omega_e|/\omega$, $R_1 \gg k_0^{-1}$ are satisfied. After that in the peripheral plasma region (nearby its border $0 \leqslant R_1 - r \ll k_0^{-1}$), solution of (1) can be found out by the aid of the method of narrow layer [20]:

$$H_z = H_z(R_1) + H_z(R_1)\int_{R_1}^{r} dr' k_\perp^2 \left[A_d + \int_{R_1}^{r'} dr \frac{H_z(r)}{H_z(R_1)}(1 + q(r))\right]. \tag{15}$$

The constant of integration A_d can be determined from the application of the above mentioned boundary conditions on the plasma–vacuum border:

$$A_d = \frac{1}{k}\frac{N_m'(kR_1)J_m'(kR_2) - N_m'(kR_2)J_m'(kR_1)}{N_m'(kR_2)J_m(kR_1) - N_m(kR_1)J_m'(kR_2)}. \tag{16}$$

Expressions for the ASW field obtained in the form of (10) and (11) in the region $R_1 - r \gg k^2k_0^{-3}$ and that obtained in the form of (15) in the region $k^2k_0^{-3} \leqslant R_1 - r \ll k_0^{-1}$ can be joined. It allows one to derive dispersion equation, which solution in the considered case (plasma density profile is linear nearby the plasma border) has the following form:

$$\omega \approx \frac{\sqrt{9k_0^4 \Delta m^2 c^2 \left(\Delta R_1^2 + V_1 R_1 k_0^{-1}\right) + V_1^2 m^2 \omega_e^2}}{3k_0^2 \left(\Delta R_1^2 + V_1 R_1 k_0^{-1}\right)} + \frac{V_1 m\,|\omega_e|}{3k_0^2} \left(\Delta R_1^2 + \frac{V_1 R_1}{k_0}\right)^{-1}, \quad (17)$$

where $V_1 = -V(0)/V'(0) \approx 1.37$. From comparison of the expressions (13) and (17), one can make the conclusion that in the case of linear type of the density profile the obtained expression for the ASW frequency as the function of the plasma parameters is similar to that one, which was obtained in the case of uniform plasma [21] when doing replacement of the ASW penetration depth δ_1 by the parameter k_0^{-1}.

In this section, the frequency ν of collisions between plasma particles has not been taken into account so far, but under the experimental conditions, its value cannot be so small frequently. Let us take it into consideration, then account of the ν in expressions for components of dielectric permeability of the plasma leads to appearance of the ASW damping. Damping rate of the ASW determined by plasma particles collisions can be calculated from dispersion equation (3) making replacement $\omega \to \omega - i\gamma_c$, then the following expression can be obtained:

$$\gamma_c \approx \frac{\nu}{3}\left[1 + \frac{2m|\omega_e|}{3R_1 k_0 \omega}\right]\left[\frac{2m|\omega_e|}{3R_1 k_0 \omega} + \frac{2k_0 m^2 \Delta}{V_1 k^2 R_1}\right]^{-1}. \quad (18)$$

Expression (18) in the limiting case $R_1 \to \infty$ turns to $\gamma_c \approx \nu/3$ that is typical for planar plasma–vacuum border [1]. Collisional damping rate of the ASW propagating along plasma–metal boundary is larger than that one is calculated in the case of plasma–dielectric boundary. It is possible to explain such behavior of the ASW by the circumstance that in the first case the ASW energy propagates only in the plasma region, while in the second case, it propagates as well in the region of dielectric coating of the waveguides' wall.

If the ASW frequency exceeds electron cyclotron frequency, then the resonance point $\mathrm{Re}(\varepsilon_1) = 0$ can be located nearby the plasma border. It leads to a substantial increasing of the ASW damping. In this case, ASW damping rate is consisted of two summands, namely, collisional and resonant damping rates $\gamma_c + \gamma_r$. To determine resonant damping rate γ_r, whose appearance is connected with the presence of the resonance point, one can take into account the imaginary part of ε_1 in the parameter $k_\perp^2$, which enters into the first element of (1). It will allow one to calculate the value of the resonant damping rate in the following form:

$$\gamma_r \approx \frac{\pi\omega m^2 k_0}{\varsigma k^2 R_1^2}\left(1 - \frac{m\,|\omega_e|}{3R_1 \omega\, k_0}\right)\left[v_1 + \frac{\Delta k_0}{k^2 R_1}\left(k^2 R_1^2 + m^2 - \frac{2m^3\,|\omega_e|}{3R_1 \omega\, k_0}\right)\right]^{-1}, \quad (19)$$

where $\varsigma = (d\varepsilon_1/dr)_{|r=R_1} > 0$. Expression (19) is also valid for the case of non-magnetized plasma. Its analysis confirms that resonant damping of the ASW becomes larger with removing the resonant point from the plasma border toward its depth and with removing plasma surface toward the metal wall of the waveguide. It corresponds to the results, which were obtained for other types of SW [1].

Thus, if plasma density is characterized by a linear profile nearby the plasma periphery, then dispersion properties of the ASW are similar to that one determined in the case of uniform plasma. Influence of the plasma non-uniformity can be taken

https://doi.org/10.1017/S0022377810000644 Published online by Cambridge University Press

into account by the aid of replacement of the ASW penetration depth into the plasma by another definite parameter. Expressions for the ASW damping rates correspond to the results obtained for the other SW; their values increase with increasing the value of ratio $|m|R_1^{-1}$.

2.2. *Influence of an external magnetic field radial non-uniformity on the ASW frequency spectrum*

Plasma confinement in various magnetic traps is performed by the aid of an external magnetic field $\vec{B}_0(r)$ with non-uniform radial profile. As a rule, this non-uniformity is relatively weak. That is why, in this subsection, just the case of a weak radial non-uniformity of the magnetic field and its influence on the ASW frequency spectrum are considered. As the influence of the plasma density non-uniformity on the ASW properties has been examined in the previous subsection, then let us assume here the case of uniform plasma density profile. It is interesting that in this case, magnetic component of the extraordinary polarized ASW field is also described by (1). And because of that it can be solved by the method of etalon equations [18], applying it one can find out that in the plasma region, the ASW magnetic field is described by the following expression:

$$H_z = A(r)I_m(b). \tag{20}$$

This solution is valid if condition (2) is satisfied. Substituting the expression (20) into (1), one can calculate correction ω_1 to the ASW frequency, which is connected with non-uniformity of external constant magnetic field $\vec{B}_0$ in the case of the wide plasma waveguide $R_1 \gg |m|\delta_1$ and large azimuthal wave number $|m| \gg 1$:

$$\omega_1 \approx \frac{-m\xi\omega^2}{2\sqrt{m^2 + R_1^2/\delta_1^2}} \left[m\omega + 2\Delta \frac{m^2\Omega_e^2}{|\omega_e(R_1)|} \right]^{-1}. \tag{21}$$

Here, the parameter of external magnetic field non-uniformity ξ is determined by the expression $\xi = R_1 d \ln B_0/dR_1$, which is calculated in the point of $r = R_1$. Under the considered conditions, the following inequality for this parameter is valid $\xi \ll R_1^2\omega/|m\omega_e(R_1)\delta_1^2|$. Expression (21) can be applied to obtain frequency corrections ω_1 in other limiting cases, namely, narrow plasma waveguide $k_\perp R_1 \ll 1$ and small azimuthal wave number $|m| \ll k_\perp R_1$.

If the thickness of dielectric layer is small enough that the inequality $\Delta < |\omega_e(R_1)|$ $\omega/|m\Omega_e^2|$ is valid, then the ASW become unidirectional waves (in this case, azimuthal wave number can be only positive $m > 0$) and in the external magnetic field, whose value diminishes with going from plasma boundary, their frequency is less than it was in the case of uniform magnetic field. This conclusion coincides with results of the paper [18] that concern dependence of the ASW frequency upon value. If the thickness of dielectric layer is so large that the following inequality $\Delta > |\omega_e(R_1)|\omega/|m\Omega_e^2|$ is valid, then the influence of external magnetic field is more pronounced for the ASW with $m < 0$, as compared with the case of opposite meaning of the azimuthal wave number. It can be explained as demonstration of the phenomenon of non-reciprocity for ASW with opposite meanings of azimuthal wave numbers.

