

WHICH INFLATION TO TARGET? A SMALL OPEN ECONOMY WITH STICKY WAGES

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There is common agreement on price inflation stabilization being one of the objectives of monetary policy. But, in an open economy, two alternative measures of inflation coexist: domestic inflation and consumer price inflation. Which of the two should be the target variable? Most of the new open economy macroeconomics (NOEM) literature suggests that the monetary authority should stabilize domestic inflation. This is in sharp contrast with the practice of many inflation-targeting central banks that are using consumer price index (CPI) inflation as target variable. The paper shows that the standard result in the NOEM literature is derived under the simplifying assumption of flexible wages. The inclusion of sticky wages in an otherwise standard small open economy model is shown to rationalize CPI inflation targeting. This conclusion is robust to changes in key parameters, including the trade elasticity.

Keywords: Inflation, Open Economy, Sticky Wages, Optimal Monetary Policy

1. INTRODUCTION

One of the main objectives of monetary policy is to stabilize price inflation. In a closed economy, inflation is well defined. However, in an open economy, two measures of inflation coexist: domestic inflation, computed using the GDP deflator, and consumer price inflation, computed using the consumer price index (CPI). For most of the OECD countries, those two variables display quite different dynamics. Therefore, the relevant question is which one of these two should be targeted by the monetary authority. This question is particularly important given that although in practice inflation-targeting central banks have adopted CPI inflation as a target variable,¹ most of the new open economy macroeconomics (NOEM) literature suggests that the monetary authority should target domestic inflation instead.²

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This paper shows that this theoretical result is based on the assumption of flexible wages. The introduction of sticky nominal wages and monopolistic competition into the labor market, in an otherwise standard open-economy model reconciles the theory with the central banks' practice. Under these assumptions, the welfare losses due to business cycles are lower under CPI inflation targeting than under domestic inflation targeting.

To understand the intuition behind this result, let us first summarize the mechanism at work in standard models. Take a small open economy model with monopolistic competition in the goods market and sticky prices à la Calvo (1983). Assume, as usual in this literature, that a fiscal subsidy eliminating the distortion due to monopolistic competition is in place, so that the decentralized flexible-price allocation is efficient. In this context, the only source of inefficiency are the sticky staggered prices that create an undesirable price dispersion across firms. Setting domestic inflation to zero in each period, the monetary authority makes the constraint on price changes not binding *de facto* and replicates the efficient allocation. In this context, because foreign shocks generate fluctuations in the consumer price level but not in the real wage (nominal wages being flexible), they do not feed into domestic inflation. Thus the need for reacting only to domestic price movements.

Introducing monopolistic competition and sticky wages in the labor market changes that result in the following way: because of monopolistic competition, workers would like to charge a constant markup over their marginal rate of substitution; this is, however, not possible because of sticky *nominal* wages [modeled à la Calvo (1983)]. Therefore, wage markups fluctuate in response to shocks (not only internal productivity shocks but also foreign shocks affecting CPI). In this context, fluctuations in CPI inflation induce undesired fluctuations in the *real* wages and therefore, in the wage markup. This in turn translates into undesired fluctuations in firms' marginal costs and domestic inflation (because of monopolistic competition in the goods market, prices are at a markup over the marginal cost). Therefore, stabilizing CPI inflation directly reduces the volatility of wage inflation (therefore reducing the inefficient wage dispersion caused by the wage staggering), and also indirectly reduces the volatility of domestic inflation (through the firms' cost channel just described).

It is important to underline that, because of sticky wages, fluctuations of the wage markup around the desired level act as an endogenous cost-push shock in the New Keynesian Phillips curve (NKPC). Given that movements in CPI inflation translate into fluctuations in the wage mark-up, the higher the volatility of CPI inflation, the higher the volatility of the endogenous cost-push shock. This translates into a stronger tradeoff faced by the monetary authority between stabilizing inflation and closing the output gap. Given that, it is clearly difficult to stabilize domestic inflation and output gap without also stabilizing CPI inflation.

We show that by reacting to changes in CPI inflation instead of focusing on targeting domestic inflation, the monetary authority obtains better results in terms of stabilizing wage inflation, domestic price inflation, and output gap, thus attaining

higher welfare. Results are presented first for a baseline calibration, and then are shown to be robust to changes in key parameters. One such parameter of particular interest is the trade elasticity (i.e., the elasticity of substitution between domestic and foreign produced varieties). The introduction of wage rigidity makes CPI inflation targeting better than domestic inflation targeting even for low values of trade elasticity (including the case of complementarity between domestic and foreign goods). This is in stark contrast with previous studies such as Sutherland (2006) and De Paoli (2009a) that, although emphasizing the role played by this parameter, needed relatively high substitutability to get away from the desirability of domestic inflation targeting. This new result is entirely due to the presence of sticky wages.

Regarding the assumptions on which the results of the paper are built, there is strong micro empirical evidence of nominal wage rigidity in the economy.³ Also, as underlined by Christiano et al. (2005) and by Smets and Wouters (2003), the introduction of wage rigidity is a crucial assumption to improve the ability of New Keynesian models to match the data. Consequently, there is macro empirical evidence in favor of the importance of modeling wage rigidity also in order to obtain more reliable dynamics.

The structure of the paper is as follows: Section 2 presents the related literature, Section 3 introduces the open economy model, Section 4 explains the main mechanism at work, Section 5 presents the welfare analysis for a special but commonly used calibration (log utility and unitary trade elasticity), and Section 6 extends the welfare analysis to a general specification and show robustness of the results to different choices of the trade elasticity. Finally, Section 7 concludes.

2. RELATED LITERATURE

The present paper is closely related to Clarida et al. (2001) and Galí and Monacelli (2005). Clarida et al. (2001) analyze a small open economy model with price rigidities and exogenous variations in wage markup. They find that, as long as there is perfect exchange rate pass-through, the central bank should target domestic inflation. However, they do not explicitly model frictions in the labor market. They just assume an exogenous stochastic process for the wage markup. This is a crucial difference from the present paper because, even if assuming an exogenous process for the wage markup makes price stability no closer to optimal, the relationship between fluctuations in the wage markup and fluctuations in CPI inflation is missing. A similar result is obtained in Galí and Monacelli (2005), in which strict domestic inflation targeting is the optimal monetary policy, outperforming a CPI inflation-targeting rule. Aoki (2001) shows that in a two-sector closed economy model with different price rigidities, more weight should be attributed to the inflation of the stickier sector. He also shows that the extension of this result to a small open economy context implies that the monetary authority should target domestic inflation. Clarida et al. (2002) and Benigno (2004) extend the previous results to, respectively, a two-country model and a currency-area model.

Most of the previous results have been derived under a set of simplifying assumptions, and part of the literature has focused on the consequences of relaxing them. In particular, two key parameters are the elasticity of substitution between home- and foreign-produced goods (trade elasticity) and the intertemporal elasticity of substitution. Sutherland (2006) solves the model for the general case of nonunitary trade elasticity. In this context, CPI inflation targeting is to be preferred to domestic inflation targeting but only when home and foreign goods are highly substitutable. Kirsanova et al. (2006) instead show that even after allowing the intertemporal elasticity of substitution in consumption to be different from one and introducing international risk-sharing shocks, it is still desirable to target domestic inflation rather than CPI inflation. De Paoli (2009a) allows both elasticities to differ from one and finds that a fixed-exchange rate regime performs better than domestic inflation targeting, but only when domestic and foreign goods are highly substitutable.

Two things are worth mentioning at this point. First, the debate on the value of the trade elasticity is still very much open, with empirical estimates from the trade literature being much higher than the aggregate estimates used by the macro literature. That makes it difficult to use the previous models to choose one inflation-targeting regime over the other, given that results depend heavily on the specific value assigned to the trade elasticity. Second, some recent contributions show that the ability of the international business cycle models to match some main international business cycle facts also depends on the choice of the trade elasticity. In particular, Corsetti et al. (2008) estimate the trade elasticity to be below unit and find that a low value, together with incomplete markets, is crucial to reconciling standard international business cycle models with the Backus–Smith puzzle. Similarly, Thoenissen (2011) shows that for an open economy model to be reconciled with several international business cycle facts, the trade elasticity has to be very low. Thus, once the model is calibrated to match key international business cycle facts, the desirability of domestic inflation targeting stands.

The main contribution of this paper is to show that the welfare superiority of CPI inflation targeting over domestic inflation targeting rests on a much simpler change in the model: the introduction of wage rigidity. The fact that wage rigidity is enough to justify CPI inflation targeting makes the case in favor of CPI inflation targeting much stronger than the findings of the previous literature. Indeed, the role played by wage rigidity is so important that CPI inflation targeting is better than domestic inflation targeting even when domestic and foreign goods are complements, whereas this is not the case under flexible wages.

