Environment and Development Economics 12: 11–31 © 2007 Cambridge University Press
doi:10.1017/S1355770X06003366 Printed in the United Kingdom

Can NNP be used for welfare comparisons?

GEIR B. ASHEIM
Department of Economics, University of Oslo, P.O. Box 1095 Blindern, NO-0317 Oslo, Norway. E-mail: g.b.asheim@econ.uio.no

ABSTRACT. This paper contains a critical assessment of the claim that NNP can be used for welfare comparisons. The analysis assumes that national accounts are comprehensive (in particular 'greened' by taking into account environmental amenities and natural resource depletion), but does not assume optimal resource allocation. The general conclusion is that greater NNP corresponds to welfare enhancement only if net investment flows are revalued. Real utility-NNP and real measurable NNP made comparable across time by means of a consumer price index allow for such revaluation, and thus indicate welfare improvement.

1. Introduction

Among the different purposes that net national product (NNP) may serve, the following has been a prime concern in the theoretical literature on national accounting: can NNP be used for welfare comparisons if national accounts are made comprehensive by including the effects of environmental amenities and natural resource depletion as well as technological progress?

In a perfectly competitive economy with comprehensive national accounting, NNP represents the maximized value of the flow of goods and services that are produced by the productive assets of an economy. If NNP increases, then the economy's capacity to produce has increased, and – one might think – the economy is better off. Although such an interpretation is often made in public debate, the assertion has been subject to controversy in the economic literature. While Samuelson (1961: 51) writes that '[o]ur rigorous search for a meaningful welfare concept has led to a rejection of all *current* income concepts', Weitzman (1976), in his seminal contribution, shows that greater NNP indicates higher welfare if:

(a) dynamic welfare equals the sum of utilities discounted at a constant rate (i.e., discounted utilitarianism), and
(b) current utility equals the value of the consumed goods and services (i.e., a linearly homogeneous utility function).

Weitzman's result is remarkable – as it means that changes in the stock of forward-looking welfare can be picked up by changes in the flow of the value of current net product – but, unfortunately, strong

I thank two anonymous referees for useful comments, Bengt Kriström, Karl-Gustav Löfgren and other participants at the 12th Ulvön Conference on Environmental Economics for helpful discussions, Cornell University for hospitality, and the Research Council of Norway for financial support.

assumptions are invoked. More recently, Asheim and Weitzman (2001) have established that assumption (b) can be relaxed when concerned with whether welfare is increasing locally in time: real NNP growth corresponds to welfare improvement even when current utility does *not* equal the value of current consumption, as long as NNP is deflated by a consumer price index. Moreover, Asheim and Buchholz (2004) have shown that there are conditions under which even assumption (a) need not be invoked.

These findings are not, however, uncontroversial. Dasgupta and Mäler (2000) and Dasgupta (2001) warn against using NNP for welfare comparisons, while Weitzman (2001) and Li and Löfgren (2006) point out that there are other ways to deflate NNP (or argue that no NNP deflator is needed at all). Moreover, Heal and Kriström (2005b) present a critical assessment of the usefulness of NNP for making welfare comparisons. Here I contribute to this debate in the following two ways:

1 In section 3, I give an interpretation of the basic insights and results of welfare accounting, as developed by Samuelson (1961), Weitzman (1970, 1976), and Dixit *et al.* (1980). In particular, Samuelson (1961) argues that welfare changes should be measured by the present value of future changes in consumption, an insight that Heal and Kriström have brought to our attention through various contributions over recent years (see Heal and Kriström, 2005a,b). Moreover, Weitzman (1970, 1976) shows that there are conditions under which welfare changes can be measured by changes in utility-NNP, and, through this, establishes the link between welfare improvement and a positive value of net investments (cf. Weitzman, 1976, equation (14) above). Finally, Dixit *et al.* (1980) demonstrate the relationship between a positive value of net investments, on the one hand, and a positive present value of future consumption growth, on the other.
2 In section 4, I build on these insights (a) to warn against using NNP for measuring the welfare effects of capital stock perturbations, and (b) to derive the result that real NNP growth in variable consumption and net investment prices can be used to indicate welfare improvement, as reported in Asheim and Weitzman (2001) and Asheim and Buchholz (2004). *I summarize the results in section 2, where the following overall conclusion is stated:* NNP *can be used for welfare comparisons only if net investment flows are revalued.* I argue for the relative merits of using a consumer price index as an NNP deflator, when compared with the alternative of measuring real NNP changes in fixed consumption and net investment prices. I reconcile my findings with Li and Löfgren's (2006) demonstration that welfare improvement can be related to real NNP growth, measured in fixed consumption and net investment prices. Throughout I use the Dasgupta–Heal–Solow model (Dasgupta and Heal, 1974, 1979; Solow, 1974) to illustrate the analysis and results.

In my analysis, I invoke weak assumptions concerning how dynamic welfare is derived – by not necessarily assuming discounted utilitarianism – and how the economy functions – by not necessarily assuming an optimal resource allocation mechanism. Instead, I assume sufficient differentiability to derive my results.

Throughout I am concerned with local comparisons, either 'small' perturbations, or local-in-time comparisons. I also assume that national accounts are comprehensive (i.e., they are 'greened'). Global comparisons and non-comprehensive national accounting give rise to other issues that will not be addressed here.

2. Summary of results and relevant literature

Following Weitzman (1970, 1976, 2001), I consider a dynamic economy where the instantaneous well-being of the economy is measured by a one-dimensional indicator U, which will be referred to as *utility*. Assume that the instantaneous well-being of the economy depends on a vector of non-negative *consumption flows* $\mathbf{C}$ that includes also environmental amenities. Let U be an increasing, concave, smooth, and time-invariant *utility function* that assigns utility $U(\mathbf{C})$ to any consumption vector. Here, $\mathbf{C}$ is comprehensive, containing all variable determinants of current instantaneous well-being. This implies that an economy's instantaneous well-being is increased by moving from $\mathbf{C}'$ to $\mathbf{C}''$ if and only if $U(\mathbf{C}') < U(\mathbf{C}'')$.

Let the instantaneous net productive capacity of the economy depend on a vector of non-negative *capital stocks* $\mathbf{K}$ that includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other durable productive assets. Moreover, let $\mathbf{I}$ $(= \dot{\mathbf{K}})$ stand for the corresponding vector of *net investment flows*. The net investment flow of a natural resource is negative if the extraction rate exceeds its natural growth.

Say that the consumption – net investment pair $(\mathbf{C}, \mathbf{I})$ is *attainable* given $\mathbf{K}$ if and only if $(\mathbf{C}, \mathbf{I})$ is in $S(\mathbf{K})$, where $S(\mathbf{K})$ is a set that constitutes current *instantaneous net productive capacity*. Here, $\mathbf{K}$ is comprehensive, containing all variable determinants of current net productive capacity. This implies that society's productive capacity is changed by moving from $\mathbf{K}'$ to $\mathbf{K}''$ if and only if $S(\mathbf{K}') \neq S(\mathbf{K}'')$. Assume that the set of feasible triples

$$\{(\mathbf{C}, \mathbf{I}, \mathbf{K}) \mid (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\}$$

is a convex, smooth, and time-invariant set, with free disposal of consumption and net investment flows.

Assume for now that the economy maximizes dynamic welfare and that dynamic welfare is discounted utilitarian; i.e. the economy maximizes the sum of utilities, discounted at a constant rate ρ. Let *real utility*-NNP be defined as

$$Y_u := U(\mathbf{C}^*) + \mathbf{Q}_u \mathbf{I}^*,$$

where $\mathbf{Q}_u$ is a vector of net investment prices in terms of utility (formally, the vector of current value co-state variables) and $(\mathbf{C}^*, \mathbf{I}^*)$ maximizes $U(\mathbf{C}) + \mathbf{Q}_u \mathbf{I}$ over all attainable consumption – net investment pairs. By identifying Weitzman's (1976) one-dimensional composite consumption good (which 'might be calculated ... as any cardinal utility function', Weitzman, 1976: 156–157) with the indicator of instantaneous well-being $U(\mathbf{C})$, Weitzman's (1976) main result entails that real utility-NNP is the *stationary welfare equivalent* of the path of future utilities. This means that under these assumptions real utility-NNP has the following welfare significance:

Dynamic welfare is increased by moving from a situation with real utility-NNP *equal to* Y'_u *to a situation with real utility*-NNP *equal to* Y''_u *if and only if* $Y'_u < Y''_u$.

