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# REVIEW

***David Hilbert's lectures on the foundations of arithmetic and logic 1917–1933.*** Edited by William Ewald and Wilfried Sieg. Springer, Berlin, Heidelberg and New York, 2013, xxv + 1062 pp.

This hefty volume is the third of a projected six of David Hilbert's lectures on the foundations of mathematics and physics given between the years 1891 and 1933. There is a very clear line of division in Hilbert's foundational work, namely the effect of the *Principia Mathematica* from 1917 on (*PM* below). It made Hilbert realize, as the editors put it (p. 35), that "the axiomatic method he had developed in the work on geometry in the late 1890's could be extended to the logic of the *Principia* and that the latter could provide the foundation for all of mathematics." This volume reveals in detail the development of logic in the Hilbert school from the decisive Russellian impulse to the demise of logic in Göttingen in 1933. That year is significant in that it presents the final point of Hilberts' influence and the beginning of another end: the chasing out of Paul Bernays and the destruction of Göttingen mathematics by Nazism. How this glorious period ended receives almost no attention in this volume.

The volume is structured upon four lecture series of Hilbert's in the German original, often with improvements over the years following the original idea of a lecture course, and introductory and complementary materials. The latter include Hilbert's last publications on the topic, between 1917 and 1931, a reproduction of the Hilbert–Ackermann book *Grundzüge der theoretischen Logik* of 1928 (*H-A* below) and, most remarkably, the first full publication of Bernays' Habilitation thesis of 1918. Here is a summary of the divisions of the book: After an editorial introduction, Chapter 1 contains lectures from 1917–18 with the title *Lectures on the Principles of Mathematics*. Chapter 2 is *Lectures on Logic*, from 1920, Chapter 3 *Lectures on Proof Theory*, from 1921–22 and 1923–24, and finally, Chapter 4 *Lectures on the Infinite*, from 1924–25, 1931, and 1933.

Not all of Hilbert's lectures on logic and foundations have been preserved, and those that have been delivered to us come in different vestiges. Some are notes in Hilbert's hand, others clear elaborations of the lectures, even typewritten, and prepared by Bernays. Still others are notes by participants in the courses. Such notes by Hellmuth Kneser are included in Chapter 3, and one gets by comparisons to Bernays' elaborate version an impression of how the original lectures were.

It would be impossible to discuss the whole book in a short review. I find the most interesting general question to be the role of Bernays in the Hilbert program. He joined Hilbert in 1917 and soon developed a mastery of the formalism of *PM* that far superseded anything Russell and Whitehead had accomplished. Here and there in Hilbert's papers and books on logic and foundations, the former always signed only by him, discoveries are credited to Bernays, such as the discovery of the definability of the quantifiers by the $\tau$-operator and its dual the $\varepsilon$-operator and the discovery of the formal rules of predicate logic as presented in *H-A*. My general feeling after many years of study is that Bernays did not get the credit he deserved. He just got fired as "non-Aryan" in April 1933, at the time of finishing the first volume of the big *Grundlagen der Mathematik*. There is a story by which Hilbert paid his salary for some time, repeated even in this book, but shown false by Eckart Menzler's study of the Bernays–Hilbert correspondence in Zurich.

Richard Zach has suggested that the formal development of logic in the Hilbert school was Bernays' achievement. The editors use a lot of space to rebut this suggestion (on pp. 51–53), though I don't think they do justice to Bernays. When once, in a discussion with his nephew Ludwig Bernays I took up this topic, the reaction was that Paul's sister sometimes made


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exclamations such as "you did all the work and that Hilbert took all the credit," whereas "uncle Paul" would at once have toned down any such claims. Personal merit seems not to have been an issue for him. Reading the editorial introduction to *H-A*, page 49, we find that the book was in practice written by Bernays on the basis of Hilbert's 1917/18 lectures: "But in fact virtually the whole book beginning with Section 10 of Chapter One is taken, often verbatim, from Part B of Bernays's 1917/18 typescript. Sections 1–9 of Chapter One are taken similarly from Bernays's typescript from the Winter Semester 1920. . . ." Why is the second author Ackermann, not Bernays? As the editors note about the former (p. 50), his "role seems to have been more that of textual editor than of co-author." The only comment about *H-A* on the part of Bernays that I have found is well hidden: Namely, the expression *theoretische Logik* in the title of *H-A* strikes one as an oddity and is indeed nowhere to be found in Hilbert's lectures (by Amazon's "Search Inside This Book" function). Some months before Hilbert signed the preface, Bernays delivered a talk in Göttingen with the title *Probleme der theoretischen Logik*, (September 1927). The coincidence of the title with *H-A*, except for the first word, cannot be accidental. Bernays shows a witty sense of humor by starting his essay with:

> The theme of the talk as well as how it is named has been chosen in Hilbert's sense. As theoretical logic is indicated what is usually named symbolic logic, mathematical logic, algebra of logic, or logical calculus.