An alternative way of breaking the *inward-looking* characteristic of monetary policy is introducing incomplete exchange rate pass-through, as shown by Corsetti and Pesenti (2005). If the export prices of domestic firms are set in the foreign currency, they are exposed to exchange rate fluctuations. In this setup, the monetary authority should also give some weight to exchange rate stabilization in the monetary policy rule. However, note that the desirability of exchange rate targeting depends crucially on the currency used to set the price of exported

goods. If for example U.S. (E.U.) firms set their prices in dollars (euros), then the FED (ECB) should still target domestic price inflation. Another interesting extension is that by De Paoli (2009b), in which alternative asset market structures are considered. She finds that, whereas under complete markets an exchange rate peg is better than domestic inflation targeting when the trade elasticity is very high, the opposite is true under incomplete markets. Finally, two papers using CPI inflation targeting are Svensson (2000) and Monacelli (2005). But in neither of the papers is the desirability of CPI inflation targeting derived from first principles. They both assume an ad hoc loss function that also includes CPI inflation, basing this modeling choice on the observation that inflation-targeting central banks do target CPI inflation.

3. THE MODEL

Following the standard setup laid out by Galí and Monacelli (2005), the world consists of a continuum $[0, 1]$ of small, identical countries. Each country is populated by a continuum $[0, 1]$ of households deriving utility from consumption and disutility from work. Households consume both domestically produced and imported goods. As is standard in this literature, labor is immobile across countries. Monopolistic competition and price stickiness are assumed in the goods market. Differently, from the original model, the labor market is modeled as monopolistically competitive and workers’ optimal decisions over wages are made under the assumption of Calvo staggering. Because complete markets and separable utility are assumed, households differ in the amount of labor supplied (a consequence of the presence of sticky wages) but share the same consumption. It is also assumed that the law of one price holds for individual goods at all times. From now on “ h ” refers to a particular household, “ i ” to a particular country, and “ j ” to a specific sector. When no index is specified, the variables refer to the home country. Nominal variables are expressed in the currency of the home country when not otherwise stated.

3.1. Households

Household h maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where σ represents the relative risk aversion coefficient, φ is the inverse of the labor supply elasticity, $N_t(h)$ is the labor supply, and C_t is a consumption index that aggregate bundles of domestic and imported goods,

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $1 - \alpha$ represents the home bias in consumption and $C_{H,t}$ and $C_{F,t}$ are two aggregate consumption indices, respectively for domestic and imported goods,

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}, \quad C_{F,t} \equiv \left[\int_0^1 C_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad (3)$$

$$C_{i,t} \equiv \left[\int_0^1 C_{i,t}(j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}, \quad (4)$$

where $C_{H,t}(j)$ is the consumption of domestically produced good j , $C_{i,t}(j)$ is the consumption of variety j produced in country i , and $C_{i,t}$ is the total consumption of goods produced in country i . The parameter $\theta_p > 1$ represents the elasticity of substitution between two varieties of goods produced in the same country, whereas the parameter $\eta > 0$ represents the trade elasticity. Following Benigno and Benigno (2006), we define domestic and foreign goods to be complements (substitutes) in utility when $\sigma\eta < 1$ ($\sigma\eta > 1$), i.e., when the marginal utility of one good increases (decreases) as consumption of the other increases. Each household h maximizes (1) subject to a sequence of budget constraints,

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t [Q_{t,t+1}D_{t+1}] \leq D_t + (1 + \tau_w)W_t(h)N_t(h) + T_t, \quad (5)$$

where $Q_{t,t+1}$ is the stochastic discount factor, D_t is the payoff in t of the portfolio held at the end of $t - 1$, T_t is a lump-sum transfer (or tax) which also includes profits resulting from firms' ownership, τ_w is a subsidy to labor income, $P_{H,t}(j)$ is the price of domestic variety j , and $P_{i,t}(j)$ is the price (expressed in the home country currency) of variety j produced in country i .

Expenditure minimization problem. Solving the expenditure minimization problem yields

- demand for variety j produced at home:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta_p} C_{H,t} \quad (6)$$

- demand for variety j produced in country i :

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\theta_p} C_{i,t} \quad (7)$$

- demand for bundle of varieties produced in country i :

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\eta} C_{F,t} \tag{8}$$

- demand for home and foreign bundles:

$$C_{H,t} = (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \tag{9}$$

where the price indices are defined as

$$P_t \equiv \left[(1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{10}$$

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}, \quad P_{F,t} \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \tag{11}$$

$$P_{i,t} \equiv \left[\int_0^1 P_{i,t}(j)^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}. \tag{12}$$

Intratemporal consumption decision. Using the results of the previous section, it is possible to rewrite the budget constraint in aggregate terms:

$$P_t C_t + E_t [Q_{t,t+1} D_{t+1}] \leq D_t + (1 + \tau_w) W_t(h) N_t(h) + T_t. \tag{13}$$

Maximizing (1) with respect to consumption and asset holdings subject to (13) leads to the standard Euler equation

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right\} = 1, \tag{14}$$

with $R_t = \frac{1}{E_t[Q_{t,t+1}]}$ the gross nominal interest rate and $\pi_t \equiv \frac{P_t}{P_{t-1}}$.

Wage decisions. Household h supplies a differentiated labor service in each sector j , so that the total labor supplied is given by $N_t(h) = \int_0^1 N_{t,h}(j) dj$. Consequently, the household maximizes (1) w.r.t. $W_t(h)$ subject to the labor demand and the budget constraint. Given that production in each sector j is given by $Y_t(j) = A_t N_t(j)$, with $N_t(j) \equiv \left[\int_0^1 N_{t,j}(h)^{\frac{\theta_w-1}{\theta_w}} dh \right]^{\frac{\theta_w}{\theta_w-1}}$, the firm’s cost-minimization problem yields the labor demand

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\theta_w} N_t, \tag{15}$$

where $\theta_w > 1$ represents the elasticity of substitution between workers, and the aggregate wage index is given by $W_t \equiv \left[\int_0^1 W_t(h)^{1-\theta_w} dh \right]^{\frac{1}{1-\theta_w}}$.

Following Erceg et al. (2000), in each period only a fraction $(1 - \xi_w)$ of households can reset wages optimally, whereas for the others the wage is constant at $W_t(h) = W_{t-1}(h)$. The optimal wage of a household that can reoptimize in t solves

$$E_t \sum_{T=0}^{\infty} (\beta \xi_w)^T \left[C_{t+T}^{-\sigma} \frac{\tilde{W}_t(h)}{P_{t+T}} (1 - \Phi_w) - \tilde{N}_{t+T}(h)^\varphi \right] \tilde{N}_{t+T}(h) = 0, \tag{16}$$

where the optimal wage $\tilde{W}_t(h)$ is the same for all households, $\tilde{N}_{t+T}(h) = [\tilde{W}_t(h)/W_{t+T}]^{-\theta_w} N_{t+T}$, and $1 - \Phi_w \equiv \frac{1+\tau_w}{\mu_w}$, where $\mu_w = \frac{\theta_w}{\theta_w-1}$ is the desired wage markup. When $\tau_w = \frac{1}{\theta_w-1}$, $\Phi_w = 0$ and the fiscal policy completely eliminates the distortion caused by the presence of monopolistic competition in the supply of labor. Following Schmitt-Grohe and Uribe (2006), it is possible to write (16) recursively. In particular, let $w_t \equiv \frac{W_t}{P_t}$ be the aggregate real wage and $\tilde{w}_t \equiv \frac{\tilde{W}_t}{P_t}$ be the real wage of the optimizers. Then (16) can be written as $f_t^1 = f_t^2$, where

$$f_t^1 = (1 - \Phi_w) \tilde{w}_t C_t^{-\sigma} \left(\frac{w_t}{\tilde{w}_t} \right)^{\theta_w} N_t + \beta \xi_w E_t \left\{ \pi_{t+1}^{\theta_w-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\theta_w-1} f_{t+1}^1 \right\} \tag{17}$$

and

$$f_t^2 = \left(\frac{w_t}{\tilde{w}_t} \right)^{\theta_w(1+\varphi)} N_t^{1+\varphi} + \beta \xi_w E_t \left\{ \pi_{t+1}^{\theta_w(1+\varphi)} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\theta_w(1+\varphi)} f_{t+1}^2 \right\}. \tag{18}$$

Using the definition of the aggregate wage index, we can derive the following relation:

$$w_t^{1-\theta_w} = (1 - \xi_w) \tilde{w}_t^{1-\theta_w} + \xi_w w_{t-1}^{1-\theta_w} \pi_t^{\theta_w-1}. \tag{19}$$

Finally, after defining $\pi_{w,t} \equiv W_t/W_{t-1}$, we have

$$\pi_{w,t} = \frac{w_t}{w_{t-1}} \pi_t. \tag{20}$$

3.2. Firms

The production function of a domestic firm in sector j is given by

$$Y_t(j) = A_t N_t(j) \tag{21}$$

with $a_t \equiv \log(A_t)$ and

$$a_{t+1} = \rho_a a_t + \varepsilon_{a,t}, \tag{22}$$

where $\varepsilon_{A,t}$ is an i.i.d. shock with zero mean. The aggregate domestic output is given by