Summarizing the above mentioned, one can conclude that radial non-uniformity of an external magnetic field affects much stronger on these ASWs, which propagate along the direction of electrons' rotation in the magnetic field B_0 nearby the plasma surface. For the fixed value of parameter of external magnetic field non-uniformity ξ, correction of the ASW frequency diminishes with increasing of the dielectric layer thickness, absolute value of azimuthal wave number, radius of plasma, density of plasma, and value of the utilized magnetic field $B_0(R_1)$.

2.3. *Influence of toroidal magnetic field non-uniformity on the ASW frequency spectrum, which propagate in metal waveguides completely filled by plasma*

In gaseous plasma, which is confined in toroidal experimental devices, influence of toroidicity of the utilized magnetic field on the eigen transverse electromagnetic waves [22] reveals at first, via spatial non-uniformity of an external constant toroidal magnetic field; at second, via toroidal shift of magnetic surfaces [23]; and at third, via distinguishing the forms of poloidal cross sections of magnetic surfaces from the circular cross section. Influence of the second and third factors on the eigen electromagnetic waves' dispersion properties, which propagate across an uniform axial magnetic field along a small azimuthal angle is particularly examined in [24]. This subsection is devoted to the study of the first factor's influence. By doing that, the theory of ASW propagating in the uniform magnetic field is applied as the zeroth approximation.

In the case of absence of the dielectric layer which separates plasma from a metal wall of the utilized chamber, an external uniform magnetic field affects on the ASW dispersion properties by the strongest manner [2]. That is why, let us examine just the case of toroidal metal waveguide of circular cross section with radius a that is completely filled by plasma. In the right quasi-toroidal coordinates' system, the chamber is assumed to be symmetric along the circular axis, so $\partial/\partial\zeta = 0$. Poloidal angle ϑ is counted off along direction on the center of symmetry of the torus. It is assumed as well that the plasma filling is uniform, and utilized external constant toroidal magnetic field is described by the following expression:

$$B_{0\zeta} = B_0/[1-(r/R)\cos\vartheta], \tag{22}$$

where R is large radius of the torus. Let us study the propagation of extraordinary polarized modes with the components of electromagnetic field E_r, E_ϑ, H_ζ. In the physics of semiconductors, such SW is entitled as magnetoplasma polaritons and the noted orientation of an external magnetic field related to plasma–metal surface is entitled as Voight geometry [25].

In the framework of magnetic hydrodynamics, vectors of the electric induction and strength of the electric field are connected with each other by the tensor ε_{ij} of the dielectric permeability of cold magnetoactive plasma without taking into account the collisions between plasma particles. Under the case of small toroidicity (parameter $\varepsilon_t = a/R \ll 1$), one can represent expressions for the components of the tensor ε_{ij} as a series over the small parameter of toroidicity:

$$\varepsilon_{1,2} = \varepsilon_{1,2}^{(0)} + \varepsilon_{1,2}^{(1)}\cos\vartheta + \varepsilon_{1,2}^{(2)}. \tag{23}$$

The main summands in the series (23) do not depend on the coordinates $\varepsilon_{1,2}^{(0)} = \varepsilon_{1,2}(B_{0\zeta} = B_0)$. The summands of the first order of smallness over small radius of the torus r are as follows:

$$\varepsilon_1^{(1)} = -2\frac{r}{R}\sum_\alpha \frac{\Omega_\alpha^2\omega_\alpha^{(0)2}}{\left(\omega^2-\omega_\alpha^{(0)2}\right)^2}, \qquad \varepsilon_2^{(1)} = -\frac{r}{R}\sum_\alpha \frac{\omega_\alpha^{(0)}\Omega_\alpha^2\left(\omega^2+\omega_\alpha^{(0)2}\right)}{\omega\left(\omega^2-\omega_\alpha^{(0)2}\right)^2}. \tag{24}$$

The corrections of the second order of smallness are quadratic with respect to the radius r. In the series (23), summands of the second order of smallness, which are proportional to $\exp(2i\vartheta)$ are not taken into account because they make contribution into the correction of the ASW frequency, which is larger than the second order of smallness. Cyclotron frequency in expression (24) is determined by the strength of toroidal magnetic field (22) in the zeroth approximation: $\omega_\alpha^{(0)} = \omega_\alpha(\varepsilon_t = 0)$.

Toroidal component of the ASW magnetic field can be calculated from the following equation, in which one can derive from Maxwell set of equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\varepsilon_1}{\varepsilon_\perp}\frac{\partial H_\zeta}{\partial r}\right) + \frac{i}{r}\frac{\partial}{\partial\vartheta}\left(\frac{\varepsilon_2}{\varepsilon_\perp}\right)\frac{\partial H_\zeta}{\partial r} + \frac{\omega^2}{c^2}H_\zeta + \frac{1}{r^2}\frac{\partial}{\partial\vartheta}\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{\partial H_\zeta}{\partial\vartheta}\right)$$
$$-\frac{i}{r}\frac{\partial}{\partial r}\left(\frac{\varepsilon_2}{\varepsilon_\perp}\right)\frac{\partial H_\zeta}{\partial\vartheta} = 0, \tag{25}$$

where $\varepsilon_\perp = \varepsilon_1^2 - \varepsilon_2^2$. In the zeroth approximation (25) describes independent propagation of the ASW with different values of azimuthal wave numbers. But in the case of toroidal non-uniformity of an external magnetic field, one can take into consideration the satellite harmonics of the ASW field. Taking into account the symmetry of the problem [see (23), (24)], solution of (25) can be represented in the following wave packet form:

$$H_\zeta = \left[H_\zeta^{(0)}(r) + H_\zeta^{(2)}(r) + H_\zeta^{(+1)}(r)\,e^{i\vartheta} + H_\zeta^{(-1)}(r)\,e^{-i\vartheta}\right]\exp(im\vartheta - i\omega t). \tag{26}$$

Components of the ASW electric fields can be represented in the form that is similar to expression (26) form, where the next (with respect to the main harmonic $\propto \exp(im\vartheta)$) two satellite harmonics $\propto \exp[i(m \pm 1)\vartheta]$ are taken into account. The fields' correction $H_\zeta^{(2)}$, which is determined by the waveguides' toroidicity, appears as summand of the second-order smallness. To derive dispersion equation of the ASW propagating in toroidal chamber, which is completely filled by magnetoactive plasma, one can find connection of the poloidal component of the ASW electric field E_ϑ with H_ζ. Then they can satisfy the standard boundary conditions (see Sec. 2.1).

For the zeroth approximation, one can apply theory of ASW propagating along azimuth in metal cylindrical waveguide with radius a, which is completely filled by uniform plasma [2]. Amplitudes of the satellite harmonics can be found out as solutions of the considered task in the first approximation ($H_\zeta^{(\pm1)}(r)$ are assumed to be the parameters of the first order of smallness). They can be substituted into (25), taking into account expression (23), and summands of the first-order smallness will appear there with the factor $\propto \exp[i(m \pm 1)\vartheta]$. That is why, the account of summands of the first order of smallness does not change amplitudes of the main harmonic, and consequently the solution of the considered problem in the first approximation does not determine the ASW frequency correction. These summands form non-uniform Bessel equation for satellite harmonics $H_\zeta^{(\pm j)}(r)$ with the known right part. Constants of integration for this equation can be determined by the application of the above mentioned boundary conditions.

Correction of the ASW frequency connected with toroidicity of the utilized constant magnetic field can be calculated in the second approaching of the considered task. Summands of the second order of smallness in (26) are proportional to the summands of the following phases multipliers: either $\exp(im\vartheta)$ or $\exp[i(m \pm 2)\vartheta]$. Summands, which have factor $\exp[i(m \pm 2)\vartheta]$, make up the equation for amplitudes of the second satellite harmonics. Thus these summands in series (26) can be neglected because their contribution to the frequency corrections of the transversal SW appears to be the values of such order of smallness, which is higher than the second. The summands of the second order of smallness, which are proportional to the factor $\exp(im\vartheta)$, make up the non-uniform differential Bessel equation for the correction $H_\zeta^{(2)}(r)$ to the amplitude of the basic harmonic. Solution of this equation that satisfies condition on finite value of the wave field on the circular axis of the torus can be found out by the method of constant variation. It can be written down as the sum of partial solution of the non-uniform differential equation and common solution of the corresponding uniform differential equation. In the obtained expression for the axial component of the waves' magnetic field, the common solution $C_2 I_m(k_\perp r)$ of the studied uniform equation and amplitude $H_\zeta^{(0)}(r)$ appear to be similar elements. Therefore, constant of integration C_2 plays role of correction to the normalized multiplier, which in the zeroth approach was equated to unit. That is why, constant C_2 can be found not from the above mentioned boundary conditions, but by the aid of the following condition, which is similar to the condition of normalizing the wavefunction in quantum mechanics [26]:

$$\int_0^a \left[2H_\zeta^{(2)} I_m(k_\perp r) + \left(H_\zeta^{(+1)}\right)^2 + \left(H_\zeta^{(-1)}\right)^2\right] r dr = 0. \tag{27}$$

Equation (27) expresses by itself the following physical condition: energy of the magnetic field of the wave, which is calculated by taking into account the summands of the second order of smallness, can coincide with the value, which is calculated in the zeroth approximation. At the same time, we can underline that type of dispersion equation, which is derived by taking into account the summands of the second order of smallness, does not depend on the value of the integration constant C_2.

