Macroeconomic Dynamics, **22**, 2018, 1076–1095. Printed in the United States of America.
doi:10.1017/S1365100516000572

TEMPTATION AND SELF-CONTROL IN A MONETARY ECONOMY

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We construct a microfounded model of money with Gul–Pesendorfer preferences. In each period, agents are tempted to spend all their money by the end of the period, and they suffer from the forgone utility that could have been obtained by adopting the tempting choice. We find that the Friedman rule may not be optimal. A positive nominal interest rate improves welfare because it reduces the real money balances and renders the temptation less attractive. The welfare gained by deviating from the rule is equivalent to 0.67% of consumption.

Keywords: Temptation, Search, Friedman Rule

1. INTRODUCTION

A number of authors study the desirability of the Friedman rule, which sets the nominal interest rate to zero. Chari and Kehoe (1999) show that the rule is optimal in reduced-form monetary models such as the money-in-the-utility-function model and the shopping-time model. These models are sometimes criticized, however, because the role of money is uncertain. To overcome this criticism, Lagos and Wright (2005), henceforth LW, construct a search-theoretic model in which money is essential and find that the Friedman rule is still optimal because a deviation from the rule reduces the match surplus. Their framework has been extended in many directions. Aruoba and Wright (2003) combine the LW model with the neoclassical growth model. Lagos and Rocheteau (2005) consider endogenous search intensity. Faig and Huangfu (2007) study a competitive search model based on LW. Interestingly, the optimality of the rule is unchanged in their models. These findings indicate that the rule is robustly optimal in the standard monetary models.

One limitation of the existing monetary models is their assumption of constant discounting, which is inconsistent with experimental evidence. As Thaler (1981) and Benzion et al. (1989) find, an agent's preference is actually reversed as time passes. To explain the preference reversals, Gul and Pesendorfer (2001) construct a model in which the agent compromises between his commitment utility and

This research is financially supported by Grant-in-Aid for Specially Promoted Research (No. 23000001). We thank Takashi Shimizu and Ryoichi Imai for their very valuable comments. We also thank the seminar participants at Osaka University. We thank two anonymous referees for their valuable comments. Address correspondence to: Ryoji Hiraguchi, Faculty of Law and Economics, Chiba University. 1-33, Yayoi-cho, Inage-ku, Chiba, Japan; e-mail: ryojih@chiba-u.jp.

temptation utility but suffers from the forgone utility that could have been obtained by adopting the tempting choice.[1] This is called the self-control cost; the agent's utility is the maximized sum of the commitment and temptation utilities *minus* the maximum temptation level. Recent experimental and empirical results, including Ashraf et al. (2006), Ameriks et al. (2007), and Bucciol (2012), find support for the Gul–Pesendorfer preferences.

In this paper, I incorporate the Gul–Pesendorfer preferences into the LW model. Each date is divided into two subperiods, day and night. The day market is decentralized and the night market is centralized. I first assume that the price mechanism is competitive in the two markets. The role of money is to facilitate trade in the day market, where buyers and sellers are anonymous and the buyers need money to make payments. I set the temptation function proportional to the period utility, in line with DeJong and Ripoll (2007). In each period, the agent is tempted to consume all his money by the end of the period, but he holds back some money for his next-period transactions. Thus, he is tempted to work less and enjoy more leisure in the night market than he actually does, owing to the quasi-linearity of the utility function. In our model, the extra utility from enjoying leisure is the self-control cost, and it is proportional to the real value of the end-of-period money balances.

I first show that if the relative risk aversion coefficient of the utility function in the day market is less than or equal to 1, the optimal policy deviates from the Friedman rule. In our model, an agent facing a self-control problem has to pay a cost to carry a positive amount of money with him. This self-control cost does not exist in Lagos and Wright (2005). A deviation from the Friedman rule reduces the end-of-period money balances and lowers the cost. As in Lagos and Wright (2005), a positive nominal interest rate reduces consumption and generates inefficiency in the decentralized market. However, its first-order effect is zero around the rule because the rule maximizes match surplus. Thus, the reduction in self-control cost dominates the surplus loss.

I next measure the welfare gained by deviating from the Friedman rule. The estimates of the temptation parameter vary with the literature. Following DeJong and Ripoll (2007), the optimal nominal interest rate is 5.9% and the welfare gained by deviating from the Friedman rule is equivalent to a 0.67% consumption. These results do not change significantly in a recent estimate by Bucciol (2012).

As a robustness check, I finally consider three situations: (i) the terms of trade in the decentralized market are determined by Nash bargaining, (ii) the agent faces the self-control problem in each subperiod, and (iii) the temptation function is quasigeometric, as in Krusell et al. (2010). We demonstrate that the nonoptimality of the Friedman rule continues to hold in these cases.

The Gul–Pesendorfer preferences are then incorporated into the various types of macroeconomic models. DeJong and Ripoll (2007) demonstrate that the self-control preferences partially resolve the equity premium puzzle in asset pricing models. Miao (2008) studies the option exercise decision problem of an agent facing temptation. Krusell et al. (2010) incorporate self-control preferences into the Ramsey model and find that the famous zero capital tax principle does not hold. Bucciol (2011), Kumru and Thanopoulos (2011), and Kumru and Tran (2012) construct an overlapping generations model to investigate the social security system. Nakajima (2012) studies the consumer debt. However, these studies are nonmonetary. As far as we know, our paper is the first to deal with the self-control problem in a monetary economy.

Several authors study the monetary models with nonconstant discounting.[2] Graham and Snower (2008, 2013) incorporate hyperbolic discounting into the New Keynesian model. Since hyperbolic discounting generates time-inconsistency, they solve the individual's problem as a game between current self and future self. They find that inflation has a long-run effect on a real variable and that the Friedman rule is not optimal. Their conclusion is similar to ours, but the mechanism through which the deviation from the rule works differs. In Graham and Snower (2013), the deviation works because agents are excessively myopic. Here, the agents' preferences are time-consistent, but it is still desirable to have preferences because it operates as a temptation-reducing device, just as the savings subsidy in Krusell et al. (2010) and the Pay-As-You-Go (PAYG) social security in Kumru and Thanopoulos (2011) do.

Some literature finds that the optimal policy may deviate from the Friedman rule in the LW framework. Craig and Rocheteau (2008) incorporate nominal frictions into the LW model. He et al. (2008) consider theft and banking. Andolfatto (2013) studies a case in which agents cannot commit to pay tax. In this paper, we provide another reason for why a deviation from the rule can be good. As Walsh (2003) and Bhattacharya et al. (2009) point out, most of the central banks pursue nonnegative inflation rates, and this is inconsistent with the Friedman rule, which implies deflation. Our results suggest that by introducing empirically and experimentally plausible preferences into monetary models, we can rationalize the actual monetary policies. It is well known that a deviation from the Friedman rule reduces a cost to carry money. In this paper, the cost is generated by the self-control problem.

The remainder of the paper is organized as follows. Section 2 provides our model. Section 3 studies the optimal policy and quantifies the welfare gained by deviating from the Friedman rule. Section 4 studies the robustness of our result. Section 5 concludes the paper. The appendix provides the proofs of propositions.

2. MODEL

In this section, we describe our model. In the following, we denote any next-period variable z as z_{+1}.

2.1. The Setup

The setup is very close to the LW model. Time is discrete and goes from $t = 0$ to $+\infty$. There is a continuum of agents with unit measure who discount the future by $\delta \in (0, 1)$. Each date is divided into two subperiods, day and night. While trade occurs in a decentralized market during the day subperiod, a centralized market is open during the night subperiod. We refer to the day market as the DM and the night market as the CM. Here, we assume that the pricing mechanism in the DM is competitive (Walrasian). A competitive version of the LW model is also investigated in Berentsen et al. (2005).