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}. \tag{23}$$

Calvo price staggering is assumed, with $(1 - \xi_p)$ being the probability of changing price in each period. Given the elasticity of substitution between different varieties, $\theta_p > 1$, the markup that each firm would like to charge is $\mu_p = \frac{\theta_p}{\theta_p - 1}$. Assuming the presence of a subsidy τ_p to the firm's output, optimal price setting by a home firm j implies

$$E_t \sum_{T=0}^{\infty} \xi_p^T Q_{t,t+T} \tilde{Y}_{t+T}(j) [(1 - \Phi_p) \tilde{P}_{H,t}(j) - MC_{t+T}] = 0, \tag{24}$$

where $MC_t = \frac{W_t}{A_t}$ represents the nominal marginal cost, $Q_{t,t+T} \equiv \beta^T (\frac{C_{t+T}}{C_t})^{-\sigma} \frac{P_t}{P_{t+T}}$, $\tilde{Y}_{t+T}(j) \equiv (\frac{\tilde{P}_{H,t}}{P_{H,t+T}})^{-\theta_p} Y_{t+T}$ represents the demand function, and $1 - \Phi_p \equiv (1 + \tau_p)^{\frac{\theta_p - 1}{\theta_p}}$. If the fiscal authority chooses τ_p in order to exactly offset the monopoly distortion, then $\Phi_p = 0$. Following Schmitt-Grohe and Uribe (2007), it is possible to rewrite (24) as $(1 - \Phi_p) F_t = K_t$, where

$$F_t = \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{1-\theta_p} Y_t + \xi_p \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left(\frac{\tilde{P}_{H,t} P_{H,t+1}}{P_{H,t} \tilde{P}_{H,t+1}} \right)^{1-\theta_p} \pi_{H,t+1}^{\theta_p} F_{t+1} \right\} \tag{25}$$

and

$$K_t = \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\theta_p} Y_t RMC_t + \xi_p \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left(\frac{\tilde{P}_{H,t} P_{H,t+1}}{P_{H,t} \tilde{P}_{H,t+1}} \right)^{-\theta_p} \pi_{H,t+1}^{1+\theta_p} K_{t+1} \right\} \tag{26}$$

with $RMC_t \equiv \frac{MC_t}{P_{H,t}} = \frac{w_t}{A_t} \frac{P_t}{P_{H,t}}$. Using the definition of the price index, we can derive the following relation:

$$1 = (1 - \xi_p) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{1-\theta_p} + \xi_p \pi_{H,t}^{\theta_p - 1}. \tag{27}$$

3.3. Equilibrium Conditions

The law of one price holds for individual goods, so that $P_{i,t}(j) = \epsilon_{i,t} P_{i,t}^i(j)$, where $\epsilon_{i,t}$ is the bilateral nominal exchange rate and $P_{i,t}^i(j)$ is the good j 's price expressed

in country i 's currency. The good market clearing condition requires that

$$\begin{aligned}
 Y_t(j) &= C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) \\
 &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\theta_p} \left[(1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta} C_t^i di \right].
 \end{aligned}
 \tag{28}$$

Using the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\theta_p-1}{\theta_p}}\right]^{\frac{\theta_p}{\theta_p-1}}$, we have

$$Y_t = (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \int_0^1 Q_{i,t}^\eta C_t^i di,
 \tag{29}$$

where $Q_{i,t} \equiv \epsilon_{i,t} P_t^i / P_t$ is the bilateral real exchange rate. Let $c_t^* \equiv \int_0^1 \log C_t^i di$ be the exogenous (log) world consumption and $q_t \equiv \int_0^1 \log Q_{i,t} di$ be the (log) effective real exchange rate. As in Galí and Monacelli (2005), the assumption of complete markets implies the following risk-sharing condition:

$$C_t = Q_{i,t}^{\frac{1}{\sigma}} C_t^i.
 \tag{30}$$

Taking logs and integrating over i , we have

$$c_t = \frac{1}{\sigma} q_t + c_t^*,
 \tag{31}$$

where $c_t \equiv \log C_t$. Let $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$ be the bilateral terms of trade, whereas the effective terms of trade are defined as $S_t \equiv \frac{P_{E,t}}{P_{H,t}}$. Combining this definition with the one of the consumer price level P_t , we have

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} = 1 - \alpha + \alpha S_t^{1-\eta}.
 \tag{32}$$

Using the definition of the bilateral real exchange rate, taking logs and integrating over i , we have

$$q_t = \int_0^1 \log S_{i,t} di + \log P_{H,t} - \log P_t.
 \tag{33}$$

Substituting (30) into (29), it is possible to write the market clearing condition as

$$Y_t = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[1 - \alpha + \alpha \int_0^1 Q_{i,t}^{\eta-\frac{1}{\sigma}} di \right].
 \tag{34}$$

The labor market clearing condition is given by $N_t = \int_0^1 N_t(j) dj$. Combining it with (21) and (23), we obtain

$$N_t = \frac{Y_t}{A_t} \Delta_t, \tag{35}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\theta_p} dj$ measures the output loss due to price dispersion. Δ_t can be written recursively:

$$\Delta_t = (1 - \xi_p) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\theta_p} + \xi_p \pi_{H,t}^{\theta_p} \Delta_{t-1}. \tag{36}$$

Up to a first-order approximation, (35) simplifies to

$$y_t = a_t + n_t, \tag{37}$$

with lowercase letters indicating logs. Finally, from the definition of CPI inflation, we have

$$\pi_t = \pi_{H,t} \frac{P_t}{P_{H,t}} \frac{P_{H,t-1}}{P_{t-1}}. \tag{38}$$

Log-linearizing (32), (33), and (34) around the steady state, we have⁴

$$\alpha s_t = \log P_t - \log P_{H,t}, \tag{39}$$

$$q_t = (1 - \alpha) s_t, \tag{40}$$

$$y_t = c_t + \alpha s_t, \tag{41}$$

where $s_t \equiv \log S_t$. Note that these last three equations hold exactly when $\eta = \sigma = 1$.

4. WAGE INFLATION EQUATION AND THE NEW KEYNESIAN PHILLIPS CURVE

Log-linearizing (16) around the steady state and using the equilibrium conditions just derived, we obtain the wage inflation equation,

$$\pi_{w,t} = -\lambda_w \hat{\mu}_{w,t} + \beta E_t[\pi_{w,t+1}], \tag{42}$$

where $\lambda_w = \frac{1-\xi_w}{\xi_w} \frac{1-\beta\xi_w}{1+\varphi\theta_w}$, whereas $\hat{\mu}_{w,t} = \log(W_t) - \log(P_t) - \log(MRS_t) + \log(1 - \Phi_w)$ represents fluctuations in the wage markup. Note that CPI enters the expression for the wage markup. In particular, if workers expect a future rise in the price level (positive CPI inflation) that will contract their wage markup via a reduction in the real wage, they optimally choose to demand a higher nominal wage today, thus pushing up wage inflation. Current wage inflation $\pi_{w,t}$ also depends positively on the expected future wage inflation.

From the log-linear approximation of (24) around the steady state we obtain

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{43}$$

with $\lambda \equiv \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$ and where \widehat{mc}_t denotes (log) deviations of the real marginal cost from its level in the absence of nominal rigidities [i.e., $\widehat{mc}_t = mc_t - mc$ with $mc = \log(1 - \Phi_p)$]. The relation between domestic inflation and real marginal cost is not affected by the presence of sticky wages. However, the presence of sticky wages does lead to an additional term in the equation relating the marginal cost with the output gap,⁵

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)x_t + \widehat{\mu}_{w,t}, \tag{44}$$

where \bar{y}_t represents the natural level of (log) output, i.e., output, when both prices and wages are flexible and $x_t \equiv y_t - \bar{y}_t$.

When wages are fully flexible, $\widehat{\mu}_{w,t} = 0$ and we have the standard NKPC:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\sigma_\alpha + \varphi)x_t. \tag{45}$$

In this context there is no trade-off between closing the output gap and inflation stabilization. If the fiscal authority sets the subsidies to eliminate the steady state distortions, the monetary authority can reach the first, best allocation by setting domestic inflation to zero in every period [Galí and Monacelli (2005)].

When wages are sticky, the wage markup fluctuates over the cycle and the NKPC for a small open economy with both price and wage rigidities becomes

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\sigma_\alpha + \varphi)x_t + \lambda \widehat{\mu}_{w,t}. \tag{46}$$

Even when the only distortions left in the economy are the ones generated by the presence of nominal rigidities (i.e., the fiscal authority sets the subsidies to eliminate the steady state distortions), clearly, as Erceg et al. (2000) showed for the closed economy case, it is not possible to stabilize domestic inflation, wage inflation, and output gap at the same time. The flexible allocation is no longer a feasible target. More importantly, as will be clear from the welfare analysis, the simple introduction of sticky wages is enough to make CPI inflation targeting preferred to domestic inflation targeting. The intuition goes as follows. Because nominal wages are sticky, fluctuations in CPI inflation translate into fluctuations of the real wage and, therefore, into fluctuations of $\widehat{\mu}_{w,t}$; i.e., the more volatile is CPI inflation, the more volatile will be $\widehat{\mu}_{w,t}$. Because this variable acts like an endogenous cost-push shock in the NKPC, reducing the volatility of CPI inflation helps reduce the trade-off faced by the monetary authority. Looking jointly at equations (42) and (46) makes it clear that reducing CPI inflation volatility first reduces the volatility of wage inflation and second reduces the trade-off faced by the monetary authority (by reducing the volatility of the endogenous cost-push shock), therefore making it easier to stabilize domestic inflation and output gap. This is not the case when the monetary authority targets domestic inflation. To show that CPI inflation targeting is indeed a better policy than domestic inflation

targeting when wages are rigid, we compare the welfare performance of different policy rules for the special case $\sigma = \eta = 1$ (Section 5) and for the general specification of the utility function (Section 6).

