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Filamentation instability in a collisional magnetoplasma with thermal conduction

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Abstract. This paper presents an analysis of the spatial growth of a transverse instability, corresponding to the propagation of an electromagnetic beam, with uniform irradiance along the wavefront in a collisional plasma, along the direction of a static magnetic field; expressions have been derived for the rate of growth, the maximum value of the rate of growth and the corresponding value of the wave number of the instability. The instability arises on account of the ejection of electrons from regions where the irradiance of the perturbation is large. The energy balance of the electrons taking into account ohmic heating and the power loss of electrons on account of (i) collisions with ions and neutral species and (ii) thermal conduction has been taken into account for the evaluation of the perturbation in electron temperature, which determines the subsequent growth of the instability. Further, the relationship between the electron density and temperature, as obtained from the kinetic theory, has been used. The filamentation instability becomes enhanced with the increase of the static magnetic field for the extraordinary mode while the reverse is true for the ordinary mode. Dependence of growth rate on irradiance of the main beam, magnetic field and a parameter proportional to the ratio of power loss of electrons by conduction to that by collisions has been numerically studied and illustrated by figures. The dependence of the maximum growth rate and the corresponding optimum value of the wave number of the instability on the irradiance of the main beam has also been studied. The paper concludes with a discussion of the numerical results, so obtained.

1. Introduction

A nonlinear medium is susceptible to filamentation instability, which is characterized by growing electron density and irradiance fluctuations, transverse to the direction of propagation. There are two complementary approaches to the study of the filamentation instability in a plasma. Sodha and Sharma (2007) have recently made a comparison of the two approaches, referring to 60 important papers in the process. In the first approach, one considers an instability $E_1 \exp[ik_\perp x + ik_\parallel z]$, superposed on a beam $E_0 \exp[i(\omega t - kz)]$; the suffixes $\parallel$ and $\perp$ refer to the components of the wave number k of the instability parallel and perpendicular to the direction of propagation, viz. z; the instability grows or does not grow as the beam propagates depending on whether $k_{\|}$ is imaginary or real. Inspired by the direct (Chiligarayan 1968; Abbi and Mahr 1971) and indirect (Loy and Shen 1969) evidence that filamentation in a nonlinear medium is caused by the presence of intensity spikes, normal to the direction of propagation, Sodha and his associates (Sodha et al. 1979, 1981, 1992), Sharma et al. (2004), Pandey and Tripathi (1990) and Asthana et al. (1999) followed the other approach, viz. the study of the growth of a Gaussian ripple on a plane (uniform or Gaussian) wavefront of an electromagnetic beam. The second approach has however not been followed in this paper.

In the presence of a high-power electromagnetic beam the electrons of the plasma are redistributed in the transverse direction on account of the ponderomotive force and non-uniform heating, arising from the gradient of the non-uniform irradiance (Sodha et al. 1974; Litvak 1970; Tewari et al. 1973). Stenzel (1976) reported experimental results regarding filamentation of a high-power whistler in a laboratory plasma. Giving special attention to low-frequency whistlers ($\omega \ll \omega_c$, ω and ω_c being wave and electron cyclotron frequencies), Sodha and Tripathi (1977) discussed various aspects of self focusing and growth of the instability and revealed that for short time scales $t \ll \tau_\varepsilon$ (dominant ponderomotive nonlinearity), where τ_ε is the energy relaxation time, the self focusing of whistlers is possible for all frequencies while for long time scales $t > \tau_\varepsilon$ (dominant collisional nonlinearity) only those whistlers can be self focused whose frequency is larger than a certain value; this is in conformance with the experimental results of Stenzel (1976). Temporal growth of the filamentation instability has been investigated by Sodha et al. (1978) in collisional and collisionless plasmas, while the same has been studied by Sharma et al. (1981) for the whistler mode and the upper hybrid mode of propagation in a strongly ionized magnetoplasma. Thermal conduction has been included in the analyses by some researchers (Perkins and Valeo 1974; Cornolti and Lucchesi 1989) but they ignored the change in electron temperature on account of the main beam. Ott et al. (1974) studied the self-focusing instability in a magnetized plasma when the applied magnetic field was weak and concluded that the growth rate is unaffected by the magnetic field. Many papers have been devoted to the study of self-focusing and filamentation instabilities in a plasma embedded in a magnetic field with (e.g. Sodha et al. 2007) and without (e.g. Sodha and Sharma 2008) thermal conduction. Sodha et al. (2007) have investigated the filamentation instability in the ionospheric plasma, but the analysis was restricted by the assumption that the wave frequency is much larger than the electron cyclotron frequency and thus the circularly polarized nature of the beam was ignored. Further, the direction of the magnetic field was assumed to coincide with the transverse component of the wave number of the perturbation, so that its effect on thermal conductivity was justifiably neglected. However, thermal conduction can be neglected only when $\delta r_0^2/l^2 \gg 1$, where δ is the fraction of excess energy lost by an electron in a collision with heavy particles and l is the mean free path of the electrons; this inequality is not valid over a range of parameters for plasmas.