During the day subperiod, individuals participates in a decentralized market with bilateral matching. For two individuals i and j, the probability that the individual i consumes what j produces but not vice versa equals 0.5, and the individual j consumes what i produces but not vice versa also equals 0.5. For simplicity, we rule out the possibility of double coincidence of wants. At the CM, agents trade a general good that they produce and consume. The utility over the period is

$$\mathcal{U}(q_b, q_s, x, h) = u(q_b) - c(q_s) + U(x) - h, \tag{1}$$

where $u(q)$ is the utility from consuming q units of the DM good, $c(q)$ is the cost of producing q units of the DM good, $U(x)$ is the utility from consuming x units of the CM good, and h is the cost of producing h units of the CM good. The labor productivity in the CM equals 1.

The functions u, U, and c are twice continuously differentiable and satisfy $c(0) = u(0) = 0$, $u' > 0$, $U' > 0$, $c' > 0$, $u'' < 0$, $U'' < 0$, $c'' \geq 0$, $u'(0) = U'(0) = \infty$, and $u'(\infty) = U'(\infty) = 0$. We denote the efficient quantity in the DM as q^*. It satisfies $u'(q^*) = c'(q^*)$. Similarly, let x^* denote the efficient quantity in the CM, which satisfies $U'(x^*) = 1$. The maximized surplus in the CM is $S^* \equiv U(x^*) - x^*$.

Money has the role to facilitate trade in the DM market, where buyers and sellers are anonymous. Money is divisible and storable but intrinsically useless. The stock of money M evolves according to $M_{+1}/M = 1 + \pi$, where the growth rate of money π is constant. New money is injected into the CM as a lump-sum transfer, $T = \pi M$.

2.2. Seller's Problem

In each period, the seller supplies q_s units of labor in the DM, consumes x units of the general good, supplies h units of labor in the CM, and holds m_{+1} units of money at the end of the period. He does not consume in the DM; his utility over the period is denoted by $\mathcal{U}(0, q_s, x, h)$. Now, let $\mathbf{z}_s \equiv (q_s, x, h, m_{+1})$ be a vector of these variables and $\hat{\mathbf{z}}_s = (\hat{q}_s, \hat{x}, \hat{h}, \hat{m}_{+1})$ be a vector of the variables the agent is tempted to choose.

In the following, we denote the value functions of a seller (a buyer) in the DM who holds m units of money as $V^s(m)$ $[V^b(m)]$ and the expected value for the agent entering the DM as $V(m) = \frac{1}{2}[V^s(m) + V^b(m)]$. The problem of the seller

https://doi.org/10.1017/S1365100516000572 Published online by Cambridge University Press

is

$$V^s(m) = \max_{\mathbf{z}_s \in A}[\mathcal{U}(0, q_s, x, h) + \delta V_{+1}(m_{+1}) + W^s(q_s, x, h, m_{+1})] - \max_{\hat{\mathbf{z}}_s \in A}[W^s(\hat{q}_s, \hat{x}, \hat{h}, \hat{m}_{+1})]. \quad (2)$$

Here, $A = [(q, x, h, m_{+1}) \in \mathbf{R}^4_+ : x = h + \phi(pq + m + T - m_{+1})]$ is the budget set, ϕ is the price of the money in terms of the general good, p is the price of the DM goods, and W^s is the temptation function. The first part of equation (2) describes the actual choice of the seller, who maximizes the sum of the commitment utility and temptation utility. It represents a *compromise* between the utility under commitment and the cost of self-control. The second part of equation (2) represents the utility from the most tempting choice. The seller suffers from the lack of the forgone utility from the tempting choice.

The temptation utility is usually assumed to be more tilted toward current consumption than commitment utility. Following DeJong and Ripoll (2007), we set the temptation function proportional to the period utility function:

$$W^s(q_s, x, h, m_{+1}) = \lambda \mathcal{U}(0, q_s, x, h), \quad (3)$$

where $\lambda > 0$ indicates the strength of temptation. We simplify equation (2) on the basis of equation (3) as

$$V^s(m) = \max_{\mathbf{z}_s \in A}[(1+\lambda)\mathcal{U}(0, q_s, x, h) + \delta V_{+1}(m_{+1})] - \lambda \max_{\hat{\mathbf{z}}_s \in A}[\mathcal{U}(0, \hat{q}_s, \hat{x}, \hat{h})]. \quad (4)$$

We denote the first part of equation (4), representing the compromise, as V^s_{com}, and the second part, representing the temptation, as V^s_{tem}. From the quasilinearity of the utility function in the CM, the two parts reduce to

$$V^s_{\text{com}} = (1+\lambda)\{\max_{q_s}[\phi p q_s - c(q_s)] + \max_x[U(x) - x] + \phi(m+T)\} + \max_{m_{+1} \geq 0}[-(1+\lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})], \quad (5)$$

$$V^s_{\text{tem}} = \lambda\{\max_{\hat{q}_s}[\phi p \hat{q}_s - c(\hat{q}_s)] + \max_{\hat{x}}[U(\hat{x}) - \hat{x}] + \phi(m+T)\} + \lambda \max_{\hat{m}_{+1} \geq 0}(-\phi \hat{m}_{+1}). \quad (6)$$

From equations (5) and (6), labor supply in the DM and consumption in the CM, which the seller is tempted to choose, coincide with the actual choices, and trade in the CM is efficient. Therefore, $\hat{q}_s = q_s$ and $\hat{x} = x = x^*$.

The only difference between the actual choice and the temptation choice is on the next period's nominal balances. The seller takes a positive amount of money into the next period in case he becomes a buyer in that period. However, as is clear from equation (6), he is tempted to choose $\hat{m}_{+1} = 0$. This is because he is tempted to gain utility solely from the current consumption. From these results, we simplify the seller's value function $V^s(m) = V^s_{\text{com}} - V^s_{\text{tem}}$ as

$$V^s(m) = \max_q[\phi pq - c(q)] + S^* + \phi(m+T)$$
$$+ \max_{m_{+1}}[-(1+\lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \quad \textbf{(7)}$$

The first-order conditions on the output and future money balances are

$$\phi p = c'(q), \quad \textbf{(8)}$$

$$(1+\lambda)\phi = \delta V'_{+1}(m_{+1}). \quad \textbf{(9)}$$

When the seller supplies one unit of labor additionally in the DM, his disutility from labor increases by $c'(q)$ units, but he saves his labor supply in the CM by ϕp units; from equation (8), the cost and the benefit are the same. On the other hand, the left-hand side of equation (9), $(1+\lambda)\phi = \phi + \lambda\phi$, gives the cost of holding one unit of money for the next period. The first term ϕ indicates the marginal disutility of labor, while the second term $\lambda\phi$ gives the marginal cost of self-control. The agent is tempted to enjoy leisure today and hold no money for tomorrow. Thus, when he holds one unit of money additionally, he suffers from the forgone utility from leisure by $\lambda\phi$ units. This is the cost of self-control. Equation (9) indicates that the sum of the marginal costs is equal to the marginal value of holding money, $\delta V'_{+1}(m_{+1})$. As in the LW model, the choice of holding future money is independent of the current level of money balances.