Before we move to the next section, it is worth noticing that the simple introduction of an exogenous cost-push shock such as that in Clarida et al. (2001), does not do the same job. Indeed, although it does introduce a trade-off in the NKPC so that strict inflation targeting is not optimal, such a trade-off is exogenous and therefore not related to the behavior of CPI inflation. In this context, an interest rate rule targeting domestic inflation outperforms the one targeting CPI inflation. This is also the case in De Paoli (2009a), in which CPI inflation targeting (or an exchange rate peg) is a better policy than domestic inflation targeting only if η and/or σ are bigger than 2, even though the economy is hit by cost-push shocks. Thus, the use of exogenous cost-push shock in an open economy framework can not really be considered a short cut to sticky wages if we want to derive monetary policy prescriptions.

5. A SPECIAL CASE $\sigma = \eta = 1$

Galí and Monacelli (2005) show the optimality of domestic inflation targeting when $\eta = \sigma = 1$ and wages are flexible. Under this calibration the *income effect* and the *expenditure-switching effect* generated by terms-of-trade movements exactly offset each other and welfare does not depend on terms of trade. More generally, those two parameters are key because they jointly determine how strong the terms of trade externality is.⁶ In this section we evaluate the performance of different monetary policy rules under this specification and show how the introduction of wage rigidity challenges this result. Section 6 extends the analysis to the more general specification of the utility function and shows the interplay between wage rigidity and terms-of-trade externality.

5.1. Welfare Function

In the present model there are five distortions: monopolistic power in both goods and labor markets; nominal rigidities in both wages and prices; and incentives to generate an exchange rate appreciation. The first four would be present in a closed economy as well. The last one is specific to the open economy framework and was first emphasized by Corsetti and Pesenti (2001). As a consequence, whereas in a closed economy framework it is enough to require $\Phi_w = \Phi_p = 0$ to ensure that the flexible allocation coincides with the optimal one (from the single-country point of view), this is no longer true in an open economy. We therefore need to solve the planner's problem and then set the subsidies accordingly. Under the assumption $\sigma = \eta = 1$, (31), (40), and (41) hold exactly, and maximizing (1) under these constraints plus the production function $Y_t = A_t N_t$ leads to the following first-order condition:

$$-\frac{U_N}{U_C} = (1 - \alpha)A^{1-\alpha}N^{-\alpha}(C^*)^\alpha. \quad (47)$$

The solution is a constant, optimal level of employment $N = (1 - \alpha)^{\frac{1}{1+\varphi}}$. Under flexible prices and wages, in every period $\widehat{\mu}_{w,t} = \widehat{m}c_t = 0$. Combining these two conditions together with the equilibrium conditions, it is possible to derive

$$N_t^{1+\varphi} \frac{\mu_w}{1 + \tau_w} = \frac{1 + \tau_p}{\mu_p}. \tag{48}$$

Once having substituted for the optimal level of N , (48) tells us how the two subsidies should be set in order to attain the optimal allocation in the flexible equilibrium.⁷

As in Erceg et al. (2000), all households have the same level of consumption but different levels of labor. For this reason, when computing the welfare function, we need to average the disutility of labor across agents:

$$W_t = U(C_t) + \int_0^1 V(N_t(h))dh. \tag{49}$$

The details of the derivation of the welfare function as a second-order approximation of the utility of the representative consumer can be found in Appendix B. The expected welfare loss in a small open economy with both price and wage rigidities is given by

$$L = -\frac{1 - \alpha}{2} \left[(1 + \varphi)\text{Var}(x_t) + \frac{\theta_p}{\lambda}\text{Var}(\pi_{H,t}) + \frac{\theta_w}{\lambda_w}\text{Var}(\pi_{w,t}) \right]. \tag{50}$$

Under price rigidity only, the loss depends on output gap and domestic inflation volatility. The introduction of wage rigidity adds a new term, the volatility of wage inflation.

In the next section we compute the fully optimal monetary policy under commitment, whereas in Section 5.3 we compare the performance of different interest rate rules, using the results under optimal monetary policy as benchmark.

5.2. Optimal Monetary Policy under Commitment

In this section, the fully optimal monetary policy under commitment is computed following Clarida et al. (1999), Giannoni and Woodford (2002), and Benigno and Benigno (2006).

The system of equations fully characterizing the model can be reduced to the following equations:

$$\alpha(x_t + a_t - c_t^*) = \alpha(x_{t-1} + a_{t-1} - c_{t-1}^*) + \pi_t - \pi_{H,t}, \tag{51}$$

$$\pi_{w,t} = w_t + \pi_t - w_{t-1}, \tag{52}$$

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \times \left[w_t - \alpha c_t^* - (1 - \alpha) a_t - (1 + \varphi - \alpha) \left(x_t + \frac{\log(1 - \alpha)}{1 + \varphi} \right) \right], \tag{53}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda \alpha x_t + \lambda \times \left[w_t - \alpha c_t^* - (1 - \alpha) a_t - (1 + \varphi - \alpha) \frac{\log(1 - \alpha)}{1 + \varphi} \right], \tag{54}$$

plus the monetary policy rule and the shock processes for a_t and c_t^* . To compute the optimal monetary policy under commitment, the central bank has to choose $\{x_t, \pi_{H,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^\infty$ in order to maximize⁸

$$W = -\frac{1 - \alpha}{2} E_0 \sum_{t=0}^\infty \beta^t \left[(1 + \varphi) x_t^2 + \frac{\theta_p}{\lambda} \pi_{H,t}^2 + \frac{\theta_w}{\lambda_w} \pi_{w,t}^2 \right] \tag{55}$$

subject to the sequence of constraints defined by equations (51)–(54). The solution⁹ to this optimization problem gives rise to the following optimal inflation-targeting rule:

$$\lambda_w \theta_p \pi_{H,t} - \lambda_w (x_{t-1} - x_t) + \beta \theta_w (\pi_{w,t} - E_t \pi_{w,t+1}) + \theta_w (\pi_{w,t} - \pi_{w,t-1}) + \lambda \theta_w \pi_{w,t} + (1 + \lambda + \beta) (x_t - x_{t-1}) + \beta (x_t - E_t x_{t+1}) - (x_{t-1} - x_{t-2}) = 0. \tag{56}$$

Equations (51)–(54) plus the last equation fully characterize the behavior of the economy under optimal monetary policy. A few things are worth noticing. Given the form of the loss function, there is no direct reason that the central bank should stabilize open economy variables such as the exchange rate, the terms of trade, or the CPI inflation. Because of the presence of a trade-off in the economy, due to both price and wage rigidity, the optimal policy implies a balance between stabilizing domestic inflation, wage inflation, and the output gap. As usual, the optimal monetary policy may be difficult to implement in practice. For this reason, and as is common in this literature, in the next section we concentrate our attention on a few simple interest rate rules and evaluate their performance using the optimal monetary policy as a benchmark.

5.3. Interest Rate Rules

Now we can go back to the original question, i.e., assuming that the monetary authority follows a simple rule, once wage rigidity is introduced into a small open economy, is it better to choose domestic inflation as target variable, or is it preferable to target CPI inflation? We are thus mainly interested in the relative performance of CPI versus domestic inflation–targeting rule. For completeness,

we also report the performance of a wage inflation–targeting rule. We therefore concentrate on the following three interest rate rules:

$$\begin{aligned} r_t &= \rho + \phi_p \pi_t, \\ r_t &= \rho + \phi_{p,H} \pi_{H,t}, \\ r_t &= \rho + \phi_w \pi_{w,t}. \end{aligned} \tag{57}$$

The values of ϕ_p , $\phi_{p,H}$, and ϕ_w are chosen to minimize the welfare loss over a grid of parameters. In that way each rule has the best chance of performing well. The grid ranges from 1 to 10 with intervals of 0.25. For each rule we also consider the case of strict inflation targeting ($\phi_i = 100$). We first present the results under a baseline parameterization and then show robustness checks for relevant parameters.

5.4. Baseline Parametrization

Preferences. We set the Frisch elasticity of the labor supply to $\frac{1}{3}$ by choosing $\varphi = 3$, a value commonly used in the real business cycle literature. In the robustness checks we allow φ to vary between 1 and 10. The discount factor is $\beta = 0.99$, which implies a riskless annual return in the steady state of 4%.

Goods and labor markets. Following the estimates of Basu and Fernald (1997) for the United States, it is common in the literature to set the elasticity of substitution between different goods as $\theta_p = 6$, implying a steady state price markup $\mu = 1.2$. In order not to introduce asymmetries between goods and labor markets in the baseline calibration, we set the elasticity of substitution between different workers as $\theta_w = 6$. In the robustness checks we allow those elasticities to vary between 4 and 12, to cover the range of values used in the literature. The average contract duration is set to four quarters in the baseline version, i.e., $\xi_p = \xi_w = 0.75$. However, we check the robustness of the results over the whole range $[0, 1]$.

Open economy. Following Galí and Monacelli (2005), we set $\alpha = 0.4$ in order to match the import/GDP ratio for Canada, which is used as proxy for a small open economy.