Note that in this comparison the net investment prices in terms of utility $\mathbf{Q}'_u$ used to calculate Y'_u are different from those $\mathbf{Q}''_u$ used to calculate Y''_u. Hence, Weitzman's (1976) welfare significance of NNP is based on comparisons of NNP in *variable* net investment prices expressed *in terms of utility*: The net investment prices vary between the situations compared. In particular, in comparisons over time, dynamic welfare improves if and only if

$$0 < \dot{Y}_u(t) = \frac{d}{dt}(U(\mathbf{C}^*(t)) + \mathbf{Q}_u(t)\mathbf{I}^*(t))$$
$$= \nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) + \mathbf{Q}_u(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_u(t)\mathbf{I}^*(t),$$

where $\nabla U(\mathbf{C})$ denotes the vector of partial derivatives of $U(\mathbf{C})$ (and likewise for other functions). Hence, growth in real utility-NNP indicates welfare improvement if net investment prices are allowed to vary over time. It does not follow from Weitzman's analysis that growth in NNP in *fixed* net investment prices, $\nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) + \mathbf{Q}_u(t)\dot{\mathbf{I}}^*(t)$, has welfare significance.

Weitzman's (1976) main result is often identified with the case in which consumption is a scalar C and the utility function is linear: $U(C) = P_u C$, where P_u is a constant factor and utility is derived from consumption. Since in this case utility is equal to the value of consumption, dynamic welfare improves if and only if $Y_u(t) = P_u C^*(t) + \mathbf{Q}_u(t)\mathbf{I}^*(t)$ grows. More generally, utility equals the value of consumption, provided that U is a linearly homogeneous function so that, by Euler's theorem, $U(\mathbf{C}) = \mathbf{P}_u\mathbf{C}$ and $Y_u(t) = \mathbf{P}_u(t)\mathbf{C}^*(t) + \mathbf{Q}_u(t)\mathbf{I}^*(t)$, where $\mathbf{P}_u = \nabla U(\mathbf{C})$. If U is a linearly homogeneous, then by comparing

$$\frac{d}{dt}U(\mathbf{C}^*(t)) = \frac{d}{dt}(\mathbf{P}_u(t)\mathbf{C}^*(t)) = \mathbf{P}_u(t)\dot{\mathbf{C}}^*(t) + \dot{\mathbf{P}}_u(t)\mathbf{C}^*(t)$$

(by Euler's theorem and the rules of differentiation) with

$$\frac{d}{dt}U(\mathbf{C}^*(t)) = \nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) = \mathbf{P}_u(t)\dot{\mathbf{C}}^*(t),$$

it follows that $\dot{\mathbf{P}}_u(t)\mathbf{C}^*(t) = 0$ in this case, meaning that the consumer price changes weighted by the consumption flows are equal to zero. As will be pointed out in the appendix, this is a defining property of a (Divisia) *consumer price index*.

As argued by Asheim and Weitzman (2001) and Sefton and Weale (2000, 2006), NNP has welfare significance if real prices are determined through a consumer price index, even in the case when U is not linearly homogeneous. Let *real measurable* NNP *deflated by means of a consumer price index* be defined as

$$Y_c := \mathbf{P}_c\mathbf{C}^* + \mathbf{Q}_c\mathbf{I}^*,$$

where $\mathbf{P}_c$ is proportional to $\nabla U(\mathbf{C}^*)$, $\mathbf{Q}_c$ is measured in the same numeraire, and where the development of prices over time are determined by a consumer price index, defined by the property that $\dot{\mathbf{P}}_c\mathbf{C}^* = 0$ at all times. Then, provided that the real consumption interest rate is positive, it follows from Asheim and Weitzman (2001) that dynamic welfare improves if and only if

$$0 < \dot{Y}_c(t) = \frac{d}{dt}(\mathbf{P}_c(t)\mathbf{C}^*(t) + \mathbf{Q}_c(t)\mathbf{I}^*(t)) = \mathbf{P}_c(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_c(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_c(t)\mathbf{I}^*(t).$$

Hence, since $\dot{\mathbf{P}}_c(t)\mathbf{C}^*(t) = 0$ by construction, the use of a consumer price index allows for the revaluation of net investment flows needed for NNP to have welfare significance, while canceling out the effects of variable consumer prices.

Another possibility – a seemingly natural method, which appears to be employed in practise – is to make measurable NNP at different times comparable by means of a (Divisia) NNP *price index*, defined by the property that $\dot{\mathbf{P}}_n\mathbf{C}^* + \dot{\mathbf{Q}}_n\mathbf{I}^* = 0$ at all times. Let *real measurable* NNP *deflated by means of an* NNP *price index* be defined by

$$Y_n := \mathbf{P}_n\mathbf{C}^* + \mathbf{Q}_n\mathbf{I}^*.$$

Then, since $\dot{\mathbf{P}}_n(t)\mathbf{C}^*(t) + \dot{\mathbf{Q}}_n(t)\mathbf{I}^*(t) = 0$ by construction of the NNP price index

$$\dot{Y}_n(t) = \frac{d}{dt}(\mathbf{P}_n(t)\mathbf{C}^*(t) + \mathbf{Q}_n(t)\mathbf{I}^*(t)) = \mathbf{P}_n(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_n(t)\dot{\mathbf{I}}^*(t).$$

Hence, the use of an NNP price index cancels out not only the effects of variable consumer prices but also the effects of the variable investment prices. As will be demonstrated through the analysis of sections 3 and 4, this means that growth in real measurable NNP in variable consumption and net investment prices, deflated by means of an NNP price index, cannot be used to measure welfare improvement along a discounted utilitarian optimal path. For the same reason, growth in NNP in fixed consumption and net investment prices cannot be used to measure welfare improvement along a discounted utilitarian optimal path. Finally, change in NNP in fixed consumption and net investment prices cannot be used to measure the welfare effects of capital stock perturbations.

In the analysis of later sections, I will *not* insist on the assumptions that the economy implements a welfare-maximizing path and that dynamic welfare is discounted utilitarian. The weaker assumptions that I invoke will be explained in section 3 and stated in proposition 2. The analysis will lead to the following results:

Proposition 1. *Under the assumptions stated in proposition 2, the following holds:*

(a) *Provided that the real utility interest rate (= supporting utility discount rate) is positive and real utility-*NNP *is measurable, growth in real utility-*NNP *in variable net investment prices can be used to measure welfare improvement along the implemented path.*

(b) *Provided that the real consumption interest rate is positive, growth in real measurable* NNP *(deflated by means of a consumer price index) in variable consumption and net investment prices can be used to measure welfare improvement along the implemented path.*

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

(c) *Growth in real measurable* NNP *(deflated by means of an* NNP *price index) in variable consumption and net investment prices cannot be used to measure welfare improvement along the implemented path.*
(d) *Growth in* NNP *in fixed consumption and net investment prices cannot be used to measure welfare improvement along the implemented path.*
(e) *Change in* NNP *in fixed consumption and net investment prices cannot be used to measure the welfare effects of capital stock perturbations.*

As mentioned above, part (a) of proposition 1 is a generalization of a result shown by Weitzman (1976) (which in turn is based on Weitzman, 1970). Utility-NNP is *not* a linear index of the produced goods and services, since the first term equals the utility derived from the consumption flows. Hence, in order to calculate the change in utility, one must not only calculate the change in the value of consumption in utility terms, but also the change in 'consumers' surplus' $U(\mathbf{C}^*(t)) - \nabla U(\mathbf{C}^*(t))\mathbf{C}^*(t)$. Weitzman (2001) argues that the change in such 'consumers' surplus' is in principle observable. His analysis has been further developed by Li and Löfgren (2002).