The talk and article were meant for the "Conference of German philologists and schoolmen," yet, Bernays succeeds in introducing in a very accessible way the elements of logic: Propositional logic is explained through truth values and then the rules of inference of logic explained. "The rules of inference have to be chosen so that they eliminate logical thinking, for otherwise we would have to have logical rules anew for how the first rules are applied." Axioms are given in separate groups for each of the connectives, with "the advantage that the separation of *positive logic* is made possible." It is well known that two years earlier, Bernays had succeeded by this method to axiomatize intuitionistic logic, five years before Heyting. I find that the message in the direction of Hilbert is subtle but clear: The purely classical *H-A*-axioms present an outdated approach to logic. Was the publication of a book that covered just classical logic perhaps an attempt on the part of Hilbert to wipe intuitionistic logic under the carpet?

The ambivalence of *H-A* is implicitly noted even by the editors who write on the one hand (p. 2) that it "presents mathematical logic in its definitive modern form," and on the other hand (p. 23) that "this book does not reflect the proof-theoretic work of the 1920s." With the two-volume *Grundlagen* of 1934 and 1939, the editors take a simple view (p. 2) by which it was "written with Bernays." On p. 800 we read that in volume I, "Hilbert and Bernays discuss the methodological standpoint of Brouwer's intuitionism." In this passage, Bernays calls intuitionism into help to save Hilbert's proof theory from the dead end it had reached. I doubt seriously that Hilbert had ever read this praise of the methods of his arch-enemy. With the second volume, there can be no doubt that Hilbert never read it: In a letter of January 1936, Hilbert's newly nominated assistant Gentzen asks Bernays about the second volume of the *Grundlagen*: "How far are you with your book?" Authorship apart, after the first volume was completed in March 1933, proof theory was radically changed from 1933 on by Gentzen's discovery of natural deduction and sequent calculus, by the proof of the consistency of arithmetic, and by the creation of ordinal proof theory. As with the *H-A* book of 1928, one could say that the second volume does not reflect the proof-theoretic work of the 1930s; it is the first instance of what Haskell Curry, even a Hilbert-student, described in 1963 as follows:

> The German writers tend to shy away from the Gentzen technique and to devise ways of modifying the ordinary formulations so as to obtain its advantages without its formal machinery. This is much the same as if one attempted to develop group theory without introducing the abstract group operation. . . .

The editors state (p. 430) that "there are no lecture notes of Hilbert's from the second half of the decade [1920s] concerned with foundational issues or proof theory." In a footnote, they mention lecture courses on foundations of mathematics, 1927/28, and on set theory, 1929, and that detailed notes of the latter in the hand of Lothar Collatz exist. The Collatz notes are very clearly written, with three parts: 1. Algebraic numbers 2. The concept of a set, and 3. Paradoxes and mathematical logic. In the third part, the topics presented include the language of logic, the formalization of arithmetic, and the problem of its consistency. Concepts and methods that became of crucial importance in the later development of foundational study, such as ordinal arithmetic and transfinite induction on constructive ordinals up to $\varepsilon_0$, are developed in detail.

The drive to accomplish a work of this magnitude clearly involves a grade of admiration of Hilbert's achievement, at places turned into a veneration and into some blindness about who actually did what. I find that this volume would have become even more beautiful had it undergone the scrutiny of someone with a critical eye towards Hilbert. Not everyone admired him; here is what Thoralf Skolem wrote about the famous 1925 article "On the infinite," the published outcome of lectures contained in Chapter 4 of the present volume (my translation from the 1934 Norwegian original *Den matematiske grunnlagsforskning*):

> At the mathematical conference in Copenhagen 1925 it was spoken of as something epoch-making. . . . It surprises me greatly that mathematicians. . . didn't understand right from the beginning how problematic—to use a mild expression—the essential contents of this paper were. Now it has come clear to everyone that it is not good. When in Vienna last autumn, I heard one mathematician use the expression "comprometting" about it.

In sum, this volume does not reveal remarkable results that would not have been made available in the 1920s and 30s, but it is instead indispensable as a source material for further studies of the development of logic and foundations.

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