Exogenous shocks. The productivity shock follows an AR(1) process with $\rho_a = 0.66$. The exogenous shock to productivity is an i.i.d. with zero mean and standard deviation $\sigma_a = 0.0071$. Galí and Monacelli (2005) compute those numbers using the GDP of Canada as proxy for the output of a small open economy. The world is like a closed economy; thus equilibrium requires equality between output and consumption. As a proxy for world output, they use U.S. GDP, assume an AR(1) process, and estimate $\rho_y = 0.86$ and $\sigma_y = 0.0078$. They also estimate the correlation between the two exogenous shocks to be $\text{corr}_{a,y} = 0.3$.

TABLE 1. Welfare losses and standard deviations associated with each policy rule when $\sigma = \eta = 1$

	Optimal MP	Interest Rate			$\phi_p = 100$	$\phi_{p,H} = 100$
		π	π_H	π_w		
Welfare Losses	0.0024	$\frac{\phi_p = 5.25}{0.0265}$	$\frac{\phi_{p,H} = 4.75}{0.0563}$	$\frac{\phi_w = 100}{0.0027}$	0.0357	0.2961
$\sigma(\pi)$	0.3484	0.0634	0.1450	0.3291	0.0000	1.4106
$\sigma(\pi_H)$	0.1045	0.1047	0.0900	0.1047	0.1024	0.0819
$\sigma(\pi_w)$	0.0047	0.0542	0.0886	0.0000	0.0643	0.0819
$\sigma(x)$	0.0461	1.0228	1.3950	0.1842	1.1911	4.7252

Note: Welfare losses are measured as percentage units of steady state consumption. Standard deviations are also expressed in %. The coefficients of the policy rule minimizing the welfare losses are also reported. A coefficient of 100 stands for strict inflation targeting. In the last two columns we show the welfare losses associated with two simple targeting rules.

5.5. Performance of Different Monetary Policy Rules

Table 1 reports the welfare loss (measured as percentage units of steady state consumption) and the standard deviations of the main variables for both the fully optimal and the simple optimal rules. The last two columns also report the welfare losses associated with two simple rules: strict CPI inflation and strict domestic inflation targeting.¹⁰

Under the baseline parameterization the rule performing best is strict wage inflation targeting. But given that in practice central banks do not have an explicit target for wage inflation, what is most interesting is the comparison between the two inflation-targeting rules. The result under the baseline calibration is that an interest rate rule reacting to domestic inflation delivers welfare losses considerably higher than an interest rate rule reacting to CPI inflation targeting. This result becomes even stronger if, instead of considering optimal simple interest rate rules, one focuses on simple strict targeting rules (the last two columns). The ability of an interest rate rule reacting to CPI inflation to outperform a rule reacting to domestic inflation is particularly interesting given that the volatility of CPI inflation does not enter into the loss function. As explained previously, because of sticky wages, fluctuations in CPI inflation translate into undesired movements of the wage markup, acting as an endogenous cost-push shock in the economy. Reducing the volatility of CPI inflation reduces such cost-push shock and therefore reduces the trade-off faced by the monetary authority. For this reason, it makes it easier to stabilize wage inflation, domestic inflation, and output gap. Indeed, from Table 1 it is clear that the rule reacting to CPI inflation delivers much better results in terms of reducing the volatility of wage inflation and output gap than the rule reacting to domestic inflation, and this is why the associated welfare losses are lower.

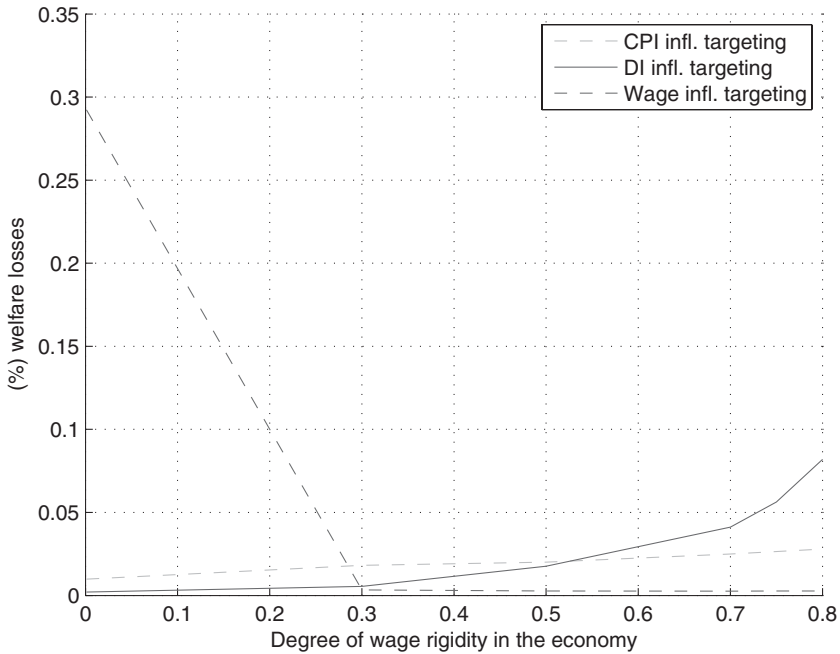


FIGURE 1. Wage stickiness. Comparison among different MP rules for different degrees of wage rigidity.

Therefore, the presence of wage rigidity rationalizes CPI inflation targeting even in the special case $\sigma = \eta = 1$. This is an important novelty with respect to the previous literature, where the preferability of CPI or exchange rate targeting has always been limited to economies where domestic and foreign goods are highly substitutable. In the next section we study how the relative performance of the rules is affected by some crucial parameters.

5.6. Robustness Checks

The parameters over which a robustness check is performed are the degree of wage stickiness ξ_w ; the inverse of the labor supply elasticity φ ; the elasticity of substitution between different types of labor θ_w ; and the elasticity of substitution between different goods θ_p . To understand the role played by each component, only one of the parameters is changed in each experiment, whereas the others are kept at their baseline values. The next section instead generalizes the present results under the assumption of complementarity ($\sigma \eta < 1$) and substitutability ($\sigma \eta > 1$) between domestic and foreign goods.

Wage stickiness. The wage rigidity parameter determines the weight of wage inflation stabilization in the loss function. The more wages are rigid, the higher is

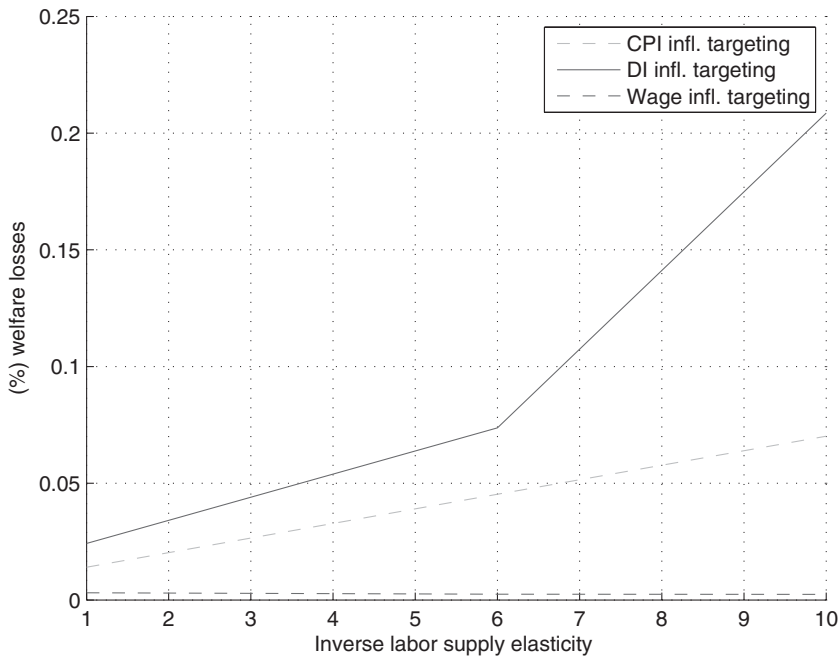


FIGURE 2. Inverse labor elasticity. Comparison among different MP rules for different labor supply elasticity.

the weight of wage inflation volatility in the loss. Figure 1 shows that, whereas the performance of the rules reacting to wage inflation and domestic inflation crucially depends on the level of wage rigidity in the economy, this is not the case for the rule reacting to CPI inflation. Indeed, the wage inflation–targeting rule performs really badly when there is no wage stickiness and is overall the worst rule for levels of wage rigidity below 0.3, whereas it becomes the best rule afterward.

The opposite is true for the domestic inflation–targeting rule, which coincides with the optimal monetary policy when $\xi_w = 0$ and is the best rule for low levels of wage rigidity. The threshold value in the comparison between the two price inflation–targeting rules is $\xi_w = 0.5$. Whenever the level of wage rigidity is above this value, the rule targeting CPI inflation outperforms the one targeting domestic inflation. The level of price rigidity under which the experiment is run is the baseline value $\xi_p = 0.75$; i.e., the CPI inflation–targeting rule is better than the domestic inflation–targeting rule even when the level of price rigidity in the economy is higher than the degree of wage rigidity. Also, $\xi_w = 0.5$ implies an average duration of wage contracts of six months, well below the average one-year duration usually found in the literature estimating DSGE models.¹¹

Frisch labor supply elasticity. Figure 2 reports the welfare losses associated with the three rules for values of φ ranging from 1 to 10.