2.3. Buyer's Problem

The buyer chooses a vector of variables, $\mathbf{z}_b \equiv (q_b, x, h, m_{+1})$, where q_b is the consumption in the DM, and x, h, and m_{+1} are the same as for the seller. We denote the vector that the buyer is tempted to choose as $\hat{\mathbf{z}}_b \equiv (\hat{q}_b, \hat{x}, \hat{h}, \hat{m}_{+1})$. Since he does not work in the DM, his period utility is denoted as $\mathcal{U}(q_b, 0, x, h)$.

We assume that the temptation utility of the buyer is the same as that of the seller and is given as $W^b(q_b, x, h, m_{+1}) = \lambda\mathcal{U}(q_b, 0, x, h)$. The buyer's value function is

$$V^b(m) = \max_{\mathbf{z}_b \in B}[(1+\lambda)\mathcal{U}(q_b, 0, x, h) + \delta V_{+1}(m_{+1})] - \lambda \max_{\hat{\mathbf{z}}_b \in B}[\mathcal{U}(\hat{q}_b, 0, \hat{x}, \hat{h})]. \quad \textbf{(10)}$$

Here, $B = [(q, x, m_{+1}, h) \in \mathbf{R}^4_+ : x = h + \phi(m + T - m_{+1} - pq)$ and $pq \leq m]$ is the budget set; the second constraint requires that the buyer cannot pay more money than he has. The first and the second parts of equation (10), say V^b_{com} and V^b_{tem}, respectively, reduce to

$$V^b_{\text{com}} = (1+\lambda)\{\max_{pq_b \leq m}[u(q_b) - \phi pq_b] + \max_x[U(x) - x] + \phi(m+T)\}$$
$$+ \max_{m_{+1}}[-(1+\lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})], \quad \textbf{(11)}$$

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$$V^b_{\text{tem}} = \lambda[\max_{p\hat{q}_b \le m} (u(\hat{q}_b) - \phi p\hat{q}_b) + \max_{\hat{x}}\{U(\hat{x}) - \hat{x}\} + \phi(m + T)] + \lambda \max_{\hat{m}_{+1}}\{-\phi\hat{m}_{+1}\}. \quad (12)$$

By definition, we have $V^b(m) = V^b_{\text{com}} - V^b_{\text{tem}}$. As with the seller's case, trade in the CM is efficient, the quantity q_b is the same as the hypothetical temptation quantity $\hat{q}_b$ in the DM, and $\hat{m}_{+1} = 0$. Function V^b thus simplifies to

$$V^b(m) = \max_{pq \le m}[u(q) - \phi pq] + S^* + \phi(m + T) + \max_{m_{+1} \ge 0}[-(1 + \lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \quad (13)$$

From equations (7) and (13), it follows that the value function of the buyer differs from that of the seller only in the surplus term in the DM. The first-order conditions on the next-period money balances are given by equation (9).

2.4. Competitive Equilibrium

In this section, we obtain the equilibrium allocation in which $q_b = q_s$. From equations (7) and (13), it is clear that the DM value function V satisfies the following Bellman equation:

$$V(m) = \frac{1}{2}\max_{pq \le m}[u(q) - \phi pq] + \frac{1}{2}\max_{q}[\phi pq - c(q)] + \phi(m + T) + S^* + \max_{m_{+1}}[-(1 + \lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \quad (14)$$

We observe λ only in the third term of equation(14) because the temptation choice in the DM is the same as the actual choice. The following lemma characterizes the real interest rate R.

LEMMA 1. $R = \frac{1+\lambda}{\delta}$.

Proof. See Appendix A.

From equation (14), we have $V(m) = 1/2\max_{pq \le m}[u(q) - \phi pq] + \phi m + V(0)$. By substituting this into equation (9) lagged one period, we obtain

$$\max_{m \ge 0}\left\{-i\phi m + \frac{1}{2}\max_{pq \le m}[u(q) - \phi pq]\right\}, \quad (15)$$

where $i = R\phi_{-1}/\phi_t - 1$ is the nominal interest rate. In equation(15), the term $i\phi m$ represents the cost of holding m units of money and the term $u(q) - \phi pq$ represents the surplus from buying q units of the special good. This problem is well defined, if and only if the nominal interest rate is nonnegative. If $i > 0$, the constraint $pq \le m$ binds.

For the time being, we follow the LW model and focus on equilibria at the Friedman rule, which are the limits of equilibria, as the nominal interest rate goes to 0. Under this assumption, the buyer's constraint binds even if $i = 0$. We then show that under the Friedman rule, the equilibrium at which the constraint binds welfare dominates the equilibria at which the agents hold more money than needed.

From equation (8), the first-order condition on m in equation (15) reduces to $i = 1/2[u'(q)/c'(q) - 1]$. In the steady state, the inflation rate is equal to the money growth rate $1 + \pi$. Thus, $i = (1 + \lambda)(1 + \pi)/\delta - 1$, and the quantity q solves

$$\frac{1}{2}\left[\frac{u'(q)}{c'(q)} - 1\right] = i = \frac{(1 + \lambda)(1 + \pi)}{\delta}. \quad \textbf{(16)}$$

Hence, the quantity q is strictly decreasing in i and trade is efficient (i.e., $q = q^*$) under the Friedman rule.

At the equilibrium with $m = M$, the real value of money balances, $\phi M = \phi pq = c'(q)q$, is a strictly increasing function of q. Moreover, from equation (16), q is decreasing in the temptation parameter λ. Therefore, given the inflation rate, ϕM is decreasing in λ. When the degree of temptation becomes higher, the agent suffers from the self-control cost more severely than before. As we show in the next section, the cost is proportional to the money balances. Hence, the agent tries to reduce his money holdings.

3. OPTIMAL MONETARY POLICY

In this section, we focus on the monetary policy that maximizes stationary welfare.

3.1. Steady-State Welfare

In equilibrium, equation (14) implies $V(M) = [u(q) - c(q)]/2 - \lambda\phi M_{+1} + \delta V_{+1}(M_{+1}) + S^*$ since $m + T = (1 + \pi)M = M_{+1}$. Stationary welfare, say v, is expressed as

$$(1 - \delta)v = \frac{u(q) - c(q)}{2} - \lambda\phi M_{+1} + S^*. \quad \textbf{(17)}$$

The welfare function is very similar to that of the LW model. The only difference is that it negatively depends on the real value of the end-of-period money balances ϕM_{+1}. The term $\lambda\phi M_{+1}$ represents the value of the extra leisure enjoyed by adopting the tempting choice. At the end of each period, the agent holds M_{+1} units of money for future transactions. However, the agent is tempted to consume all the money by the end of the period and enjoy extra leisure by ϕM_{+1} units of time owing to the quasilinearitiy of the utility function in the CM. The extra utility from leisure reduces the agent's welfare because he suffers from the forgone utility that could have been obtained by adopting the tempting choice. This is what Gul and Pesendorfer (2001) call the self-control cost.

So far, we assumed that the buyer's constraint always binds. Strictly speaking, under the Friedman rule, agents may hold more money than what is needed.

Overaccumulation of money does not affect welfare in the LW model, but here it does, because real balances enter the welfare function. In any equilibria under the Friedman rule, if the quantity in the DM is q^*, the price of the DM good is $p^* = c'(q^*)/\phi$, and the inflation rate is $\phi/\phi_{+1} = \delta/(1+\lambda)$, then the stationary welfare under the rule, say v_{FR}, is

$$(1-\delta)v_{\mathrm{FR}} = \frac{u(q^*) - c(q^*)}{2} - \frac{\lambda\delta}{1+\lambda}\phi_{+1}M_{+1} + S^*.$$

Stationary welfare is decreasing in the real balances, $\phi_{+1}M_{+1}$, which are equal to or greater than $\phi p^* q^* = q^* c'(q^*)$. The best equilibrium under the Friedman rule is the one in which the agent has the smallest amount of money. Therefore, when we study the optimal policy, we focus on the equilibria where the buyer's constraint binds.