Part (b) of proposition 1 is a generalization of a result shown by Asheim and Weitzman (2001) (and which is further developed by Asheim and Buchholz, 2004). When a consumer price index is used, the change in utility is proportional to the change in the value of consumption. Hence, there is no need to be concerned with the problem of calculating the change in 'consumers' surplus'.

The result underlying part (d) of proposition 1 is a generalization of insights demonstrated by Li and Löfgren (2006). They, however, interpret this result in a different manner than I have done here. I will discuss their interpretation in section 4. The problem of indicating welfare improvement by means of NNP growth in fixed consumption and net investment prices has also been observed and discussed by Dasgupta and Mäler (2000) and Dasgupta (2001).

Part (c) of proposition 1 is a restatement of part (d), designed to make the point that – even though the result of part (d) does not depend on a particular choice of price index – NNP growth in fixed consumption and net investment prices is equivalent to NNP growth in variable consumption and net investment prices using an NNP price index. A comparison of the negative result of part (c) with the positive result of part (b) yields a theory for deflating NNP. In order for real measurable NNP to have local-in-time welfare significance, NNP must be deflated by a consumer price index.

Part (e) of proposition 1 is a generalization of a result reported in Asheim (2001). It has been a key result in the critical assessment of the usefulness of NNP for making welfare comparisons that Heal and Kriström have presented through various contributions over recent years (see Heal and Kriström, 2005a,b).

3. Theory of welfare comparisons in a dynamic economy

In this section, I present and illustrate a general result on the welfare significance of genuine savings. This result will be used in section 4 to prove proposition 1.

3.1. The welfare significance of the genuine savings indicator

If *dynamic welfare* is welfarist, forward-looking, and numerically representable, then dynamic welfare, denoted V, is a function of the flow of future utilities

$$V^*(t) = \mathcal{F}(\{U^*(s)\}_{s=t}^{\infty}, t).$$

I assume throughout that the functional $\mathcal{F}$ is concave, time-invariant, and smooth, and satisfies a condition of *independent future*: If $\{U'(t)\}_{t=0}^{\infty}$ and $\{U''(t)\}_{t=0}^{\infty}$ coincide during the interval $[0, \tau]$, then

$$\mathcal{F}(\{U'(t)\}_{t=0}^{\infty}, 0) < \mathcal{F}(\{U''(t)\}_{t=0}^{\infty}, 0) \quad \Leftrightarrow \quad \mathcal{F}(\{U'(t)\}_{t=\tau}^{\infty}, \tau) < \mathcal{F}(\{U''(t)\}_{t=\tau}^{\infty}, \tau).$$

Since $\mathcal{F}$ is smooth and satisfies the condition of independent future, there exists, for any path of utility flow $\{U^*(t)\}_{t=0}^{\infty}$, a path of supporting utility discount factors $\{\mu(t)\}_{t=0}^{\infty}$, unique up to a choice of numeraire, such that, for all t

$$\lambda(t)\, dV^*(t) = \int_t^{\infty} \mu(s)\, dU^*(s) \tag{1}$$

for some $\lambda(t) > 0$. Since, in addition, $\mathcal{F}$ is time-invariant, local welfare comparisons across time for a given path of utility flow $\{U^*(t)\}_{t=0}^{\infty}$ depend on the present value of future growth in utility

$$\lambda(t)\dot{V}^*(t) = \int_t^{\infty} \mu(s)\dot{U}^*(s)\, ds. \tag{2}$$

By means of the time-invariant utility function U that maps any vector of consumption flows into a utility flow, dynamic welfare can be expressed as a function of the path of the vector of future consumption flows

$$V^*(t) = \mathcal{G}\big(\{\mathbf{C}^*(s)\}_{s=t}^{\infty}, t\big),$$

where $\mathcal{G}(\{\mathbf{C}^*(s)\}_{s=t}^{\infty}, t) = \mathcal{F}(\{U(\mathbf{C}^*(s))\}_{s=t}^{\infty}, t)$. The assumptions on $\mathcal{F}$ and U imply that the functional $\mathcal{G}$ is concave, time-invariant, and smooth, and satisfies a condition of independent future. Since $\mathcal{G}$ is smooth and satisfies independent future, there exists, for any path of the vector of consumption flows $\{\mathbf{C}^*(t)\}_{t=0}^{\infty}$, a path of supporting present value consumer prices $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ satisfying, for all t

$$\mu(t)\nabla U(\mathbf{C}^*(t)) = \mathbf{p}(t). \tag{3}$$

This means that

$$\lambda(t)\, dV^*(t) = \int_t^{\infty} \mathbf{p}(s)\, d\mathbf{C}(s). \tag{4}$$

Furthermore, since $\mathcal{G}$ is time-invariant

$$\lambda(t)\dot{V}^*(t) = \int_t^{\infty} \mathbf{p}(s)\dot{\mathbf{C}}(s)\, ds. \tag{5}$$

Assume that the economy's actual decisions are taken according to a *resource allocation mechanism* (RAM) that assigns some attainable consumption – net investment pair $(\mathbf{C}, \mathbf{I})$ to any vector of capital stocks $\mathbf{K}$. Hence, for any vector of capital stocks $\mathbf{K}$, the RAM determines consumption

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

and the net investment flows. The net investment flows in turn map out the development of the capital stocks. The resource allocation mechanism thereby implements a feasible path of consumption flows, net investment flows, and capital stocks, for any initial vector of capital stocks.

Since the function S that assigns the *set* of all attainable consumption – net investment pairs to any vector of capital stocks $\mathbf{K}$ is time-invariant, one can assume that the RAM in the economy depends only on $\mathbf{K}$ (i.e. is *Markovian*) and is time-invariant. Hence, the RAM assigns to any vector of capital stocks $\mathbf{K}$ a consumption – net investment pair $(\mathbf{C}(\mathbf{K}), \mathbf{I}(\mathbf{K}))$ satisfying $(\mathbf{C}(\mathbf{K}), \mathbf{I}(\mathbf{K})) \in S(\mathbf{K})$. I assume that there exists a unique solution $\{\mathbf{K}^*(t)\}_{t=0}^{\infty}$ to the differential equation $\dot{\mathbf{K}}^*(t) = \mathbf{I}(\mathbf{K}^*(t))$ that satisfies the initial condition $\mathbf{K}^*(0) = \mathbf{K}^0$, where $\mathbf{K}^0$ is given. Hence, $\{\mathbf{K}^*(t)\}_{t=0}^{\infty}$ is the capital path that the RAM implements. For all t, write $\mathbf{C}^*(t) := \mathbf{C}(\mathbf{K}^*(t))$ and $\mathbf{I}^*(t) := \mathbf{I}(\mathbf{K}^*(t))$. I do not assume that the RAM implements an efficient path.

