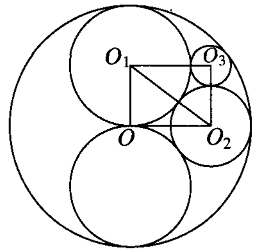
Correspondence

DEAR EDITOR,

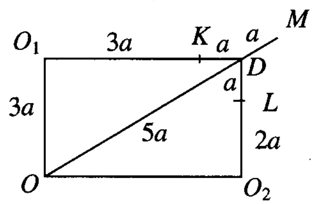
Re: The curious rectangles of Rollett and Rees, *Math, Gaz.* July 2001, pp. 208-225.

With reference to Figure 21 (p. 224) (reproduced alongside), if one removes O_3 and its associated circle and, instead, completes the rectangle O_2OO_1D, then it is a simple matter to show that that a circle centre D and radius a (where $OO_1 = 3a$) will touch each of the circles centred at O, O_1, O_2. Hence D coincides with O_3.



In the diagram, $OO_2 \perp OO_1$ by symmetry. Let $OO_1 = 3a$, then, by Pythagoras' theorem applied to ΔO_1OO_2, we find $O_1O_2 = 5a$ and $OO_2 = 4a$ (as mentioned by Mr Rees (p. 224)).

Let OD meet circle centre O at M, then we have



(i) $DK = a$ and $DK \perp$ tangent at K.

(ii) $DL = a$ and $DL \perp$ tangent at L.

(iii) $DM = a$ and $DM \perp$ tangent at M.

Yours sincerely,

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DEAR EDITOR,

Femtodays, the importance of the Inch?

The metric system was designed and adopted by law in France in the 1790s shortly after the time of the French Revolution. Previously and elsewhere, many measures were used by different trades, some still familiar to us today, like fathom for *depths* at sea, nautical miles for *horizontal distances*. To standardise and to make calculations easier, one measure of length was used, one measure of mass, and a set of prefixes for much longer or shorter lengths.

I have often wondered why they did not also decimalise time. The second, the official SI unit, is an arbitrary fraction, $1/(24 \times 60 \times 60)$, of a day. Because of the variation in the spinning of the Earth, this is now defined in terms of atomic vibration, but is still the same for all practical purposes. They could easily have decimalised time, and so we could be having coffee breaks of one centiday (approximately a quarter of an hour), lessons in schools of 4 centidays.

In the metric system, quantities which are essentially the same, are measured in the same units. Originally heat was measured in calories, now it tends to be measured in joules, the unit for work or energy, after it was realised that heat is a form of energy. In relativity theory, Einstein showed that length and time are essentially the same, so that the same units could be used for both. We in fact talk of light years as a measure of distance. If metrication were being considered now for the first time, we might therefore decide to use the same units for length and time (i.e. make the speed of light one). Choosing a day as the standard measure (this has the disadvantage of not being a universal measure, but is of practical importance to everyone living on Earth).

For ordinary linear measurements the day (i.e. the distance travelled by light in a day) is far too big. The speed of light is 299,792,458 m s^{-1} so 2.590207×10^{13} metres in a day. So a nano-day would be 25.9 km, this could be useful for describing distances between cities.

A pico-day is 25.9 metres, so that a school hall may be a pico-day square. Perhaps of most interest is a femto-day. which is 0.0259m = 2.59cm. This is almost exactly an inch, which is approximately 2.54 cm.

Yours sincerely,

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DEAR EDITOR,

I enjoyed Phil Colville's Note 86.21 in the March 2002 *Gazette* on approximating π on a spreadsheet. It is, perhaps, worth remarking that, for ant irrational number θ (such as π^n), there are infinitely many whole numbers p for which $\theta + \frac{p}{\theta}$ is as close as you like to a whole number q. Noting that $\theta + \frac{p}{\theta} \approx q$ is equivalent to $\theta^2 \approx q\theta - p$, we recall the standard result that 1, together with θ, generate a dense subgroup of $\mathbb{R}$, [1, p. 251]. It follows that there are integers q', p' for which $\theta^2 \approx q'\theta - p'$. If q', p' are positive, we are done; otherwise there are (large) whole numbers Q, P with $0 \approx Q\theta - P$ and, subtracting, i.e. $\theta^2 \approx (Q - q')\theta - (P - p')$ with $Q - q', P - p'$ both positive.

Reference

1. A. F. Beardon, A disc rolling in a tray, *Math. Gaz.* **86** (July 2002).

Yours sincerely,

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