Let $e(q) \equiv [u'(q) + c'(q)]q$. From equation (8), it follows that $\phi M_{+1} = (1+\pi)\phi M = \frac{1}{2}\frac{\delta}{1+\lambda}e(q)$. Substituting this equality into equation (17), we express v as a function of q:

$$(1-\delta)v(q) = \frac{u(q) - c(q)}{2} - \frac{1}{2}\frac{\lambda\delta}{1+\lambda}e(q) + S^*. \tag{18}$$

If the degree of temptation λ is zero, the welfare function coincides with that in the LW model.

3.2. Nonoptimality of the Friedman Rule

A deviation from the Friedman rule is welfare-improving if $v'(q^*) < 0$, since q is decreasing in i, and $q = q^*$ if $i = 0$. The first term of equation (18), $[u(q) - c(q)]/2$, represents the match surplus and is maximized under the rule. The first-order effect of a positive nominal interest rate on the surplus is zero around the rule. Hence, $v'(q^*) < 0$ if $e'(q^*) > 0$. Now, we have the following proposition.

PROPOSITION 1. *If the relative risk aversion coefficient of the function u is less than or equal to 1, the Friedman rule is not the optimal policy.*

Proof. Since $(qc')' > 0$, the function $e = qu' + qc'$ is strictly increasing in q if $(qu')' \geq 0$. We can easily see that this holds if and only if $\frac{-qu''}{u'} \leq 1$. ■

For example, if the utility function is of the constant relative risk aversion (CRRA) type $u(q) = \frac{q^{1-\eta}}{1-\eta}$ with $\eta > 0$ and the coefficient of relative risk aversion η is equal to or less than 1, $e(q) = qc'(q) + q^{1-\eta}$ is clearly increasing in q. If we further assume that the cost function is of the standard power form $c(q) = \frac{q^{1+\chi}}{1+\chi}$ with $\chi \geq 0$, we can analytically show that the optimal inflation rate is positive when the cost parameter χ is sufficiently large.

PROPOSITION 2. *Suppose that* $u(q) = \frac{q^{1-\eta}}{1-\eta}$ *and* $c(q) = \frac{q^{1+\chi}}{1+\chi}$. *The optimal monetary policy deviates from the Friedman rule if* $\chi > \eta - 2$. *If this holds, the optimal nominal interest rate is* $i = \frac{1}{2}\frac{\lambda\delta(\chi-\eta+2)}{1+\lambda(1-\delta+\delta\eta)}$. *The optimal inflation rate is positive if* $\chi > \frac{2(1+\lambda)(1+\lambda+\lambda\eta)}{\lambda\delta^2} - \eta$.

Proof. See Appendix B.

The condition $\chi > \eta - 2$ in Proposition 2 implies that the Friedman rule may not be optimal even if the relative risk aversion coefficient exceeds 1.

The agent facing a self-control problem experiences utility loss from not adopting the tempting choice. Here, the utility loss is proportional to the real value of the end-of-period money balances because the agent is tempted to spend all his money by the end of each period. Hence, if a positive nominal interest rate reduces the real balances, it improves welfare. A deviation from the Friedman rule generates inefficiency in the DM, but the first-order effect is zero at the rule. Therefore, the welfare improvement obtained from the reduction of the temptation utility outweighs the surplus loss. In our model, a deviation from the Friedman rule works as a temptation-reducing device, just as the saving subsidy in Krusell et al. (2010) and the PAYG social security in Kumru and Thanopoulos (2011) do.

3.3. Numerical Analysis

We now calculate the welfare gain obtained from the positive nominal interest rate. We parameterize our model by closely following the LW model and set $u(q) = \frac{q^{1-\eta}}{1-\eta}$ with $\eta = 0.163$, $c(q) = q$, $U(x) = A \ln x$ with $A = 1.968$, and $\delta = 0.96$. In terms of the temptation parameter λ, DeJong and Ripoll (2007) estimate the parameter of 0.075, while Bucciol (2012) finds the parameter of 0.052. A recent paper of Huang et al. (2015) estimate the parameter of 0.01. At this point, there is no consensus on the value of λ and we quantify the welfare gain for the two cases. We measure the welfare gained ω by deviating from the Friedman rule by how much consumption the agents would increase by following the rule instead of having a positive nominal interest rate:

$$(1-\delta)v[q(i)] = \frac{1}{2}\{u[(1+\omega)q^*] - c(q^*)\} - \frac{1}{2}\frac{\lambda\delta}{1+\lambda}e(q^*) + U[(1+\omega)x^*] - x^*.$$

Here, $q(i)$ is the quantity traded when the nominal interest rate is i and it solves equation(16). Figure 1 shows a graph of welfare gain as a function of i for the two cases. If we follow DeJong and Ripoll (2007) and set $\lambda = 0.075$, the optimal nominal interest rate is 5.9% and the welfare gained by deviating from the Friedman rule is 0.67%. On the other hand, if we set $\lambda = 0.052$, the optimal nominal rate of interest is 4.2% and the welfare gain is 0.38%. In both cases, the nominal interest rate is significantly away from zero. The welfare gains are similar to Aruoba and Schorfheide (2011), but are small when compared with Liu et al.

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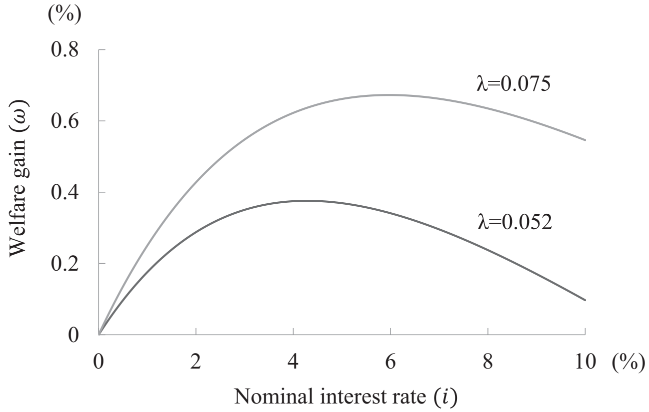

FIGURE 1. Nominal interest rate and welfare.

(2011). In Aruoba and Schorfheide (2011), the welfare gain is less than 0.6%, but in Liu et al. (2011), the welfare gain is about 2%.

4. EXTENSION

In this section, we modify our model in the following points: (i) terms of trade in the DM, (ii) timing of temptation, and (iii) the functional form of the temptation function.

4.1. Nash Bargaining

In the preceding section, we considered a competitive pricing mechanism. Here, we consider a case in which the terms of trade in the DM are determined by Nash bargaining as in the LW model. For simplicity, we assume that the buyer has all the bargaining power. A buyer makes a take-it-or-leave-it offer (q, d) to a seller in the DM, where q is the quantity and d the money transferred, and the seller determines whether to accept the offer or not.