Application of the boundary condition on the metal wall of the considered toroidal waveguide for the basic harmonic of the poloidal component of the waves' electric field with taking into account the summands of the second order of smallness leads to obtaining dispersion equation in the form $D^{(0)} + D^{(2)} = 0$, where $D^{(0)}$ describes ASW in the case of uniform constant magnetic field [2] and the second term describes correction $D^{(2)} \propto E_\vartheta^{(2)}(a)$, which is determined by toroidicity of the considered magnetic field $\vec{B}_{0\zeta}$. That is why, correction to the eigen frequency $\omega_m^{(0)}$ of the studied transversal SW appears to be the values of the second order of smallness: $\omega_m^{(2)} \propto \varepsilon_t^2$. The condition of applicability of the applied method of determination of the transversal SW dispersion properties, which propagate in toroidal metal waveguides, consists in the validity of the inequality $|\omega_m^{(2)}| \ll \omega_m^{(0)}$.

Therefore, the basic mathematical achievement of this result can be formulated in the following way: the task about two-dimensional non-uniformity of an external constant magnetic field in the waveguides' poloidal cross section has been reduced to one-dimensional non-uniformity (it means that finally we have only radial distribution of the considered external magnetic field). But of course, the obtained solutions are of cumbersome form.

In the limiting case of a wide waveguides $R \gg a \gg |m| k_\perp^{-1}$, eigen frequency of the considered transversal SW can be represented at the LF range by the following asymptotic expressions:

$$\omega_m^{(0)} = m\delta|\omega_e| a^{-1}. \tag{28}$$

$$\omega_m^{(2)} = -0, \quad 25\varepsilon_t^2 \omega_m^{(0)}. \tag{29}$$

At the HF range, their eigen frequency can be written with the same accuracy by the following way:

$$\omega_m^{(0)} = \sqrt{\omega_{\rm UH}^2 + m^2 c^2 a^{-2}}. \tag{30}$$

$$\omega_m^{(2)} = \frac{\varepsilon_t^2}{4} \frac{c^2 m^2}{\omega^2 a^2} \left(1 + \frac{3|\omega_e| k_\perp a}{\omega |m|}\right) \omega_m^{(0)}. \tag{31}$$

For the transversal SWs, which propagate in a narrow $|m| \gg ak_\perp$ toroidal waveguides at the LF range, one can derive the following asymptotic expressions:

$$\omega_m^{(0)} = |\omega_e| \left[1 - \frac{1}{2m(m+1)} \left(\frac{a}{\delta}\right)^2\right], \tag{32}$$

$$\omega^{(2)} = \frac{2m^2 - m + 1}{4m^2(m+1)} \varepsilon_t^2 \omega_m^{(0)}. \tag{33}$$

At the HF range, expression of the transverse SW eigen frequency in the zeroth approximations described by formula (30), and frequency correction to it is equal to approximately:

$$\omega^{(2)} = \frac{2m^2 + m + 1}{2m^2} \varepsilon_t^2 \left(\frac{m\delta}{a}\right)^4 \frac{\Omega_e}{|\omega_e|} \omega_m^{(0)}. \tag{34}$$

It should be mentioned that spatial distributions of the satellite harmonics in these two limiting cases are substantially different, namely, in the limiting case of narrow waveguide $H_\zeta^{(\pm1)}(r) \propto r^{m\pm1}$, but their values are approximately equal to each other on a metal wall of the considered waveguide.

To carry on comparative analysis of the obtained results, let us write down here asymptotical expressions for correction $\omega_{el}^{(2)}$ to the eigen frequency of the transversal SW that is determined by elliptisity of transverse cross section of the metal waveguide, which is completely filled by plasma. For transversal SWs, which propagate in a wide waveguides at the LF range, the frequency correction $\omega_{el}^{(2)}$ is as follows:

$$\omega_{\rm el}^{(2)} = -0.25\varepsilon_{\rm el}^2 (4 - m^2 + 2a\delta^{-1}) \omega_m^{(0)}. \tag{35}$$

In this case, the form of the cross section of chamber is determined by parameter of ellipticity ε_{el} by the following way: $r(\vartheta) = a(1 + \varepsilon_{el} \cos(2\vartheta - \pi))$. In the limiting case of a narrow waveguides:

$$\omega_{\rm el}^{(2)} = -\frac{\varepsilon_{\rm el}^2}{2m} \frac{2 - 4m^2 - 4m^2}{2 - 4m^2} \frac{a^2}{\delta^2} \omega_m^{(0)}. \tag{36}$$

Thus after comparing expressions (33) and (34) with expressions (35) and (36), one can make the following conclusions: first, influence of toroidal non-uniformity of the external magnetic field on the spectrums of transversal SW is stronger than the influence of ellipticity of the waveguide cross section in the case of rarefied plasma density; second, comparison of the obtained results with influence of toroidal change of magnetic surfaces is redundant, because in experimental practice this change is too small nearby the surface of the metal chamber, where the considered waves are localized.

Consequently, the account of toroidal non-uniformity of the external magnetic field leads to the fact that transversal SWs propagate in toroidal waveguides in the form of wave packet. Amplitudes A_{m+N} of satellite harmonics in such packets are small enough as compared with amplitude of basic harmonic A_m so that $A_{m+N} \sim \varepsilon_t^N A_m$. Influence of the toroidal non-uniformity of the external magnetic field on amplitude of basic harmonic becomes apparent in the second order of smallness over the parameter of toroidicity ε_t. Represented results can be useful for practical application in the branch of plasma electronics, and also for interpretation of experimental results concerned studying phenomena in peripheral plasma [27] confined by magnetic field in fusion devices.

2.4. Propagation of azimuthal surface waves around a metal ring in a non-uniform toroidal magnetic field

In this subsection, within the framework of the method of successive approximation, extraordinarily polarized electromagnetic SW with components E_r, E_ϑ, H_ζ propagating across the constant toroidal magnetic field around a metal ring is studied. Small radius of metal ring with ideally conductivity is assumed to be small as compared with the large radius of the ring, so that inequality $\varepsilon_t = a/R \ll 1$ is valid. The constant toroidal magnetic field $\vec{B}_0 = B_{0\zeta}\vec{e}_\zeta$ is created by a conductor with a direct current, which is located along the direct axis of the ring, perpendicularly to the plane of symmetry of the ring, and is expressed by formula (22). Plasma, which is surrounded by the metal ring, is assumed to be cold and uniform [28], and all other suppositions, which have been done in Sec. 2.3, are valid in this subsection as well.

Components of dielectric permeability tensor ε_{ik} can be written in the form of series over small parameter of toroidicity ε_t like expression (23). In the present case, toroidal $H_\xi(r)$ component of the waves' magnetic field can be derived from (25) in the form like expression (26). Solution of (25) can satisfy to the boundary conditions, which are particularly discussed in the previous subsections. Amplitude $H_\xi^{(0)}(r)$ of basic harmonic of the considered transverse SW in the zeroth approximation in this case can be expressed by the aid of the MacDonald function [19] $K_m(k_\perp r)$. Determination of amplitudes $H_\zeta^{(\pm j)}(r)$ of satellite harmonics is executed by the method, which is similar to the one that is applied in the previous subsection, because of the similarity of these tasks. Thus solving of the present task in the first approximation does not make contribution into correction to the ASW eigen frequency. Expressions for the fields of satellite harmonics are obtained in [28]; they are very cumbersome so we do not represent them here.