In this communication the authors have investigated the filamentation instability superposed on a uniform beam in a magnetoplasma, characterized by dominant collisional nonlinearity, which is manifested in a period of the order of $1/\delta\nu_e$, where ν_e is the electron collision frequency. Expressions for the growth rate of the instability and the condition for the instability to occur have been obtained and the maximum value of the growth rate and the corresponding value of $q_\perp$ have been specifically investigated for both the modes of propagation. The effect of a magnetic field has been graphically presented for the neutral atom collision-dominated plasma (constant mean free path). The energy balance of the electrons taking into account the energy loss on account of collisions and thermal conduction has been considered for the evaluation of the electron temperature, which determines the subsequent nonlinearity. Growth rate of the instability has been plotted taking into account the variation of the parameter proportional to the ratio of power loss of electrons by conduction to that by collisions. Finally, a discussion of numerical results has been presented.

2. Analysis

2.1. Expression for spatial growth rate of instability

Let the electric field of a beam of uniform illumination and that of a small perturbation (filament) superimposed on the beam be represented by $E_{0\pm}$ and $E_{1\pm}$, respectively. The total field $E_\pm$ (corresponding to extraordinary or ordinary modes) propagating in the z-direction through a magnetoplasma can be expressed as

$$E_\pm = (E_x \pm iE_y) = (E_{0\pm} + E_{1\pm}) = (A_{0\pm} + A_{1\pm}) \exp i(\omega t - k_\pm z), \tag{1}$$

where $A_{0\pm}$, without loss of generality, is a real positive constant and $A_{1\pm}$ ($|A_{1\pm}| \ll A_{0\pm}$) is complex, $k_\pm$ is the wave number defined later and ω is the wave frequency. Neglecting the small contribution $A_1 A_1^*$ as compared to other terms, one can write

$$\boldsymbol{E}_\pm \cdot \boldsymbol{E}_\pm^* = A_{0\pm}^2 + A_{0\pm}(A_{1\pm} + A_{1\pm}^*). \tag{2}$$

The effective dielectric function of the plasma depends on $E_\pm E_\pm^*$ and hence can be expressed as

$$\varepsilon_\pm(z, E_\pm E_\pm^*) = \varepsilon_{0\pm}(z) + \varepsilon_{2\pm}(z) A_{0\pm}(A_{1\pm} + A_{1\pm}^*), \tag{3}$$

where

$$\varepsilon_{2\pm} = \left[\frac{\partial \varepsilon_\pm}{\partial (E_\pm E_\pm^*)}\right]_{E_\pm E_\pm^* = A_{0\pm}^2}.$$

The wave equation for the total field can be separated for $A_{0\pm}$ and $A_{1\pm}$. On choosing $k_\pm = (\omega/c)\sqrt{\varepsilon_{0\pm}}$ and dropping the subscript $\pm$ for convenience one can write the wave equation for A_0, which yields a solution

$$A_0 = \text{constant}.$$

The wave equation for A_1, on neglecting $\partial^2 A_1/\partial z^2$ (assuming A_1 to be slowly varying) and $A_1 A_1^*$, reduces to

$$-2ik\frac{\partial A_1}{\partial z} + \nabla_\perp^2 A_1 + \frac{\omega^2}{c^2}\varepsilon_2 A_0^2 (A_1 + A_1^*) = 0, \tag{4}$$

in the JWKB approximation.

One can express the complex amplitude A_1 of the perturbation as

$$A_1 = A_{1\mathrm{r}} + iA_{1\mathrm{i}}, \tag{5}$$

where $A_{1\mathrm{r}}$ and $A_{1\mathrm{i}}$ are real and $\nabla_\perp^2 = \nabla^2 - (\partial^2/\partial z^2)$. In earlier analyses $A_{1\mathrm{r}}$ and $A_{1\mathrm{i}}$, which are real, have been assumed to be proportional to the complex quantity $\exp\{i(q_\perp x + q_\parallel z)\}$, which is not consistent. However, the results so obtained are the same as the ones based on the following considerations, free of any objection. Assuming A_1 to be independent of y and proportional to $\exp(iq_\perp x + iq_\parallel z)$, one

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has $\nabla_\perp^2 A_1 = -q_\perp^2 A_1$. With this assumption and using (5), one obtains two homogeneous equations in $A_{1\mathrm{r}}$ and $A_{1\mathrm{i}}$ (after equating the real and imaginary parts). Thus,

$$2k\frac{\partial A_{1\mathrm{i}}}{\partial z} - \Lambda^2 A_{1\mathrm{r}} = 0, \tag{6a}$$

$$2k\frac{\partial A_{1\mathrm{r}}}{\partial z} + q_\perp^2 A_{1\mathrm{i}} = 0, \tag{6b}$$

where $\Lambda^2 = (2\omega^2/c^2)\varepsilon_2 A_0^2 - q_\perp^2$.