After the seller accepts or declines the offer, he chooses a vector of variables $\mathbf{y}_s = (x, h, m_{+1})$. Let $\hat{\mathbf{y}}_s=(\hat{x}, \hat{m}_{+1}, \hat{h})$ be the vector of variables the seller is tempted to choose. If the seller accepts or is tempted to accept the offer (q, d), his problem is

$$V^s(m) = \max_{\mathbf{y}_s \in E}[(1+\lambda)\mathcal{U}(0, q, x, h) + \delta V_{+1}(m_{+1})] - \lambda \max_{\hat{\mathbf{y}}_s \in E} \mathcal{U}(0, q, \hat{x}, \hat{h}), \quad \mathbf{(19)}$$

where $E = [\mathbf{y}_s \in \mathbf{R}^3_+ : h + \phi(d + m + T - m_{+1}) = x]$ is the budget set. We rewrite $\mathcal{U}$ as $\mathcal{U}(0, q, x, h) = \phi d - c(q) + U(x) - x + \phi(m + T - m_{+1})$, where $\phi d - c(q)$ represents the surplus from the offer. Thus, the seller accepts the offer (q, d) if and only if $\phi d \geq c(q)$. Clearly, he is tempted to accept the offer under

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the same condition and to choose $\hat{m}_{+1} = 0$. Thus, equation (19) reduces to

$$V^s(m) = \phi d - c(q) + \phi(m + T) + S^* + \max_{m_{+1}}[-(1 + \lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \quad \textbf{(20)}$$

The first-order condition on m_{t+1} is the same as before and is given by equation (9).

Now, let us turn to the buyer's problem. Let $\mathbf{y}_b = (q, d, x, m_{+1}, h)$ be the vector of variables the buyer chooses. Similarly, let $\hat{\mathbf{y}}_b = (\hat{q}_b, \hat{d}, \hat{x}, \hat{m}_{+1}, \hat{h})$ denote the vector that he is tempted to choose. His problem is

$$V^b(m) = \max_{\mathbf{y}_b \in F}[(1 + \lambda)\mathcal{U}(q, 0, x, h) + \delta V_{+1}(m_{+1})] - \lambda \max_{\hat{\mathbf{y}}_b \in F} \mathcal{U}(\hat{q}_b, 0, \hat{x}, \hat{h}), \quad \textbf{(21)}$$

where $F = [\mathbf{y}_b \in \mathbf{R}^5_+ : h + \phi(m + T - m_{+1} - d) = x, c(q) \leq \phi d, \text{and } d \leq m]$ is the budget set. This is simplified as

$$V^b(m) = \max_{(q,d)}[u(q) - \phi d] + \phi(m + T) + S^* + \max_{m_{+1}}[-(1 + \lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})], \quad \textbf{(22)}$$

where the offer (q, d) must satisfy $c(q) \leq \phi d$ and $d \leq m$. The buyer chooses (q, d) so that the seller's participation constraint binds. As before, we focus on the equilibria where the constraint $d \leq m$ also binds.

From equations (20) and (22), it is clear that the DM value function is

$$V(m) = \frac{1}{2} \max_{c(q) \leq \phi m} [u(q) - c(q)] + \phi(m + T) + S^* + \max_{m_{+1}}[-(1 + \lambda)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \quad \textbf{(23)}$$

As in Section 3, the real interest rate is $R = (1+\lambda)/\delta - 1$, the first-order condition on q is equation (16), and the stationary welfare is equation (17). Thus, $q = q^*$ if $i = 0$ and $dq/di < 0$. However, the real value of money balances ϕM is related differently to q from the preceding section. Here, according to the binding participation constraint of the seller, $\phi M = c(q)$. Previously, it was equal to $c'(q)q$. It follows from equation (17) that stationary welfare is

$$(1 - \delta)v_1(q) = \frac{1}{2}[u(q) - c(q)] - \frac{1}{2}\frac{\lambda\delta}{1 + \lambda}\left[\frac{u'(q)}{c'(q)} + 1\right]c(q) + S^*. \quad \textbf{(24)}$$

We have the following proposition.

PROPOSITION 3. *Suppose that the trade in the DM is determined by Nash bargaining. If a function $u'(q)c(q)/c'(q)$ is nondecreasing in q, the Friedman rule is not optimal.*

Proof. See Appendix C.

The assumption in Proposition 3 holds if $u(q) = \frac{q^{1-\eta}}{1-\eta}$ with $\eta \in (0, 1]$ and $c(q) = \frac{q^{1+\chi}}{1+\chi}$. If $c(q) = q$, the welfare function $v_1(q)$ coincides with $v(q)$ in Section 3. Therefore, the numerical results on the welfare gained by deviating from the Friedman rule in Section 3.3 continue to hold if the parametric values are the same.

In LW, the Friedman rule achieves efficiency, if and only if the buyer has all the bargaining power, but regardless of the bargaining power, the rule is optimal. In our model, the buyer has all the bargaining power by assumption and then the Friedman rule achieves efficiency. However, the Friedman rule may not be optimal. An agent facing a self-control problem has to pay a cost to carry a positive amount of money with him, and a deviation from the Friedman rule reduces a cost. Thus it improves welfare. The benefit of reducing the self-control cost dominates the loss from reducing consumption, the effect of which is only second order.

When the seller has some bargaining power, the equilibrium is not efficient even under the Friedman rule just as LW. Therefore, reduction of consumption by deviating the rule is a first-order effect on welfare. In that case, welfare analysis becomes harder, but we believe that in some cases, the welfare loss dominates the welfare gain from reducing the self-control cost, and the Friedman rule becomes optimal.

4.2. Self-Control in Each Subperiod

Here, we assume that the agent faces the self-control problem in each subperiod. The temptation functions in the day and night subperiods are, respectively, $\lambda[u(q_b) - c(q_s)]$ and $\mu[U(x) - h]$. The problems of the seller and buyer entering the DM and the agent entering the CM are, respectively,

$$V^s(m) = \max_{q_s \geq 0}[-(1+\lambda)c(q_s) + W(m + pq_s)] - \lambda \max_{\hat{q}_s \geq 0}[-c(\hat{q}_s)], \tag{25}$$

$$V^b(m) = \max_{p\hat{q}_b \leq m}[(1+\lambda)u(q_b) + W(m - pq_b)] - \lambda \max_{p\hat{q}_b \leq m}[u(\hat{q}_b)], \tag{26}$$

$$W(m) = \max_{\mathbf{g} \in G}[(1+\mu)\{U(x) - h\} + \delta V_{+1}(m_{+1})] - \mu \max_{\hat{\mathbf{g}} \in G}[U(\hat{x}) - \hat{h}], \tag{27}$$

where $\mathbf{g} = (x, h, m_{+1})$ is the vector that the agent chooses, $\hat{\mathbf{g}} = (\hat{x}, \hat{h}, \hat{m}_{+1})$ is the vector that the agent is tempted to choose, $G = [\mathbf{g} \in \mathbf{R}^3_+ : h + \phi(m + T - m_{+1}) = x]$ is a budget set, and $V = (V^b + V^s)/2$ is the DM value function.