As a consequence of $\mathcal{G}$ being time-invariant and the RAM being Markovian and time-invariant, the dynamic welfare of the implemented path

$$V^*(t) = V(\mathbf{K}^*(\mathbf{t})),$$

is time-invariant and a function solely of the current vector of capital stocks $\mathbf{K}$. The *state valuation function* V satisfies $V(\mathbf{K}^*(t)) = \mathcal{G}(\{\mathbf{C}^*(s)\}_{s=t}^{\infty})$. Assume that, combined with a smooth $\mathcal{G}$, the RAM makes V differentiable. Hence, there exists a vector of net investment prices $\mathbf{q}(t)$ at time t satisfying

$$\lambda(t)\nabla V(\mathbf{K}^*(t)) = \mathbf{q}(t). \tag{6}$$

This means that

$$\lambda(t)\, dV^*(t) = \mathbf{q}(t)\, d\mathbf{K}^*(t). \tag{7}$$

Since V is time-invariant, local welfare comparisons across time for a given implemented path $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ depend on the value of net investments

$$\lambda(t)\dot{V}^*(t) = \mathbf{q}(t)\mathbf{I}^*(t). \tag{8}$$

By comparing (1), (4), and (7), on the one hand, and (2), (5), and (8), on the other hand, the following result is obtained:

Proposition 2. *Let (1) dynamic welfare be numerically representable by a welfarist, forward-looking and time-invariant function of the path of future utilities, satisfying a condition of independent future, (2) utility be a time-invariant function of the vector of consumption flows, and (3) the RAM be Markovian and time-invariant. Then, under the assumptions that the welfare functionals* $\mathcal{F}$ *and* $\mathcal{G}$ *are smooth and the state valuation function* V *is differentiable, there exist paths of discount factors* $\{\mu(t)\}_{t=0}^{\infty}$*, present value consumer prices* $\{\mathbf{p}(t)\}_{t=0}^{\infty}$*, and present value net investment prices* $\{\mathbf{q}(t)\}_{t=0}^{\infty}$ *such that the implemented path* $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ *satisfies:*

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

(a) A perturbation of the vector of capital stocks at time t increases welfare if and only if

$$\int_t^\infty \mu(s)\, dU^*(s) = \int_t^\infty \mathbf{p}(s)\, d\mathbf{C}^*(s) = \mathbf{q}(t)\, d\mathbf{K}^*(t) > 0.$$

(b) Welfare improves along the implemented path at time t if and only if

$$\int_t^\infty \mu(s)\dot{U}^*(s)ds = \int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)\, ds = \mathbf{q}(t)\mathbf{I}^*(t) > 0.$$

In section 4, the results of proposition 1 will be used to prove proposition 2. Note that $\mathbf{q}(t)\mathbf{I}^*(t)$ represents the value of net investments, and it is often referred to as the 'genuine savings indicator' (a term coined by Hamilton, 1994: 166). Hence, proposition 2 shows the welfare significance of genuine savings under conditions that are general in two respects:

- Proposition 2 does not assume that the RAM implements a welfare-maximizing (or even efficient) path. Hence, there need not be an infinite dimensional hyperplane separating the feasible paths from those that lead to greater welfare.
- Proposition 2 does not assume that dynamic welfare is discounted utilitarian. Hence, the analysis allows for other kinds of forward-looking and time-invariant welfare functions, as long as they satisfy a condition of independent future. Such possibilities are investigated by, for example, Koopmans (1960), Beals and Koopmans (1969), and Asheim *et al.* (2006).

In the remainder of this section, I first consider two special cases which are still more general than the usual analysis of an economy-maximizing discounted utilitarian welfare. The first case, presented in subsection 3.2, assumes that dynamic welfare is discounted utilitarian, but does not make the assumption that an optimal path is implemented. The second case, presented in subsection 3.3, is based on the assumption that the RAM implements a path maximizing dynamic welfare, but does not require that dynamic welfare be given by the sum of discounted utilities. I then turn to a review of relevant literature in subsection 3.4.

3.2. Discounted utilitarian welfare

Let the welfare function be given as

$$\mathcal{F}\big(\{U^*(s)\}_{s=t}^\infty, t\big) = \int_t^\infty e^{-\rho(s-t)}U^*(s)\, ds.$$

Then it follows that

$$\frac{d}{dt}\left(\int_t^\infty e^{-\rho(s-t)}U^*(s)\, ds\right) = -U^*(t) + \rho\int_t^\infty e^{-\rho(s-t)}U^*(s)\, ds \qquad (9)$$
$$= e^{\rho t}\int_t^\infty e^{-\rho s}\dot{U}^*(s)\, ds,$$

where the second equality follows by integrating by parts. This verifies (2) in the case of discounted utilitarianism by setting, for all t, $\lambda(t) = \mu(t) = e^{-\rho t}$.

Equation (9) can be rewritten as

$$\nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t) = -U(\mathbf{C}^*(t)) + \rho V(\mathbf{K}^*(t))$$

or

$$U(\mathbf{C}^*(t)) + \nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t) = \rho V(\mathbf{K}^*(t)).$$

Differentiating once more w.r.t. time yields

$$\nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) + \frac{d\nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t)}{dt} = \rho \nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t),$$

or equivalently, by (3) and (6)

$$\mu(t)\dot{U}^*(t) = \mathbf{p}(t)\dot{\mathbf{C}}^*(t) = -\frac{d\mathbf{q}(t)\mathbf{I}^*(t)}{dt} \tag{10}$$

as $d\nabla V(\mathbf{K}^*)\mathbf{I}^*/dt = d\big(\mathbf{q}\mathbf{I}^*/\lambda\big)/dt = \big(d(\mathbf{q}\mathbf{I}^*)/dt - \rho\mathbf{q}\mathbf{I}^*\big)/\lambda$. This means that the equalities in proposition 2(b) follow through integration, provided that the following net investment value transversality condition holds

$$\lim_{t\to\infty} \mathbf{q}(t)\mathbf{I}^*(t) = 0. \tag{11}$$

3.3. Welfare optimum implemented as a competitive path

Following Dixit *et al.* (1980), say that the RAM $(\mathbf{C}(\cdot), \mathbf{I}(\cdot))$ implements a path $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ that is *competitive* with respect to the path of discount factors $\{\mu(t)\}_{t=0}^{\infty}$ if there exist paths of present value consumer prices $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ and present value net investment prices $\{\mathbf{q}(t)\}_{t=0}^{\infty}$ such that, for all t:

C1 $\mathbf{C}^*(t)$ maximizes $\mu(t)u(\mathbf{C}) - \mathbf{p}(t)\mathbf{C}$ over all $\mathbf{C}$,
C2 $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)) = (\mathbf{C}(\mathbf{K}^*(t)), \mathbf{I}(\mathbf{K}^*(t)), \mathbf{K}^*(t))$ maximizes $\mathbf{p}(t)\mathbf{C} + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}$ over all $(\mathbf{C}, \mathbf{I}, \mathbf{K})$ satisfying $(\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})$.

By standard arguments, it follows from the concavity of $\mathcal{F}$, U, and $\{(\mathbf{C}, \mathbf{I}, \mathbf{K}) \mid (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\}$ that the competitive path $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$ implemented by the RAM $(\mathbf{C}(\cdot), \mathbf{I}(\cdot))$ is a welfare optimum if:

(a) $\{\mu(t)\}_{t=0}^{\infty}$ supports $\{U(\mathbf{C}^*(t))\}_{t=0}^{\infty}$ under the welfare functional $\mathcal{F}$,
(b) $\int_0^{\infty} \mu(t)U(\mathbf{C}^*(t))\,dt$ exists, and
(c) a capital value transversality condition holds: $\lim_{t\to\infty} \mathbf{q}(t)\mathbf{K}^*(t) = 0$.

Furthermore, it follows from the smoothness of U and $\{(\mathbf{C}, \mathbf{I}, \mathbf{K}) \mid (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\}$ that, for all t

$$\mu(t)\nabla U(\mathbf{C}^*(t)) = \mathbf{p}(t), \tag{12}$$

$$\mathbf{p}(t)\nabla_{\mathbf{K}}\mathbf{C}(\mathbf{K}^*(t)) + \mathbf{q}(t)\nabla_{\mathbf{K}}\mathbf{I}(\mathbf{K}^*(t)) = -\dot{\mathbf{q}}(t). \tag{13}$$

Since (13) entails that $\mathbf{p}\dot{\mathbf{C}}^* = -\mathbf{q}\dot{\mathbf{I}}^* - \dot{\mathbf{q}}\mathbf{I}^*$, expression (10) is again obtained, showing again that the equation of proposition 2(b) follows through integration, provided that the net investment value transversality condition (11) is satisfied.