Differentiating (6a) and (6b) with respect to z and substituting for $\partial A_{1\mathrm{r}}/\partial z$ and $\partial A_{1\mathrm{i}}/\partial z$ from (6b) and (6a), respectively, one obtains

$$\frac{\partial^2 A_{1\mathrm{i}}}{\partial z^2} = \frac{\Lambda^2 q_\perp^2}{4k^2} A_{1\mathrm{i}} \tag{7a}$$

and

$$\frac{\partial^2 A_{1\mathrm{r}}}{\partial z^2} = \frac{\Lambda^2 q_\perp^2}{4k^2} A_{1\mathrm{r}}. \tag{7b}$$

Hence, A_1 will grow exponentially with z, with a growth rate

$$\Gamma = iq_\parallel = \frac{\Lambda q_\perp}{2k} = \frac{q_\perp}{2k}\left\{\frac{2\omega^2}{c^2}\varepsilon_2 A_0^2 - q_\perp^2\right\}^{1/2}. \tag{8}$$

From the above equation one obtains the condition for the growth of the instability (Γ being real) as the beam propagates, viz.

$$\frac{2\omega^2}{c^2}\varepsilon_2 A_0^2 > q_\perp^2.$$

2.2. *Evaluation of $\varepsilon_{0\pm}$ and $\varepsilon_{2\pm}$*

2.2.1. Energy balance. The redistribution of electron density in a magnetoplasma by the plane uniform irradiance beam is determined by the ohmic heating of electrons and subsequent loss of energy by collisions with heavy particles and by thermal conduction. The energy balance of the electrons in a magnetoplasma may be expressed as

$$\frac{e^2 N_\mathrm{e}\nu_\mathrm{e}}{8m}\left\{\frac{A_+A_+^* + A_-A_-^*}{\nu_\mathrm{e}^2 + (\omega-\omega_\mathrm{c})^2} + \frac{A_+A_+^* + A_-A_-^*}{\nu_\mathrm{e}^2 + (\omega+\omega_\mathrm{c})^2}\right\}$$
$$= \frac{3}{2}N_\mathrm{e}k_\mathrm{B}(T_\mathrm{e} - T_0)\nu_\mathrm{e}\delta - \frac{\partial}{\partial x}\left(\chi_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial x}\right), \tag{9a}$$

where ν_e is the electron collision frequency, k_B is Boltzmann's constant, χ_e is the electronic thermal conductivity, e and m are the electronic charge and mass, respectively, and $\omega_\mathrm{c} = eB/mc$ is known as the electron cyclotron frequency. The left-hand side represents the ohmic loss (Shkarofsky et al. 1966) per unit volume or the power lost per unit volume by the electric field to the electrons; the first term on the right-hand side represents the power lost by the electrons in collisions per unit volume while the second corresponds to the power lost by the electrons on

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account of thermal conduction. Equation (9a) can be simplified to

$$\frac{T_e}{T} - 1 = \frac{2}{3k_B T\delta}\frac{1}{N_e \nu_e}\frac{\partial}{\partial x}\left(\chi_e \frac{\partial T_e}{\partial x}\right) + \left[\frac{\alpha(E_+E_+^* + E_-E_-^*)}{\nu_e^2/\omega^2 + (1-\omega_c/\omega)^2} + \frac{\alpha(E_+E_+^* + E_-E_-^*)}{\nu_e^2/\omega^2 + (1+\omega_c/\omega)^2}\right], \tag{9b}$$

where $\alpha = e^2/12mk_B T\delta\omega^2$.

2.2.2. Distribution of electron density. The non-uniformity in the electric field and consequent non-uniform ohmic heating of electrons on account of the instability results in a non-uniform electron temperature distribution. The non-uniform electron density distribution is determined by balancing the force on an electron on account of (i) pressure gradient and (ii) space-charge field; charge neutrality of the plasma is assumed. Thus, following the rigorous kinetic treatment based on Boltzmann's transfer equation for the velocity distribution of carriers (Sodha et al. 1976), the electron density–temperature relationship can be written as

$$\frac{N_e}{N_0} = \left(\frac{2T_0}{T_e + T_0}\right)^{1-s/2}, \tag{10}$$

where N_0 and T_0 are the equilibrium concentration and temperature of the electrons in the absence of the field, respectively, the parameter s in the exponent characterizes the nature of collisions; in the case of collisions of electrons with neutral particles, $s = 1$ and when collisions with ions predominate, $s = -3$. The dependence of electron temperature on the irradiance of the beam and the instability is thus governed by the energy balance equation.