From equation (25), the seller is tempted to choose $\hat{q}_s = 0$ since the disutility $-c(q)$ is decreasing in q. From equation (26), the buyer is tempted to spend all his money on trade and chooses $\hat{q}_b = m/p$. At night, the seller as well as the buyer are tempted to choose $\hat{m}_{+1} = 0$. The CM problem simplifies to

$$W(m) = S^* + \phi(m + T) + \max_{m_{+1} \geq 0}[-(1+\mu)\phi m_{+1} + \delta V_{+1}(m_{+1})]. \tag{28}$$

Function $W(m) = \phi m + W(0)$ is linear and therefore equations (25) and (26) reduce to

$$V^s(m) = \phi m + W(0) + \max_{q_s \geq 0}[-(1+\lambda)c(q_s) + \phi p q_s], \quad \textbf{(29)}$$

$$V^b(m) = \phi m + W(0) + \max_{pq_b \leq m}[(1+\lambda)u(q_b) - \phi p q_b] - \lambda u(m/p). \quad \textbf{(30)}$$

The first-order conditions on q_s and m_{t+1} are $(1+\lambda)c'(q_s) = \phi p$ and $(1+\mu)\phi = \delta V'_{+1}(m_{+1})$, respectively. It is straightforward to show that the real interest rate in this economy is equal to $(1+\mu)/\delta$. From equations (28), (29), and (30), the choice of m solves

$$\max_m \left\{ \frac{1}{2} \max_{pq \leq m}[(1+\lambda)u(q) - \phi p q] - i\phi m - \frac{1}{2}\lambda u\left(\frac{m}{p}\right)\right\}. \quad \textbf{(31)}$$

In the stationary equilibrium, equation (31) implies

$$\frac{1}{2}\left[\frac{u'(q)}{(1+\lambda)c'(q)} - 1\right] = i = \frac{(1+\mu)(1+\pi)}{\delta}.$$

We now have the following proposition on the nonoptimality of the Friedman rule.

PROPOSITION 4. *Suppose that each agent faces a self-control problem in each subperiod. The steady-state welfare v_2 is expressed as a function of quantity:*

$$(1-\delta)v_2(q) = \frac{1}{2}[u(q) - (1+\lambda)c(q)] - \frac{1}{2}\frac{\mu\delta}{1+\mu}e_2(q) + S^*, \quad \textbf{(32)}$$

where $e_2(q) = [u'(q) + (1+\lambda)c'(q)]q$. The Friedman rule is not optimal if the relative risk aversion coefficient of $u(q)$ is less than or equal to 1.

Proof. See Appendix D.

4.3. Quasi-Geometric Temptation

So far, the temptation function was proportional to the period utility. Here, we assume that it is quasigeometric as in Krusell et al. (2010). The problem of the seller is

$$V^s(m) = \max_{\mathbf{z}_s \in A}[(1+\lambda)\mathcal{U}(0, q_s, x, h) + \delta(1+\lambda\beta)V_{+1}(m_{+1})]$$
$$- \lambda \max_{\hat{\mathbf{z}}_s \in A}[\mathcal{U}(0, \hat{q}_s, \hat{x}, \hat{h}) + \delta\beta V_{+1}(\hat{m}_{+1})], \quad \textbf{(33)}$$

where β represents temptation impatience and vector $\mathbf{z}_s$ and the set A are the same as those in Section 2. The buyer's problem is similarly defined. The only difference between temptation utility and commitment utility is the importance of current consumption relative to future consumption. If $\beta = 1$, the tempting choice is the same as the actual choice and the self-control problem disappears. We assume that

$\beta < 1$ and the temptation utility is tilted more toward current consumption than the commitment utility. The model in Section 2 corresponds to a case with $\beta = 0$.

LEMMA 2. *Let* $\bar{\beta} \equiv \frac{1+\lambda\beta}{1+\lambda}$. *The value function entering the DM satisfies*

$$V(m) = \frac{1}{2}\max_{pq\leq m}[u(q) - \phi pq] + \frac{1}{2}\max_{q\geq 0}[\phi pq - c(q)] + \phi(m+T) + S^* + (1+\lambda)\max_{m_{+1}}[-\phi m_{+1} + \bar{\beta}\delta V_{+1}(m_{+1})] - \lambda\max_{\hat{m}_{+1}}[-\phi\hat{m}_{+1} + \beta\delta V_{+1}(\hat{m}_{+1})]. \quad \textbf{(34)}$$

Proof. See Appendix E.

From equation (34), the term $\bar{\beta}\delta$ represents a compromise between temptation impatience and commitment impatience. The temptation choice differs from the actual choice only at the next-period nominal balances. The first-order conditions on m_{+1} and $\hat{m}_{+1}$ are

$$\phi = \bar{\beta}\delta V'_{+1}(m_{+1}) = \beta\delta V'_{+1}(\hat{m}_{+1}). \quad \textbf{(35)}$$

From equation (35), it is clear that $\hat{m}_{+1} < m_{+1}$. As in the preceding section, the temptation choice is tilted less toward future money balances than the actual choice. In the steady state, $\phi/\phi_{+1} = 1+\pi$ and $\phi p = c'(q)$. Thus, equation (35) implies that

$$1+\pi = \frac{\bar{\beta}\delta}{2}\left[\frac{u'(q)}{c'(q)} + 1\right] = \frac{\beta\delta}{2}\left[\frac{u'(\hat{q})}{c'(q)} + 1\right], \quad \textbf{(36)}$$

where $\hat{q} = \frac{\hat{m}}{p}$. This can be interpreted as the hypothetical temptation quantity. In the preceding section, $\beta = 0$ and therefore $\hat{q} = 0$. It is straightforward to show that the real interest rate is equal to $(\bar{\beta}\delta)^{-1}$ and that under the Friedman rule, $q = q^*$.

PROPOSITION 5. *If the temptation function is quasigeometric, the stationary welfare is*

$$(1-\delta)v_3(q,\hat{q}) = \frac{1}{2}[u(q) - c(q)] - \frac{\lambda\beta\delta}{2}e_3(q,\hat{q}) + S^*. \quad \textbf{(37)}$$

where q and $\hat{q}$ solve equation (36), *and* $e_3(q,\hat{q}) = u'(\hat{q})(q-\hat{q}) + u(\hat{q}) - u(q)$. *The Friedman rule is not optimal if the utility function is in the form* $u(q) = \frac{q^{1-\eta}}{1-\eta}$ *with* $\eta \in (0, 1]$.

Proof. See Appendix F.

As in the preceding section, the Friedman rule may not be optimal in this case.

https://doi.org/10.1017/S1365100516000572 Published online by Cambridge University Press

5. CONCLUSION

In this paper, I incorporate self-control preferences into the microfounded model of money and study the resulting optimal monetary policy. In each period, the agent is tempted to enjoy leisure and spend all his money by the end of the period and suffers from the forgone utility that could have been obtained by adopting the most tempting choice. I prove that for some utility functions, a deviation from the Friedman rule works as a temptation-reducing device and is welfare-improving.

NOTES

1. Laibson (1997) uses hyperbolic discounting to explain the preference reversals.
2. Lahiri (2007) finds that the Friedman rule is optimal in a model with endogenous discounting.

REFERENCES

Ameriks, John, Andrew Caplin, John Leahy, and Tom Tyler (2007) Measuring self-control problems. *American Economic Review* 97, 966–972.

Aruoba, S. Borağan and Frank Schorfheide (2011) Sticky prices versus monetary frictions: An estimation of policy trade-offs. *American Economic Journal: Macroeconomics* 3, 60–90.

Aruoba, Boragan, Christopher Waller, and Randall Wright (2011) Money and capital. *Journal of Monetary Economics* 58, 98–116.

Aruoba, Boragan and Randall Wright (2003) Search, money, and capital: A neoclassical dichotomy. *Journal of Money, Credit and Banking* 35, 1085–1105.

Ashraf, Nava, Dean Karlan, and Wesley Yin (2006) Tying odysseus to the mast: Evidence from a commitment savings product in the Philippines. *Quarterly Journal of Economics* 121, 635–672.

Benzion, Uri, Amnon Rapoport, and Joseph Yagil (1989) Discount rates inferred from decisions: An experimental study. *Management Science* 35, 270–284.

Berentsen, Aleksander, Gabriele Camera, and Christopher Waller (2005) Money, credit and banking. *Journal of Economic Theory* 135, 171–195.