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

3.4. Review of relevant literature

Samuelson (1961: 51–52) states that, 'in complete analogy with the static one-period case', welfare comparisons should be made by comparing the present value of future changes in consumption, as stated in proposition 2 of this section. In his notation, $\sum P^a Q^a$ and $\sum P^b Q^b$ are the present value of future consumption in two different situations, A and B. Samuelson stresses that a comparison of $\sum P^b Q^b \gtreqless \sum P^a Q^a$ is meaningless; rather the comparisons should be of $\sum P^b(Q^b - Q^a) \gtreqless 0$ or $\sum P^a(Q^b - Q^a) \gtreqless 0$. Samuelson (1961: 52) states that 'there is no meaning in comparing *money* wealth in one situation (i.e. time and place) with that of another situation', unless 'we use the *same prices and interest rates* in the comparison'. In the present notation this translates into the proposition that over-time comparisons should *not* be of

$$\frac{d}{dt}\left(\int_t^\infty \mathbf{p}(s)\mathbf{C}^*(s)\,ds\right) \gtreqless 0,$$

but rather of

$$\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)\,ds \gtreqless 0$$

as reported in proposition 2. Samuelson (1961; 53) refers to the latter as comparisons of 'wealth-like magnitudes'. Recently, Samuelson's insights have been brought to our attention by the analysis that Heal and Kriström have presented in various contributions (see Heal and Kriström, 2005a,b), under the assumptions of discounted utilitarianism and an optimal RAM.

Any direct attempt to estimate the present value of future changes in consumption would seem futile. In the words of Samuelson (1961; 53): 'We are left with the pessimistic conclusion that there is so much "futurity" in any welfare evaluation of any dynamic situation as to make it exceedingly difficult for the statistician to approximate to the proper wealth comparisons.' Fortunately, Weitzman (1970, 1976) and later contributions show that Samuelson was overly pessimistic. By combining his (10) with the equation prior to his (14), one can see as Weitzman (1976) proves (but does not emphasize) the result that, under the assumptions of discounted utilitarianism and an optimal RAM, welfare is improving if and only if the value of net investments is positive, as reported in proposition 2(b) above.

Equation (10) above is shown by Dixit *et al.* (1980, proof of theorem 1) in the case of a competitive path, but without assuming discounted utilitarianism. Hence, Dixit *et al.* thereby tie together the welfare results reported by Samuelson (1961) and Weitzman (1976), since – by integration – equation (10) implies that the present value of future consumption growth equals the value of net investments.

Dasgupta and Mäler (2000), Dasgupta (2001), and Arrow *et al.* (2003) have introduced the concept of a RAM into the literature on welfare comparisons based on national accounting aggregates. Under the assumption that dynamic welfare is discounted utilitarian, but without assuming that the RAM implements the discounted utilitarian optimum, Arrow *et al.* (2003) report the results of proposition 2 through their theorems 2 and 4. I have followed them by assuming the state valuation function is differentiable, instead of establishing this property from more primitive assumptions. Through proposition 2 I have generalized their results by *not* imposing that dynamic welfare is discounted utilitarian.

In the second special case of subsection 3.3, where the path is competitive, the present value consumption and net investment prices may correspond to market prices in a perfect market economy. In the general case of proposition 2 (and also in the special case of discounted utilitarianism as analyzed in subsection 3.2), the present value consumption and net investment prices are accounting prices, which need not be directly observable. Externalities, monopolistic competition, and distortionary taxation give rise to an inefficient RAM, and hence welfare measurement under such conditions (see, e.g., Aronsson *et al.*, 2004, for an extended treatment of such welfare measurement) can be imbedded into the present analysis. However, it is not trivial to calculate the relevant accounting prices under such conditions. Guidelines for practical calculation of accounting prices, as well as doing cost – benefit analysis on the basis of such prices, are outside the scope of the present paper. Arrow *et al.* (2003) consider some of the problems that arise within the present framework and is a useful reference.

4. Proof of proposition 1

I now apply proposition 2 to prove the results on the welfare significance of NNP as stated in proposition 1. First, the concept of NNP must be defined within the framework and notation presented in section 3. For a given implemented path $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$, *utility*-NNP can be defined as

$$\mu(t)U(\mathbf{C}^*(t)) + \mathbf{q}(t)\mathbf{I}^*(t),$$

and *measurable* NNP, y, can be defined as

$$y(t) := \mathbf{p}(t)\mathbf{C}^*(t) + \mathbf{q}(t)\mathbf{I}^*(t).$$

In contrast to utility-NNP, y is a *linear* index of the produced goods and services. Utility-NNP does not have this property unless U is linearly homogeneous so that $\mu(t)U(\mathbf{C}^*(t)) = \mathbf{p}(t)\mathbf{C}^*(t)$. If the path is competitive, then it follows from C2 that $y(t)$ is the maximized value of the current net product, given the price vectors $\mathbf{p}(t)$ and $\mathbf{q}(t)$, and the set of attainable consumption – net investment vectors $S(K^*(t))$.

4.1. Welfare effects of capital stock perturbations

While not the central concern of this paper, it turns out to be instructive to start with the question of whether NNP can measure welfare effects of capital stock perturbations. Let the assumptions of proposition 2 be satisfied, and differentiate the expressions in part (a) of the proposition with respect to time. This yields

$$-\mu(t)dU^*(t) = -\mathbf{p}(t)\,d\mathbf{C}^*(t) = \mathbf{q}(t)\,d(d\mathbf{K}^*(t))/dt + \dot{\mathbf{q}}(t)\,d\mathbf{K}^*(t).$$

Since $d(d\mathbf{K}^*(t))/dt = I(\mathbf{K}^*(t))d\mathbf{K}^*(t) = d\mathbf{I}^*(t)$, the following result is obtained

$$\mu(t)\,dU^*(t) + \mathbf{q}(t)\,d\mathbf{I}^*(t) = \mathbf{p}(t)\,d\mathbf{C}^*(t) + \mathbf{q}(t)\,d\mathbf{I}^*(t) = -\dot{\mathbf{q}}(t)\,d\mathbf{K}^*(t). \qquad (14)$$

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

Hence, the change in utility-NNP or measurable NNP in fixed consumption and net investment prices, as a result of perturbation of the vector of capital stocks, is equal to the value of the perturbation of the capital stocks using $-\dot{\mathbf{q}}(t)$ as relative prices. Compare this with proposition 2(a), which states that a perturbation of the capital stocks is welfare enhancing if and only if the value of the perturbation is positive using $\mathbf{q}(t)$ as relative prices.

If the economy is in a steady state, so that the rate of decline of the present value prices is constant, then $-\dot{\mathbf{q}}(t)$ is proportional to $\mathbf{q}(t)$, with a constant and positive real interest rate being the proportionality factor. In this case, a positive change in utility-NNP or measurable NNP in fixed consumption and net investment prices as a result of a perturbation of the vector of capital stocks indicates that the perturbation is welfare enhancing. This conclusion does not hold in general. While $-\dot{\mathbf{q}}(t)$ measures the *instantaneous* net marginal productivity of the vector of capital components *as stocks*, $\mathbf{q}(t)$ measures the present value of the *future* marginal contributions that the capital components make, *both as stocks and flows*. Both $-\dot{\mathbf{q}}(t)$ and $\mathbf{q}(t)$ are measured relative to the possibly inefficient RAM.

The case of a non-renewable resource is a prime example of a capital component where instantaneous net marginal productivity as a stock need *not* correspond to the present value of future contributions both as a stock and a flow.