Corrections of the transversal SW eigen frequencies in this case can be found out by the same method, as it was done in Sec. 2.3. Non-uniform differential Bessel equation for the second-order correction $H_\zeta^{(2)}(r)$ to amplitude of the basic harmonic of the ASW, and its solution is similar to the corresponding correction, which has been obtained in the previous subsection. That is why, one can apply the method of constant variation once again for the present task.

In the limiting case of the thick ring $R \gg a \gg |m|/k_\perp$, eigen frequency of transversal SW coincides with expression (29) within accuracy of a sign. At the HF range, the wave frequency correction becomes negative, but it coincides with (32) over its absolute value. Studied transversal SW can exist in the HF range if the following inequality is valid for the considered plasma parameters $(\delta m/a)^2 < |\omega_e|\Omega_e^{-1}$.

For the eigen frequencies of the considered transversal SWs, which propagate around the thin $a \ll |m|k_\perp^{-1} \ll R$ metal ring at the LF range, it is possible to derive the following asymptotic expression for the frequency correction:

$$\omega^{(2)} = -\varepsilon_t^2(2m^2 + m + 3)\omega_m^{(0)}/8m^2. \tag{37}$$

In this case, the ASW frequency $\omega_m^{(0)}$ is determined by a formula (32). At the HF range, value of the ASW eigen frequency in the zeroth approximation $\omega_m^{(0)}$ is determined by formula (30), and the frequency correction in this case is approximately equal to:

$$\omega^{(2)} = -\varepsilon_t^2(2m^2 - m + 2)\left(\delta k_\perp^{-1} a^2\right)^2 m(m-1)\omega_m^{(0)}. \tag{38}$$

Radial dependences of satellite harmonics for these transversal SW in this limiting case (thin metal ring) are substantially differ from each other: $H_\zeta^{(\pm 1)}(r) \propto r^{-|m\pm 1|}$, but these amplitudes appear to be of the same order of smallness on the surface of the metal ring.

Consequently, the account of toroidal non-uniformity of the external magnetic field results in the fact that transversal SWs propagate in the form of wave packet. Amplitudes of satellite harmonics in such packets are of small values as compared with the amplitude of the basic harmonic: $A_{m+N} \sim \varepsilon_t^N A_m$. Influence of toroidal non-uniformity of the external magnetic field on the amplitude of basic harmonic appears to be of the second order of smallness over the parameter ε_t of toroidicity. The eigen frequency corrections for these transversal SWs, which are determined by toroidal non-uniformity of the external magnetic field, appear to be values of the second order of smallness. They are of the same order of smallness as it was found out for the frequency corrections in the previous subsection, but they have the opposite sign as compared with the results obtained in the previous subsection. Concerning practical application of the results presented in this subsection, one can see that such type of metal ring can be an aerial, which it is intended for excitation of the transversal SWs with the purpose to sustain a gas discharge.

2.5. Azimuthal surface waves in a weakly corrugated magnetic field

An external magnetic field that confines plasma in laboratory devices is often of a corrugated type. For instance, adiabatic traps have a corrugated magnetic field [29], corrugated confining magnetic field in tokamaks is connected with discreteness of the coils of toroidal magnetic field, corrugated magnetic field is typical for such toroidal systems as ELMO BUMPY TORUS [30, 31]. It is planned [32] that the so-called 'mirror' non-uniformity will be predominant in the confining magnetic field of modular stellarator Helias. If in the expression of the confining magnetic field derived for the Helias reactor, which is offered in [32], anybody replaces flow coordinates by cylindrical coordinates, then this expression will coincide with expression of the corrugated magnetic field applied in this subsection. In [33], the eigen disturbances of thermonuclear plasma of modular stellarator Helias are foreseen theoretically. Theory of propagation, conversion, and absorption of magnetohydrodynamic waves in a plasma column, which is located in a weakly corrugated magnetic field, has been developed in [34–37]. The waves in a corrugated magnetic field propagate in the form of wave packets, which in addition to a basic harmonic contain infinite quantity of satellite spatial harmonics.

Let us consider propagation of transversal electromagnetic waves nearby the border of the plasma column separated by a vacuum layer from circular metal chamber cylinder with ideal conductivity (geometry of the system is particularly described in Sec. 2.3) [38, 39]. An external confining magnetic field $\vec{B}_0 = B_{0z}\vec{e}_z + B_{0r}\vec{e}_r$ is assumed to be weakly corrugated:

$$B_{0r} = B_{00}\varepsilon_m' k_m^{-1}\sin(k_m z), \qquad B_{0z} = B_{00}[1+\varepsilon_m(r)\cos(k_m z)], \tag{39}$$

where $\varepsilon_m' \equiv d\varepsilon_m/dr$, $k_m = 2\pi/L$, in which L is the period of corrugation. The parameter of corrugation ε_m is usually (in all modern fusion reactors) small. For example, it is ~ 0.05 nearby the border of plasma in tokamak ASDEX-U, Germany [38]. In the Helias configuration [33], 'mirror' non-uniformity is planned to be $\varepsilon_m \sim 0.13$.

One can see that expression (39) does not automatically provide implementation of the fundamental equation $\operatorname{div}\vec{B}_0 = 0$. Therefore, one can substitute expressions (39) to this fundamental equation and then obtain equation like the modified Bessel equation, which determines dependence ε_m on the radial coordinate. Its solution is proportional to the modified Bessel function of the zero order. Consequently, if the period of corrugation is larger as compared with the small radius of chamber of the fusion device, then ε_m can be considered as a practically constant value. In opposite case, if the period of corrugating is small, the ε_m value diminishes approximately according to exponential law with going away from the plasma interface to the plasma center. Thus non-uniformity of the confining magnetic field is substantial only in a narrow layer within k_m^{-1} thickness nearby the metal wall of the chamber.

As it is known, the magnetic field fluxes are parallel to the vector of the confining magnetic field induction in every point of the coordinate space. The vector form of this condition allows one to derive the next equation for the flux lines in cylinder coordinates:

$$\frac{dr}{B_{0r}} = \frac{dz}{B_{0z}}. \tag{40}$$

Integrating (40) after the substitution explicit expressions (39) for confining magnetic field components into it, one can obtain equation for magnetic surface taking into account that properties of corrugation parameter $\varepsilon_m(r)$ are described by the modified Bessel functions:

$$r_0 \approx r + \frac{\cos(k_m z)}{r}\int_0^r r\varepsilon_m dr = r - \cos(k_m z)\left(1 - \frac{\varepsilon_m}{2}\cos(k_m z)\right)\varepsilon_m' k_m^{-2} + o\left(\varepsilon_m^3\right). \tag{41}$$

Equilibrium plasma density here is the function of the magnetic surface: $n(z,r) = n(r_0)$.

Let us apply the system of coordinates $\vec{e}_1, \vec{e}_2, \vec{e}_3$ that is related to the flux lines of the magnetic field induction $\vec{B}_0$, namely, the first vector is perpendicular to the magnetic surface in every point of the coordinate space $\vec{e}_1 = \nabla_{r_0}/|\nabla_{r_0}|$, the second vector coincides with azimuthal basic vector in cylinder coordinates, the third vector is parallel to magnetic field flux lines $\vec{e}_3 = \vec{B}_0/|\vec{B}_0|$. In this system of coordinates, the components of the waves' electric field induction $\vec{D}$ and electric field strength $\vec{E}$ are coupled by the tensor of permeability of a cold plasma without the collisions in the simplest way:

$$\vec{D} = \varepsilon_1(E_1\vec{e}_1 + E_2\vec{e}_2) + \varepsilon_3 E_3\vec{e}_3 - i\varepsilon_2\vec{e}_3\times\vec{E}. \tag{42}$$

In approximation of small value of collisional frequency, the components of the cold plasma permeability tensor applied in (42) have the following forms:

$$\varepsilon_1 = 1 - \sum_i \Omega_i^2(r_0)/\left(\omega^2 - \omega_i^2\right), \quad \varepsilon_2 = -\sum_i \Omega_i^2(r_0)\omega/\left(\left(\omega^2 - \omega_i^2\right)\omega_i\right),$$

$$\varepsilon_3 = 1 - \Omega_i^2(r_0)\omega^{-2}. \tag{43}$$

Value of cyclotron frequency in (43) is determined by the complete magnetic field $B_0(r, z)$.