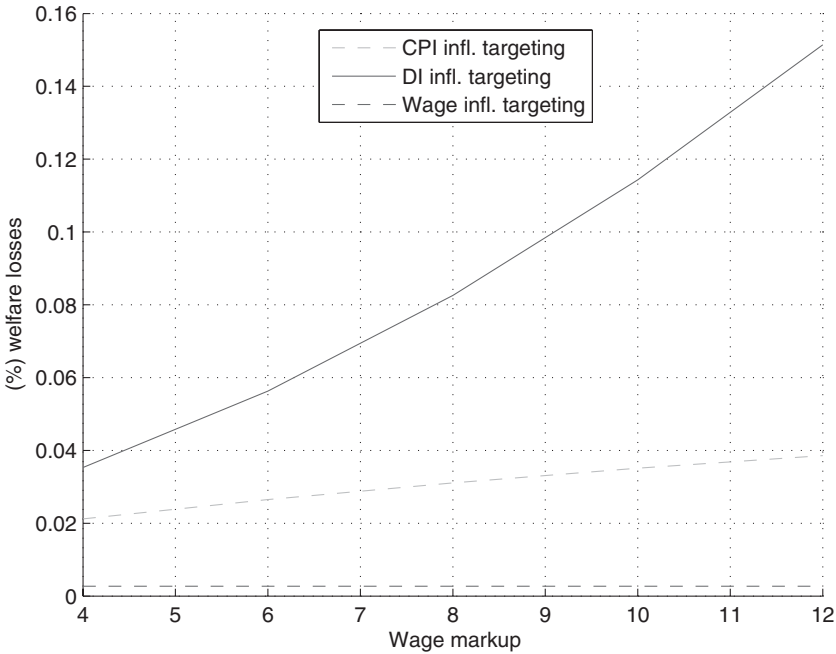


FIGURE 3. Elasticity of substitution across labor types. Comparison among different MP rules for different degree of wage markup.

This is the same range used by Canzoneri et al. (2007). They report estimates for the Frisch elasticity in the United States ranging from 0.05 to 0.35, which coincide with $\varphi \in [3, 20]$. Independent of the level of labor supply elasticity, domestic inflation targeting is always worse than CPI inflation targeting. Reducing the elasticity (i.e., increasing φ) amplifies the distance between the two rules. This is reasonable given that a lower elasticity implies a greater penalization of both output gap and wage inflation variability in the loss function, and we saw under the baseline calibration that domestic inflation targeting fails to contain the variability of those two variables. For a value of $\varphi = 10$, which is well below the maximum value estimated for the United States, the rule targeting domestic inflation delivers a welfare loss of 0.2% units of steady state consumption.

Wage markup. Another parameter for which different calibrations can be found in the literature is the elasticity of substitution between different labor types, θ_w . In the baseline calibration it has been set to 6, implying a wage markup of 1.2. In Figure 3 it is allowed to vary between 4 and 12.

As for the labor elasticity, changing this parameter does not alter the ranking of the rules. However, for values of θ_w above 6, the performance of the domestic inflation–targeting rule progressively worsens. This is again due to the fact that a high elasticity of substitution implies a high weight of wage volatility in the loss function.

Other checks. We have done other robustness checks, for which pictures are not reported for brevity. Allowing θ_p to vary between 4 and 12 changes the weight of domestic inflation volatility in the loss function but does not change the ranking of the rules. In the baseline calibration we set $\xi_p = 0.75$, a relatively high value, especially given the new empirical findings of Bils and Klenow (2004). Decreasing the degree of price rigidity strongly worsens the performance of domestic inflation targeting, thus reinforcing our results.

6. GENERAL SPECIFICATION: TERMS OF TRADE EXTERNALITY AND WAGE RIGIDITY

To derive the loss function (50) from a second-order approximation to the utility function, we used equations (40) and (41), which, however, hold exactly only when $\sigma = \eta = 1$. In the more general case they hold only up to a first-order approximation. Thus, (50) cannot be used to evaluate the performance of different monetary policy rules under a more general parameterization, because in that case it would not be accurate to second order. For the general specification we thus follow Schmitt-Grohe and Uribe (2004), (2007) and compute welfare numerically. Using the labor demand equation (15), we can rewrite the per-period welfare (49) as

$$W_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(\Delta_{w,t} N_t)^{1+\varphi}}{1+\varphi}, \tag{58}$$

where $\Delta_{w,t}^{1+\varphi} = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\theta_w(1+\varphi)} dh$ represents the inefficiency due to wage dispersion and can be written recursively:

$$\Delta_{w,t}^{1+\varphi} = (1 - \xi_w) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\theta_w(1+\varphi)} + \xi_w \pi_{w,t}^{\theta_w(1+\varphi)} \Delta_{w,t-1}^{1+\varphi}. \tag{59}$$

Let us define lifetime utility as $V_0 \equiv E_0 \sum_{i=0}^{\infty} \beta^i W_i$. Then

$$V_t = W_t + \beta E_t V_{t+1}. \tag{60}$$

In deviation from the nonstochastic steady state ($\bar{V} = \frac{1}{1-\beta} \bar{W}$), we have

$$\tilde{V}_t = W_t + \beta E_t \tilde{V}_{t+1} - (1 - \beta) \bar{V}. \tag{61}$$

Using Schmitt-Grohe and Uribe (2004), it is possible to derive a second order-accurate solution for the welfare (61) and all the equilibrium conditions.

To keep comparability with Sutherland (2006) and De Paoli (2009a), we restrict the analysis in this section to four simple targeting rules: domestic inflation targeting ($\phi_{p,H} = 100$), CPI inflation targeting ($\phi_p = 100$), wage inflation targeting ($\phi_w = 100$), and exchange rate peg. Results in the previous sections have been derived under the assumption of unitary elasticity ($\sigma \eta = 1$). We now generalize them by allowing domestic and foreign goods to be either complements ($\sigma \eta < 1$) or substitutes ($\sigma \eta > 1$). We do so by setting $\sigma = 1$ and $\eta \in \{0.5, 3.5\}$.¹²

TABLE 2. Welfare losses (gains) for the general specification of the model

	$\eta = 0.5$			$\eta = 3.5$		
	$\phi_p = 100$	$\phi_w = 100$	$\phi_e = 100$	$\phi_p = 100$	$\phi_w = 100$	$\phi_e = 100$
Flexible wages	-0.0225	-0.0037	-0.0633	-0.0010	-0.0006	-0.0032
Sticky wages	1.1138	1.1608	1.0928	0.8667	0.8756	0.8628

Note: Welfare losses (gains) are measured as percentage units of steady state consumption lost (gained) when moving from strict domestic inflation targeting ($\phi_{p,H} = 100$) to any of the other three rules considered. We retain the assumption of log utility ($\sigma = 1$) and consider two values for the trade elasticity: $\eta = 0.5$, representing the case in which domestic and foreign goods are complements, and $\eta = 3.5$, representing the case in which domestic and foreign goods are substitutes.

Let V_0^A (V_0^B) be the welfare associated with the policy regime A (B). Let Λ be the fraction of regime B 's consumption that households are willing to give up in order to be as well off under regime A as under regime B ; i.e., Λ measures the welfare cost in terms of consumption of regime A relative to regime B . Formally, Λ must be such that

$$V_0^A = E_0 \sum_{i=0}^{\infty} \beta^i \left[\frac{((1 - \Lambda)C_i^B)^{1-\sigma}}{1 - \sigma} - \frac{(\Delta_{w,i}^B N_i^B)^{1+\varphi}}{1 + \varphi} \right]. \tag{62}$$

Let regime B represent domestic inflation targeting and regime A be one of the alternative rules. It is easy to show that when $\sigma = 1$,

$$\Lambda = 1 - \exp \{ (1 - \beta) (V_0^A - V_0^B) \} \tag{63}$$

Table 2 reports welfare losses (gains) in terms of steady state consumption of moving from domestic inflation targeting to one of the alternative rules. It does so for both the case of flexible and sticky wages.

When wages are flexible, switching from strict domestic inflation targeting to any of the other rules induces welfare losses. This is so for both the case of complementarity and the case of substitutability between domestic and foreign goods. This result is consistent with both Sutherland (2006) and De Paoli (2009a). If wages are sticky instead, the opposite is true, because welfare gains can be achieved by switching from strict inflation targeting to any of the other rules. In particular, switching to a strict CPI-targeting rule implies a welfare gain of 0.86% or 1.11% of steady state consumption, depending on whether the goods are substitutes or complements. This result is new compared to those of Sutherland (2006) and De Paoli (2009a), in which strict domestic inflation targeting outperformed strict CPI inflation targeting when domestic and foreign goods were complements ($\sigma \eta < 1$). In both papers a high degree of substitutability (i.e., a large role for the terms-of-trade effect) was needed to make CPI inflation targeting or exchange rate peg desirable. Thus, the introduction of sticky wages makes CPI inflation

targeting preferable to domestic inflation targeting even when the terms-of-trade effect would not be strong enough to justify this policy.