Therefore, to show that $-\dot{\mathbf{q}}(t)$ need not be proportional to $\mathbf{q}(t)$ and thereby proving the claim made in proposition 1(e), consider the Dasgupta–Heal–Solow (DHS) model with a Cobb–Douglas production function (Dasgupta and Heal, 1974, 1979; Solow, 1974). In this model, the consumption flow is one-dimensional, and the net investment flows and capital stocks are two-dimensional, having both a man-made (M) and a natural (N) component. The latter is a non-renewable resource which is not productive as a stock. For positive stocks of man-made and natural capital, the set of attainable consumption – net investment pairs is

$$S(K_M, K_N) = \{(C, I_M, I_N) \mid C \leq K_M^{\alpha}(-I_N)^{\beta} - I_M; C \geq 0; I_N \leq 0\}.$$

It is worth noting that $S(K_M, K_N)$ does not depend on K_N, as long as K_N is positive. Therefore, along a competitive path $\{C^*(t), I_M^*(t), I_N^*(t), K_M^*(t), K_N^*(t)\}_{t=0}^{\infty}$, where all variables are positive, it follows from C2 that

$$\begin{aligned} q_M &= p > 0 \\ q_N &= p\beta X^*/(-I_N^*) > 0 \\ -\dot{q}_M &= p\alpha X^*/K_M^* > 0 \\ -\dot{q}_N &= 0, \end{aligned}$$

where X denotes output (i.e., $X = K_M^{\alpha}(-I_N)^{\beta}$), and where the references to time have been suppressed. The constancy of q_N in present value terms (i.e., $-\dot{q}_N = 0$) corresponds to the Hotelling rule and it reflects that natural capital (the non-renewable resource) has zero net productivity as a stock. On the other hand, $q_N > 0$, reflecting that the flow of resource extraction is productive. Thus, by proposition 1, a perturbation of the stock of natural capital has an effect on welfare, while, as shown above, such a perturbation

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

does not change utility-NNP or measurable NNP in fixed consumption and net investment prices.

Since $-\dot{q}_M > 0$, reflecting that man-made capital is productive as a stock, one can construct examples of a welfare-enhancing perturbation of capital stocks, with a relatively small *negative* dK_M^* and a relatively large *positive* dK_N^* so that $q_M dK_M^* + q_N dK_N^* > 0$, which at the same time decreases utility-NNP and measurable NNP in fixed consumption and net investment prices since $-(\dot{q}_M dK_M^* + \dot{q}_N dK_N^*) = -\dot{q}_M dK_M^* < 0$.

4.2. Welfare improvement

Turn now to the question of measuring local-in-time changes in welfare along the implemented path. Let, as before, the assumptions of proposition 2 be satisfied, and differentiate the expressions in part (b) of the proposition with respect to time. This yields

$$-\mu(t)\dot{U}^*(t) = -\mathbf{p}(t)\dot{\mathbf{C}}^*(t) = \mathbf{q}(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{I}^*(t),$$

and leads to the following result

$$\mu(t)\dot{U}^*(t) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = \mathbf{p}(t)\dot{\mathbf{C}}^*(t) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = -\dot{\mathbf{q}}(t)\mathbf{I}^*(t). \tag{15}$$

Hence, the change in utility-NNP or measurable NNP in fixed consumption and net investment prices along the implemented path is equal to the value of the net investments in the capital stocks using $-\dot{\mathbf{q}}(t)$ as relative prices. Compare this with proposition 2(b) which states that welfare improves along the implemented path if and only if the value of the net investments is positive using $\mathbf{q}(t)$ as relative prices.

Thus, this leads to the same conclusion as in the case of capital stock perturbations just considered: since $-\dot{\mathbf{q}}(t)$ need not be proportional to $\mathbf{q}(t)$, the growth in utility-NNP or measurable NNP in fixed consumption and net investment prices along the implemented path does not, in general, indicate welfare improvement.

Again, the DHS model can be used to illustrate this negative result. Consider the discounted utilitarian optimum that is implemented by means of a competitive path. Any such path has an eventual phase with decreasing consumption. It follows from proposition 2(b) that welfare and the value of net investments (using $(q_M(t), q_N(t))$ as relative prices) are negative in this eventual phase. One can, however, choose the parameters of the model such that this eventual phase is proceeded by an initial phase in which welfare and the value of net investments are positive (cf. Pezzey and Withagen, 1998). Hence, initially $q_M I_M^* + q_N I_N^* > 0$, while later $q_M I_M^* + q_N I_N^* < 0$, since the path will reach the eventual phase with decreasing consumption. Since all variables develop in a continuous manner and $(q_M, q_N) \gg 0$ and $I_N^* < 0$ throughout, there exists some interval of time just following the point in time at which $q_M I_M^* + q_N I_N^* = 0$, in which welfare is decreasing, while $I_M^* > 0$. In this interval, $-(\dot{q}_M I_M^* + \dot{q}_N I_N^*) > 0$, since $-\dot{q}_M > 0$ and $-\dot{q}_N = 0$ throughout. Hence, just after welfare has started to decrease, NNP in fixed consumption and net investment prices are still growing. This proves the claim made in proposition 1(d).

The result underlying proposition 1(d) is demonstrated by Li and Löfgren (2006) under the assumption that the RAM implements an optimal path and dynamic welfare is discounted utilitarian – assumptions that are not made in the present analysis. They, however, interpret this result in a different manner than I have done here. They rewrite the second equation of (15) as

$$\mathbf{p}(s)\dot{\mathbf{C}}^*(s) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = -\frac{\dot{\mathbf{q}}(t)\mathbf{I}^*(t)}{\mathbf{q}(t)\mathbf{I}^*(t)}\mathbf{q}(t)\mathbf{I}^*(t),$$

thereby establishing the result that growth in NNP in fixed consumption and net investment prices indicates welfare improvement, provided that $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ is positive. They refer to $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ as 'the genuine rate of return on investment' (Li and Löfgren, 2006; 257). This terminology may not be appropriate, as $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)$ captures only the instantaneous net marginal productivity of the capital components as stocks, where for each such component j, $-\dot{q}_j/q_j$ is the component's *own rate of interest*. The rate of return on investment for each capital component does not depend only on its own rate of interest, but also on its anticipated capital gains, reflecting the future contributions that the capital component makes both as a stock and a flow. This is the essence of the no-arbitrage equation, which holds along an efficient path. Moreover, as the above analysis of the DHS model demonstrates, $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ need not positive. In the DHS model with a discounted utilitarian optimum, the rate of return on investment for each of the two capital components is positive, both in terms of consumption and utility. For the natural capital component (the non-renewable resource), the positive returns are solely in terms of capital gains, which are not captured by its zero own rate of interest.

Hence, in order to get further, one must follow Weitzman (1976) and consider NNP in variable prices. Moreover, for local-in-time comparisons to be meaningful, NNP must be measured in *real* (not *nominal*) prices. A price index $\{\pi(t)\}_{t=0}^{\infty}$ turns nominal prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into *real* prices $\{\mathbf{P}(t), \mathbf{Q}(t)\}_{t=0}^{\infty}$ by imposing $\mathbf{P}(t) = \mathbf{p}(t)/\pi(t)$ and $\mathbf{Q}(t) = \mathbf{q}(t)/\pi(t)$ at each t; cf. the appendix. This implies that the *real* interest rate, $R(t)$, at time t is given by $R(t) = -\dot{\pi}(t)/\pi(t)$, since the nominal prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ are present-value prices. However, what kind of price index will entail that real NNP growth indicates welfare improvement?

NNP price index. One possibility is to make measurable NNP at different times comparable by means of a (Divisia) NNP *price index* $\{\pi_n(t)\}_{t=0}^{\infty}$, which weights price changes by consumption and net investment flows and satisfies, for all t

$$\frac{\dot{\pi}_n(t)}{\pi_n(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t) + \dot{\mathbf{q}}(t)\mathbf{I}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t) + \mathbf{q}(t)\mathbf{I}^*(t)}.$$

It turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_n(t), \mathbf{Q}_n(t)\}_{t=0}^{\infty}$ satisfying $\dot{\mathbf{P}}_n\mathbf{C}^* + \dot{\mathbf{Q}}_n\mathbf{I}^* = 0$. Let *real measurable* NNP *deflated by means of an* NNP *price index* be defined as

$$Y_n(t) := \mathbf{P}_n(t)\mathbf{C}^*(t) + \mathbf{Q}_n(t)\mathbf{I}^*(t).$$

Then – since $\dot{\mathbf{P}}_n\mathbf{C}^* + \dot{\mathbf{Q}}_n\mathbf{I}^* = 0$ by construction of the NNP price index – it follows from (15) that growth in real NNP in variable prices is given by

$$\dot{Y}_n(t) = \mathbf{P}_n(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_n(t)\dot{\mathbf{I}}^*(t) = -\frac{\dot{\mathbf{q}}(t)}{\pi_n(t)}\mathbf{I}^*(t).$$

Hence, $\dot{Y}_n(t)$ is positive if and only if NNP in fixed prices is increasing. By proposition 1(d), growth in real measurable NNP in variable prices deflated by an NNP price index does *not* indicate welfare improvement, thereby establishing proposition 1(c).