2.2.3. Electron temperature. The electron temperature in the plasma in the presence of the main beam as well as the perturbation given by (1), (6a) and (6b) can be expressed as

$$T_e = T_{e0} + T_{e1} \tag{11}$$

where $T_{e1} = T_{e10}\exp[i(q_\parallel z + q_\perp x)]$, $T_{e1} \ll T_{e0}$ and $q_\perp \gg q_\parallel$. The temperature dependence of the electron collision frequency and the electronic thermal conductivity is given by

$$\nu_e = \nu_0 (T_e/T_0)^{s/2} \simeq \nu_0\left[1 + (s/2)\frac{T_e - T_0}{T_0}\right]; \quad \frac{T_e - T_0}{T_0} \ll 1 \tag{12}$$

and

$$\chi_e = \frac{5k_B^2 N_e T_e}{m}\frac{\nu_e}{\nu_e^2 + \omega_c^2}.$$

Using expressions for electron density and collision frequency from (10) and (12), the expression for electronic thermal conductivity can be written as

$$\chi_e = \chi_0 \frac{T_{p0}^{1+s/2}((1+T_{p0})/2)^{s/2-1}T_{p1}}{T_{p0}^s + \omega_c^2/\nu_0^2} \times \left\{1 + \frac{((s/2)-1)}{1+T_{p0}} + \frac{(1+(s/2))}{T_{p0}} - \frac{sT_{p0}^{s-1}}{T_{p0}^s + \omega_c^2/\nu_0^2}\right\}, \tag{13}$$

where $\chi_0 = 5k_B^2 N_0 T_0/m\nu_0$, $T_{p0} = T_{e0}/T_0$ and $T_{p1} = T_{e1}/T_0$.

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The term for thermal conduction in the energy balance equation can be written as

$$\frac{\partial}{\partial x}\left(\chi_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial x}\right)=\frac{\partial \chi_{\mathrm{e}}}{\partial T_{\mathrm{e}}}\left(\frac{\partial T_{\mathrm{e}}}{\partial x}\right)^{2}+\chi_{\mathrm{e}}\frac{\partial^{2} T_{\mathrm{e}}}{\partial x^{2}}. \tag{14}$$

From (11), one obtains

$$\frac{\partial T_{\mathrm{e}}}{\partial x}=iq_{\perp}T_{\mathrm{e}1} \quad \text{and} \quad \frac{\partial^{2} T_{\mathrm{e}}}{\partial x^{2}}=-q_{\perp}^{2}T_{\mathrm{e}1}.$$

Therefore, after neglecting the second-order perturbation terms, equation (14) further reduces to

$$\frac{\partial}{\partial x}\left(\chi_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial x}\right)=\frac{-(q_{\perp}^{2}T_{\mathrm{e}1})\chi_{0}T_{\mathrm{p}0}^{1+s/2}((1+T_{\mathrm{p}0})/2)^{s/2-1}}{T_{\mathrm{p}0}^{s}+\omega_{\mathrm{c}}^{2}/\nu_{0}^{2}}. \tag{15}$$

Further, the irradiance term present in the energy balance equation can be written as

$$\begin{aligned}
&\left[\frac{\alpha(E_{+}E_{+}^{*}+E_{-}E_{-}^{*})}{\nu_{\mathrm{e}}^{2}/\omega^{2}+(1-\omega_{\mathrm{c}}/\omega)^{2}}+\frac{\alpha(E_{+}E_{+}^{*}+E_{-}E_{-}^{*})}{\nu_{\mathrm{e}}^{2}/\omega^{2}+(1+\omega_{\mathrm{c}}/\omega)^{2}}\right]\\
&\quad=\alpha\{A_{0\pm}^{2}+A_{0\pm}(A_{1\pm}+A_{1\pm}^{*})\}\left[\left\{(1-\Omega_{\mathrm{c}})^{2}+\frac{\nu_{0}^{2}}{\omega^{2}}T_{\mathrm{p}0}^{s}\left(1+s\frac{T_{\mathrm{p}1}}{T_{\mathrm{p}0}}\right)\right\}^{-1}\right.\\
&\qquad\left.+\left\{(1+\Omega_{\mathrm{c}})^{2}+\frac{\nu_{0}^{2}}{\omega^{2}}T_{\mathrm{p}0}^{s}\left(1+s\frac{T_{\mathrm{p}1}}{T_{\mathrm{p}0}}\right)\right\}^{-1}\right]\\
&\quad=\alpha\{A_{0\pm}^{2}+A_{0\pm}(A_{1\pm}+A_{1\pm}^{*})\}\left[\left\{\frac{1}{D_{1}}+\frac{1}{D_{2}}\right\}-s\frac{\nu_{0}^{2}}{\omega^{2}}T_{\mathrm{p}0}T_{\mathrm{p}1}\left\{\frac{1}{D_{1}^{2}}+\frac{1}{D_{2}^{2}}\right\}\right],
\end{aligned} \tag{16}$$

where

$$D_{1}=(1-\Omega_{\mathrm{c}})^{2}+\frac{\nu_{0}^{2}}{\omega^{2}}T_{\mathrm{p}0}^{s},$$

$$D_{2}=(1+\Omega_{\mathrm{c}})^{2}+\frac{\nu_{0}^{2}}{\omega^{2}}T_{\mathrm{p}0}^{s},$$

$\Omega_{\mathrm{c}}=\omega_{\mathrm{c}}/\omega$ and terms with $T_{\mathrm{p}1}^{2}$ and higher powers of $T_{\mathrm{p}1}$ have been neglected.