Bhattacharya, Joydeep, Joseph Haslag, and Antoine Martin (2009) Optimal monetary policy and economic growth. *European Economic Review* 53, 210–221.

Bucciol, Alessandro (2011) A note on social security welfare with self-control problems. *Macroeconomic Dynamics* 15, 579–594.

Bucciol, Alessandro (2012) Measuring self-control problems: A structural estimation. *Journal of the European Economic Association* 10, 1084–1115.

Chari, V. V. and Patrick J. Kehoe (1999) *Optimal Fiscal and Monetary Policy*. Handbook of Macroeconomics, vol. 1C. Elsevier.

Craig, Ben and Guillaume Rocheteau (2008) State–dependent pricing, inflation, and welfare in search economies. *European Economic Review* 52, 441–468.

DeJong, David and Marla Ripoll (2007) Do self-control preferences help explain the puzzling behavior of asset prices? *Journal of Monetary Economics* 54, 1035–1050.

Faig, Miquel and Xiuhua Huangfu (2007) Competitive search equilibrium in monetary economies. *Journal of Economic Theory* 127, 709–718.

Graham, Liam and Dennis J. Snower (2008) Hyperbolic discounting and the Phillips curve. *Journal of Money, Credit and Banking* 40, 427–448.

Graham, Liam and Dennis J. Snower (2013) Hyperbolic discounting and positive optimal inflation. *Macroeconomic Dynamics* 17, 591–620.

Gul, Faruk and Wolfgang Pesendorfer (2001) Temptation and self-control. *Econometrica* 69, 1403–1436.

He, Ping, Lixin Huang, and Randall Wright (2008) Money, banking, and monetary policy. *Journal of Monetary Economics* 55, 1013–1024.

Huang, Kevin, Zheng Liu, and John Zhu (2015) Temptation and self-control: Some evidence and applications. *Journal of Money, Credit and Banking* 47, 581–615.

Krusell, Per, Burhanettin Kuruşçu, and Anthony A. Smith (2010) Temptation and taxation. *Econometrica* 78, 2063–2084.

Kumru, Cagri and Athanasios Thanopoulos (2011) Social security reform with self-control preferences. *Journal of Public Economics* 95, 886–899.

Kumru, Cagri and Chung Tran (2012) Temptation and social security in a dynastic framework. *European Economic Review* 56, 1422–1445.

Lagos, Ricardo and Guillaume Rocheteau (2005) Inflation, output, and welfare. *International Economic Review* 46, 495–522.

Lagos, Ricardo and Randall Wright (2005) A unified framework for monetary theory and policy analysis. *Journal of Political Economy* 113, 463–484.

Lahiri, Radhika (2007) Liquidity effects, variable time preference, and optimal monetary policy. *B.E. Journal of Macroeconomics (Topics)* 7, Article 6.

Laibson, David (1997) Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112, 443–477.

Liu, Lucy, Liang Wang and Randall Wright (2011) On the \hot potato" effect of inflation: Intensive versus intensive margins. *Macroeconomics Dynamics* 15, 191–216.

Miao, Jianjun (2008) Option exercise with temptation. *Economic Theory* 34, 473–501.

Nakajima, Makoto (2012) Rising indebtedness and temptation: A welfare analysis. *Quantitative Economics* 3, 257–288.

Thaler, Richard (1981) Some empirical evidence on dynamic inconsistency. *Economic Letters* 8, 201–207.

Walsh, Carl (2003) Accountability, transparency, and inflation targeting. *Journal of Money, Credit and Banking* 35, 829–849.

APPENDIX A: PROOF OF LEMMA 1

Following Aruoba and Wright (2003) and Aruoba et al. (2011), we introduce capital. We assume that agents have access to a concave technology for producing general goods $f(k)$ with $f(0) = 0$, $f'(0) = +\infty$, $f'(+\infty) = 0$, $f'(k) > 0$, and $f''(k) < 0$. Capital is illiquid and is available only in the CM. At night, the seller chooses the next-period capital k_{+1} in addition to m_{+1}. Let $\mathbf{y} = (q_s, x, h, m_{+1}, k_{+1})$ denote the vector of variables that the seller chooses. We denote the vector of variables that he is tempted to choose as $\hat{\mathbf{y}} = (\hat{q}_s, \hat{x}, \hat{h}, \hat{m}_{+1}, \hat{k}_{+1})$. The seller's problem is

$$V^s(m,k) = \max_{\mathbf{y}\in H}[(1+\lambda)\mathcal{U}(0,q_s,x,h) + \delta V_{+1}(m_{+1},k_{+1})] - \lambda \max_{\hat{\mathbf{y}}\in H}[\mathcal{U}(0,\hat{q}_s,\hat{x},\hat{h})], \quad \textbf{(A.1)}$$

where $H = [\mathbf{y} \in \mathbf{R}^5_+ : x + k_{+1} = f(k) + h + \phi(pq + m + T - m_{+1})]$ is the budget set. From the quasilinearity of the utility function in the CM, the function $V^s(m,k)$ reduces to

$$\begin{aligned} V^s(m,k) &= \max_q[\phi pq - c(q)] + f(k) + S^* + \phi(m+T) \\ &+ \max_{m_{+1},k_{+1}}[-(1+\lambda)\phi m_{+1} - (1+\lambda)k_{+1} + \delta V_{+1}(m_{+1},k_{+1})] \\ &- \lambda \max_{\hat{m}_{+1},\hat{k}_{+1}\geq 0}[-\phi\hat{m}_{+1} - \hat{k}_{+1}]. \end{aligned} \quad \textbf{(A.2)}$$

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From equation (A.2), it is clear that the agent is tempted to choose $\hat{m}_{+1} = \hat{k}_{+1} = 0$. As in the preceding section, the value function of the buyer differs from that of the seller, V^s, only in surplus term. The surplus of the buyer is given by $\max_{pq \le m}[u(q) - \phi pq]$. The value function of the agent at the beginning of each period, $V(m, k)$, reduces to

$$V(m,k) = f(k) + \frac{1}{2}\max_{pq \le m}[u(q) - \phi pq] + \frac{1}{2}\max_{q}[\phi pq - c(q)] + \phi(m+T) + \max_{m_{+1},k_{+1}}[-(1+\lambda)\phi m_{+1} - (1+\lambda)k_{+1} + \delta V_{+1}(m_{+1}, k_{+1})] + S^*. \quad \textbf{(A.3)}$$

From equation (A.3), it follows that $V(m, k) = f(k) + V(m, 0)$, and the first-order condition with respect to k_{+1} is $\frac{1+\lambda}{\delta} = \frac{\partial V_{+1}}{\partial k_{+1}}$, which is equal to $f'(k_{+1})$. Therefore, $R = \frac{1+\lambda}{\delta}$. ■

APPENDIX B: PROOF OF PROPOSITION 2

We have $e(q) = q^{1-\eta} + q^{1+\chi}$, and $q^* = 1$. Thus, $e'(q^*) = 2 + \chi - \eta > 0$ if $\chi > \eta - 2$. We also have $2(1-\delta)v(q) = u(q) - c(q) - \alpha e(q) + 2S^*$ with $\alpha = \frac{\lambda\delta}{1+\lambda}$. Thus,

$$2(1-\delta)v'(q) = [1 - \alpha(1-\eta)]q^{-\eta} - [1 + \alpha(1+\chi)]q^{\chi}.$$