Consumer price index. Another possibility is to make measurable NNP at different times comparable by means of a (Divisia) *consumer price index* $\{\pi_c(t)\}_{t=0}^{\infty}$, which weights price changes by consumption flows only and satisfies, for all t

$$\frac{\dot{\pi}_c(t)}{\pi_c(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)}.$$

It turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_c(t), \mathbf{Q}_c(t)\}_{t=0}^{\infty}$ satisfying $\dot{\mathbf{P}}_c\mathbf{C}^* = 0$. By (3) this entails that instantaneous well-being is increasing if and only if the real value of consumption $\mathbf{P}_c\mathbf{C}^*$ is increasing

$$\mu\dot{U}^* = \mu\nabla U(\mathbf{C}^*)\dot{\mathbf{C}}^* = \mathbf{p}\dot{\mathbf{C}}^* = \pi\mathbf{P}_c\dot{\mathbf{C}}^* = \pi(\dot{\mathbf{P}}_c\mathbf{C}^* + \mathbf{P}_c\dot{\mathbf{C}}^*) = \pi\frac{d}{dt}(\mathbf{P}_c\mathbf{C}^*).$$

Let *real measurable* NNP *deflated by means of a consumer price index* be defined as

$$Y_c(t) := \mathbf{P}_c(t)\mathbf{C}^*(t) + \mathbf{Q}_c(t)\mathbf{I}^*(t).$$

Then – since $\dot{\mathbf{P}}_c\mathbf{C}^* = 0$ by construction of the consumer price index – it follows from (15) that growth in real NNP in variable prices is given by

$$\begin{aligned}\dot{Y}_c(t) &= \mathbf{P}_c(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_c(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_c(t)\mathbf{I}^*(t) \\ &= \left(-\frac{\dot{\mathbf{q}}(t)}{\pi_c(t)} - \frac{\dot{\pi}_c(t)}{\pi_c(t)}\frac{\mathbf{q}(t)}{\pi_c(t)} + \frac{\dot{\mathbf{q}}(t)}{\pi_c(t)}\right)\mathbf{I}^*(t) = R_c(t)\mathbf{Q}_c(t)\mathbf{I}^*(t).\end{aligned}$$

Hence, $\dot{Y}_c(t)$ is positive if and only if $\mathbf{Q}_c(t)\mathbf{I}^*(t)$ is positive, provided that the real consumption interest rate $R_c(t)$ is positive. This implies that the use of a consumer price index leads to the result that growth in real measurable NNP in variable prices indicates welfare improvement, under the provision that the real consumption interest rate is positive, thereby proving proposition 1(b).

Note that in the DHS model, real consumption interest equals the net marginal productivity of man-made capital, which is positive throughout. Therefore, in the DHS model, growth in real measurable NNP deflated by means of a consumer price index indicates welfare improvement.

Note also that the consumer price index $\{\pi_c(t)\}_{t=0}^{\infty}$ can be calculated from observable consumer prices and quantities. Hence, welfare improvement can be indicated from the change in an observable linear index of the produced goods and services, namely real measurable NNP deflated by means of a consumer price index.

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

Utility price index. A third possibility is to follow Weitzman (1976, 2001) and make utility-NNP at different times comparable by means of a *utility price index* $\{\pi_c(t)\}_{t=0}^{\infty}$, which satisfies, for all t

$$\pi_u(t) = \mu(t),$$

with $\{\mu(t)\}_{t=0}^{\infty}$ being the path of supporting utility discount factors introduced in section 3. A utility price index turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_u(t), \mathbf{Q}_u(t)\}_{t=0}^{\infty}$ measured in terms of utility. Let *real utility*-NNP be defined as

$$Y_u(t) := U(\mathbf{C}^*(t)) + \mathbf{Q}_u(t)\mathbf{I}^*(t).$$

Then, since $\nabla U(\mathbf{C}^*) = \mathbf{P}_u$ by invoking (3), it follows from (15) that growth in real utility-NNP in variable net investment prices is given by

$$\begin{aligned}\dot{Y}_u(t) &= \mathbf{P}_u(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_u(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_u(t)\mathbf{I}^*(t)\\ &= \left(-\frac{\dot{\mathbf{q}}(t)}{\pi_u(t)} - \frac{\dot{\pi}_u(t)}{\pi_u(t)}\frac{\mathbf{q}(t)}{\pi_u(t)} + \frac{\dot{\mathbf{q}}(t)}{\pi_u(t)}\right)\mathbf{I}^*(t) = R_u(t)\mathbf{Q}_u(t)\mathbf{I}^*(t).\end{aligned}$$

Hence, $\dot{Y}_u(t)$ is positive if and only if $\mathbf{Q}_u(t)\mathbf{I}^*(t)$ is positive, provided that the real utility interest rate $R_u(t) = -\dot{\mu}(t)/\mu(t)$ (= supporting utility discount *rate*) is positive. This implies that, by measuring net investment prices in terms of utility, growth in real utility-NNP in variable net investment prices indicates welfare improvement, under the provision that the real utility interest rate is positive.

Note that along a discounted utilitarian path in the DHS model, the real utility interest rate equals the constant utility discount rate ρ, which is positive throughout. Therefore, along a discounted utilitarian path in the DHS model, real utility-NNP growth indicates welfare improvement.

Note also that local-in-time comparisons by means of real utility-NNP require that changes in utility be measurable.

5. Concluding remarks

In this paper, I have shown under weak assumptions that a change in NNP in fixed consumption and net investment prices equals the value of net investments in the capital stocks, using their instantaneous net marginal productivities as weights. Welfare enhancement is, however, measured by the value of net investment, using the net investment prices as weights. The net investment prices reflect the present value of the future marginal contributions that the capital components make, both as stocks and flows. Outside a steady state, the two kinds of weights need not be proportional, implying that changes in NNP in fixed consumption and net investment prices do not have welfare significance.

Under competitive conditions, NNP is the maximized value of the economy's instantaneous net productive capacity. Depleting a stock of a non-renewable resource does not change the economy's instantaneous net productive capacity. The welfare-decreasing effects of such depletion can therefore *not* be captured by changes in NNP in fixed consumption and net investment prices. Rather, the effect is captured by the property that, in accordance with the Hotelling rule, the price of resource inputs is increasing (in terms of consumption or utility). This positive change in the net investment price of the non-renewable resource *ceteris paribus* decreases the maximized value of the economy's instantaneous net productive capacity, since the net investment flow of the resource is negative.

Changes in NNP in fixed consumption and net investment prices do not allow for such revaluation of net investment flows and, hence, welfare improvement is not properly indicated. This same holds for growth in real measurable NNP in variable consumption and net investment prices, when NNP is deflated by means of an NNP price index. When instead NNP is deflated by means of a consumer price index, the net investment flows are appropriately revalued, leading to the conclusion that growth in real measurable NNP in variable consumption and net investment prices indicates welfare improvement, as long as the real consumption interest rate is positive.

Hence, *a consumer price index – rather than an* NNP *price index – endows real measurable* NNP *in variable consumption and net investment prices with welfare significance. This yields a theory for deflating* NNP. When applying a consumer price index in models with environmental amenities, it is important to take the relative price changes of such amenities into account.

Growth in real utility-NNP in variable net investment prices also allows for the revaluation of net investment flows, implying that it indicates welfare improvement, as long as the supporting utility discount rate is positive. This indicator requires that changes in utility-NNP can be measured.