Applying symmetry of the task, the components of the plasma permeability tensor can be written as series over the small parameter ε_m. Let us write down those of them, which describe the extraordinary modes taking into account the summands of the second order of smallness over the parameter of corrugation:

$$\varepsilon_{1,2} = \varepsilon_{1,2}^{(0)}(r) + \varepsilon_{1,2}^{(1)}\cos(k_m r) + \varepsilon_{1,2}^{(2)}(r) + O(\varepsilon_m^3), \tag{44}$$

where $\varepsilon_{1,2}^{(0)}(r)$ are the basic terms, $\varepsilon_{1,2}^{(1)}\cos(k_m r)$ are the corrections of the first order of smallness $|\varepsilon_{1,2}^{(1)}| \sim |\varepsilon_m \varepsilon_{1,2}^{(0)}|$. In the second approximation over the small parameter of the task, one can take into account the only corrections to the basic summands $|\varepsilon_{1,2}^{(2)}| \sim |\varepsilon_m^2 \varepsilon_{1,2}^{(0)}|$, because they make contribution into the dispersion equation [39] of these waves.

Applying symmetry of the task, one can search into the solution of Maxwell set of equations for the ASW field as a wave packet, which contains two nearest satellite harmonics in addition to the basic harmonic. Radial dependence of amplitudes of the basic harmonic (in the case of large period of corrugation, when the inequality $k_m c < \omega$ is valid; let us indicate that it is concerned the satellite harmonics as well) of the wave in a vacuum region is expressed through the Bessel functions of the first type and the Neumann functions [19]. As it is known from [40], the ordinary modes' properties do not depend on the utilized constant magnetic field [see expression for component of the permeability tensor ε_3 in (43)]. That is why, we shall consider influence of corrugation of the confining magnetic field on properties of the surface X-modes in a plasma cylinder with a non-uniform radial profile of the plasma density. Let us assume that only the mth harmonic of the field $H_z^{(0)}(r)$ differs in the zeroth approximation from zero, thus for the following component of the ASW field, one can write down: $E_z^{(0)}(r) = H_r^{(0)}(r) = 0$, $H_\vartheta^{(0)}(r) = 0$.

In the zeroth approximation, dispersion equation for the transversal surface X-modes can be derived by the aid of application of the above mentioned boundary conditions. By doing that, we consider that eigen frequency value is known in the zeroth approximation. The account of summands of the first order of smallness at the solution of Maxwell equations in the plasma region does not change amplitudes of the basic harmonics of the X-mode field components, but it results in the appearance of the satellite harmonics, which are proportional to $\exp[i(m\vartheta \pm k_m z - \omega t)]$. Weak corrugation of the external confining magnetic field is the reason of the coupling between amplitudes of $E_z^{(\pm)}$satellite harmonics of the axial electric field and amplitudes of the basic harmonic of radial electric $E_r^{(0)}$ and axial magnetic $H_z^{(0)}$ fields of the surface X-modes. Differential equation, which links these fields' components of the X-modes, has the form of non-uniform Bessel equation. That is why, it is convenient to search into its solution by the method of constant variation. Appearance of the axial electric field of the X-modes, which is not equal to zero, is connected also with its weak coupling with the radial electric field because of the application of the boundary condition for tangential electric field on the metal wall of the fusion chamber. Amplitudes of the satellite harmonics of axial magnetic and also radial and poloidal electric fields of the surface X-modes are symmetrical: $E_r^{(+)} = E_r^{(-)}$, $E_\vartheta^{(+)} = E_\vartheta^{(-)}$, $H_z^{(+)} = H_z^{(-)}$, and amplitudes of axial electric and also radial and poloidal magnetic fields are antisymmetrical: $H_r^{(+)} = -H_r^{(-)}$, $H_\vartheta^{(+)} = -H_\vartheta^{(-)}$, $E_z^{(+)} = E_z^{(-)}$.

The account of summands of the second order of smallness enables one to determine values of the corrections of the second order of smallness to the amplitude of basic harmonic of axial magnetic field and also radial and poloidal electric fields of the wave. Thus the spatial distribution of the X-modes fields can be determined by the aid of the theory of successive approximations from the spatial distribution of the axial magnetic field of the waves that is known in the zeroth approximation within accuracy up to summands of the second order of smallness inclusively. Applying the above mentioned boundary conditions, one can derive dispersion equation for the surface X-modes as it has been done in the previous subsections. It can be analytically examined only in some limiting cases. For example, in the case of uniform dense plasma ($\Omega_e^2 \gg \omega_e^2$) with a large radius ($a \gg |m|\delta$), which is separated from a metal wall of the fusion chamber by a narrow vacuum layer ($R_2 R_1^{-1} - 1 \ll 1$) that is immersed into the constant magnetic field with a large period of corrugation ($k_m R_1 \ll 1$), the surface X-modes frequency can be determined by this method: $\omega = \omega^{(0)} + \Delta\omega$, where $\omega^{(0)}$ is value of the ASW frequency in the zeroth approximation:

$$\omega^{(0)} \approx \frac{(1-\Delta)\delta^2 m\,|\omega_e| + \sqrt{(1-2\Delta)\delta^4 m^2\omega_e^2 - (2a^2\Delta - a^2\delta^2 + 2a\delta)m^2c^2\Delta(3\Delta-2)}}{2a^2\Delta - a^2\delta^2 + 2a\delta} \tag{45}$$

and $\Delta\omega$ is correction to its value, which is connected with the weak corrugation of the confining magnetic field of the fusion device:

$$\frac{\Delta\omega}{\omega^{(0)}} \approx -\frac{(\varepsilon_m')^2}{4k_m^4}\left[\frac{k^2\delta}{a}\left(1-\frac{m\delta\,|\omega_e|}{a\omega^{(0)}}\right)\left(\frac{m^2}{a^2k^2}-1\right)+\frac{\Delta}{\delta^2}-\frac{\Delta m^2\delta^{-2}}{a^2k^2}-\frac{3m^2}{k^2a^4}+\frac{\delta+a}{a^2\delta}\right.$$
$$\left.-\frac{m\,|\omega_e|}{a^2\omega^{(0)}}\right]\left[(1-\Delta)\frac{m\delta^2\,|\omega_e|}{a^2\omega^{(0)}}+(2-3\Delta)\frac{\Delta m^2}{k^2a^2}\right]^{-1}. \tag{46}$$

Summarizing materials represented here, we can indicate that similar mathematical approach has been applied at researching beam–plasma instabilities, which happen in waveguides with a periodic dielectric inserts and/or in waveguides with the corrugated walls [41]. In these cases, dispersion equations contain the small correcting summands, which are proportional to the square of the corrugation depth of corrugating of devices' environment. The similar structure is realized for the dispersion equation that describes the radiation of the free electron lasers as well. Experimental study of the processes of modes conversion in a waveguide with corrugated walls proves that for practical aims it is enough to apply the analytical calculations, which have been done within the framework of the theory of successive approximations [41]. Consequently, such conclusions can be considered as another independent confirmation of correctness of the results represented in this section.

3. Gases discharges sustained by ASW propagation

The results of theoretical investigation of plasma sources, which operate due to propagation of ASW, are presented in this section. Construction of such type plasma sources can be designed either as a cylinder metal waveguide with a plasma filling or cylinder metal antenna that is immersed into plasma. It is suggested that ASW can be utilized for sustaining microwave gas discharges in such devices, gas discharges will be created due to the processes of volume ionization, which happen due to collisions between plasmas' electrons and neutral particles of the operating gas. Microwave gases discharges, which are sustained by the SWs, are of great interest for successful development of plasma technologies because of their advantages as compared with the case of bulk waves' utilization [12, 13, 42]. In this section, we consider both cases of a magnetoactive plasma production and of a non-magnetized plasma production. Two different types of plasma sources' are examined here, namely, first, plasma maintains close contact with dielectric, which separates plasma from metal wall of the discharge device and second, plasma maintains complete contact with metal boundary of the discharge device. The ASW powers' transfer into the sustained plasma can be realized due to the collisional and resonant mechanisms, both of them are studied here.

3.1. Electrodynamical model of gas discharges sustained by ASWs, which propagate along a plasma–metal boundary

Analyzing dispersion properties of ASW, which are studied in [2], it is possible to conclude that the plasma sources utilized propagation of ASW as operating mode will be characterized by such advantages. First, ASW energy practically cannot be lost out of discharge chamber. Second, plasma density, which can be produced during such gas discharges, will be homogeneous along azimuthal direction due to the spatial distribution of the ASW field. Carried out theoretical study testifies high efficiency of the ASW utilization, possibility to obtain plasma with large enough density and with high level of its uniformity.