7. CONCLUSIONS

Although inflation-targeting central banks are all using CPI inflation as their target variable, domestic inflation is the preferred target for price inflation in most of the NOEM literature. In previous studies, CPI inflation targeting has been found to be preferable to domestic inflation targeting only when domestic and foreign goods are highly substitutable, i.e., for medium/high values of the trade elasticity η . The lack of consensus on the *true* value of the trade elasticity makes it difficult to express a judgment between the two inflation targets based on those studies. Using a small open economy with both price and wage rigidities and deriving the welfare function from the second-order approximation of the utility of the representative consumer, we showed that the simple introduction of wage rigidities makes CPI inflation targeting superior to domestic inflation targeting even for low values of trade elasticity. This result is new to the literature and is particularly interesting because it is robust to changes in the baseline calibration. Therefore, wage stickiness provides a rationale for CPI inflation targeting.

NOTES

1. As stressed by Bernanke and Mishkin (1997), starting from 1990, the following countries have adopted an explicit target for CPI inflation: Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden, and the United Kingdom. To this list we can add more recently Norway and Hungary. In the EMU, the European Central Bank has the objective of stabilizing the Harmonized Index of Consumer Prices (HCPI) below 2%.

2. A detailed review of the related literature is provided in the next section.

3. For a review of the micro evidence of nominal wage stickiness and of the importance of modeling wage rigidities together with price rigidities, see Taylor (1998).

4. See Galí and Monacelli (2005) for the derivations.

5. See the derivation in Appendix A.

6. For earlier contributions on the role of the terms-of-trade externality and on the interplay between these two parameters see Obstfeld and Rogoff (1998), Corsetti and Pesenti (2001), and Benigno and Benigno (2003, 2006).

7. In the simulations, $\Phi_w = 0$ and consequently, $1 - \Phi p = 1 - \alpha$.

8. See Appendix B for the derivation of (55).

9. See Appendix C for the derivation.

10. Strict wage-inflation targeting turns out to be the optimal simple rule. Thus, results for this case are already reported in the fifth column.

11. Christiano et al. (2005) estimate an average contract duration of three quarters for wages using U.S. data, whereas Smets and Wouters (2003) find an average wage duration of one year for the Euro Area.

12. All the other parameters are set at their baseline values.

13. See Galí and Monacelli (2005) for the derivations.

14. Note that in the presence of taxes that exactly offset the monopoly distortions, the wedge between the real wage and the mr_s_t is due only to the presence of stickiness, whereas when $\Phi_w > 0$, $\mu_{w,t}$ reflects both the presence of stickiness and the presence of monopoly power.

15. The reference for the results in this section is Erceg et al. (2000).

16. With flexible prices and wages there are no differences across workers and firms, so $\text{Var}_h = 0$.

17. In general, if X assumes value X_1 with probability α and X_2 with probability $(1 - \alpha)$, then $E(X^2) = (\alpha) * X_1^2 + (1 - \alpha)X_2^2$, but the fraction of workers that cannot reoptimize in t will all have a different wage. This is why, as in Erceg et al. (2000), we need to take expectations again.

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APPENDIX A: DERIVATION OF \widehat{mc}_t

Taking a first-order approximation to the equilibrium conditions,¹³ it is possible to rewrite the real marginal cost as follows:

$$\begin{aligned}
 mc_t &= w_t - p_{H,t} - a_t \\
 &= mrs_t + \log(\mu_{w,t}) + p_t - p_{H,t} - a_t \\
 &= \sigma y_t^* + (1 - \alpha)s_t + \varphi(y_t - a_t) + \alpha s_t - a_t + \log(\mu_{w,t}) \\
 &= (\sigma - \sigma_\alpha)y_t^* + (\sigma_\alpha + \varphi)y_t - (1 + \varphi)a_t + \log(\mu_{w,t}), \tag{A.1}
 \end{aligned}$$

where $\mu_{w,t}$ represents the actual markup charged in each period.¹⁴ From equation (A.1) we can express the level of output as

$$y_t = \frac{mc_t}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1 + \varphi}{\sigma_\alpha + \varphi} a_t - \frac{\log(\mu_{w,t})}{\sigma_\alpha + \varphi}. \tag{A.2}$$

Let us define \bar{y}_t as the natural level of output, i.e., the level of output in the absence of nominal rigidities:

$$\bar{y}_t = \frac{mc}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1 + \varphi}{\sigma_\alpha + \varphi} a_t + \frac{\log(1 - \Phi_w)}{\sigma_\alpha + \varphi}. \tag{A.3}$$

Then

$$y_t - \bar{y}_t = \frac{\widehat{m}c_t}{\sigma_\alpha + \varphi} - \frac{\widehat{\mu}_{w,t}}{\sigma_\alpha + \varphi}, \tag{A.4}$$

which is exactly equation (44).

APPENDIX B: DERIVATION OF THE WELFARE FUNCTION

All the results in this section are derived under the assumption that $\sigma = \eta = 1$.

B.1. STEP 1: $W_t - \bar{W}$

From now on all the variables of type \widehat{a}_t represent log deviations from the steady state. We will substitute the following expression of the second-order derivative: $V_{NN} = \varphi V_N N^{-1}$. We will also use the fact that

$$\frac{X_t - \bar{X}}{\bar{X}} = \widehat{x}_t + \frac{1}{2} \widehat{x}_t^2 + o(\|a\|^3). \tag{B.1}$$

The first step is to compute a second-order approximation around the steady state of (49). Up to a second-order approximation it is true that

$$U(C_t) = +U(\bar{C}) + \bar{U}_C(C_t - \bar{C}) + \frac{1}{2} \bar{U}_{CC}(C_t - \bar{C})^2 + o(\|a\|^3). \tag{B.2}$$

Using (B.1) and the relations between consumption and output defined in (3.3), this equation becomes

$$\begin{aligned} U(C_t) - U(\bar{C}) &= \widehat{c}_t + o(\|a\|^3) \\ &= (1 - \alpha) \widehat{y}_t + o(\|a\|^3). \end{aligned} \tag{B.3}$$

In an analogous way it is true that

$$E_h V(N_t(h)) = V(\bar{N}) + E_h[\bar{V}_N(N_t - \bar{N})] + \frac{1}{2} E_h[\bar{V}_{NN}(N_t - \bar{N})^2] + o(\|a\|^3). \tag{B.4}$$

Using (B.1) and the relation between first-order and second-order derivatives leads to

$$E_h[V(N_t(h))] = V(\bar{N}) + \bar{V}_N \bar{N} E_h \left[\widehat{n}_t(h) + \frac{1 + \varphi}{2} \widehat{n}_t^2(h) \right] + o(\|a\|^3). \tag{B.5}$$

Combining (B.3) and (B.5) leads to

$$W_t - \bar{W} = (1 - \alpha) \widehat{y}_t + \bar{V}_N \bar{N} E_h \left[\widehat{n}_t(h) + \frac{1 + \varphi}{2} \widehat{n}_t^2(h) \right] + o(\|a\|^3). \tag{B.6}$$

The second step is to compute the approximation of the two expected values.

B.2. STEP 2: DERIVATION OF $E_h[\hat{n}_t(h)]$ AND $E_h[\hat{n}_t^2(h)]$

Because in general, for $A = [\int_0^1 A(i)^\phi di]^{1/\phi}$, it is true that¹⁵ $\hat{a}_t = E_t[\hat{a}(i)] + \frac{1}{2}\phi \times \text{Var}_i[\hat{a}(i)] + o(\|a\|^3)$, given the way in which aggregate labor has been defined, it is possible to write

$$\hat{n}_t = E_h[\hat{n}_t(h)] + \frac{1}{2} \frac{\theta_w - 1}{\theta_w} \text{Var}_h[\hat{n}_t(h)] + o(\|a\|^3). \tag{B.7}$$

Following Erceg et al. (2000), it is useful to write \hat{n}_t as a function of the aggregate demand for labor by firms $N_t = \int_0^1 N_t(j) dj$:

$$\hat{n}_t = E_j[\hat{n}_t(j)] + \frac{1}{2} \text{Var}_j[\hat{n}_t(j)] + o(\|a\|^3). \tag{B.8}$$

Clearly, because $\hat{y}_i(j) = a_t + \hat{n}_t(j)$, $\text{Var}_j[\hat{n}_t(j)] = \text{Var}_j[\hat{y}_i(j)]$ and $E_j[\hat{n}_t(j)] = E_j[\hat{y}_i(j)] - a_t$. Also, given the expression for aggregate output, $E_j[\hat{y}_i(j)] = \hat{y}_t - \frac{1}{2} \frac{\theta_p - 1}{\theta_p} \text{Var}_j[\hat{y}_i(j)] + o(\|a\|^3)$. Therefore, we can write

$$\begin{aligned} E_h[\hat{n}_t(h)] &= \hat{n}_t - \frac{1}{2} \frac{\theta_w - 1}{\theta_w} \text{Var}_h[\hat{n}_t(h)] + o(\|a\|^3) \\ &= E_j[\hat{y}_i(j)] - a_t + \frac{1}{2} \text{Var}_j[\hat{y}_i(j)] - \frac{1}{2} \frac{\theta_w - 1}{\theta_w} \text{Var}_h[\hat{n}_t(h)] + o(\|a\|^3) \\ &= \hat{y}_t - a_t + \frac{1}{2\theta_p} \text{Var}_j[\hat{y}_i(j)] - \frac{1}{2} \frac{\theta_w - 1}{\theta_w} \text{Var}_h[\hat{n}_t(h)] + o(\|a\|^3). \end{aligned} \tag{B.9}$$