Substituting the relevant values from (11), (15) and (16) in the energy balance equation (9b) and equating the terms with and without $\exp(iq_{\parallel}z+iq_{\perp}x)$, one obtains

$$\frac{T_{\mathrm{e}0}}{T_{0}}-1=\frac{\alpha A_{0\pm}^{2}}{D_{1}}+\frac{\alpha A_{0\pm}^{2}}{D_{2}} \tag{17}$$

and

$$\frac{T_{\mathrm{e}1}}{T_{0}}=\frac{\alpha A_{0\pm}(A_{1\pm}+A_{1\pm}^{*})}{F(q_{\perp})}, \tag{18}$$

where

$$F(q_{\perp})=\left[\frac{(1+(DT_{\mathrm{p}0}q_{\perp}^{2}/(T_{\mathrm{p}0}^{s}+\omega_{\mathrm{c}}^{2}/\nu_{0}^{2}))+sT_{\mathrm{p}0}^{s-1}(\nu_{0}^{2}/\omega^{2})\alpha A_{0\pm}^{2}(1/D_{1}^{2}+1/D_{2}^{2}))}{(1/D_{1}+1/D_{2})}\right]$$

and

$$D=10k_{\mathrm{B}}T_{0}\omega^{2}/3mc^{2}\delta c^{2}.$$

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2.2.4. Dielectric function. For electromagnetic wave propagation along the direction of the magnetic field, the dielectric function $\varepsilon_{\pm}$ in a plasma (assuming the medium to be non-absorptive) corresponding to the two modes $(E_x \pm iE_y)$ can be expressed as

$$\varepsilon_{\pm} = 1 - \Omega_{\rm p}^2 \frac{N_{\rm e}}{N_0} \frac{[1 \mp \omega_{\rm c}/\omega]}{[(\nu_{\rm e}/\omega)^2 + (1 \mp \omega_{\rm c}/\omega)^2]},$$

where $\Omega_{\rm p} = \omega_{\rm p0}/\omega$; $\omega_{\rm p0} = (4\pi N_0 e^2/m)^{1/2}$ is the electron plasma frequency in the absence of the beam.

Substituting for $N_{\rm e}/N_0$ in terms of the electron temperature from (10) and using (12),

$$\varepsilon_{\pm} = 1 - \frac{\Omega_{\rm p}^2(1 \mp \Omega_{\rm c})}{M}\left(\frac{1+T_{\rm p0}}{2}\right)^{s/2-1} + \frac{(1 \mp \Omega_{\rm c})\Omega_{\rm p}^2}{M}\left(\frac{1+T_{\rm p0}}{2}\right)^{s/2-1}\left\{\frac{1-s/2}{1+T_{\rm p0}} + \frac{s(\nu_0/\omega)^2(T_{\rm p0}-1)}{M}\right\} \times \frac{\alpha A_{0\pm}(A_{1\pm}+A_{1\pm}^*)}{F(q_\perp)}. \tag{19a}$$

Comparing (3) and (19a), one obtains

$$\varepsilon_{0\pm}(z) = 1 - \frac{\Omega_{\rm p}^2(1 \mp \Omega_{\rm c})}{M}\left(\frac{1+T_{\rm p0}}{2}\right)^{s/2-1}, \tag{19b}$$

$$\varepsilon_{2\pm}(z) = \frac{\alpha\Omega_{\rm p}^2(1 \mp \Omega_{\rm c})}{MF(q_\perp)}\left(\frac{1+T_{\rm p0}}{2}\right)^{s/2-1}\left[\frac{1-s/2}{1+T_{\rm p0}} + \frac{s(\nu_0/\omega)^2(T_{\rm p0}-1)}{M}\right], \tag{19c}$$

where

$$M = \left[(1 \mp \Omega_{\rm c})^2 + (\nu_0/\omega)^2\left\{1 + \frac{s}{2}(T_{\rm p0}-1)^2\right\}\right].$$