Let us consider discharge chamber modeled by a cylinder metal waveguide of radius R_c that is completely filled by magnetoactive gaseous plasma. Metal walls of the chamber are assumed to be characterized by an ideal electric conductivity, external constant magnetic field $\vec{B}_0$ is directed along the axis of waveguide parallel to $\vec{z}$, and discharge is uniform along the $\vec{z}$ axis. As plasmas' temperature in technological gas discharges is low enough, then let us apply hydrodynamic approach to describe waves' processes in the produced plasma. Using the Fourier method, one can solve set of Maxwell equations, which describe ASW fields. By doing that, dependence of ASW fields on azimuthal angle φ and time t is assumed to be as follows:

$$A(r, \varphi, t) = A_0(r, \varphi) \exp(im\varphi - i\omega t). \tag{47}$$

It is supposed that the A_0 value satisfies the following inequality: $\frac{1}{A_0}\frac{\partial A_0}{\partial \varphi} \ll |m|$, which means that assumption on weak changing of the gas parameters along the azimuthal direction is valid. It is supposed also that the process of ionization in the sustained plasma volume takes place due to collisions between electrons and neutral gas particles filled the chamber. These collisions can be described by an effective frequency ν of transmission of impulse, whose value is assumed much less than the ASW eigen frequency value.

Electrodynamical properties of such model during stationary stage of gas discharge are described by dispersion equation for ASW, power balance equation, and correlation between energy that is absorbed along an azimuth angle of the discharge chamber and local density of the sustained plasma [43]. The mentioned correlation characterizes the considered type of gas discharge and is determined by the conditions of the carried out experiment, namely, by geometry of the discharge chamber, by pressure and type of operating gas, and so on. Applying results of the papers [44, 45], the power balance equation for the modes, which propagate in a discharge can be written in the following form:

$$\frac{dS_\varphi}{rd\varphi} = -Q, \tag{48}$$

where S_φ is density of the ASW energy flow along an azimuthal angle, Q is the ASW energy, which is absorbed at unit of the discharge length along an azimuthal angle. It is known that correlation between plasma density that is obtained in the discharge and the SW energy that is absorbed at unit of the discharge length is different for each other discharge. In [45], it was obtained within the framework of phenomenological approach, so let us write down here the expression as:

$$Q = G(\beta)N^{1+\beta}, \tag{49}$$

where $N = \Omega_e^2/\omega^2$ is the normalized plasma density, $G(\beta)$ is the constant for each discharge, it does not depend on a coordinate φ and it can be obtained from equations on density of electrons and temperature in a gas discharge [45, 46]. Equation (49) is valid just for the stationary stage of gas discharge, when SW energy transforms into energy of thermal motion of plasma electrons then in its turn it spends for excitation, ionization, heating of neutral gas, and on other elementary processes. Scopes of (49) validity for magnetoactive plasma was determined in [47]. Value of the parameter β is determined by conditions of experiment. For example, the value $\beta = 0$ relates the diffuse regime of the discharge, during this regime diffusion of the charged particles is the main mechanism of electrons' losses from a discharge volume. It can be realized for the discharge models where plasma is produced mainly due to one-step ionization. In this case, the quantity of SW energy that is absorbed in discharge is directly proportional to the value of the produced plasma density. If discharge is carried out under the regime of two-step ionization, removal of the excitation state for plasma particles and/or volume recombination, then β value satisfies the inequality $0 < \beta \leqslant 2$, and this regime of the discharge is entitled by regime of the volume recombination.

To calculate the ASW power flow S_φ and the ASW energy Q, which is absorbed by plasma at the unit of length along direction of ASW propagation, one can find out spatial distribution of the ASW field (it has been done in the previous section). Then the obtained expressions for S_φ and Q can be averaged over the discharge chamber radius. Since one can obtain the following analytical expressions [43]:

$$S_\varphi = \frac{c}{4\pi\psi_0} C_1^2 \int_0^a \left\{ \frac{\varepsilon_2}{\varepsilon_1} I_m(\xi) I_m'(\xi) + \frac{m}{\xi} I_m^2(\xi) \right\} dr, \tag{50}$$

$$Q = \frac{\omega}{4\pi\varepsilon_1} \mathrm{Im}(\varepsilon_1) C_1^2 \int_0^a \left\{ \left(I_m'(\xi)\right)^2 - \frac{m^2}{\xi^2} I_m^2(\xi) \right\} rdr, \tag{51}$$

where $\mathrm{Im}(\varepsilon_1) = \nu\Omega_e^2(\omega^2+\omega_e^2)/\omega(\omega^2-\omega_e^2)^2$, $\xi = k\psi_0 r$, $\psi = \sqrt{\varepsilon_2^2-\varepsilon_1^2/\varepsilon_1}$, $\xi_c = k\psi R_c$, C_1 is the constant that determines ASW fields' amplitude on the inner metal wall of the discharge chamber.

Let us consider the case of gas discharge sustained by ASW propagating in HF range (5). Then, in the limiting case of plasma chamber modeled by a narrow ($\xi_c \ll 1$), plasma cylinder expressions for the ASW power flow and the ASW energy, which is absorbed at unit length of the discharge due to Ohmic damping of the waves, can be written in the following forms [43]:

$$S_\varphi = -\frac{\omega a^2\,(\varepsilon_2+\varepsilon_1)}{8\pi\varepsilon_1 2^{2|m|}(|m|!)^2} C_1^2\xi_0^{2|m|-2} \tag{52}$$

$$Q = \frac{\nu\omega^2\left(\omega^2+\omega_e^2\right)a^2}{4\pi\left(\omega^2-\omega_e^2\right)^2}\cdot\frac{(\varepsilon_1+\varepsilon_2)^2}{\varepsilon_1^2 2^{2|m|}\,|m|!(|m|-1)!}C_1^2\xi_0^{2|m|-2}. \tag{53}$$

For the practical purposes just the case of gas discharge, which produces plasma with high density, is the most interesting one [12, 13, 42, 44, 47]. That is why, further research is devoted to the case of a dense plasma production. In this case, one can obtain the following expression for the produced plasma density as function of azimuthal angle by the aid of power balance equation (48):

$$N \approx N_0(1-\varphi/\varphi_0), \tag{54}$$

where $N_0 = N(\varphi = 0)$, φ_0 is effective angular length of the discharge. In the limiting case of narrow discharge chamber $\xi_c \ll 1$, the approximate expression of φ_0 is as follows:

$$\varphi_0 \approx \frac{(|m|-1)(\Omega^2-1)^2}{2|m|Y(\Omega^2+1)(\Omega-1)}, \tag{55}$$

where $Y = \nu/|\omega_e|$, $\Omega = \omega/|\omega_e|$.

Analysis of the ASW dispersion equation proves that their frequency slowly depends on the effective wavenumber $k_{\mathrm{ef}} = mc/a\omega_e$ at the HF range (5). It means that changing of the discharge chamber radius is not effective manner to control the gas discharge parameters in this frequency range. For the LF ASW, formulae (52) – (55) are entirely suitable for the application. But, in this case, the ASW frequency substantially increases with increasing of k_{ef}; that is why, there are good possibilities for controlling the gas discharge parameters by changing radius of the discharge chamber. From analysis of expression (55), one can see that φ_0 value grows with increasing the applied constant magnetic field and the ASW frequency at the LF range (4).