The real consumption interest rate is positive in the growth models that economists analyze. The only interesting exception is the *cake-eating* model (where the consumption interest rate is zero), which however should be considered as a pedagogical tool rather than a model of empirical interest. The supporting utility discount rate is positive and constant under discounted utilitarianism. If the utility function is strictly concave, then it is a general result – not being dependent on a discounted utilitarian welfare function – that the consumption interest rate exceeds the supporting utility discount rate when utility is increasing; this is the Ramsey rule.

In the present paper, I have only been concerned with local comparisons – i.e., small perturbations of the capital stocks and local-in-time comparisons. Global comparisons raise other issues, some of which are analyzed in Asheim (2003, 2005). Also, I have only considered comprehensive national accounting. There are obvious problems associated with applying the theory of welfare measurement using national accounting aggregates, as presented here, if the changes in some consumption and capital components cannot be measured or valued.

References

Aronsson, T., K.-G. Löfgren, and K. Backlund (2004), *Welfare Measurement in Imperfect Markets: A Growth Theoretical Approach*, Cheltenham: Edward Elgar.

Arrow, K., P.S. Dasgupta, and K.-G. Mäler (2003), 'Evaluating projects and assessing sustainable development in imperfect economies', *Environmental and Resource Economics* **26**: 647–685.

Asheim, G.B. (2001), 'Welfare effects of small perturbations in optimal multi-sector growth', mimeo, Department of Economics, University of Oslo.
Asheim, G.B. (2003), 'Green national accounting for welfare and sustainability: a taxonomy of assumptions and results', *Scottish Journal of Political Economy* **50**: 113–130.
Asheim, G.B. (2005), 'Welfare comparisons between societies with different population sizes and environmental characteristics', memorandum 25/2005, Department of Economics, University of Oslo.
Asheim, G.B. and W. Buchholz (2004), 'A general approach to welfare measurement through national income accounting', *Scandinavian Journal of Economics* **106**: 361–384.
Asheim, G.B., T. Mitra, and B. Tungodden (2006), 'Sustainable recursive social welfare functions', memorandum 18/2006, Department of Economics, University of Oslo.
Asheim, G.B. and M.L. Weitzman (2001), 'Does NNP growth indicate welfare improvement?', *Economics Letters* **73**: 233–239.
Beals, R. and T.C. Koopmans (1969), 'Maximizing stationary utility in a constant technology', *SIAM Journal of Applied Mathematics* **17**: 1001–1015.
Dasgupta, P.S. (2001), 'Valuing objects and evaluating policies in imperfect economies', *Economic Journal* **111**: C1–C29.
Dasgupta, P.S. and G.M. Heal (1974), 'The optimal depletion of exhaustible resources', *Review of Economic Studies* (Symposium): 3–28.
Dasgupta, P.S. and G.M. Heal (1979), *Economic Theory and Exhaustible Resources*, Cambridge: Cambridge University Press.
Dasgupta, P.S. and K.-G. Mäler (2000), 'Net national product, wealth, and social well-being', *Environment and Development Economics* **5**: 69–93.
Divisia, F. (1925–6), 'L'indice monétaire et la théorie de la monnaie', *Revue D'Economie Politique* **39**, 842–864, 980–1008, 1121–1151, **40**: 49–81.
Dixit, A., P.J. Hammond, and M. Hoel (1980), 'On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion', *Review of Economic Studies* **47**: 551–556.
Hamilton, K. (1994), 'Green adjustments to GDP', *Resources Policy* **20**: 155–168.
Heal, G. and B. Kriström (2005a), 'National income and the environment', in K.-G. Mäler and J. Vincent (eds), *Handbook of Environmental Economics* **3**, Amsterdam: North-Holland, pp. 1147–1217.
Heal, G. and B. Kriström (2005b), 'Income, wealth and welfare in representative-agent economies', mimeo, Columbia Business School and SLU Umeå.
Hulten, C.R. (1987), 'Divisia index', in J. Eatwell, M. Milgate, and P. Newman (eds), *The New Palgrave: A Dictionary of Economics*, Basingstoke: Macmillan Press, pp. 899–901.
Koopmans, T.C. (1960), 'Stationary ordinal utility and impatience', *Econometrica* **28**: 287–309.
Li, C.-Z. and K.-G. Löfgren (2002), 'On the choice of metrics in dynamic welfare analysis: utility versus money measures', Umeå Economic Studies, No. 590, University of Umeå.
Li, C.-Z. and K.-G. Löfgren (2006), 'Comprehensive NNP, social welfare, and the rate of return', *Economics Letters* **90**: 254–259.

Pezzey, J.C.V. and C.A. Withagen (1998), 'The rise, fall and sustainability of capital-resource economies', *Scandinavian Journal of Economics* **100**: 513–527.
Samuelson, P. (1961), 'The evaluation of "social income": capital formation and wealth', in F.A. Lutz and D.C. Hague (eds), *The Theory of Capital*, New York: St Martin's Press, pp. 32–57.
Sefton, J.A. and M.R. Weale (2000), 'Real national income', Discussion paper, NIESR.
Sefton, J.A. and M.R. Weale (2006), 'The concept of income in a general equilibrium', *Review of Economic Studies* **73**: 219–249.
Solow, R.M. (1974), 'Intergenerational equity and exhaustible resources', *Review of Economic Studies* (Symposium): 29–45.
Weitzman, M.L. (1970), 'Aggregation and disaggregation in the pure theory of capital and growth: a new parable', Cowles Foundation Discussion Paper 292.
Weitzman, M.L. (1976), 'On the welfare significance of national product in a dynamic economy', *Quarterly Journal of Economics* **90**: 156–162.
Weitzman, M.L. (2001), 'A contribution to the theory of welfare accounting', *Scandinavian Journal of Economics* **103**: 1–23.

Appendix: Price indices in continuous time

The Divisia index is a continuous time formula due to Divisia (1925–26). Let $\mathbf{X} = (X_1, \ldots, X_N)$ be a vector of commodities and let $\mathbf{p} = (p_1, \ldots, p_N)$ be a vector of *nominal* prices. Consider paths $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ and $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ of these vectors. Then

$$\frac{\frac{d}{dt}\big(\mathbf{p}(t)\mathbf{X}(t)\big)}{\mathbf{p}(t)\mathbf{X}(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{X}(t)}{\mathbf{p}(t)\mathbf{X}(t)} + \frac{\mathbf{p}(t)\dot{\mathbf{X}}(t)}{\mathbf{p}(t)\mathbf{X}(t)},$$

where the first term on the right-hand side defines a Divisia price index and the second term a Divisia quantity index. Hence, a Divisia price index $\{\pi(t)\}_{t=0}^{\infty}$ is defined by

$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{X}(t)}{\mathbf{p}(t)\mathbf{X}(t)} \tag{A.1}$$

for all t. Nominal prices $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ can be turned into *real* prices $\{\mathbf{P}(t)\}_{t=0}^{\infty}$ by making the following transformation at each t

$$\mathbf{P}(t) = \frac{\mathbf{p}(t)}{\pi(t)}.$$

Since

$$\dot{\mathbf{P}}(t)\mathbf{X}(t) = \frac{\pi(t)\dot{\mathbf{p}}(t)\mathbf{X}(t) - \dot{\pi}(t)\mathbf{p}(t)\mathbf{X}(t)}{\pi(t)^2} = 0$$

by (A.1), $\dot{\mathbf{P}}(t)\mathbf{X}(t) = 0$ can be used as a defining property of a Divisia price index.

In the context of the present paper, we obtain a consumer price index if $\mathbf{X} = \mathbf{C}$ and an NNP price index if $\mathbf{X} = (\mathbf{C}, \mathbf{I})$. A Divisia price index

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press

depends on the path of prices, not only on the start and end points. Path independence is obtained in special cases (cf. Hulten, 1987). In the present context, the consumer price index is path independent if the utility function *U* is homothetic.

A Divisia price index can be approximated in discrete time by chain indexing procedures, e.g. from a Paasche or a Laspeyres index which is re-based in each period.

https://doi.org/10.1017/S1355770X06003366 Published online by Cambridge University Press