2.3. *Maximum growth rate of instability*

Substituting for $\varepsilon_{2\pm}(z)$ from (19c) in (8), one obtains an expression for the growth rate Γ of the perturbation in terms of q as follows:

$$\frac{c}{\omega}\Gamma = \frac{q}{2\sqrt{\varepsilon_0}}\left[\frac{2\Omega_{\rm p}^2(1 \mp \Omega_{\rm c})}{MF(q)}\left(\frac{1+T_{\rm p0}}{2}\right)^{s/2-1} \times \left\{\frac{1-s/2}{1+T_{\rm p0}} + \frac{s(\nu_0/\omega)^2(T_{\rm p0}-1)}{M}\right\}\alpha A_0^2 - q^2\right]^{1/2}, \tag{20}$$

where

$$q = (c/\omega)q_\perp,$$

$$F(q) = \left[\frac{(1 + (\beta T_{\rm p0}q^2/(T_{\rm p0}^s + \omega_{\rm c}^2/\nu_0^2)) + sT_{\rm p0}^{s-1}(\nu_0^2/\omega^2)\alpha A_{0\pm}^2(1/D_1^2 + 1/D_2^2))}{(1/D_1 + 1/D_2)}\right]$$

and $\beta = D(\omega^2/c^2) = 10k_{\rm B}T\omega^2/3mc^2\delta\nu_0^2$. For the instability to occur, $\Gamma > 0$. Thus, the critical value of $q = (c/\omega)q_\perp$, below which the instability occurs (viz. $q_{\rm critical}$), is given by putting the right-hand side of (20) equal to zero.

For Γ to be maximum, the optimum value of q, i.e. q_{opt}, is given by the extremum condition $\mathrm{d}\Gamma/\mathrm{d}q = 0$, which yields

$$q_{\text{opt}}^2[F(q_{\text{opt}})]^2 = \frac{\Omega_{\text{p}}^2(1 \mp \Omega_{\text{c}})}{M}\left(\frac{1+T_{\text{p}0}}{2}\right)^{s/2-1} \times \left\{\frac{1-s/2}{1+T_{\text{p}0}} + \frac{s(\nu_0/\omega)^2(T_{\text{p}0}-1)}{M}\right\}\alpha A_0^2 F_1, \quad (21)$$

where

$$F_1 = \frac{1 + sT_{\text{p}0}^{s-1}(\nu_0^2/\omega^2)\alpha A_{0\pm}^2(1/D_1^2 + 1/D_2^2)}{(1/D_1 + 1/D_2)}.$$

For q_{opt} given by (21), the maximum growth rate, written as $\Gamma_{\max}$, is given by

$$\frac{c}{\omega}\Gamma_{\max} = \left(\frac{q_{\text{opt}}^2}{2\varepsilon_0}\right)\left\{\frac{2F(q_{\text{opt}})}{F_1} - 1\right\}^{1/2}. \quad (22)$$

The significance of this instability is that as the plane wave propagates in a nonlinear medium in the z-direction, it may split up in filaments transverse to the z-axis. The transverse scale length for filaments (or the perturbation) is $k_{\perp}^{-1}$.

3. Numerical results and discussion

All the numerical calculations have been made for the scattering parameter $s = 1$ (electron–neutral collision-dominated plasma). Similar calculations can be carried out for $s = -3$ (electron–ion collision-dominated plasma). Figure 1 illustrates the dependence of the dimensionless growth rate Γ of the self-focusing instability with the dimensionless wave number $q = (c/\omega)q_{\perp}$ in the direction transverse to the propagation, for different values of the dimensionless background irradiance $\alpha A_{0+}^2 = 1$, 5 and 10 for the extraordinary mode. It is seen that, corresponding to a certain value of the beam irradiance, there is an optimum value of wave number, namely q_{opt}, for which the growth rate is maximum.

To study the effect of the static magnetic field on the spatial growth rate of the instability, curves have been presented for different values of ω_{c}/ω for both the modes (Fig. 2). The growth rate is seen to become enhanced with the static magnetic magnetic field for the extraordinary mode. However, curves for the ordinary mode are reverse in nature, i.e. the growth rate of the instability is found to be decrease on increasing the applied magnetic field.

Figure 3 depicts the variation of both the optimum value of the wave number q_{opt} and the maximum growth rate $\Gamma_{\max}$ on the uniform background irradiance for the extraordinary mode. It is seen that $\Gamma_{\max}$ and q_{opt} increase monotonically.

Figure 4 corresponds to the dependence of the growth of the instability with wave number for different values of β (the parameter which characterizes the ratio of the electron energy loss by thermal conduction to that by collisions) for a fixed value of irradiance of the main beam. It is seen that the growth rate of the instability decreases as the value of β increases. This can be readily explained by the fact that increasing thermal conductivity leads to enhanced energy loss by ohmically heated electrons, and consequently lower electron temperature and associated nonlinearity.