Similarity of spatial distributions of the ASW fields propagating in both metal cylinder, which is completely filled by magnetoactive plasma, and around cylinder metal antenna of radius R_a, which is immersed into non-bounded magneto-active plasma, is shown in [2]. Distinguishing feature of the case of ASW propagating around cylindrical metal antenna is the application of MacDonald functions [19] with argument $\xi_a = k\psi R_a$ for description of their fields instead modified Bessel functions, which have been applied in the case of utilization a cylindrical discharge chamber. Thus the signs of azimuthal wave-number m in both the frequency ranges (4) and (5) can be changed for the opposite ones in the case of ASW propagation along the cylindrical metal antenna as compared with the previous case. Therefore, one can find simple analytical expressions for the ASW power flow and their energy absorbed via the channel of Ohmic dissipation using approaching of cylindrical metal antenna with small ($\xi_a \ll 1$) radius. Averaged values of S_φ and Q for aerial in the limiting case $\xi_a \ll 1$ can be represented in the following forms, correspondingly:

$$S_\varphi = -\frac{\omega R_a^2 2^{2|m|-3}}{4\pi\varepsilon_1} C_2^2 \xi_a^{-2|m|-2}(\varepsilon_1 - \varepsilon_2)(|m|-1)!^2, \tag{56}$$

$$Q_{\text{oh}} = \frac{\nu\omega^2\left(\omega^2+\omega_e^2\right)R_a^2}{4\pi\left(\omega^2-\omega_e^2\right)^2} \cdot C_2^2\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 4^{|m|-1}|m|!(|m|-1)!\xi_a^{-2|m|-2}. \tag{57}$$

To determine angular length φ_0 of the discharge in this limiting case, one can put these expressions into the power balance equation (48) and obtain the following expressions for φ_0:

$$\varphi_0 \approx \frac{(|m|+1)(\Omega^2-1)^2}{2\,|m|\,Y(\Omega^2+1)(\Omega+1)}. \tag{58}$$

Comparison of expressions (55) and (58) obtained for the cases of cylinder metal discharge chamber and cylinder metal antenna immersed into discharge plasma testifies that they are alike. Moreover, difference between formulas obtained for the limiting cases $\xi_a \ll 1$ and $\xi_c \ll 1$ is unimportant. That is why, it is possible to make general conclusion that sustained plasma is uniform enough along an azimuth angle for both the discharge structures.

As it was proved analytically in [2] and by numerical method in [43], the tangential component of the ASW electric field is very small as compared with the ASW radial electric field (at the HF range their ratio: $E_r/E_\varphi \sim 100$ and at LF range $E_r/E_\varphi \sim 10$). That is why, in the expressions for the amount of ASW energy absorbed by discharge plasma due to the collisional mechanism of the waves damping, it is possible to ignore the summand, which is proportional to the value E_φ. Exactly, because of this circumstance angular component of the ASW energy flow prevails substantially upon the radial component of the flow.

Degree of radial uniformity of the plasma, which is sustained in the considered discharge, depends on the penetration $\lambda_\perp$ depth of the ASW field into the plasma. In the considered case, $\lambda_\perp$ is practically determined by value of parameter $\xi_{a,c}$. The results of numerical calculations of the $\lambda_\perp$ value prove that for the ASW in HF and LF ranges, there is a wide range of plasma parameters, where one can use an approach of radially uniform plasma.

Determination of optimum value of plasma density is an important problem for any gas discharge, because its understanding allows one to conduct plasma production under the regime of eigen modes excitation. These results have been obtained for discharges sustained by ASW numerically method in [48]. It is found out that optimum value of plasma density n_{opt} depends on the produced plasma parameters in opposite manner for ASW at the HF and LF ranges. Value of n_{opt} increases with $\vec{B}_0$ and azimuth number$m$, but diminishes with increasing R_c at the LF range. This dependence of the n_{opt} on the mentioned parameters is opposite at the HF range (5). In the limiting case of relatively narrow metal discharge chambers (its radius $R_c < 8$ cm) for the typical values of the produced plasma parameters, the optimum value of the plasma density is larger for the LF ASW. Moreover, for the produced plasma density and an external magnetic field values, which are typical for the modern technological processes [13], ξ_0 changes inside of the limits: $\xi_0 = 10^{-4} \div 10^{-2}$. It means that, for the theoretical calculations of gas discharge sustained by ASW in the most cases, it is possible to apply approach of narrow cylinder, for which angular length of the discharge is much larger than 2π. As energy of the ASW does not flow out from the volume of the produced plasma, then considered discharge can be characterized by high efficiency of the power transfer and by angular uniformity of the sustained plasma.

3.2. Peculiarity of the ASW power transfer into magnetoactive plasma under the condition of operating gas low pressure

Parameters of microwave gas discharges depend on many factors, such as geometry of a discharge chamber, kind and pressure of the operating gas, value of the utilized external magnetic field, type of operating electromagnetic waves, and so on [12, 13]. Gas discharges in cylinder metal chambers sustained by electromagnetic waves, which propagate along the cylinders' axis, were experimentally examined in the cases both magnetoactive plasma and non-magnetized plasma. These experiments show us that volumes of plasma produced during gas discharge in the presence of an external magnetic field become larger [12, 13, 42].

To develop modern plasma technologies, the special attention is paid to magnetoactive gas discharges, which operate under the condition of low pressure of the utilized working gas. In this case, Ohmic channel of the SW energy transfer into plasma becomes non-effective one because together with decreasing value of the working gas pressure just frequency of collisions between plasma particles falls substantially. That is why under these conditions, one can search for other mechanisms of SW energy transfer into discharges' plasma. Influence of resonant absorption of the different SW on the sustained gas discharges, and parameters of the produced plasma was studied in [48, 49]. It is proved there that, for the SW, whose frequency ω satisfies resonant condition $\varepsilon_1(r_0, \omega) = 0$, one can expect for sufficient enhancing of quantity of the absorbed SW energy.

Let us consider the case of metal discharge chamber with a radius $R_c + a_d$, which inner surface has thin dielectric coating and its thickness satisfies inequality $a_d \ll R_c$, coefficient of dielectric permeability of this coating is ε_d. In this case of ASW belonged to the LF range, they can satisfy indicated resonant condition. That is why, conversion of the ASW into upper-hybrid bulk mode can be realized under the condition: $\omega = \sqrt{\omega_e^2 + \Omega_e^2(r_0)}$, where radial coordinate r_0 belongs to the region of transitional plasma layer. We would like to mention as well that application of a dielectric coating allows one to increase operating life of the experimental discharge chamber, to shield produced plasma from penetration of mixtures (nanoparticles and ions of the metal, which the wall of the discharge chamber is made from). In this case, small part of the ASW energy flows in the region of dielectric coating, where the ASW fields are described by the Bessel functions of the first kind and Neumann functions [19]. In the dielectric region, the ASW magnetic field component H_z^d can be determined by solution of (1) as well, but one can change: $\mu \to 0$, $k_\perp^2 \to -k^2\varepsilon_d$ and electric field components are calculated by the aid of the following equations using the known expression for H_z^d field:

$$E_\varphi^d = \frac{-i}{\kappa_d\sqrt{\varepsilon_d}}\frac{dH_z^d}{dr}, \quad E_r^d = \frac{i}{r\kappa_d\sqrt{\varepsilon_d}}\frac{dH_z^d}{d\varphi}, \tag{59}$$

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where $\kappa_d = k\sqrt{\varepsilon_d}$. In the approach of thin dielectric coating, it is possible to find out expressions for radial components of the ASW field in the region of non-uniform plasma:

$$E_r^{pl}(r < R_c) \approx C_1/\varepsilon_1(r), \tag{60}$$

where C_1 is constant of integration. Then for calculation of amount of the ASW power Q_{res}, which is absorbed at unit of the discharge length due to resonant damping of the ASW (due to conversion ASW into bulk mode at upper-hybrid resonance), it is suitable to apply integral representation of Dirac delta function [19]. It will allow one to derive such expression for the Q_{res}:

$$Q_{\text{res}} = \frac{r_0\omega_e^2 C_1^2 k^2}{4\omega\psi^2}\frac{dr}{d\varepsilon_1}\left[I_m'(k\psi r_0) + \frac{m\varepsilon_1 I_m(k\psi r_0)}{k\psi r_0\varepsilon_2}\right]. \tag{61}$$

Comparison of the ASW energy that is absorbed due to the Ohmic heating and that one, which is lost by the ASW through resonant conversion into the bulk mode, makes it possible to find out the following relation [48]:

$$\frac{Q_{\text{res}}}{Q_{\text{oh}}} \approx \frac{2\pi\omega(\omega^2-\omega_e^2)^2}{\nu R_c\Omega_e^2(\omega^2+\omega_e^2)}\frac{dr}{d\varepsilon_1(R_c)}. \tag{62}$$

Let us estimate the value of this ratio for the typical parameters of modern microwave discharges: working gas is Ar, its pressure is near 10 mTorr, working frequency of generator is 2.45 GHz, temperature of electrons 1 eV, density of the produced plasma in central uniform region is $n_{\text{pl}} = 10^{11}\,\text{cm}^{-3}$, $B_0 = 300\text{G}$, $R_c = 6\,\text{cm}$, $\nu = 10^7\,\text{s}^{-1}$. Under these conditions: $Q_{\text{res}}/Q_{\text{oh}} \approx 21 \geqslant 1$.