This is maximized when $q = [\frac{1+\alpha(\eta-1)}{1+\alpha(1+\chi)}]^{1/(\chi+\eta)}$. From equation (16), it is clear that $i = \frac{1}{2}(q^{-\chi-\eta} - 1)$. Thus, welfare is maximized when $i = i_m \equiv \frac{1}{2}\frac{\alpha(2+\chi-\eta)}{1-\alpha(1-\eta)}$. The money growth rate π_m is

$$\pi_m = \frac{\delta}{1+\lambda}(1+i_m) - 1 = \frac{1}{2}\frac{\delta}{1+\lambda}\frac{2(1+\lambda) + \lambda\delta(\eta+\chi)}{1+\lambda(1-\delta+\delta\eta)} - 1,$$

and $\pi_m > 0$ if and only if $\frac{\eta+\chi}{2} > \frac{(1+\lambda)\{(1+\lambda)(1-\delta)+\lambda\delta\eta\}}{\lambda\delta^2}$. The right-hand side of this inequality is less than $\frac{(1+\lambda)(1+\lambda+\lambda\eta)}{\lambda\delta^2}$ because $\delta \in (0, 1)$. Thus, $\pi_m > 0$ if $\frac{\eta+\chi}{2} > \frac{(1+\lambda)(1+\lambda+\lambda\eta)}{\lambda\delta^2}$. ■

APPENDIX C: PROOF OF PROPOSITION 3

Let $e_1(q) \equiv [u'(q)/c'(q) + 1]c(q)$ denote the second term of equation (24). The optimal monetary policy deviates from the Friedman rule if $e_1'(q^*) > 0$. As $c'(q) > 0$, the function $e_1(q) = u'(q)c(q)/c'(q) + c(q)$ is an increasing function of q if $[u'(q)c(q)/c'(q)]' \ge 0$. ■

APPENDIX D: PROOF OF PROPOSITION 4

In the steady state, the value function $V = \frac{V_s+V_b}{2}$ is simplified as

$$(1-\delta)V = \frac{1}{2}[u(q) - (1+\lambda)c(q)] + S^* - \mu\phi M_{+1}. \quad \textbf{(D.1)}$$

The real balances are $\phi M = (1+\lambda)c'(q)q$. As $1 + \pi = \frac{\delta}{1+\mu}(1+i)$, the real value of the end-of-period cash balances is $\phi M_{+1} = \frac{1}{2}\frac{\delta}{1+\mu}[u'(q) + (1+\lambda)c'(q)]q$. Substituting this equation into equation (D.1) yields equation (32). As $v_2'(q^*) = -\frac{1}{2}\frac{\mu\delta}{1+\mu}e_2'(q^*)$, the Friedman

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rule is not optimal if $e_2'(q^*) > 0$. If the relative risk aversion coefficient of $u(q)$ is less than or equal to 1, $(u'(q)q)' \geq 0$ and then $e_2'(q) > 0$. ■

APPENDIX E: PROOF OF LEMMA 2

We denote the first and second terms of equation (33) as V^s_{com} and V^s_{tem}, respectively. From the quasilinearity of the utility function in the CM, these terms are, respectively, simplified as

$$(1+\lambda)^{-1} V^s_{\text{com}} = \max_{q_s}[\phi p q_s - c(q_s)] + \max_{x}[U(x) - x]$$

$$+ \max_{m_{+1}}[-\phi m_{+1} + \bar{\beta}\delta V_{+1}(m_{+1})] + \phi(m+T),$$

$$\lambda^{-1} V^s_{\text{tem}} = \max_{\hat{q}_s}[\phi p \hat{q}_s - c(\hat{q}_s)] + \max_{\hat{x}}[U(\hat{x}) - \hat{x}]$$

$$+ \max_{\hat{m}_{+1}}[-\phi \hat{m}_{+1} + \beta\delta V_{+1}(m_{+1})] + \phi(m+T).$$

As in the preceding section, $\hat{q}_s = q_s$ and $\hat{x} = x = x^*$. Therefore, the seller's value function $V^s(m) = V^s_{\text{com}} - V^s_{\text{tem}}$ reduces to

$$V^s(m) = \max_{q}[\phi p q - c(q)] + (1+\lambda)\max_{m_{+1}}[-\phi m_{+1} + \bar{\beta}\delta V_{+1}(m_{+1})]$$

$$-\lambda \max_{\hat{m}_{+1}}[-\phi \hat{m}_{+1} + \beta\delta V_{+1}(\hat{m}_{+1})] + S^* + \phi(m+T).$$

The first-order condition on the quantity q is the same as in the preceding section. The buyer's value function differs from the seller's value function only in the surplus terms. Thus, the function $V = \frac{V^s + V^b}{2}$ satisfies equation (34). ■

APPENDIX F: PROOF OF PROPOSITION 5

The substitution of $V(m) = \frac{1}{2}\max_{q \leq m/p}[u(q) - \phi p q] + \phi m + V(0)$ into equation (34) yields

$$V(0) = \frac{1}{2}\max_{q}[\phi p q - c(q)] + (1+\lambda)\max_{m_{+1},q}\left(-\phi m_{+1} + \bar{\beta}\delta\left\{\frac{1}{2}[u(q) - \phi p q] + \phi_{+1} m_{+1}\right\}\right)$$

$$-\lambda \max_{\hat{m}_{+1},\hat{q}}\left(-\phi \hat{m}_{+1} + \beta\delta\left\{\frac{1}{2}[u(\hat{q}) - \phi p \hat{q}] + \phi_{+1}\hat{m}_{+1}\right\}\right) + \phi T + S^* + \delta V_{+1}(0).$$

Let $\bar{\pi} = 1 + \pi$. In equilibrium, $m = pq$, $\hat{m} = p\hat{q}$, and $T = \pi p q$. The stationary welfare is

$$(1-\delta)v_3 = \frac{1}{2}[u(q) - c(q)] - \lambda\{-\bar{\pi}\phi p(\hat{q} - q) + \frac{\beta\delta}{2}[u(\hat{q}) - u(q) + \phi p(\hat{q} - q)]\} + S^*. \quad \textbf{(F.1)}$$

Substituting the seller's first-order condition $\phi p = c'(q)$ into equation (F.1) yields equation (37).

https://doi.org/10.1017/S1365100516000572 Published online by Cambridge University Press

It is sufficient to show that the self-control cost $e_3(q, \hat{q})$ is a decreasing function of i. Suppose that $u(q) = \frac{q^{1-\eta}}{1-\eta}$ with $\eta \leq 1$. Let $z = q/\hat{q} > 1$ and $\Omega = \bar{\beta}/\beta > 1$. Equation (36) reduces to $\Omega + (\Omega - 1)q^{\eta}c'(q) = z^{\eta}$. As $[q^{\eta}c'(q)]' > 0$ and $dq/di < 0$, $dz/di < 0$.

We first assume that $\eta = 1$. In this case, $e_3(q, \hat{q}) = (q - \hat{q})/\hat{q} + \ln \hat{q} - \ln q = z - \ln(z) - 1$ depends only on z. The function $z - \ln z$ is a strictly increasing function when $z > 1$. As $dz/di < 0$, $de_3/di < 0$. We next assume that $0 < \eta < 1$. Function $e_3(q, \hat{q})$ is written as $(1 - \eta)e_3(q, \hat{q}) = q^{1-\eta}[(1 - \eta)z^{\eta} + \eta z^{\eta-1} - 1]$. The term $\psi(z) \equiv (1 - \eta)z^{\eta} + \eta z^{\eta-1}$ is an increasing function of z for $z > 1$ because $\psi'(z) = (1 - \eta)\eta z^{\eta-2}(z - 1) > 0$. Since $dq/di < 0$ and $dz/di < 0$, $de_3/di < 0$ as long as $\eta \leq 1$. ■