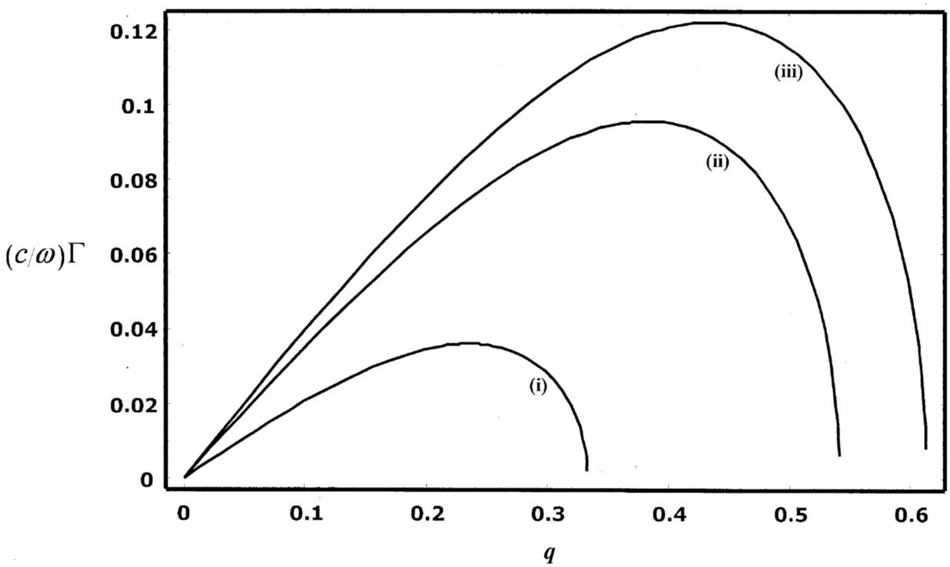

Figure 1. Dependence of dimensionless growth rate $(c/\omega)\Gamma$ of self-focusing filamentation instability of an extraordinary mode on q, the dimensionless wave number of the perturbation in the transverse direction corresponding to the dimensionless background irradiance $\alpha A_0^2 = 1$ (i), 5 (ii) and 10 (iii). The other parameters are $\Omega_\mathrm{p}^2 = \omega_\mathrm{p}^2/\omega^2 = 0.5$, $\Omega_\mathrm{c} = \omega_\mathrm{c}/\omega = 0.4$, $\nu_0^2/\omega^2 = 0.1$ and $\beta = 1$.

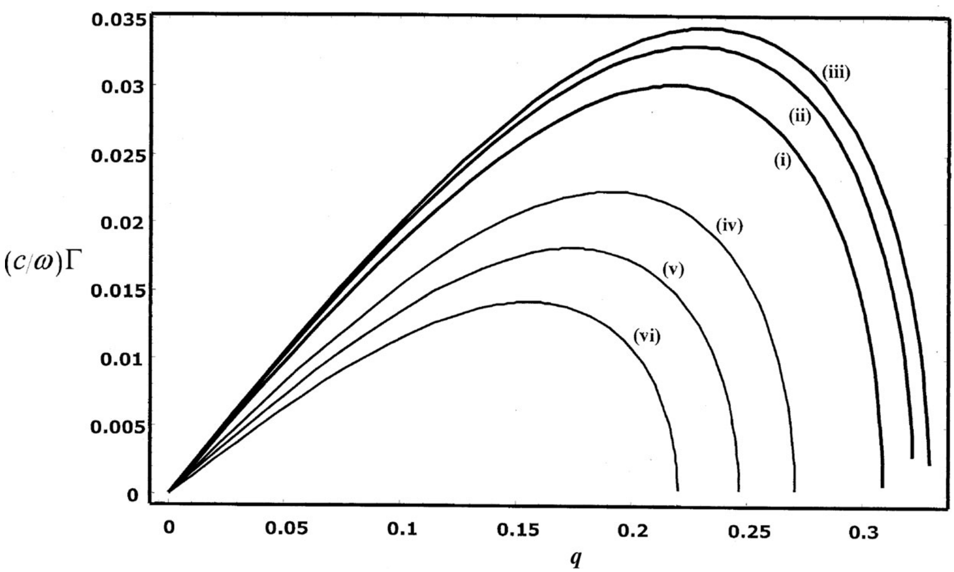

Figure 2. Variation of dimensionless spatial growth rate $(c/\omega)\Gamma$ of instability on q for extraordinary mode (---) $\omega_\mathrm{c}/\omega = 0.1$ (i), 0.2 (ii) and 0.3 (iii) and for ordinary mode (---) $\omega_\mathrm{c}/\omega = 0.1$ (iv), 0.2 (v) and 0.3 (vi); $\alpha A_0^2 = 1$, $\omega_\mathrm{p}^2/\omega^2 = 0.5$, $\nu_0^2/\omega^2 = 0.1$ and $\beta = 1$.