For the other expected value,

$$E_h[\hat{n}_t^2(h)] = \text{Var}_h[\hat{n}_t(h)] + [E_h[\hat{n}_t(h)]]^2. \tag{B.10}$$

B.3. STEP 3: DERIVATION OF $W_t - \bar{W}$

τ_p and τ_w having been chosen optimally, the following holds $-\bar{V}_N \bar{N} = (1 - \alpha)$. Then, from this relation and substituting (B.9) and (B.10): into (B.6), the second-order approximation to the welfare function around the steady state becomes

$$\begin{aligned} W_t - \bar{W} &= (1 - \alpha)a_t - \frac{(1 - \alpha)}{2\theta_p} \text{Var}_j[\hat{y}_i(j)] - \frac{(1 - \alpha)(1 + \varphi\theta_w)}{2\theta_w} \text{Var}_h[\hat{n}_t(h)] \\ &\quad - \frac{(1 - \alpha)(1 + \varphi)}{2} (\hat{y}_t - a_t)^2 + o(\|a\|^3). \end{aligned} \tag{B.11}$$

Computing the approximation around the steady state of the welfare function in the absence of nominal rigidities leads to¹⁶

$$W_t^n - \bar{W} = (1 - \alpha)a_t - \frac{(1 - \alpha)(1 + \varphi)}{2} (\hat{y}_t^n - a_t)^2 + o(\|a\|^3). \tag{B.12}$$

Consequently,

$$\begin{aligned}
 W_t - W_t^n &= -\frac{(1 - \alpha)(1 + \varphi)}{2}(\widehat{y}_t^2 - (\widehat{y}_t^n)^2) + (1 - \alpha)(1 + \varphi)(\widehat{y}_t - \widehat{y}_t^n)a_t \\
 &\quad - \frac{(1 - \alpha)}{2\theta_p}\text{Var}_j[\widehat{y}_t(j)] - \frac{(1 - \alpha)(1 + \varphi\theta_w)}{2\theta_w}\text{Var}_h[\widehat{n}_t(h)] + o(\|a\|^3).
 \end{aligned}
 \tag{B.13}$$

From the log-linearization of equation (47), $a_t = \widehat{y}_t^n$.

From (B.13):

$$\begin{aligned}
 W &\equiv \sum_{t=0}^{\infty} \beta^t (W_t - W_t^n) \\
 &= -\frac{1 - \alpha}{2} \sum_{t=0}^{\infty} \beta^t \left[(1 + \varphi)x_t^2 + \frac{1}{\theta_p}\text{Var}_j[\widehat{y}_t(j)] + \frac{1 + \varphi\theta_w}{\theta_w}\text{Var}_h[\widehat{n}_t(h)] \right],
 \end{aligned}
 \tag{B.14}$$

where $x_t = \widehat{y}_t - \widehat{y}_t^n = y_t - y_t^n$. As proved by Woodford (2001),

$$\sum_{t=0}^{\infty} \frac{\beta^t}{\theta_p} \text{Var}_j[\widehat{y}_t(j)] = \frac{\theta_p}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2.
 \tag{B.15}$$

It remains to study $\text{Var}_h[\widehat{n}_t(h)]$. Let us first write the log-linear labor demand faced by each household:

$$\widehat{n}_t(h) = -\theta_w \log(W_t(h)) + \theta_w \log(W_t) + \widehat{n}_t + o(\|a\|^2).
 \tag{B.16}$$

Consequently,

$$\text{Var}_h[\widehat{n}_t(h)] = \theta_w^2 \text{Var}_h[w_t(h)]
 \tag{B.17}$$

with $w_t(h) = \log(W_t(h))$.

The next step is to compute $\text{Var}_h[w_t(h)]$.

B.4. STEP 4: DERIVATION OF $\text{VAR}_h[w_t(h)]$

First it is useful to decompose the variance as¹⁷

$$\begin{aligned}
 \text{Var}_h[w_t(h)] &= E_h[w_t(h) - E_h w_t(h)]^2 \\
 &= \xi_w E_h[w_{t-1}(h) - E_h w_t(h)]^2 + (1 - \xi_w)[\widetilde{w}_t - E_h w_t(h)]^2.
 \end{aligned}
 \tag{B.18}$$

Using the log-linearized expression for the aggregate wage and the result by Erceg et al. (2000) that $w_t - E_h w_t(h) = o(\|a\|^2)$,

$$\begin{aligned}
 E_h[w_{t-1}(h) - E_h w_t(h)]^2 &= E_h[w_{t-1}(h) - w_t + o(\|a\|^2)]^2 \\
 &= E_h[w_{t-1}(h) - E_h w_{t-1}(h) - \pi_{w,t} + o(\|a\|^2)]^2 \\
 &= \text{Var}_h w_{t-1} + \pi_{w,t}^2 + o(\|a\|^3).
 \end{aligned}
 \tag{B.19}$$

With the same arguments, we have

$$\begin{aligned} [\tilde{w}_t - E_h w_t(h)]^2 &= [\tilde{w}_t - w_t]^2 + o(\|a\|^3) \\ &= \left[\frac{\xi_w}{1 - \xi_w} \pi_{w,t} \right]^2 + o(\|a\|^3). \end{aligned} \tag{B.20}$$

Substituting (B.19) and (B.20) into (B.18) we can write

$$\text{Var}_h[w_t(h)] = \xi_w \text{Var}_h w_{t-1}(h) + \frac{\xi_w}{1 - \xi_w} \pi_{w,t}^2. \tag{B.21}$$

As in Woodford (2001), we can define $\Delta_t^w = \text{Var}_h[w_t(h)]$. Consequently we can rewrite (B.21) as

$$\Delta_t^w = \xi_w \Delta_{t-1}^w + \frac{\xi_w}{1 - \xi_w} \pi_{w,t}^2 + o(\|a\|^3). \tag{B.22}$$

Iterating backward, the previous equation can be written as

$$\Delta_t^w = \xi_w^{t+1} \Delta_{-1}^w + \sum_{s=0}^t \xi_w^s \frac{\xi_w}{1 - \xi_w} \pi_{w,t-s}^2 + o(\|a\|^3). \tag{B.23}$$

Following Woodford (2001),

$$\sum_{t=0}^{\infty} \beta^t \Delta_t^w = \frac{\xi_w}{(1 - \beta \xi_w)(1 - \xi_w)} \sum_{t=0}^{\infty} \beta^t \pi_{w,t}^2 + \text{i.i.p.} + o(\|a\|^3). \tag{B.24}$$

B.5. FINAL EXPRESSION

Combining the results in the previous sections,

$$W = -\frac{1 - \alpha}{2} \sum_{t=0}^{\infty} \beta^t \left[(1 + \varphi)x_t^2 + \frac{\theta_p}{\lambda} \pi_{H,t}^2 + \frac{\theta_w}{\lambda_w} \pi_{w,t}^2 \right]. \tag{B.25}$$

Taking the unconditional expectation of (B.25) and letting $\beta \rightarrow 1$, the expected welfare loss is

$$L = -\frac{1 - \alpha}{2} \left[(1 + \varphi)\text{Var}(x_t) + \frac{\theta_p}{\lambda} \text{Var}(\pi_{H,t}) + \frac{\theta_w}{\lambda_w} \text{Var}(\pi_{w,t}) \right]. \tag{B.26}$$

APPENDIX C: OPTIMAL MONETARY POLICY

To compute the optimal monetary policy under commitment, the central bank has to choose $\{x_t, \pi_{H,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^{\infty}$ in order to maximize

$$W = -\frac{1 - \alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1 + \varphi)x_t^2 + \frac{\theta_p}{\lambda} \pi_{H,t}^2 + \frac{\theta_w}{\lambda_w} \pi_{w,t}^2 \right], \tag{C.1}$$

subject to the sequence of constraints defined by equations (51), (52), (53), and (54). The first-order conditions of this problem are as follows ($\Phi_{i,t}$ is the Lagrange multiplier associated with the constraint i):

- x_t :

$$-(1 - \alpha)(1 + \varphi)x_t - \alpha\Phi_{1,t} + \beta\alpha E_t\Phi_{1,t+1} + \alpha\lambda\Phi_{4,t} + \lambda_w(1 + \varphi - \alpha)\Phi_{3,t} = 0, \quad (\text{C.2})$$

- $\pi_{H,t}$:

$$-(1 - \alpha)\frac{\theta_p}{\lambda}\pi_{H,t} - \Phi_{1,t} - \Phi_{4,t} + \Phi_{4,t-1} = 0, \quad (\text{C.3})$$

- $\pi_{w,t}$:

$$-(1 - \alpha)\frac{\theta_w}{\lambda_w}\pi_{w,t} - \Phi_{2,t} - \Phi_{3,t} + \Phi_{3,t-1} = 0, \quad (\text{C.4})$$

- π_t :

$$\Phi_{1,t} + \Phi_{2,t} = 0, \quad (\text{C.5})$$

- w_t :

$$\Phi_{2,t} - \beta E_t\Phi_{2,t+1} - \Phi_{3,t}\lambda_w + \lambda\Phi_{4,t} = 0. \quad (\text{C.6})$$

By combining those equations to find an expression for the Lagrange multipliers, it is possible to derive equation (56) in the text.