Estimations of the parameter $k\psi R_c$ value confirm that approaching of narrow metal cylinder can be frequently realized for cylindrical discharge chamber in the modern experiments. In this approach, one can derive the expression for angular discharge length φ_0^{res}, which is realized under the condition of resonant mode transmission of the ASW energy into the produced plasma. For this purpose, one can substitute the obtained expression for Q_{res} (61) into the power balance equation (48). Then its solution will allow one to obtain the following formula:

$$\varphi_0^{\text{res}} \approx \frac{\Omega_e^4(\varphi_0 = 0)r_0}{\omega^3(\omega + |\omega_e|)\Delta r}, \tag{63}$$

where Δr is the thickness of transitional region of non-uniform plasma, its value according to the data of the paper [49] is about five Debye lengths.

Analyzing the obtained expressions (63), it is possible to make the conclusion that role of resonant mechanism of the ASW power loses throughout their conversion into the upper hybrid modes grows with increasing the ASW frequency, thickness of transitional region of non-uniform plasma and also with diminishing of the working gas pressure (it results in diminishing of the collisional frequency) and radius of the produced plasma column. Angular discharge length (63) grows with increasing of plasma particles concentration and radius of metal chamber that is used in the discharge.

3.3. Electrodynamic model of gas discharge sustained by ASW without application of an external magnetic field

At utilization plasma technologies, one can estimate cost of the produced plasma. One of the frequently applied methods to make cheaper plasma production is to

https://doi.org/10.1017/S0022377810000644 Published online by Cambridge University Press

refuse utilization of an external magnetic field. That is why, to carry out complex analysis of the possibility to apply ASW for plasma production, let us consider model of gas discharge sustained by ASW without application of an external magnetic field absence [50]. As it was indicated above in the case of high pressure of working gas just Ohmic dissipation is the main mechanism of the SW power transfer into plasma produced during gas discharges [13, 44, 50].

Let us consider discharge chamber with geometry, which is assumed to be the same as in the previous subsection. Ignoring radial non-uniformity of the produced plasma in the case of non-magnetized plasma, for the ASW fields, one can derive the following expressions from the set of Maxwell equations:

$$H_z = A_2 I_m(k_\perp r), \quad E_r = -\frac{N_\vartheta A_2 I_m(k_\perp r)}{\varepsilon_p}, \quad E_\vartheta = \frac{i\omega A_2 I_m'(k_\perp r)}{ck_\perp}, \tag{64}$$

where A_2 is integration constant, $k_\perp = k\sqrt{|\varepsilon_p|}$, ε_p is dielectric permeability of a cold non-magnetized plasma [40]. Then for the values of the ASW power flow S_φ and quantity of their energy Q_{oh} that is absorbed by plasma at unit of the discharge length, one can find out the following expressions in the limiting case of narrow waveguide $R_c \ll m\delta$:

$$S_\varphi \cong \frac{A_2^2 \omega \delta^2}{8\pi (m!)^2 4^m}\left(\frac{R_c}{\delta}\right)^{2m}, \quad Q_{\text{oh}} \cong \frac{\nu \Omega_e^2 m A_2^2 \delta^2}{4\pi\omega^2 (m!)^2 4^m}\left(\frac{R_c}{\delta}\right)^{2m}. \tag{65}$$

At reduction of the working gas pressure, value of collisional frequency ν diminishes and together with that efficiency of Ohmic channel of SW energy dissipation decreases. That is why, one can take into account other channel of the ASW power transfer and existence of real radial non-uniformity of plasma in a discharge chamber. Under these conditions, the resonant damping of ASW becomes the main mechanism of energy transfer in a periphery region of non-uniform plasma, where $\varepsilon_p(r_0) = 0$. Then in such transitional layer, the radial electric field becomes the main component of the ASW field:

$$E_r^{(\text{res})} = \frac{N_\vartheta I_m\left(R_c \delta^{-1}\right)}{|\varepsilon_p(r)|} A_2. \tag{66}$$

Quantity of the ASW energy that can be resonantly absorbed nearby the resonant point is determined by the following formula:

$$Q_{\text{res}} \cong \frac{c^2 m^2 I_m^2\left(R_c \delta^{-1}\right) A_2^2}{8\omega R_c}\left|\frac{dr_0}{d\varepsilon_p}\right|. \tag{67}$$

Let us execute the following numerical estimations of the ASW power that can be absorbed due to collisional and resonant damping. To do that, one can choose the following values of the discharge parameters: density of plasma $n_{pl} = 10^{11}\,\text{cm}^{-3}$, frequency of generator 2.45 GHz, temperature of electrons 1eV, $\nu = 10^8\,\text{s}^{-1}$, which are typical of modern [13] discharges sustained by SW of different types. After that one can obtain the following ratio between quantities of the ASW powers absorbed throughout these two different mechanisms:

$$\frac{Q_{\text{oh}}}{Q_{\text{res}}} \approx 0.078 R_c \text{m}^{-1} \tag{68}$$

where one has to apply value radius R_c in centimeters. Analyzing ratio (68), one can see that for the discharge chamber with radius $R_c = 10\,\mathrm{cm}$, application of ASW with $m = 2$ will be already accompanied by resonant damping as the main mechanism of the energy transfer into the produced non-magnetized plasma. It is important to also underline that angular discharge length φ_0, which can be calculated from power balance equation, appears to be enough large $\varphi_0 \sim 10\,R_c\,\mathrm{m}^{-2}$. That is why, one can consider that plasma produced at the discharge sustained by ASW is uniform enough along azimuth direction.

In the first approximation, one can apply the represented results for theory of discharge sustained by long-wave azimuthally asymmetric SW. It can be done because of presence in dispersion of these waves just such ranges of axial wave numbers k_z, where axial group velocity of these SW is approximately equal to zero. Thus, in this case, transfer of SW energy in axial direction is practically absent [50].

Consequently, possibility to apply ASW propagation under the condition of absence of an external magnetic field in cylindrical discharge chamber for sustaining gas discharge is proven. As it is indicated in [50], diminishing of the SW frequency appears to be suitable for the purpose of sustaining gas discharges. It is explained by possibility of slow waves to interact with the plasma particles produced at the discharge by more effective manner. Under the regime of low gas pressure, the main channel of the ASW power transfer into the produced plasma is their conversion into the plasma bulk mode. Thus such discharge will be characterized by sufficient uniform density profile along an azimuth angle that is very important for the practical use of such discharges in modern plasma technologies.

4. Conclusions

The results of research into the ASW properties under the conditions of their propagation in the plasma filled cylindrical non-uniform waveguides of different construction are presented here. Real experimental conditions, namely, spatial non-uniformity of the confined plasma density and of the utilized external magnetic field, are taken into account. Possible practical applications of these theoretical results are indicated here as well.

The executed research allows one to determine frequencies of the electromagnetic azimuthal waves and spatial distribution of their field, calculate their damping rates, which are connected both with collisions between the plasma particles and resonant conversion of these modes into the bulk modes. It is found out that non-uniformity of an external axial magnetic field draws the influence, which is similar (from mathematical point of view) to the case of influence of plasma density non-uniformity on these modes' frequency. The account of small toroidicity of an external magnetic field (parameter $\varepsilon_t \ll 1$) results in that azimuthal waves propagate as wave packets. Corrections to the waves' field of their basic harmonic and eigen frequency, which determined by this non-uniformity of the utilized magnetic field, are proportional to ε_t^2. Corrugation of a confining magnetic field in modern thermonuclear reactors is characterized by small parameter $\varepsilon_m \ll 1$, but it can influence on conversion and absorption of the eigen waves. In this case, azimuthal perturbations propagate as well as wave packets, in which all six fields' components are present. Nevertheless, this task can be solved by the aid of the successive approximation theory, and it shows us even to be analogous (from mathematical point of view) to the cases of beam instabilities in plasma, which is confined in waveguides with a periodic dielectric inserts or with the corrugated walls and even to the case of the theory of free electron laser. The dispersion equations obtained for the ASW in this case contain small summands, which are proportional to ε_m^2, and they can be adequately solved by the methods of theory of successive approximations.

As the samples of possible practical applications of the ASW propagation, simple electrodynamical models of gas discharges sustained by ASW are presented as well. Importance of gas discharges for modern plasma technologies is great, they are applied for treatment of solids surfaces, for creation of an active mediums for gaseous lasers, for formation of carbon nanostructures, and so on. Consequently, choosing a working surface mode for sustaining gas discharge, one can take into consideration the possibility to apply these azimuthal modes as well.

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