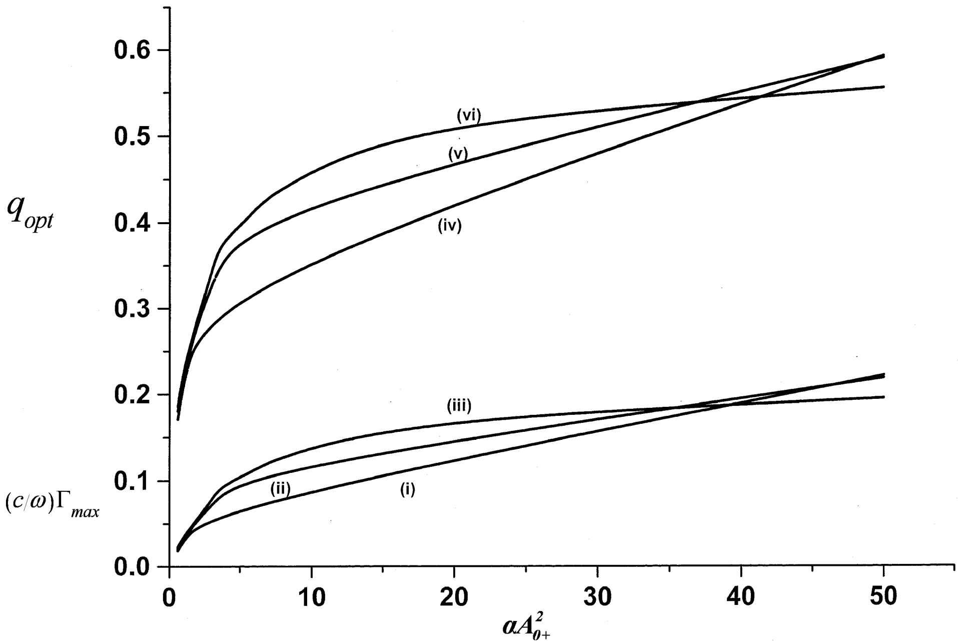

Figure 3. Variation of optimum value of the dimensionless wave number $q_{\rm opt}$ and dimensionless maximum growth rate $(c/\omega)\Gamma_{\rm max}$ with the dimensionless background irradiance αA_0^2 when $\omega_{\rm c}/\omega = 0.1$ (i), 0.2 (ii) and 0.3 (iii) {for $(c/\omega)\Gamma_{\rm max}$} and 0.1 (iv), 0.2 (v) and 0.3 (vi) {for $q_{\rm opt}$}; $\omega_{\rm p}^2/\omega^2 = 0.5$, $\nu_0^2/\omega^2 = 0.1$ and $\beta = 1$ (extraordinary mode).

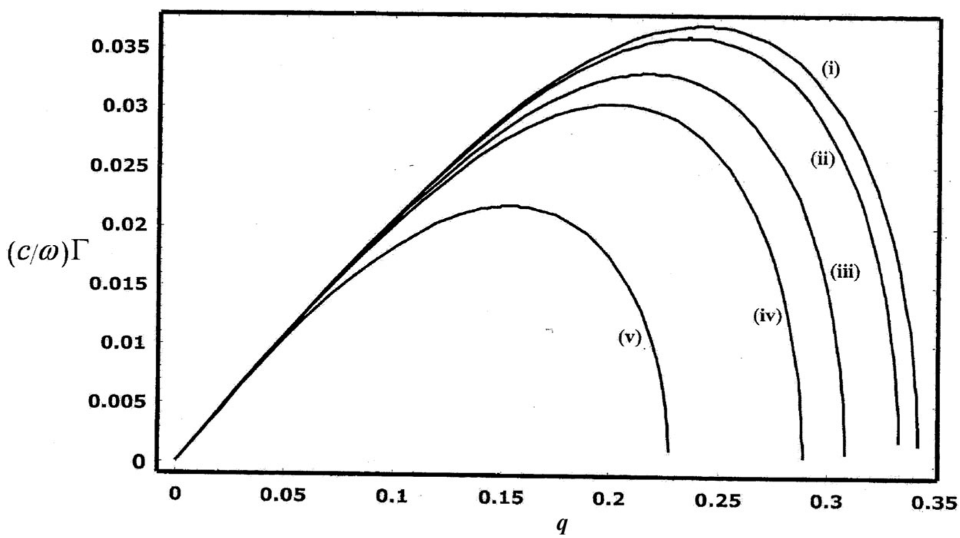

Figure 4. Variation of growth rate of instability $(c/\omega)\Gamma$ with wave number q for $\beta = 0$ (i), 1 (ii), 5 (iii), 10 (iv) and 50 (v). β is a parameter proportional to thermal conductivity. The other parameters are $\omega_{\rm p}^2/\omega^2 = 0.5$, $\nu_0^2/\omega^2 = 0.1$ and $\omega_{\rm c}/\omega = 0.4$.

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